

Agent-based modelling of the spread of financial crises conditional on the country of origin

Valentina Alto, Konrad Eilers, Leonie Intat, Anthonie Wali Kamp, Hugo Paolini

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Abstract

We propose an agent-based model of international banking crises, where banks are aggregated by country, in order to better understand systemic risk and the spread of debt default. In this model, countries are differentiated by their credit ratings and bilateral foreign claims which are both considered levers for systemic risk. Multiple financial crises are simulated where each country is set as the initially defaulted country. The speed and spread of these simulated financial crises are compared and validated by a compartmental model deployed on a network. The results show that the spread of a financial crisis mainly depends on the initially defaulted country's ability to counterbalance outstanding foreign claims with reserve capital. This capacity determines whether the initially defaulted country poses a threat to the stability of the overall banking system.

Code and data available at https://github.com/konrad1254/Simulation_Modeling.

Keywords: systemic risk, global financial crisis, foreign claims, debt default, agent-based modelling, epidemics on networks, epidemiological models, SIR modelling

1. Introduction

Due to the economic development and technological advancements over the last decades, international relations have grown in complexity leading to increased interdependence in the banking sector (Brunnermeier et al., 2010; Kaufman, 2000). Although this favors economic growth and facilitates risk sharing, it entails higher exposure and vulnerability of a nation's economy to financial crises and thus greater systemic risk (Allen and Gale, 2000; Avdjiev et al., 2019; Iori et al., 2006). Therefore, new challenges for policy makers with regard to risk measurement and management have emerged (Brunnermeier et al., 2010).

Taking into account the current economic uncertainties due to COVID-19, it is more important than ever to understand the dynamics of systemic financial risk (Goodell, 2020). The International Monetary Fund anticipates increased debt levels once the global economy begins its recovery from the pandemic and advises to act proactively (International Monetary Fund, 2020). Putting in place the right measures to protect the weakest links in the financial system will be crucial to avoid a systemic banking crisis.

Existing literature has proven the transferability of epidemiological models into other research fields (Rodrigues, 2016; Demiris et al., 2012; May et al., 2008). Hence,

our study draws on the link between epidemiology and banking—specifically how theoretical models used to study the spread of infectious diseases can serve to model financial contagion.

In this paper, two established models, a compartmental model deployed on a network (SIR-EoN: Susceptible-Infected-Recovered with the Epidemics on Networks module) and an agent-based model (ABM), are implemented to simulate the spread of a debt crisis. We use a dataset of 23 developed and emerging countries from the Bank for International Settlements (Bank for International Settlements, 2019). Multiple scenarios are simulated considering each country as the initially defaulted country. Thereby, we investigate how agent-based modelling can be used to evaluate the severity of financial crises due to debt defaults on a country by country basis.

This paper is structured as follows. Section 2 provides some further background on both systemic financial risk and the use of epidemiological models for the analysis of the spread of financial crises. Section 3 describes the data used in our model and presents the methods used to simulate the spread of financial crises. In section 4, we discuss the results of our simulations and compare selected metrics, such as the speed and severity of the crises relative to the starting country. Section 5 concludes with the

implications and potential limitations of this study and an outlook for further research.

2. Background

There is a significant amount of literature on systemic risk but no overarching definition has been established yet (Silva et al., 2016; Hurd, 2016; Kaufman, 2000). Hurd (2016) argues that it may even be dangerous to implement policies aiming to manage systemic risk without having a clear and commonly accepted definition. In this paper we draw on the definition provided by the Bank for International Settlements (BIS): “the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default, with the chain reaction leading to broader financial difficulties” (Bank for International Settlements, 1994, p.177). This chain reaction behavior of a single bank default affirms the comparison to infectious diseases.

Systemic risk usually originates from the banking sector and subsequently infects the wider economy (Cerutti et al., 2012; Kaufman, 2000). Therefore, understanding cross-border relations in the banking sector and the spread of financial crises is highly relevant for the analysis of systemic risk dynamics (Tonzer, 2015). Allen and Gale (2000) study financial contagion by looking at the spillover effect of a small shock on the debt relations between banks. This effect can be translated from individual banks to the general financial network between countries.

Looking back at the global financial crisis between 2007 and 2009, one can see the impact of interconnectivity on risk spillover in financial markets (Avdjiev et al., 2019). Cross holdings of deposits and the resulting network structure of the banking market are one of the decisive factors defining the spread of financial contagion (Allen and Gale, 2000). Depending on the completeness or incompleteness of the network, a crisis may fully spread or stay in a particular area, but in the latter case hit those affected even harder

(Allen and Gale, 2000). The focus of this paper is on complete networks and the analysis of risk spreading in incomplete networks is left open for further research.

Investigating possible methodologies to simulate the spread of a financial crisis, Rodrigues (2016) presents new areas of research where epidemiological models, and in particular SIR models, are used to simulate diffusion phenomena. Financial contagion is one those possible applications of interest (Rodrigues, 2016). Other researchers have already translated selected models into the financial environment. For instance, Garas et al. (2010) create a global economic network based on corporate ownership and international trade structures. To simulate the spread of the crisis, they then apply an SIR model on this network (Garas et al., 2010). Also, Kostylenko et al. (2019) use an epidemiological model to analyze the spread of financial contagion building up on the network characteristics of financial relations. In this paper we implement and extend this analysis with contemporary data.

As suggested by Gai and Kapadia (2010), we simulate different scenarios with each country being the initially defaulted country to compare the evolution and other characteristics of the financial crisis. The importance and influence of the starting point of a financial crisis is highlighted by Upper (2011). We expect the development of debt crises to vary depending on the starting point, which would therefore merit different policy responses. Hence, our research can support policy makers in their evaluation of potential policy responses and preventive measures (Upper, 2011). Still, one must be aware that financial contagion does not travel through one single channel but rather spreads via multiple links at the same time (Hurd, 2016). In this paper, we choose to focus on the interbank lending structure on a country by country basis. However, to guarantee the applicability of our model to the real world, it is important to consider the wider context and

further channels of risk spreading before implementing specific measures.

Given the flexibility of agent-based modelling and its capability to capture emergent phenomena, we select this method to simulate the spread of a financial crisis (Bonabeau, 2002). ABM increases heterogeneity between the entities in the system compared to equation-based modelling techniques (Hunter et al., 2018). Moreover, agent-based approaches have already been used for the simulation of interbank dynamics and the evaluation of individual entities' contribution to systemic risk (Bluhm et al., 2012).

The research discussed in this section confirms the use of epidemiological models in the banking sector and supports the application of agent-based modelling to simulate the development of financial crises.

3. Methods

Similar to Kostylenko et al. (2019), 23 developed and emerging countries are chosen from the Bank for International Settlements' "Summary of foreign claims and other potential exposures" as of September 2019 (Bank for International Settlements, 2019). This data report represents the amounts outstanding by nationality of the reporting bank on an ultimate risk basis, defined as the country in which the guarantor of a financial claim resides (Bank for International Settlements, 2019). Ultimate risk is considered to be the best measure for aggregate exposure of a banking system to a given country (Kostylenko et al., 2019).

In this paper the number of countries is held fixed at 23 and their connections are represented as follows:

$$A = \begin{bmatrix} 0 & a_{1,2} & \cdots & a_{1,23} \\ a_{2,1} & 0 & \cdots & a_{2,23} \\ \vdots & \vdots & \ddots & \vdots \\ a_{23,1} & a_{23,2} & \cdots & 0 \end{bmatrix} \quad 3.1$$

where $a_{ii} = 0$ and $a_{ij} = a_{ji}$. This matrix shows that all countries are linked to each other but not to themselves. Since A is a symmetric matrix, this results in an undirected graph. Figure 1 shows the graph where the nodes and edges represent countries and bilateral outstanding claims, respectively.

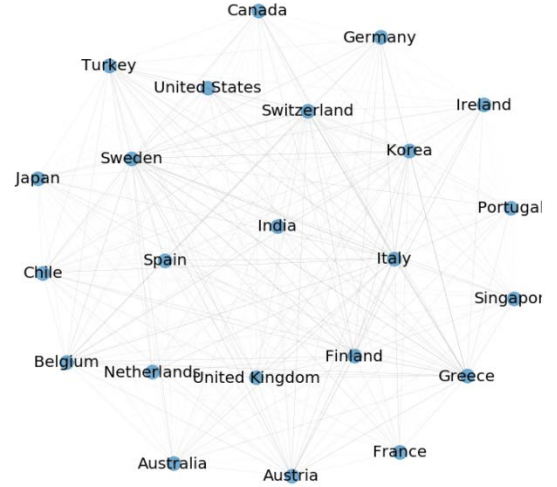


Figure 1: Graph of connectivity matrix A .

This network structure allows the diffusion process to take into consideration heterogeneous debt relationships between individual countries.

For modelling purposes, it is assumed that a financial crisis starts in a single country and spreads through the edges of the network. Therefore, we assume an initial population of 22 susceptible, one defaulted (i.e. infected), and zero recovered countries.

Kostylenko et al. (2019) provide equations for both the beta (β) and gamma (γ) parameters, which translate infectivity and recovery into the context of financial debt and systemic risk. Beta is defined as the probability of transmission:

$$\beta_i = \frac{\alpha_i}{\sum_{j=1}^{23} \alpha_j} \quad 3.2$$

where α_i represents the total claims of country i (Kostylenko et al., 2019). In other

Table 1: Country ratings, foreign debt, and parameters.

Countries	S&P Rating	S&P Numeric	Foreign Claims (\$B)	β	γ	β / γ
Australia	AAA	100	687.6	0.026	1.000	0.026
Canada	AAA	100	1,897.9	0.071	1.000	0.071
Germany	AAA	100	1,873.4	0.070	1.000	0.070
Netherlands	AAA	100	1,327.1	0.050	1.000	0.050
Singapore	AAA	100	567.6	0.021	1.000	0.021
Sweden	AAA	100	346.5	0.013	1.000	0.013
Switzerland	AAA	100	1,084.2	0.041	1.000	0.041
Austria	AA+	95	391.2	0.015	0.167	0.088
Finland	AA+	95	472.9	0.018	0.167	0.106
United States	AA+	95	3,599.6	0.135	0.167	0.808
Belgium	AA	90	233.5	0.009	0.091	0.096
France	AA	90	3,098.6	0.116	0.091	1.275
Korea	AA	90	197.8	0.007	0.091	0.081
United Kingdom	AA	90	3,605.7	0.135	0.091	1.484
Ireland	AA-	85	92.8	0.003	0.063	0.056
Chile	A+	80	14.2	0.001	0.048	0.011
Japan	A+	80	4,371.8	0.164	0.048	3.434
Spain	A	75	1,756.6	0.066	0.038	1.708
Italy	BBB	60	851.6	0.032	0.024	1.306
Portugal	BBB	60	95.6	0.004	0.024	0.147
India	BBB-	55	85.6	0.003	0.022	0.147
Greece	BB-	40	57.7	0.002	0.016	0.132
Turkey	B+	35	25.9	0.001	0.015	0.064

words, beta is the total claims of country i relative to the total claims of all countries.

Gamma, the probability of recovery, is set using Standard & Poor's (S&P) country credit ratings, which reflect a country's asset and debt volumes (Standard & Poor's, 2020). In Table 1, the S&P index, which ranges from D to AAA, is converted to a numeric scale between 5 and 100 with increments of 5 (Majnoni et al., 1999). Gamma is set directly proportional to the country credit score:

$$\gamma_i = \frac{1}{101 - C_i} \quad 3.3$$

where C_i is the credit score of country i .

The parameters specified in Table 1 were used in the SIR-EoN model and ABM. A total of 23 different simulation scenarios were run, each with a different initially

defaulted country without specifying a time unit. However, by real-world comparison the time unit can be estimated between one day and one week.

The SIR-EoN model used in this paper was developed by Kiss, Miller, & Simon, who applied the SIR differential equations to an underlying graph structure (Kiss, Miller, & Simon, 2017). This procedure extends the homogeneous version of an SIR model but does not allow for full heterogeneity as the betas are computed as edge rather than node attributes. Hence, we introduce an ABM to attribute betas to each node.

By modifying and extending the virus-on-a-network model developed by Stonedahl and Wilensky (2008) and, using the implementation of Mesa (2019), we introduce heterogeneous agents for each node using the

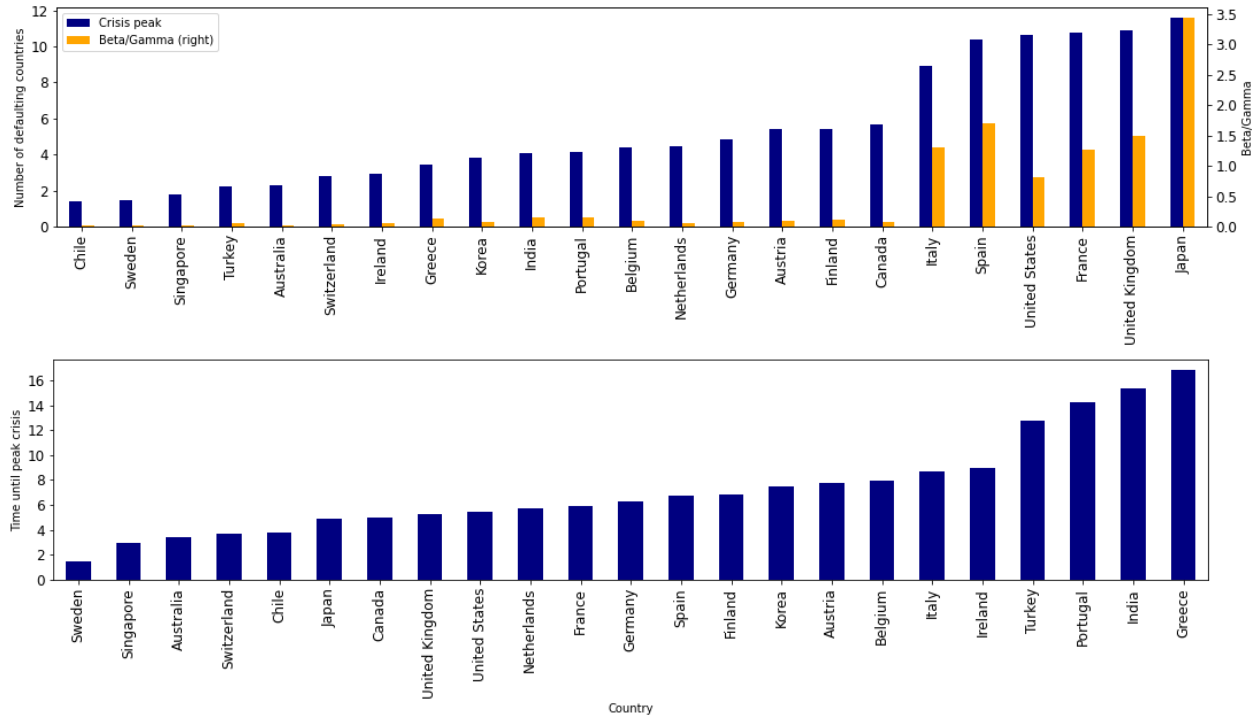


Figure 2a (top): Number of defaulted countries at the crisis peak compared to the beta/gamma ratio of the initially defaulted country. Figure 2b (bottom): Time until peak of financial crisis.

countries' betas and gammas. In this ABM, each agent is equipped with a couple of functions. First, a defaulted country attempts to infect other countries. The probability of transmission depends on each country's individual probability to default. Once defaulted, each agent attempts to recover with a success rate depending on the country's individual probability of recovery. The functionality to specify with which agent the crisis starts, i.e. which country defaults first, is also implemented. Lastly, we expand the data collection method and introduce the functionality to run multiple iterations for each simulation as well as generate bootstrapped confidence intervals.

4. Results and discussion

The severity of a financial crisis is assessed in two dimensions: spread and speed. Simulating each country in our dataset as the starting point of a financial crisis and averaging over 200 iterations, the ABM

returns a highly differentiated set of simulated crises.

Figure 2a depicts the total number of defaulted countries at the peak of the crisis, corresponding to the apex of the infected curves in Appendix A. For most simulations, the peak lies below six defaulted countries. The weakest crises originate from Chile, Sweden and Singapore, while the strongest crises are expected if France, the United Kingdom or Japan initially default on their debt obligations. The differences in the dispersion can be attributed to the ratio of beta and gamma as both contribute to the spread of the crisis, which yields a concise interpretation. As Kostylenko et al. (2019) highlight the direct link between bank reserve capital and credit ratings, we argue that the spread of a financial crisis depends on the initially defaulted country's ability to counterbalance outstanding foreign claims with reserve capital. Thus, if a country's beta to gamma ratio is large and consequently

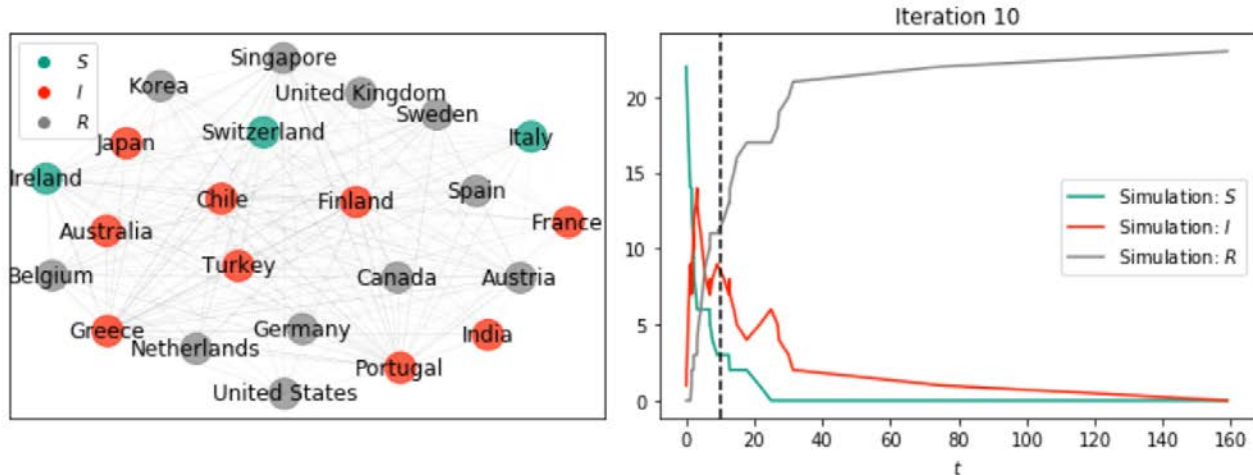


Figure 3: Left: snapshot of network diffusion process at timestamp 10 (initial default: United States). Right: Trace plot of the SIR simulation (initial default: United States).

cannot cover its debt default losses, this results in a severe financial crisis.

Figure 2b compares the time until the crisis peak is reached for each simulation. For most simulations, the time to peak lies below eight time steps. Sweden, Singapore and Austria are the quickest to reach the peak of the crisis, while Turkey, Greece, Portugal and India take the longest. Ridenhour et al. (2014) suggest that—in the case of an SIR model—the time to peak depends on the relationship between the probabilities of transmission and recovery. However, in the ABM simulations we do not observe a strong correlation between the starting country's parameters and the time to peak. Thus, given the previous result, there is no evidence for a relationship between the severity and speed of a financial crisis.

Aside from running multiple iterations of each simulation to address the inherent stochasticity of an ABM, we utilize the SIR-EoN model to validate our results. First, the SIR-EoN generally estimates a much more contagious process where the crisis peaks usually outgrow those of the ABM. Second, the SIR-EoN simulations exhibit longer time intervals before the crisis peak than the ABM. In most cases, the SIR-EoN predicts a peak around time step 20 while the ABM usually reaches the peak within ten time steps.

Nonetheless, the SIR-EoN confirms the general shape of the trace plots for most simulations. As an example, Figure 3 shows the scenario where the crisis starts in the United States which corresponds closely to the respective ABM trace plot (Appendix A). Further SIR-EoN trace plots can be found in the supplementary code.

There are some methodological differences that might explain the differences between the results of the two models. The SIR-EoN is built on the network itself, which means that the beta parameters are edge rather than node attributes. Hence, if an edge links country A and B, its corresponding beta will be the average of A's and B's beta parameters. This could lead to inconsistent and rather counterintuitive results. If the crisis starts from a country with a high beta, this should result in a faster and more widespread crisis. However, if this country is connected to a country with a low beta, the edge connecting the two will be attributed with the average of the two betas. Hence, the severity and speed of the SIR-EoN simulated crises may be under- or overestimated. The ABM circumvents this issue by considering the beta parameters as agent attributes. Thereby, a country's probability of default is only dependent on its own debt exposure and not

on the average exposure of the two connected countries.

The ABM results are consistent with the distribution of the sample countries' foreign claims and S&P ratings. Furthermore, these results largely coincide with the SIR-EoN validation model. Any differences between the two models can be attributed to the SIR-EoN model's simplification of the debt relations between countries.

5. Conclusions

The ABM developed in this paper successfully simulates the trajectory of financial crises originating from a number of different countries. The results show that countries such as France, the United Kingdom, or Japan can cause severe financial crises. This suggests that these countries bear considerable systemic risk and pose a threat to the overall banking system. Specifically, a country's inability to secure its foreign claims contributes to the spread of a financial crisis. The ratio between the probability to default and recover from the crisis is a clear and quantifiable metric for systemic risk which could be interesting for policymakers. Furthermore, no link is found between the speed and spread of a crisis.

These results are robust and are validated by comparison with the SIR-EoN results. Furthermore, the inherent stochasticity of the ABM is addressed by running 200 iterations of each simulation and bootstrapping confidence intervals.

An important limitation of this study is that the probabilities of transmission depend

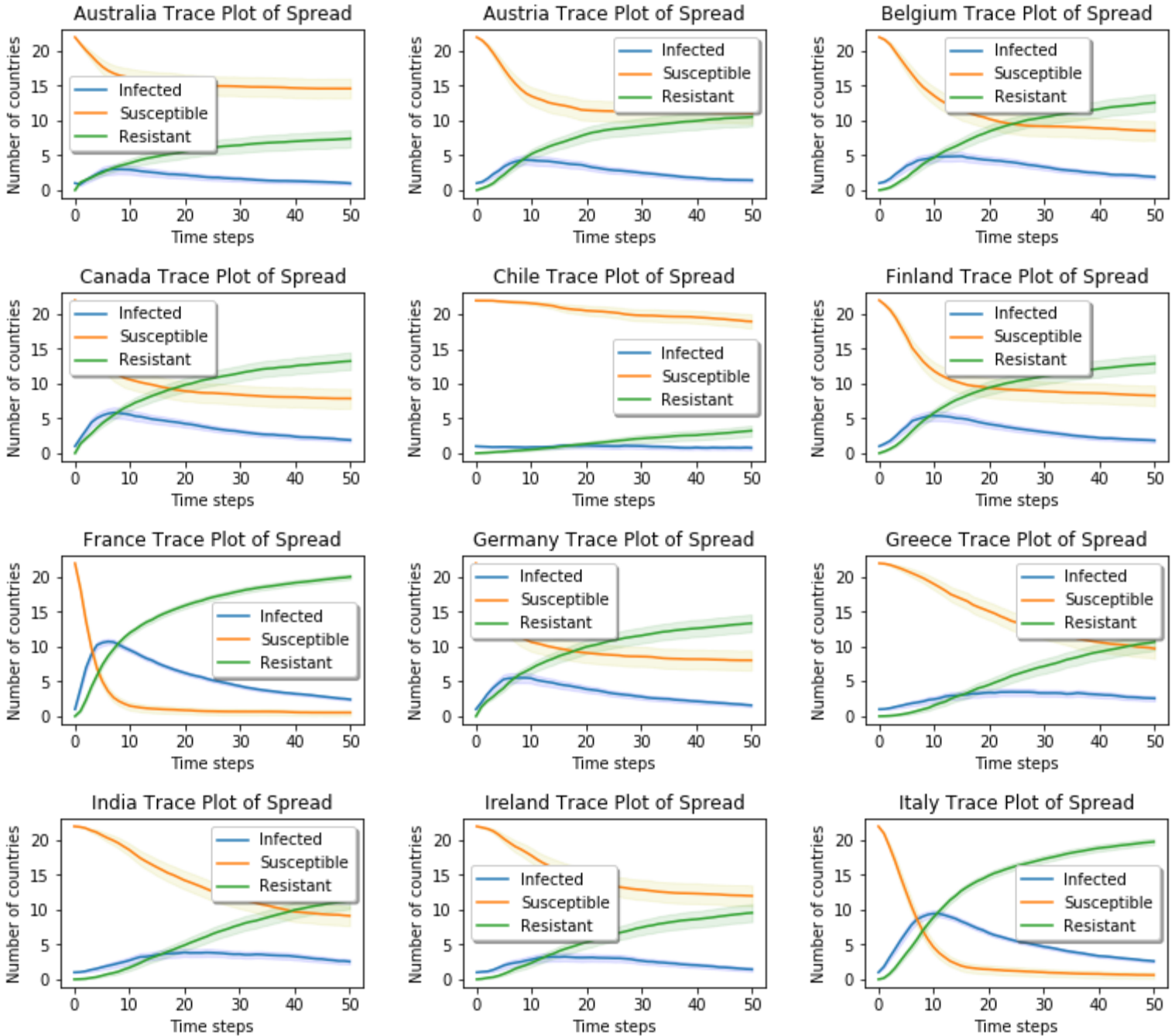
only on international debt claims. Financial crises, however, can originate from balance sheet exposures in both assets and liabilities, and spread via multiple channels simultaneously (Smaga, 2014). It is also highly likely that one defaulted channel infects other channels (Hurd, 2016). Therefore, it is important for policy makers to consider the wider context before implementing specific measures. Lastly, our conclusion heavily depends on the chosen modelling technique. Underlying data is required to confirm the model's results.

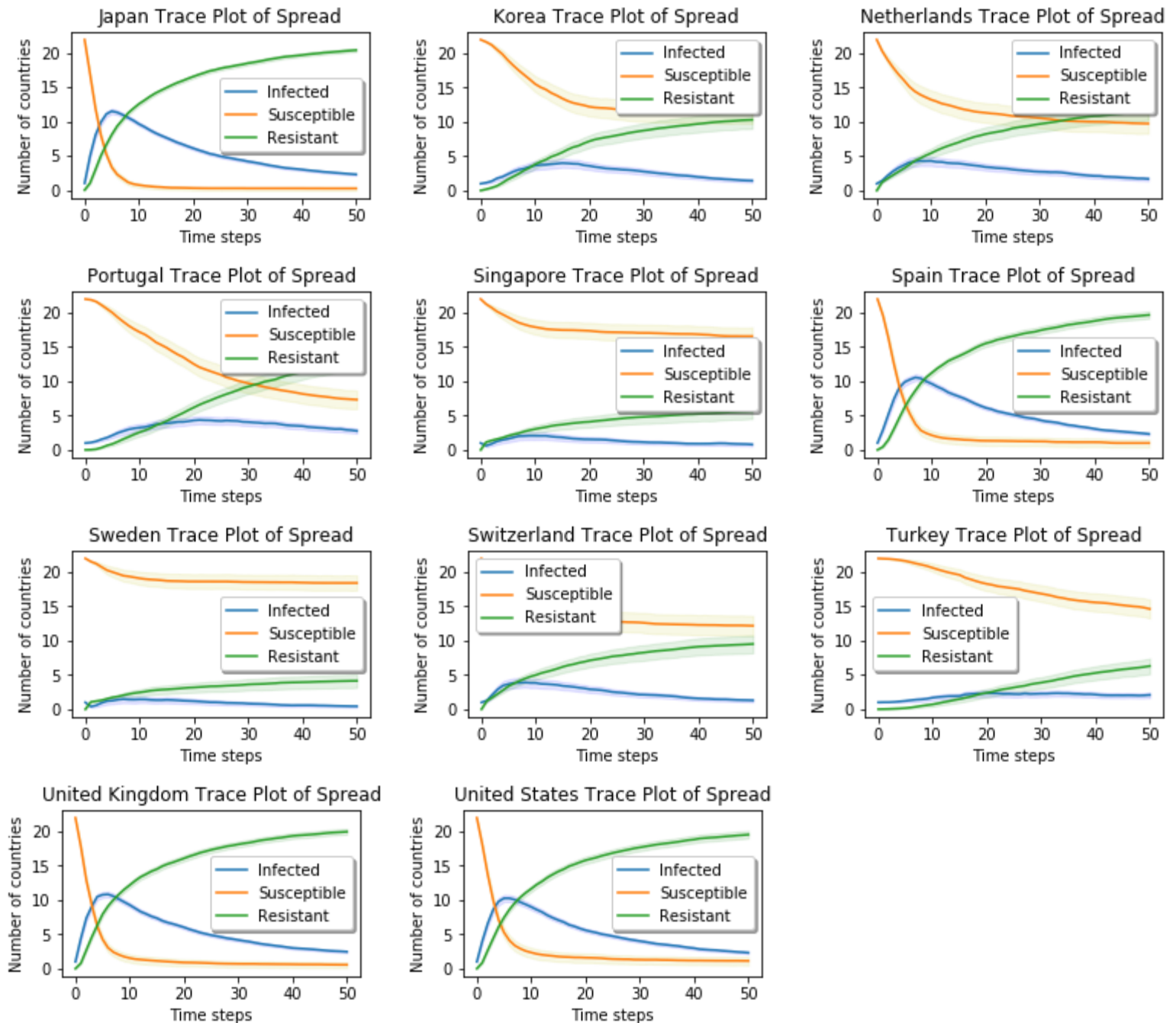
To represent our 23 countries in an SIR-EoN model, we use an undirected graph, meaning that the probability of transmission between two nodes is symmetric. The model could be improved by introducing a directed graph, where each node has inward and outward links indicating the different effects of being the recipient or the promoter of debt default, respectively. By not averaging the beta parameters of connected countries, the network is more heterogeneous and thus a more realistic representation of bilateral debt relations.

Lastly, SIR models are often found with extensions that model interventions by exogenous agents such as regulatory institutions, e.g. central banks (Kostylenko et al., 2017). Simulating the impact of regulatory interventions on crisis development could help to prevent collapse of the global banking system and provide useful insights to policy makers.

Appendix

Appendix A: Crisis spread trace plots including bootstrapped confidence intervals for each ABM simulation.





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