Mathematics for Innovations as Contribution to Energy Transition

Project: Consistent Optimization and Stabilization of Electrical Networked Systems

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS MAGDEBURG

- Model Order Reduction and Flexibility Information

energy saxony

KON SENS

TECHNISCHE UNIVERSITÄT CHEMNITZ

- Robust Model Analysis and Control
- Mixed-Integer and Semi-Definite Power Flow Optimization

TenneT Taking power further

enso NETZ

VENIOS® Uniting IT and Energy

TECHNISCHE UNIVERSITÄT ILMENAU

- Distributed Optimization and Control of Microgrids

Price-based MPC of Residential Energy Systems using Energy Storages & Controllable Loads

Philipp Sauerteig

Technische Universität Ilmenau

joint work with
Philipp Braun (University of Newcaslte), Sara Grundel (MPI Magdeburg),
Karl Worthmann (TU Ilmenau)

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the Federal Ministry of Education and Research

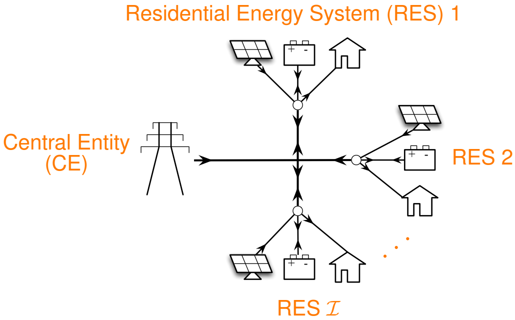
Bundesministerium
für Bildung
und Forschung

20th European Conference on Mathematics for Industry
Budapest, 22nd June 2018

Outline

- Modelling Residential Energy Systems
- Model Predictive Control
- Distributed Optimization via ADMM
- Numerical Results
- Outlook

Modelling Residential Energy Systems



Basic Setting

Given: $\mathcal{I} \in \mathbb{N}$ Residential Energy Systems (RESs)

$$\begin{aligned} x_i(k+1) \\ z_i(k) \end{aligned}$$

Notation

- State of charge $x_i(k) \geq 0$ of the battery at time $k \in \mathbb{N}_0$
- Power demand $z_i(k) \in \mathbb{R}$

Basic Setting

Given: $\mathcal{I} \in \mathbb{N}$ Residential Energy Systems (RESs)

$$x_i(k+1) \quad = \quad x_i(k) + T(\quad u_i^+(k) + u_i^-(k))$$

$$z_i(k)$$

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- State of charge $x_i(k) \geq 0$ of the battery at time $k \in \mathbb{N}_0$
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- Charging rate $u_i^+(k) \geq 0$ and discharging rate $u_i^-(k) \leq 0$

of RES $i \in \mathbb{N}_{\mathcal{I}} = \{1, \ldots, \mathcal{I}\}$

- Sampling interval length $T > 0$

Basic Setting

Given: $\mathcal{I} \in \mathbb{N}$ Residential Energy Systems (RESs)

$$
\begin{aligned}
x_i(k+1) &= x_i(k) + T(u_i^+(k) + u_i^-(k)) \\
z_i(k) &= w_i(k) + u_i^+(k) + u_i^-(k)
\end{aligned}
$$

Notation

- State of charge $x_i(k) \geq 0$ of the battery at time $k \in \mathbb{N}_0$
- Power demand $z_i(k) \in \mathbb{R}$
- Net consumption $w_i(k) = \ell_i(k) - g_i(k) \in \mathbb{R}$ (load minus generation)
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$$\begin{aligned} x_i(k+1) &= \alpha_i x_i(k) + T(\beta_i u_i^+(k) + u_i^-(k)) \\ z_i(k) &= w_i(k) + u_i^+(k) + \gamma_i u_i^-(k) \end{aligned}$$

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of RES $i \in \mathbb{N}_{\mathcal{I}} = \{1, \ldots, \mathcal{I}\}$

- Sampling interval length $T > 0$
- Losses $\alpha_i, \beta_i, \gamma_i \in (0, 1]$ due to energy transformation

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Constraints: For all $i \in \mathbb{N}_{\mathcal{I}}$ and all $k \in \mathbb{N}_0$

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$$

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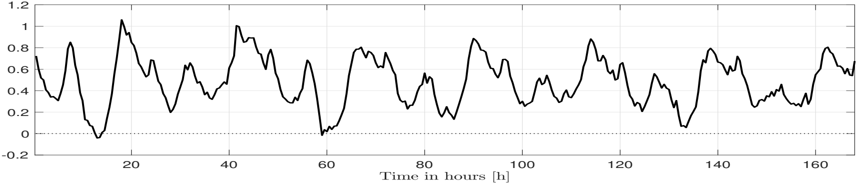
Uncontrolled Aggregated Power Demand: $\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} w_i$



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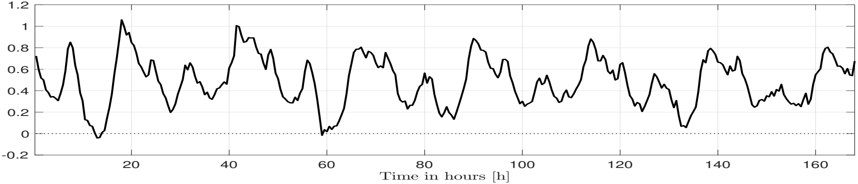

Problem: Fluctuations might lead to distributional bottlenecks

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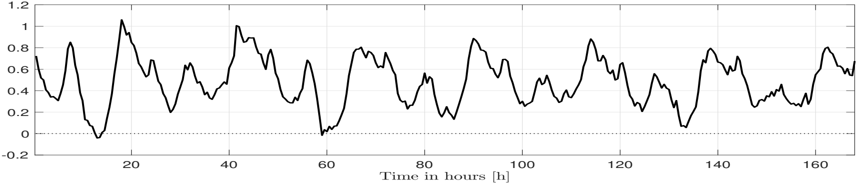

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Idea: Exploit flexibilities: storage devices, controllable loads

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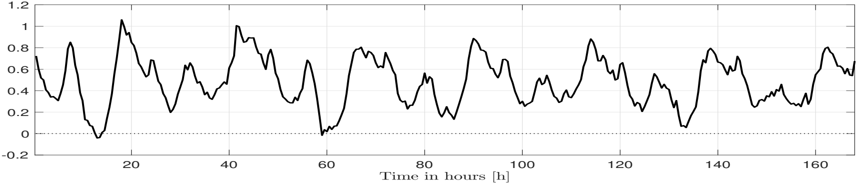

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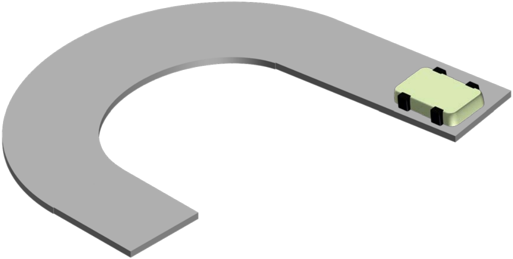
$\rightsquigarrow$ Predictions

Principle of Model Predictive Control

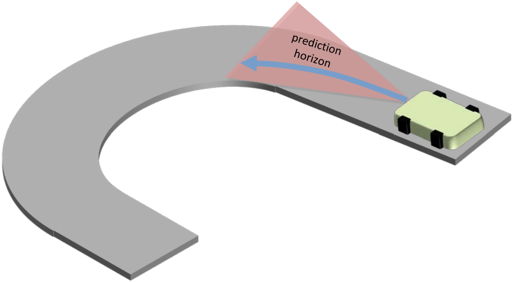
Idea: receding horizon control (MPC)



1. Obtain state measurement

Principle of Model Predictive Control

Idea: receding horizon control (MPC)



1. Obtain state measurement

2. Predict system and optimize input

Principle of Model Predictive Control

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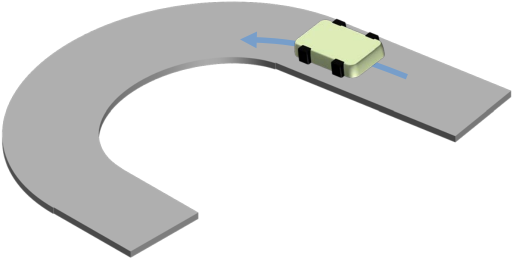

1. Obtain state measurement

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3. Apply “optimal“ input value

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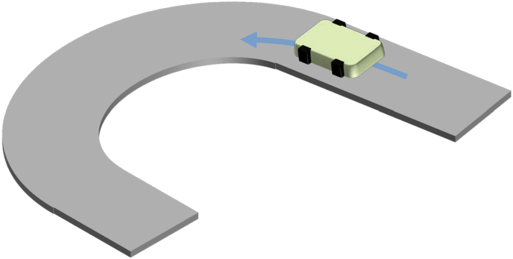

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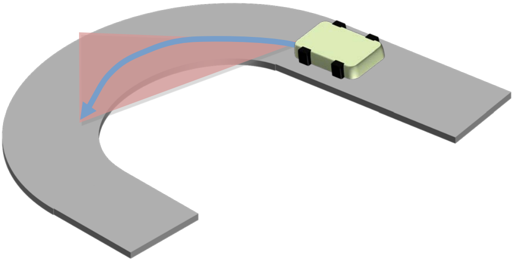

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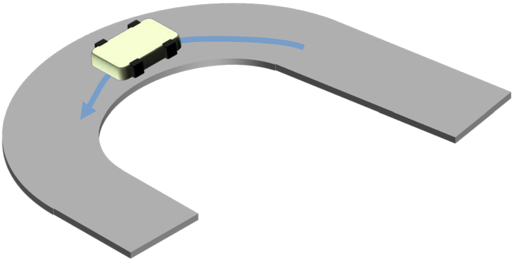

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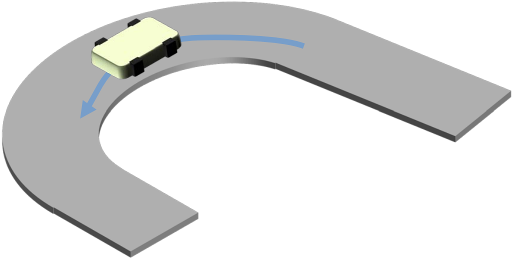

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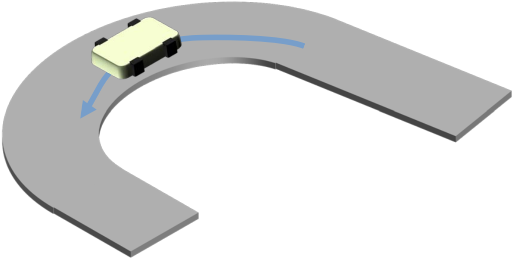

Control via “repeated” prediction & optimization

Thanks to B. Kern, OVG Universität Magdeburg.

Optimal Control of RESs

Objective: For a given prediction horizon $N \in \mathbb{N}$, minimize the deviation of the aggregated power demand from the overall average net consumption

$$\overline{\zeta}(k) = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \zeta_i(k),$$

where

$$\zeta_i(k) = \begin{cases} \frac{1}{k+1} \sum_{n=0}^{k} w_i(n) & \text{if } k < N-1, \\ \frac{1}{N} \sum_{n=k-N+1}^{k} w_i(n) & \text{if } k \geq N-1. \end{cases}$$

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Opimization Problem:

$$\min_{\mathbf{u}=(\mathbf{u}^+,\mathbf{u}^-)} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i(n) - \bar{\zeta}(n) \right)^2$$

$$\text{s.t.} \quad \text{system dynamics and constraints}$$

Extension: Controllable Loads

The net consumption is split into a static and a controllable part

$$w_i = w_i^s + w_i^c.$$

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$$w_i = w_i^s + w_i^c. \quad (w_i = \ell_i - g_i)$$

Additional Constraints:

$$0 \leq \quad u_i^c(k) \quad \leq \overline{w}_i^c$$

$$\sum_{j=0}^{k} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j) \leq \quad u_i^c(k) \quad \leq \sum_{j=0}^{k+\overline{N}-1} w_i^c(j) - \sum_{j=0}^{k-1} u_i^c(j)$$

for some constants $\overline{w}_i^c > 0$ and $\overline{N} \in \mathbb{N}$.

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System Dynamics:

$$\begin{aligned} x_i(k+1) &= \alpha_i x_i(k) + T(\beta_i u_i^+(k) + u_i^-(k)) \\ z_i(k) &= w_i(k) + u_i^+(k) + \gamma_i u_i^-(k) \end{aligned}$$

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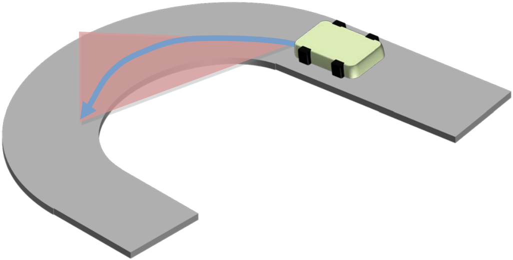
for some constants $\overline{w}_i^c > 0$ and $\overline{N} \in \mathbb{N}$.

Modified Optimization Problem:

$$\min_{\mathbf{u}=(\mathbf{u}^+,\mathbf{u}^-,\mathbf{u}^c)} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i(n) - \overline{\zeta}(n) \right)^2$$

$$\text{s.t.} \quad \text{updated system dynamics and constraints}$$

Difficulty in MPC



1. Obtain state measurement
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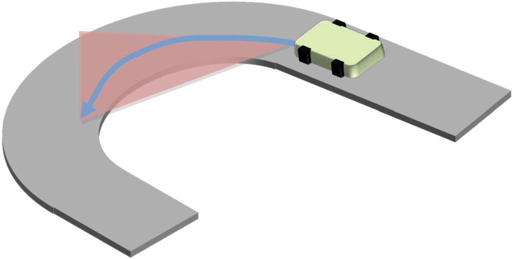
Difficulty in MPC



Step 2: Solve a large-scale finite-dimensional optimization problem with non-separable objective function in every time step

$\rightsquigarrow$ Alternating Direction Method of Multipliers (ADMM)

Reformulation of the Optimization Problem

Optimization Problem:

$$\min_{\mathbf{u}} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} z_i(n) - \overline{\zeta}(n) \right)^2$$

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$$\text{s.t.} \quad \text{system dynamics and constraints}$$

Decoupled Formulation:

$$\min_{\mathbf{u},\mathbf{a}} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\overline{\mathbf{a}}(n) - \overline{\zeta}(n) \right)^2$$

$$\text{s.t.} \quad \text{system dynamics and constraints}$$

$$z_i(n) - a_i(n) = 0 \quad \forall\, n \in \{k, \dots, k+N-1\}$$

- $\mathbf{a}_i = (a_i(k), \dots, a_i(k+N-1))^\top$, $i \in \mathbb{N}_{\mathcal{I}}$
- $\overline{\mathbf{a}} = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i, \quad \mathbf{a} = (\mathbf{a}_1, \dots, \mathbf{a}_{\mathcal{I}})^\top$

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Decoupled Formulation:

$$\min_{\mathbf{u},\mathbf{a}} \quad \frac{1}{N} \sum_{n=k}^{k+N-1} \left(\overline{\mathbf{a}}(n) - \overline{\zeta}(n) \right)^2 = \frac{1}{N} \left\| \overline{\mathbf{a}} - \overline{\zeta} \right\|_2^2$$
$$\text{s.t.} \quad \text{system dynamics and constraints}$$
$$z_i(n) - a_i(n) = 0 \quad \forall\, n \in \{k, \ldots, k+N-1\}$$

Reformulation of the Optimization Problem

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$$
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The Augmented Lagrangian: $(\rho > 0)$

$$
\mathcal{L}_\rho(\mathbf{z}, \mathbf{a}, \lambda; k) = \left\| \overline{\mathbf{a}} - \overline{\zeta} \right\|_2^2 + \sum_{i=1}^{\mathcal{I}} \left(\lambda_i^\top (\mathbf{z}_i - \mathbf{a}_i) + \frac{\rho}{2} \left\| \mathbf{z}_i - \mathbf{a}_i \right\|_2^2 \right)
$$

The ADMM Algorithm

Input: Step size $\rho > 0$, $\mathcal{I} \in \mathbb{N}$, max. number $\ell_{\max}$ of iterations.

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Loop: While $\ell \leq \ell_{\mathsf{max}}$

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$\mathcal{L}_\rho(\mathbf{z}, \mathbf{a}, \lambda; k) = \left\| \bar{\mathbf{a}} - \bar{\zeta} \right\|_2^2 + \sum_{i=1}^{\mathcal{I}} \left(\lambda_i^\top (\mathbf{z}_i - \mathbf{a}_i) + \frac{\rho}{2} \|\mathbf{z}_i - \mathbf{a}_i\|_2^2 \right)$

1. Solve (in parallel)

$$\mathbf{z}_i^{\ell+1} \in \arg\min_{\mathbf{z}_i} \; \mathbf{z}_i^\top \lambda_i^\ell + \frac{\rho}{2} \left\| \mathbf{z}_i - \mathbf{a}_i^\ell \right\|_2^2$$

   for each RES $i \in \mathbb{N}_{\mathcal{I}}$ and broadcast $\mathbf{z}_i^{\ell+1}$ to the CE.

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   for each RES $i \in \mathbb{N}_{\mathcal{I}}$ and broadcast $\mathbf{z}_i^{\ell+1}$ to the CE.

2. The CE solves

$$\mathbf{a}^{\ell+1} \in \arg\min_{\mathbf{a}} \; \|\bar{\mathbf{a}} - \bar{\zeta}\|_2^2 - \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i^\top \lambda_i^\ell + \frac{\rho}{2} \left\| \mathbf{z}_i^{\ell+1} - \mathbf{a}_\mathbf{i} \right\|_2^2 .$$

The ADMM Algorithm

Input: Step size $\rho > 0$, $\mathcal{I} \in \mathbb{N}$, max. number $\ell_{\max}$ of iterations.

Initialization: Set $\ell = 0$ and choose $\lambda^0, \mathbf{a}^0 \in \mathbb{R}^{\mathcal{I}N}$ (arbitrarily)

Loop: While $\ell \leq \ell_{\max}$

$\mathcal{L}_\rho(\mathbf{z}, \mathbf{a}, \lambda; k) = \left\|\bar{\mathbf{a}} - \bar{\zeta}\right\|_2^2 + \sum_{i=1}^{\mathcal{I}} \left(\lambda_i^\top (\mathbf{z}_i - \mathbf{a}_i) + \frac{\rho}{2} \|\mathbf{z}_i - \mathbf{a}_i\|_2^2\right)$

1. Solve (in parallel)

$$\mathbf{z}_i^{\ell+1} \in \arg\min_{\mathbf{z}_i} \ \mathbf{z}_i^\top \lambda_i^\ell + \frac{\rho}{2} \left\|\mathbf{z}_i - \mathbf{a}_i^\ell\right\|_2^2$$

   for each RES $i \in \mathbb{N}_{\mathcal{I}}$ and broadcast $\mathbf{z}_i^{\ell+1}$ to the CE.

2. The CE solves

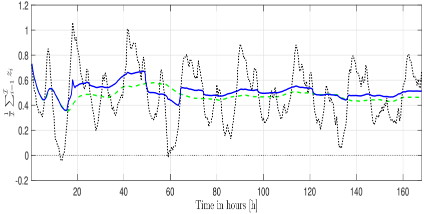
$$\mathbf{a}^{\ell+1} \in \arg\min_{\mathbf{a}} \ \|\bar{\mathbf{a}} - \bar{\zeta}\|_2^2 - \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i^\top \lambda_i^\ell + \frac{\rho}{2} \left\|\mathbf{z}_i^{\ell+1} - \mathbf{a}_\mathbf{i}\right\|_2^2 .$$

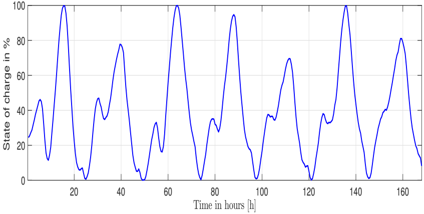
3. The CE updates the Lagrange multipliers

$$\lambda_i^{\ell+1} = \lambda_i^\ell + \rho(\mathbf{z}_i^{\ell+1} - \mathbf{a}_i^{\ell+1}) \quad \forall\, i \in \{1, \ldots, \mathcal{I}\}$$

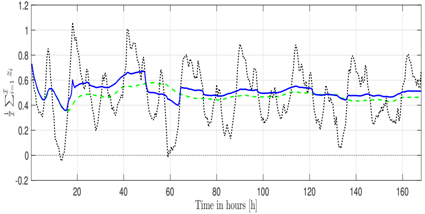
   and broadcasts $(\lambda_i^{\ell+1}, \mathbf{a}_i^{\ell+1})$ to RES $i \in \mathbb{N}_{\mathcal{I}}$. Set $\ell = \ell + 1$.

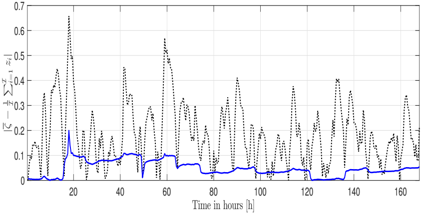
Impact of Controlled Storage Devices

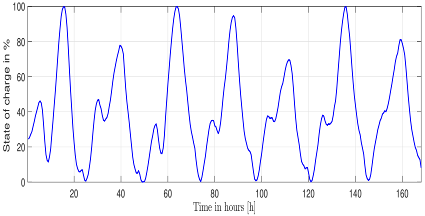




Impact of Controlled Storage Devices



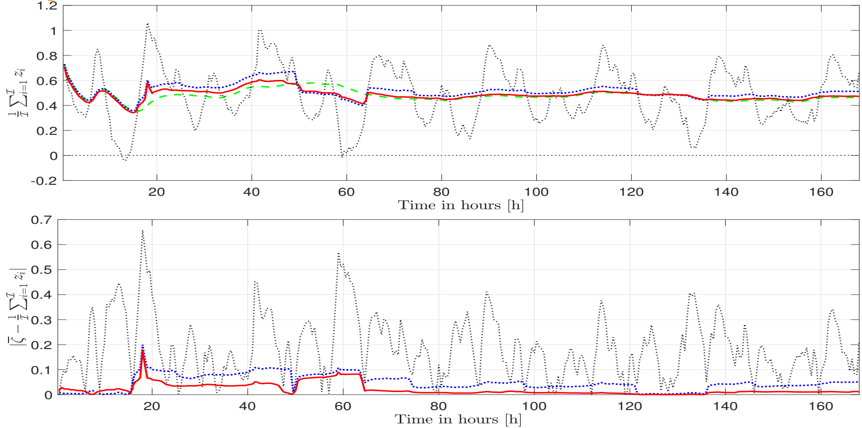




Conclusions:

- Significant peak shaving of the overall performance
- Still room for improvement due to battery capacities & (dis)charging rates

Impact of Controllable Loads

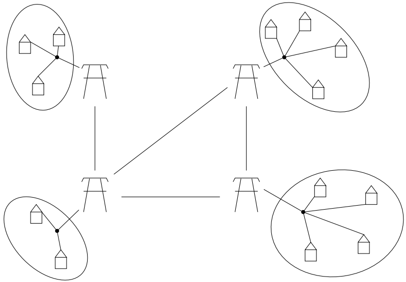


Conclusions:

- Further improvement of the overall performance
- Still no 100% accuracy

Outlook

- Coupled microgrids



First numerical simulations show potential, but no convergence analysis so far.

Outlook

- Coupled microgrids
- Surrogate models
  - For single microgrids

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    $\Rightarrow$ no strong convexity
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Outlook

- Coupled microgrids
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- Price-based control
  - Increasing price function $p : \mathbb{R} \to \mathbb{R}$ with $p(0) = 0$
  - First approach: $p(z; c) = Tcz$, $c > 0$ chosen by CE
    $\Rightarrow$ no strong convexity
    $\Rightarrow$ no convergence of ADMM guaranteed
  - Remedy: $p(z; c) = Ta\left(z + b(z - c)^2 - bc^2\right)$ with $a, b > 0$, $c \in \mathbb{R}$ (such that p is increasing)

Thank you for your attention!

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