# “By Example” introduction into the Mathematica PairwiseComparisons` Package

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To install the package on the computer start Mathematica and choose File/Install..., then select PairwiseComparisons.m as the package source and press OK.

To install the package at the Raspberry PI just copy them into /opt/Wolfram/WolframEngine/10.0/AddOns/ExtraPackages and restart the application

## Introduction

Loading the package

```
<< PairwiseComparisons`;
```

Printing function usage

```
? GeometricRank
```

GeometricRank[M] returns rank list given as geometric means of rows of the matrix M

Printing full (with implementation) function usage

```
?? GeometricRank
```

GeometricRank[M] returns rank list given as geometric means of rows of the matrix M

```
GeometricRank[PairwiseComparisons`Private`matrix_] :=
 (GeometricMean[#1] &) /@ PairwiseComparisons`Private`matrix
```



```
GeometricRank[PairwiseComparisons`Private`matrix_] :=
 (GeometricMean[#1] &) /@ PairwiseComparisons`Private`matrix
```



Print all the public functions in the package

```
? PairwiseComparisons`*
```

▼ PairwiseComparisons`

| | | | |
|---|---|---|---|
| AHP | ErrorMatrix | HREGeomRescaledRank | PrincipalEigenValue |
| AIJadd | GeometricRank | HREMatrix | PrincipalEigenValueSym |
| AIJgeom | GeometricRescaledRank | HREPartialRank | PrincipalEigenVector |
| ConsistentMatrixFromRank | GetMatrixEntry | HRERescaledRank | PrincipalEigenVectorSym |
| COP1Check | GlobalDiscrepancy | KendallTauDistance | RankOrder |
| COP1ViolationList | HarkerMatrix | KoczkodajConsistentTriad | RankToVector |
| COP2Check | HREConstantTermVector | KoczkodajIdx | RecreatePCMatrix |
| COP2ViolationList | HREFullRank | KoczkodajImproveMatrixStep | SaatyIdx |
| DeleteColumns | HREGeomConstantTermVector | KoczkodajTheWorstTriad | SaatyIdxSym |
| DeleteRows | HREGeomFullRank | KoczkodajTheWorstTriads | SetDiagonal |
| DeleteRowsAndColumns | HREGeomIntermediateRank | KoczkodajTriadIdx | VersionPC |
| EigenvalueRank | HREGeomMatrix | LocalDiscrepancyMatrix | |
| EigenvalueRankSym | HREGeomPartialRank | NormalizedKendallTauDistance | |

Assign the matrix to the variable M

$$M = \begin{pmatrix} 1 & 2 & 3 \\ \frac{1}{2} & 1 & 4 \\ \frac{1}{3} & \frac{1}{4} & 1 \end{pmatrix}$$

$$\left\{\{1, 2, 3\}, \left\{\frac{1}{2}, 1, 4\right\}, \left\{\frac{1}{3}, \frac{1}{4}, 1\right\}\right\}$$

# Eigenvalue based method

Calculate the principal eigenvalue of the matrix M

```
PrincipalEigenValue[M]
```

```
3.10785
```

Calculate the principal eigenvector of the matrix M

```
PrincipalEigenVector[M]
```

```
{0.806208, 0.558993, 0.193792}
```

Calculate the value of the Saaty (eigenvalue based) inconsistency index of M

```
SaatyIdx[M]
```

```
0.0539237
```

Calculate the eigenvalue based ranking based on M

```
EigenvalueRank[M]
```

```
{0.517134, 0.35856, 0.124306}
```

Calculate rank for the three different criteria using AHP (see Belton and Gear, On a short-coming of Saaty's method of analytic hierarchies, 1983)

$$A_1 = \begin{pmatrix} 1 & 1/9 & 1 \\ 9 & 1 & 9 \\ 1 & 1/9 & 1 \end{pmatrix};$$

$$A_2 = \begin{pmatrix} 1 & 9 & 9 \\ 1/9 & 1 & 1 \\ 1/9 & 1 & 1 \end{pmatrix};$$

$$A_3 = \begin{pmatrix} 1 & 8/9 & 8 \\ 9/8 & 1 & 9 \\ 1/8 & 1/9 & 1 \end{pmatrix};$$

$$A_{\text{criteria}} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix};$$

then the result is:

$\mathbf{AHP}[A_{\text{criteria}}, A_1, A_2, A_3]$

```
{0.451178, 0.469697, 0.0791246}
```

## Geometric Mean method (Logarithmic Least Square Method)

Calculate the geometric mean based ranking based on M

```
N[GeometricRank[M]]
```

```
{1.81712, 1.25992, 0.43679}
```

Calculate the geometric mean based ranking based on M rescaled so that the sum of entries is 1

```
N[GeometricRescaledRank[M]]
```

{0.517134, 0.35856, 0.124306}

Please note that if you want to have numeric (not symbolic) output the input data also should be "numeric".
Thus, it is not a bad idea to add a numeric conversion N@ to the input matrix. For example:

```
N[GeometricRank[N@M]]
```

{1.81712, 1.25992, 0.43679}

The above applies to all the methods in the package, except eigenvalue based methods such as PrincipalEigenValue,
PrincipalEigenVector, SaatyIdx and EigenvalueRank, which are "by design" numeric.

## Heuristic Rating Estimation Method (additive)

Calculate the HRE Matrix for the given matrix M where the unknown concepts are $\{c_1, c_2, c_3\}$ and the known concepts are $\{c_4 = 5, c_5 = 9\}$. It is assumed that the unknown concepts has value 0 whilst the known concepts have the values greater than 0.

Further references could be found in papers :
* Konrad Kułakowski, Heuristic Rating Estimation Approach to The Pairwise Comparisons Method http : // arxiv.org/abs/1309.0386
* Konrad Kułakowski, A heuristic rating estimation algorithm for the pairwise comparisons method http : // dx.doi.org/10.1007/s10100 – 013 – 0311 – x

$$\text{HREMatrix}\left[\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} & m_{1,5} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & m_{2,5} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} & m_{3,5} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} & m_{4,5} \\ m_{5,1} & m_{5,2} & m_{5,3} & m_{5,4} & m_{5,5} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 9 \end{pmatrix}\right]$$

$$\left\{\left\{1, -\frac{m_{1,2}}{4}, -\frac{m_{1,3}}{4}\right\}, \left\{-\frac{m_{2,1}}{4}, 1, -\frac{m_{2,3}}{4}\right\}, \left\{-\frac{m_{3,1}}{4}, -\frac{m_{3,2}}{4}, 1\right\}\right\}$$

Calculate the HRE constant term vector for M where $C_U$ equals $\{c_1, c_2, c_3\}$ and $C_K$ equals $\{c_4 = 5, c_5 = 9\}$

$$\text{HREConstantTermVector}\left[\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} & m_{1,5} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & m_{2,5} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} & m_{3,5} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} & m_{4,5} \\ m_{5,1} & m_{5,2} & m_{5,3} & m_{5,4} & m_{5,5} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 9 \end{pmatrix}\right]$$

$$\left\{\left\{\frac{5\, m_{1,4}}{4} + \frac{9\, m_{1,5}}{4}\right\}, \left\{\frac{5\, m_{2,4}}{4} + \frac{9\, m_{2,5}}{4}\right\}, \left\{\frac{5\, m_{3,4}}{4} + \frac{9\, m_{3,5}}{4}\right\}\right\}$$

Auxiliary function that transform an upper triangle matrix into a full and reciprocal matrix

```
M = RecreatePCMatrix[
```
$$\begin{pmatrix} 1 & \frac{3}{5} & \frac{4}{7} & \frac{5}{8} & \frac{5}{10} \\ 0 & 1 & \frac{5}{7} & \frac{5}{2} & \frac{10}{3} \\ 0 & 0 & 1 & \frac{7}{2} & 4 \\ 0 & 0 & 0 & 1 & \frac{4}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$
```
]
```

$$\left\{\left\{1, \frac{3}{5}, \frac{4}{7}, \frac{5}{8}, \frac{1}{2}\right\}, \left\{\frac{5}{3}, 1, \frac{5}{7}, \frac{5}{2}, \frac{10}{3}\right\}, \left\{\frac{7}{4}, \frac{7}{5}, 1, \frac{7}{2}, 4\right\}, \left\{\frac{8}{5}, \frac{2}{5}, \frac{2}{7}, 1, \frac{4}{3}\right\}, \left\{2, \frac{3}{10}, \frac{1}{4}, \frac{3}{4}, 1\right\}\right\}$$

Defining known and unknown alternatives. It is
assumed that $c_2$ and $c_3$ are known and equal 5 and 7 correspondingly

$$\texttt{mk} = \begin{pmatrix} 0 \\ 5 \\ 7 \\ 0 \\ 0 \end{pmatrix}$$

```
{{0}, {5}, {7}, {0}, {0}}
```

Calculate the HRE ranking vector for unknown alternatives i.e. for $c_1$, $c_4$ and $c_5$ only

```
mu = N[HREPartialRank[M, mk]]
```

```
{{2.52765}, {2.88338}, {2.61696}}
```

Calculate the full HRE ranking for the given input matrix M and the vector mk

```
mu = N[HREFullRank[M, mk]]
```

```
{2.52765, 5., 7., 2.88338, 2.61696}
```

Calculate the full HRE ranking rescaled so that all its entries sum up to 1

```
mu = N[HRERescaledRank[M, mk]]
```

```
{0.126206, 0.249651, 0.349511, 0.143967, 0.130665}
```

## Heuristic Rating Estimation Method (multiplicative/geometric)

Further references could be found in papers:
* Konrad Kułakowski, Grobler-Dębska Katarzyna, Wąs Jarosław, Heuristic rating estimation - geometric approach, http://arxiv.org/abs/1404.6981

$$\textbf{HREGeomMatrix}\left[\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} & m_{1,5} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & m_{2,5} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} & m_{3,5} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} & m_{4,5} \\ m_{5,1} & m_{5,2} & m_{5,3} & m_{5,4} & m_{5,5} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ q_4 \\ q_5 \end{pmatrix}\right]$$

```
{{4, -1, -1}, {-1, 4, -1}, {-1, -1, 4}}
```

Calculate the HRE constant term vector

$$\textbf{HREGeomConstantTermVector}\left[\begin{pmatrix} m_{1,1} & m_{1,2} & m_{1,3} & m_{1,4} & m_{1,5} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & m_{2,5} \\ m_{3,1} & m_{3,2} & m_{3,3} & m_{3,4} & m_{3,5} \\ m_{4,1} & m_{4,2} & m_{4,3} & m_{4,4} & m_{4,5} \\ m_{5,1} & m_{5,2} & m_{5,3} & m_{5,4} & m_{5,5} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ q_4 \\ q_5 \end{pmatrix}\right]$$

$$\left\{\left\{\frac{\mathrm{Log}[q_4\, q_5\, m_{1,2}\, m_{1,3}\, m_{1,4}\, m_{1,5}]}{\mathrm{Log}[10]}\right\}, \left\{\frac{\mathrm{Log}[q_4\, q_5\, m_{2,1}\, m_{2,3}\, m_{2,4}\, m_{2,5}]}{\mathrm{Log}[10]}\right\}, \left\{\frac{\mathrm{Log}[q_4\, q_5\, m_{3,1}\, m_{3,2}\, m_{3,4}\, m_{3,5}]}{\mathrm{Log}[10]}\right\}\right\}$$

Calculate the HRE geometric ranking vector for unknown alternatives i.e. for $c_1$, $c_4$ and $c_5$ only

```
mu = N[HREGeomPartialRank[M, mk]]
```

```
{{2.11273}, {2.49035}, {2.13344}}
```

Calculate the full HRE geometric ranking for the given input matrix M and the vector mk

```
mu = N[HREGeomFullRank[M, mk]]
```

```
{2.11273, 5., 7., 2.49035, 2.13344}
```

Calculate the full HRE geometric ranking, rescaled so that all its entries sum up to 1

```
mu = N[HREGeomRescaledRank[M, mk]]
```

```
{0.11276, 0.266858, 0.373602, 0.132914, 0.113866}
```

Show intermediate (before raisng up to the power) HRE geometric partial rank vector

```
N[HREGeomIntermediateRank[M, mk]]
```

```
{{0.324843}, {0.396261}, {0.329081}}
```

## Bana e Costa and Vansnick’s Condition of Order Preservation test

Let calculate the eigenvalue ranking

```
rank = EigenvalueRank[M]
```

```
{0.119092, 0.27476, 0.356526, 0.130954, 0.118669}
```

.... and check whether the first Bana e Costa and Vansnick condition “condition of order preservation - COP” is satisfied

```
COP1Check[M, mu]
```

```
True
```

then check whether the second Bana e Costa and Vansnick condition (preserving intensity of preferences postulate) is satisfied

```
COP2Check[M, mu]
```

```
False
```

Prints the list of pairs for which the 1st COP is not satisfied

```
COP1ViolationList[M, mu]
```

```
{}
```

Prints the list of pairs of pairs for which the 2nd COP is not satisfied

```
COP2ViolationList[M, mu]
```

```
{{False, {{1, 2}, {1, 5}}}, {False, {{1, 2}, {4, 2}}}, {False, {{1, 2}, {5, 2}}},
 {False, {{1, 3}, {1, 5}}}, {False, {{1, 3}, {4, 2}}}, {False, {{1, 3}, {4, 3}}},
 {False, {{1, 3}, {5, 2}}}, {False, {{1, 3}, {5, 3}}}, {False, {{1, 4}, {1, 5}}},
 {False, {{1, 4}, {2, 3}}}, {False, {{1, 5}, {1, 2}}}, {False, {{1, 5}, {1, 3}}},
 {False, {{1, 5}, {1, 4}}}, {False, {{1, 5}, {2, 3}}}, {False, {{1, 5}, {5, 4}}},
 {False, {{2, 1}, {2, 4}}}, {False, {{2, 1}, {2, 5}}}, {False, {{2, 1}, {5, 1}}},
 {False, {{2, 3}, {1, 4}}}, {False, {{2, 3}, {1, 5}}}, {False, {{2, 4}, {2, 1}}},
 {False, {{2, 4}, {3, 1}}}, {False, {{2, 5}, {2, 1}}}, {False, {{2, 5}, {3, 1}}},
 {False, {{3, 1}, {2, 4}}}, {False, {{3, 1}, {2, 5}}}, {False, {{3, 1}, {3, 4}}},
 {False, {{3, 1}, {3, 5}}}, {False, {{3, 1}, {5, 1}}}, {False, {{3, 2}, {4, 1}}},
 {False, {{3, 2}, {5, 1}}}, {False, {{3, 4}, {3, 1}}}, {False, {{3, 5}, {3, 1}}},
 {False, {{4, 1}, {3, 2}}}, {False, {{4, 1}, {5, 1}}}, {False, {{4, 2}, {1, 2}}},
 {False, {{4, 2}, {1, 3}}}, {False, {{4, 3}, {1, 3}}}, {False, {{4, 5}, {5, 1}}},
 {False, {{5, 1}, {2, 1}}}, {False, {{5, 1}, {3, 1}}}, {False, {{5, 1}, {3, 2}}},
 {False, {{5, 1}, {4, 1}}}, {False, {{5, 1}, {4, 5}}}, {False, {{5, 2}, {1, 2}}},
 {False, {{5, 2}, {1, 3}}}, {False, {{5, 3}, {1, 3}}}, {False, {{5, 4}, {1, 5}}}}
```

## Koczkodaj's Iterative Inconsistency Reduction algorithm

Calculate the value of the Koczkodaj inconsistency index

```
N[KoczkodajIdx[M]]
```

```
0.78125
```

Prints the worst Koczkodaj triad in M. As we can see it is $m_{5,3} == \frac{1}{4}$, $m_{3,1} == \frac{7}{4}$, $m_{5,1} == 2$. The value of inconsistency introduced by this triad is $\frac{25}{32}$

```
KoczkodajTheWorstTriad[M]
```

$$\left\{\{5, 3, 1\}, \left\{\frac{1}{4}, \frac{7}{4}, 2\right\}, \frac{25}{32}\right\}$$

Perform one step of the Koczkodaj inconsistency reduction algorithm. On the output there is a new slightly modified matrix M2 that is expected to be more consistent than M

```
M2 = N[KoczkodajImproveMatrixStep[M]]
```

```
{{1., 0.6, 0.344306, 0.625, 0.829827},
 {1.66667, 1., 0.714286, 2.5, 3.33333}, {2.90439, 1.4, 1., 3.5, 2.41014},
 {1.6, 0.4, 0.285714, 1., 1.33333}, {1.20507, 0.3, 0.414913, 0.75, 1.}}
```

```
KoczkodajIdx[M2]
```

```
0.585087
```

## Aggregation of Individual Judgments (AIJ)

Let us consider three different PC matrices $X$, $Y$ and $Z$ that come from three different experts

$$X = \begin{pmatrix} 1 & x_{12} & x_{13} & x_{14} \\ x_{21} & 1 & x_{23} & x_{24} \\ x_{31} & x_{32} & 1 & x_{34} \\ x_{41} & x_{42} & x_{43} & 1 \end{pmatrix};$$

$$Y = \begin{pmatrix} 1 & y_{12} & y_{13} & y_{14} \\ y_{21} & 1 & y_{23} & y_{24} \\ y_{31} & y_{32} & 1 & y_{34} \\ y_{41} & y_{42} & y_{43} & 1 \end{pmatrix};$$

$$Z = \begin{pmatrix} 1 & z_{12} & z_{13} & z_{14} \\ z_{21} & 1 & z_{23} & z_{24} \\ z_{31} & z_{32} & 1 & z_{34} \\ z_{41} & z_{42} & z_{43} & 1 \end{pmatrix};$$

Then it is possible to aggregate the results using appropriate functions.

Aggregate Individual Judgments additively (AIJadd):

```
AIJadd[X, Y, Z] // MatrixForm
```

$$\begin{pmatrix} 1 & \frac{1}{3}(x_{12}+y_{12}+z_{12}) & \frac{1}{3}(x_{13}+y_{13}+z_{13}) & \frac{1}{3}(x_{14}+y_{14}+z_{14}) \\ \frac{1}{3}(x_{21}+y_{21}+z_{21}) & 1 & \frac{1}{3}(x_{23}+y_{23}+z_{23}) & \frac{1}{3}(x_{24}+y_{24}+z_{24}) \\ \frac{1}{3}(x_{31}+y_{31}+z_{31}) & \frac{1}{3}(x_{32}+y_{32}+z_{32}) & 1 & \frac{1}{3}(x_{34}+y_{34}+z_{34}) \\ \frac{1}{3}(x_{41}+y_{41}+z_{41}) & \frac{1}{3}(x_{42}+y_{42}+z_{42}) & \frac{1}{3}(x_{43}+y_{43}+z_{43}) & 1 \end{pmatrix}$$

Aggregate Individual Judgments geometrically (AIJgeom)

```
AIJgeom[X, Y, Z] // MatrixForm
```

$$\begin{pmatrix} 1 & (x_{12}\,y_{12}\,z_{12})^{1/3} & (x_{13}\,y_{13}\,z_{13})^{1/3} & (x_{14}\,y_{14}\,z_{14})^{1/3} \\ (x_{21}\,y_{21}\,z_{21})^{1/3} & 1 & (x_{23}\,y_{23}\,z_{23})^{1/3} & (x_{24}\,y_{24}\,z_{24})^{1/3} \\ (x_{31}\,y_{31}\,z_{31})^{1/3} & (x_{32}\,y_{32}\,z_{32})^{1/3} & 1 & (x_{34}\,y_{34}\,z_{34})^{1/3} \\ (x_{41}\,y_{41}\,z_{41})^{1/3} & (x_{42}\,y_{42}\,z_{42})^{1/3} & (x_{43}\,y_{43}\,z_{43})^{1/3} & 1 \end{pmatrix}$$

Note that these two functions works also for result lists i.e.:

```
AIJgeom[{x1, x2, x3, x4}, {y1, y2, y3, y4}]
```

$$\left\{\sqrt{x_1\,y_1},\ \sqrt{x_2\,y_2},\ \sqrt{x_3\,y_3},\ \sqrt{x_4\,y_4}\right\}$$

```
AIJadd[{x1, x2, x3, x4}, {y1, y2, y3, y4}]
```

$$\left\{\frac{1}{2}(x_1+y_1),\ \frac{1}{2}(x_2+y_2),\ \frac{1}{2}(x_3+y_3),\ \frac{1}{2}(x_4+y_4)\right\}$$

## Incomplete Pairwise Comparisons Matrix - Harker Method

The idea comes from P. T. Harker, Alternative Modes of Questioning in The Analytic Hierarchy Process, Math Modeling, 1987.

Let M be an incomplete pairwise comparisons matrix.

```
M = (  1     2    □
      1/2    1    2
       □    1/2   1 );
```

Thus, to compute the ranking based on M we need to compute matrix A (hereinafter referred to as Harker matrix)

```
A = HarkerMatrix[M];
```

```
A // MatrixForm
```

$$\begin{pmatrix} 2 & 2 & 0 \\ \frac{1}{2} & 1 & 2 \\ 0 & \frac{1}{2} & 2 \end{pmatrix}$$

Then compute the eigenvalue based ranking and inconsistency index as usuall:

```
EigenvalueRank[A]
```

```
{0.571429, 0.285714, 0.142857}
```

```
SaatyIdx[A]
```

```
0
```

## Ranking errors and ranking discrepances

Let the w = {$w_1$, $w_2$, $w_3$} be the ranking vector whilst M a PC matrix. An error is defined as $e_{ij} = m_{ij}(w_j / w_i)$ and it corresponds to the "discrepancy" between individual judgment $m_{ij}$ and the ranking result. Hence, the error matrix is just E = [$e_{ij}$]. E.g.:

$$M = \text{RecreatePCMatrix}\left[\begin{pmatrix} 1 & \frac{3}{5} & \frac{4}{7} & \frac{5}{8} & \frac{5}{10} \\ 0 & 1 & \frac{5}{7} & \frac{5}{2} & \frac{10}{3} \\ 0 & 0 & 1 & \frac{7}{2} & 4 \\ 0 & 0 & 0 & 1 & \frac{4}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}\right];$$

```
ErrorMatrix[M, EigenvalueRank[M]] // MatrixForm
```

$$\begin{pmatrix} 1. & 0.722398 & 0.584559 & 1.45507 & 2.00712 \\ 1.38428 & 1. & 1.07892 & 0.839258 & 0.694602 \\ 1.71069 & 0.92685 & 1. & 0.777866 & 0.751091 \\ 0.687253 & 1.19153 & 1.28557 & 1. & 0.827639 \\ 0.498227 & 1.43967 & 1.3314 & 1.20826 & 1. \end{pmatrix}$$

The error matrix is not symmetric. An attempt to symmetrization of error matrix leads to the local discrepancy matrix D = ($d_{ij}$) where $d_{ij} = \max\{e_{ij} - 1, 1/e_{ij} - 1\}$. Another interesting property of the local discrepancy matrix is the fact that wherever $d_{ij} = 0$ this means that the local discrepancy (error) is 0. If $d_{ij} = X$ this means that the local judgment $m_{ij}$ differs from the ratio $\frac{w_i}{w_j}$ by 100%*X

```
Chop@LocalDiscrepancyMatrix[M, EigenvalueRank[M]] // MatrixForm
```

$$\begin{pmatrix} 0 & 0.384278 & 0.71069 & 0.455068 & 1.00712 \\ 0.384278 & 0 & 0.0789237 & 0.191529 & 0.439673 \\ 0.71069 & 0.0789237 & 0 & 0.285569 & 0.331397 \\ 0.455068 & 0.191529 & 0.285569 & 0 & 0.208257 \\ 1.00712 & 0.439673 & 0.331397 & 0.208257 & 0 \end{pmatrix}$$

The greatest entry of the local discrepancy matrix can be found by using the GlobalDiscrepancy function

```
GlobalDiscrepancy[M, EigenvalueRank[M]]
```

```
1.00712
```