Asymptotic Notations

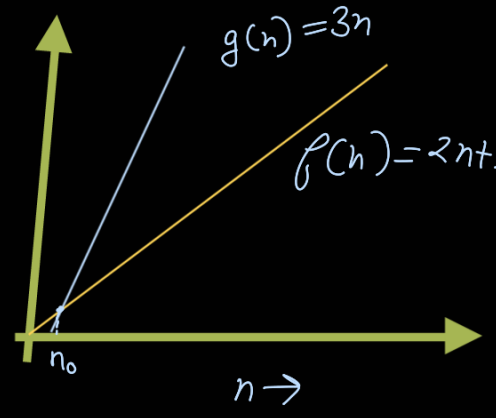
Big O : Exact or Upper
Theta : Exact
Omega : Exact or linear

→ Big O Notation

↳ Direct way:
- ↳ Ignore lower order terms
- ↳ Ignore leading term constant

→ $3n^2 + 5n + 6$ — $O(n^2)$
→ $3n + 10n\log n + 3$ — $O(n \log n)$
→ $10n^3 + 40n + 10$ — $O(n^3)$

We say $f(n) = O(g(n))$ iff there exist constant c and n. such that $f(n) \leq g(n)$
For all $n \geq n_0$



Example

$f(n) = 2n+3$
can be written as $O(n)$ $[g(n)=n]$
Let us take, $c=3$

$2n+3 \leq 3n$

$3 \leq n$

We got $n_0=3$

$\{100, \log 200, (10)^4, \ldots\} \in O(1)$

$\cup \{\frac{n}{4} + 2n+3, \frac{n}{100} + \log n, n+10000, \log n+10, \ldots\} \in O(n)$

$\cup \{n^2+n, 2n^2, n^2+1000n, n^2+n\log n+n, \frac{n^2}{10000}, \ldots\} \in O(n^2)$

Big O notation works for multiple variables also

$100n^2 + 1000m + n$: $O(n^2+m)$

$1000m^2 + 200mn + 30m + 20n$: $O(m^2+mn)$

Applications

↳ Used when we have an upper bound.

```
bool isPrime(int n)
{
    if (n==1)
      return false;
    if (n==2 || n==3)
      return true;
    if (n%2==0 || n%3==0)
      return false;
    for(int i=5; i*i<=n; i=i+6)
      if (n%i==0 || n%(i+2)==0)
        return false;
    return true;
}
```

→ Big Omega Notation

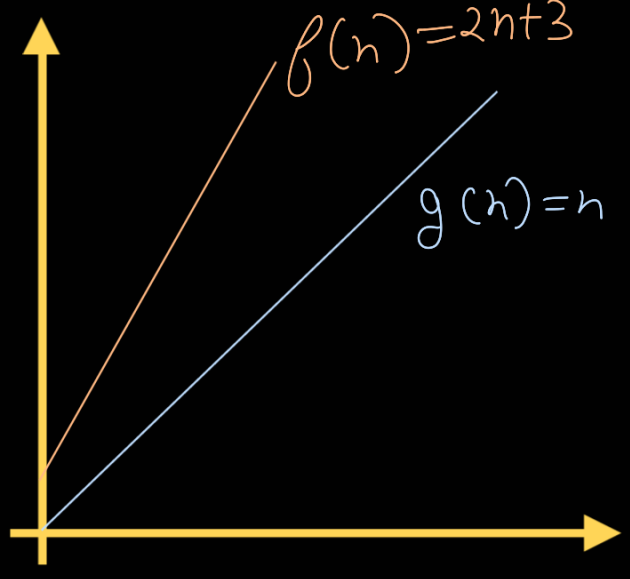
$f(n) = \Omega(g(n))$ iff there exist constants c, (where $c > 0$) and n_0 (where $n_0 \geq 0$) such that $cg(n) \leq f(n)$ for all $n \geq n_0$.

$f(n) = 2n+3 = \Omega(n)$

$c = 1$

$n \leq 2n+3$ | $n_0 = 0$

$-3 \leq n$



$\{ n^2, 2n^2+5, 1000n^2, 2n^3+n, \ldots \} \in \Omega(n^2)$

$\cup \{ 2n+3, 100n+\log n, \ldots \} \in \Omega(n)$

$\cup \{ 5000, (10)^5, \log 2000, \ldots \} \in \Omega(1)$

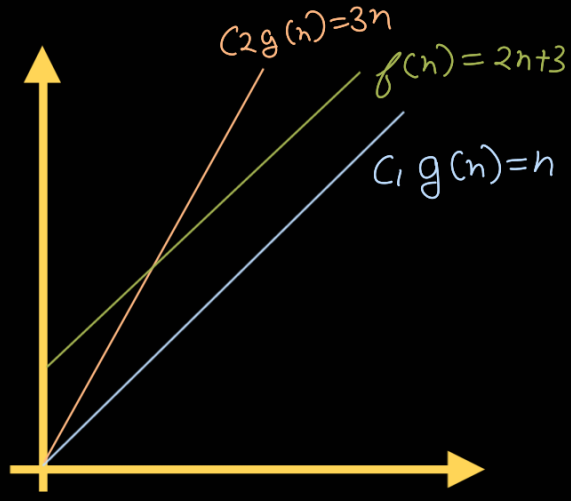
If $f(n) = O(g(n))$

$g(n) = \Omega(f(n))$

→ Theta Notation

$f(n) = \Theta(g(n))$ iff there exist constants c_1, c_2 (where $c_1 > 0$ and $c_2 > 0$) and n_0 (where $n_0 \geq 0$) such that

$c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0$



Example : $f(n) = 2n+3$: $\Theta(n)$

$c_1 = 1$, $c_2 = 3$

$1 \times n \leq 2n+3 \leq 3n$

$n \geq 3$ $\quad$ $n \geq 0$ $\quad$ $n_0 = 3$

Direct Method

$100 \overset{x}{o} n^2 + 100 \overset{x}{n} \log n + 2 \overset{x}{n}$: $\Theta(n^2)$

$200 \overset{x}{n}^3 + 30 \overset{x}{n} + 5^{x}$: $\Theta(n^3)$

$2000 \overset{x}{n} + 2 \log \overset{x}{n}$: $\Theta(n)$

↳ If $f(n) = \Theta(g(n))$

then $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$

and $g(n) = O(f(n))$ and $g(n) = \Omega(f(n))$

↳ Represents exact bound

↳ $\{ 100, 10^5, \log 2000, \text{---} \} \in \Theta(1)$

$\{ 100n, 2n + \log n, 5n+3, \text{---} \} \in \Theta(n)$

$\{ 2n^2, \frac{n^2}{4} + 5n \log n, \text{---} \} \in \Theta(n^2)$