Algorithms Analysis - Handout 1

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Exercise 1

1. $x^2 = O(x^3)$
2. $x^3 = O(x^2)$
3. $5x^3 + 3x^2 = \Omega(x^4)$
4. $3x^2 + 5x + 2 = \Theta(x^2)$
5. $2^{n+1} = O(2^n)$
6. $3^n = O(2^n)$
7. $3^n = \Omega(2^n)$

1. True.

Big-O notation represents an upper bound on the growth of a function. Since x^3 grows faster than x^2, x^2 is $O(x^3)$.

$$x^2 \leq x^3 \cdot c$$

divide by x^3

$$\frac{1}{x} \leq c$$

As $x \to \infty$, $\frac{1}{x} \to 0$, so the equation is true for any $c \geq 0$ and $x_0 \geq 1$.

2. False.

x^3 grows faster than x^2, so x^3 cannot be bounded above by x^2.

$$x^3 \leq x^2 \cdot c$$

divide by x^2

$$x \leq c$$

This inequality is false for large x because x grows without bound.

3. False.

The term $\Omega(x^4)$ suggests a lower bound, but $5x^3 + 3x^2$ grows slower than x^4. Hence, it is not bounded below by xˆ4.

$$5x^3 + 3x^2 \geq c \cdot x^4$$

divide by x^4

$$\frac{5}{x} + \frac{3}{x^2} \geq c$$

As $x \to \infty$, $\frac{5}{x} + \frac{3}{x^2} \to 0$, so there is no such c that inequality would be true.

4. True.

The polynomial $3x^2 + 5x + 2$ has a dominant term of x^2 as $x \to \infty$. Therefore, it has both upper and lower bounds that are proportional to xˆ2, making it $\Theta(x^2)$.

$$c_1 \cdot x^2 \leq 3x^2 + 5x + 2 \leq c_2 \cdot x^2$$

We can ignore terms $5x + 2$ for large x, for which both lower and upper bounds can be satisfied by choosing constants.

For example $c_1 = 1, c_2 = 4$

5. False.

$2^{n+1} = 2 \cdot 2^n$, which grows faster than 2^n. Thus, 2^{n+1} is not $O(2^n)$.

$$2 \cdot 2^n \leq c \cdot 2^n$$

divide by 2^n

$$2 \leq c$$

Which implies that the statement is false.

6. False.

3^n grows exponentially faster than 2^n, so it cannot be bounded above by 2^n. Therefore, the statement is false.

$$3^n \leq c \cdot 2^n$$

divice by 2^n

$$(\frac{3}{2})^n \leq c$$

$(\frac{3}{2})^n$ grows exponentially, so this inequality would be false for large n

7. True.

Since 3^n grows faster than 2^n, 3^n is indeed bounded below by 2^n, making this statement true.

$$3^n \geq c \cdot 2^n$$

divice by 2^n

$$(\frac{3}{2})^n \geq c$$

$(\frac{3}{2})^n$ grows exponentially, which makes this inequality true for lagre n and any $c \geq 1$.

Exercise 2

```
MAX-HEAPIFY(A, i)
l ← LEFT(i)
r ← RIGHT(i)
if l ≤ heap-size[A] and A[l] > A[i]
   then largest ← l
   else largest ← i
if r ≤ heap-size[A] and A[r] > A[largest]
   then largest ← r
if largest ≠ i
   then exchange A[i] ↔ A[largest]
        MAX-HEAPIFY(A, largest)
```

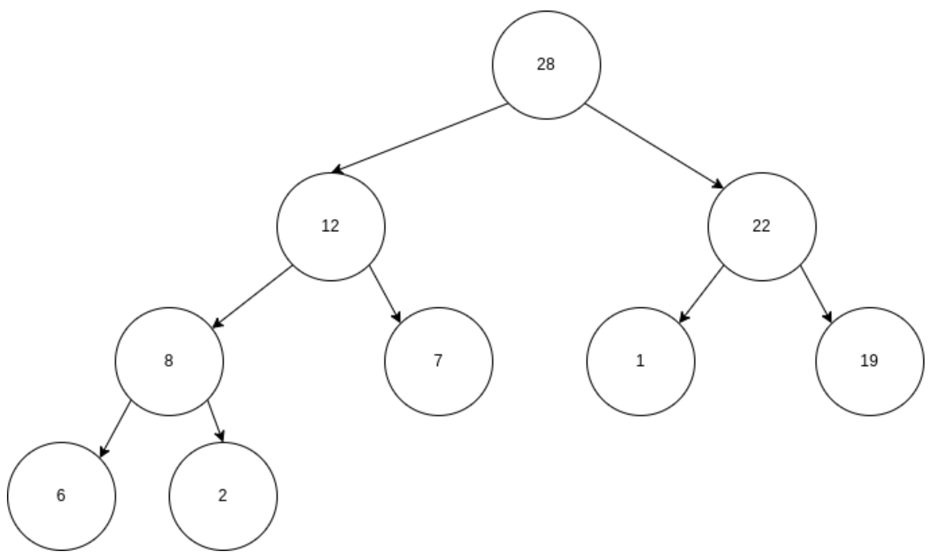
Figure 1: alt text

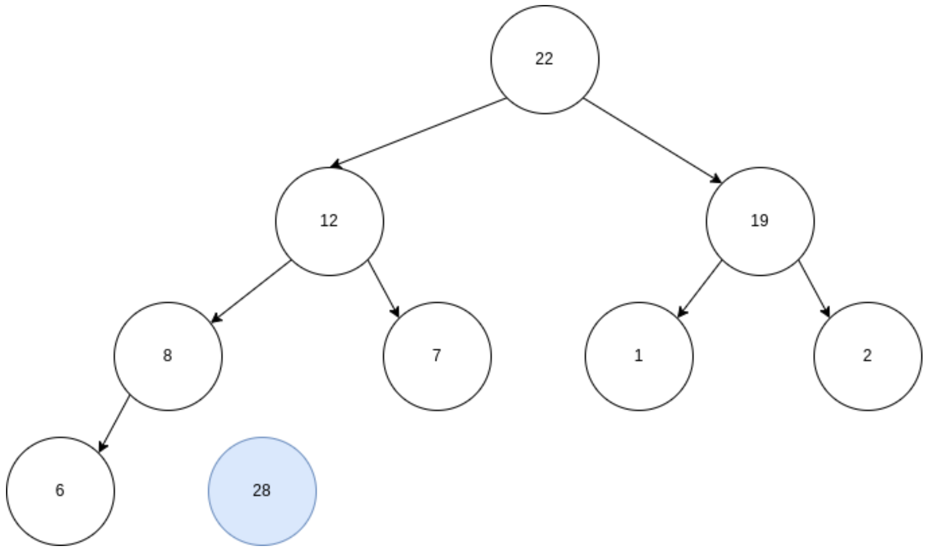
```
MAX-HEAPIFY(A, i)
while (true)
    l = LEFT(i)
    r = RIGHT(i)
    if l <= heap-size[A] and A[l] > A[i]
```

```
        largest = l
    else
        largest = i
    if r <= heap-size[A] and A[r] > A[largest]
        largest = r
    if largest != i
        exchange A[i] with A[largest]
        i = largest
    else
        break
end
```

Exercise 3

The following pictures show sorting an array using heap sort.




22
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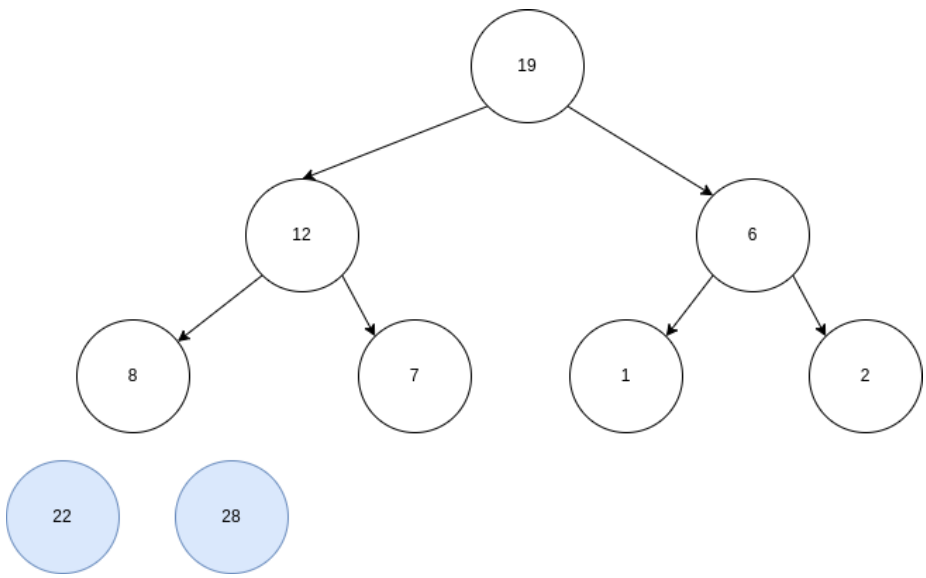
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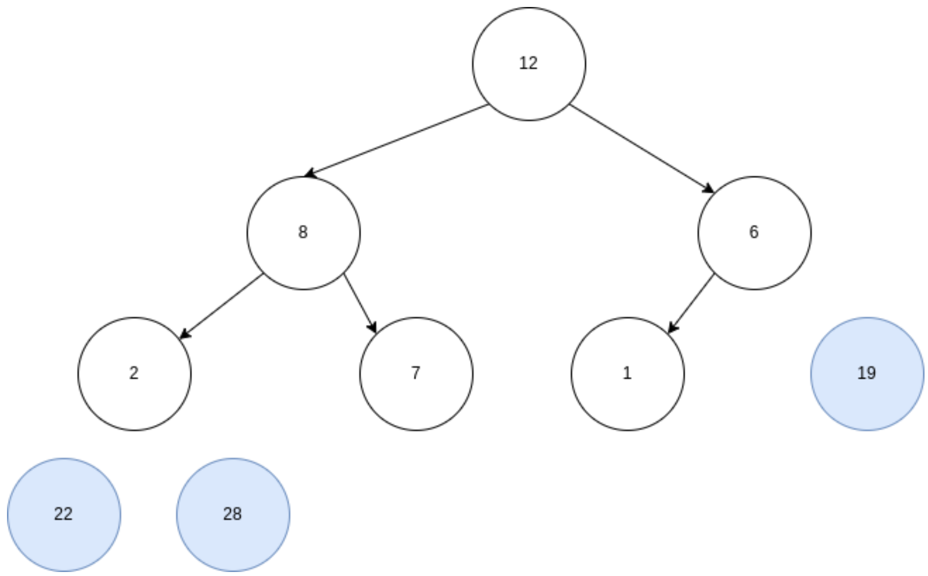
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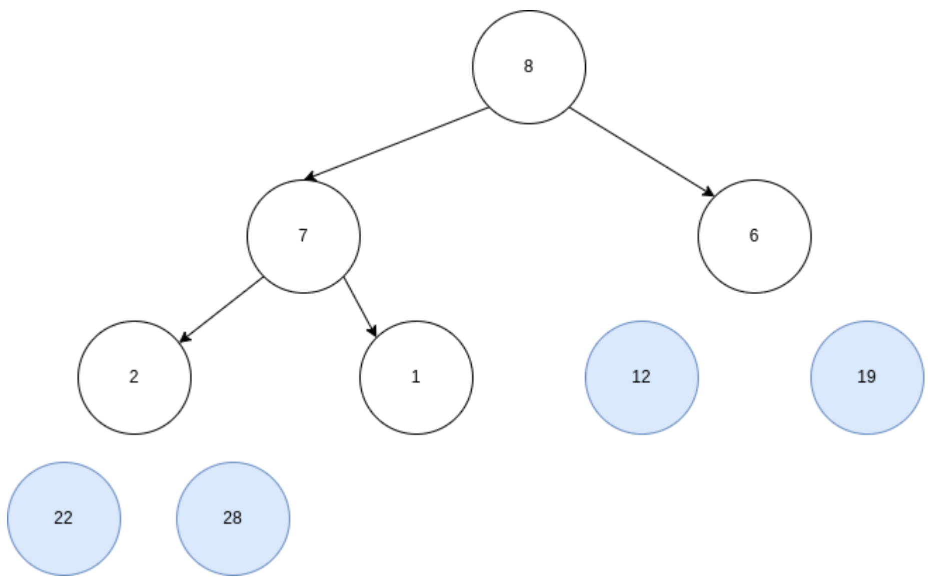
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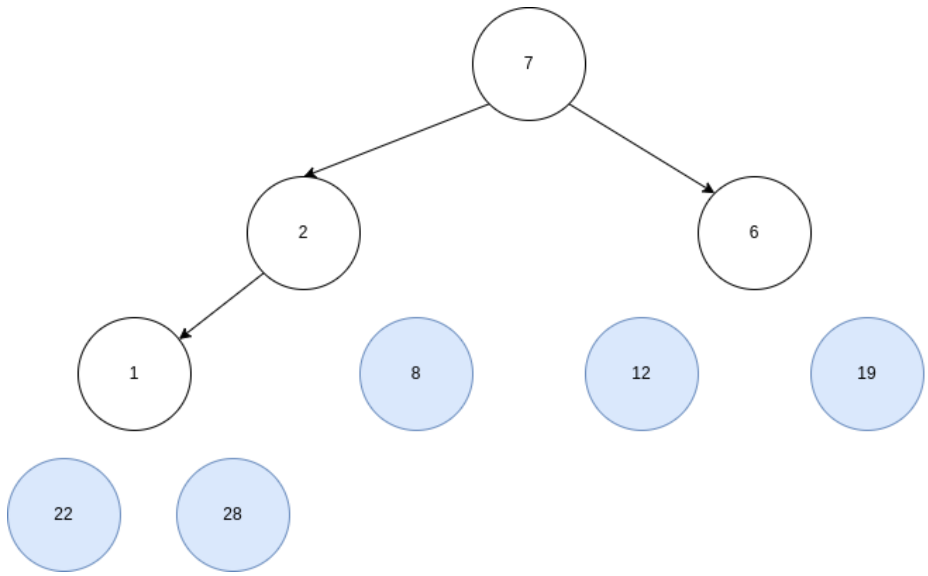
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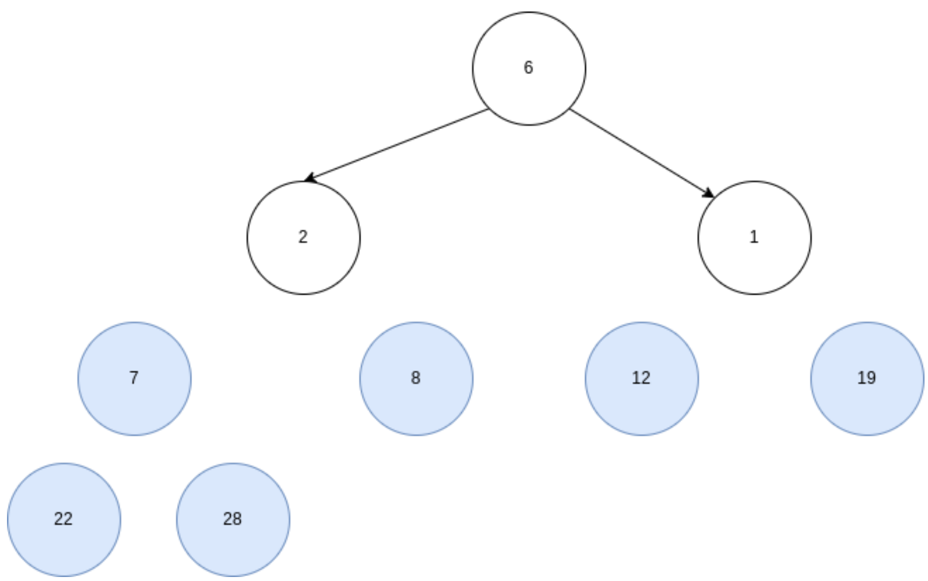
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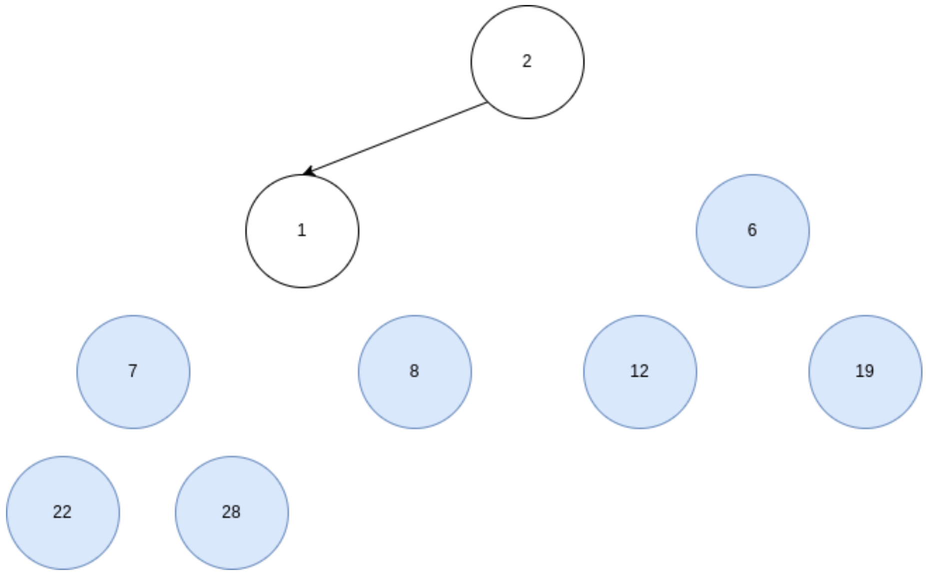

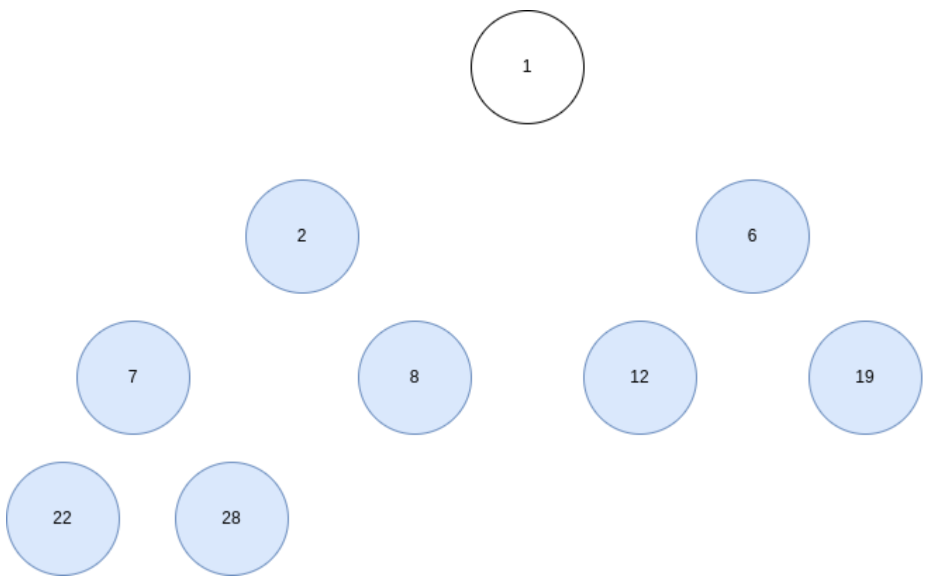

Sorted array is: **[1,2,6,7,8,12,19,22,28]**