Current Research Topics in Distributed Optimization of Smart Grids

Philipp Sauerteig
Technische Universität Ilmenau

joint work with
Karl Worthmann (TU Ilmenau)

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of Education
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ShanghaiTech University, 10 June 2019

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	Shanghai
size [km^2]	6 340
population	26 000 000
students	35 000

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	Shanghai	Ilmenau
size [km^2]	6 340	200
population	26 000 000	26 000
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	Shanghai	Ilmenau	Thuringia
size [km^2]	6 340	200	16 000
population	26 000 000	26 000	2 150 000
students	35 000	6 000	33 000

KONSENS: Konsistente Optimierung uNd Stabilisierung Elektrischer NetzwerkSysteme

Federal Ministry of Education and Research

Mathematics for Innovations as Contribution to Energy Transition

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Mathematics for Innovations as Contribution to Energy Transition

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS MAGDEBURG

- Model Order Reduction and Flexibility Information

TECHNISCHE UNIVERSITÄT CHEMNITZ

- Robust Model Analysis and Control
- Mixed-Integer and Semi-Definite Power Flow Optimization

KON SENS

TECHNISCHE UNIVERSITÄT ILMENAU

- Distributed Optimization and Control of Microgrids

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TenneT Taking power further

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energy saxony

TECHNISCHE UNIVERSITÄT ILMENAU

- Distributed Optimization and Control of Microgrids

enso NETZ

VENIOS® Uniting IT and Energy

Outline

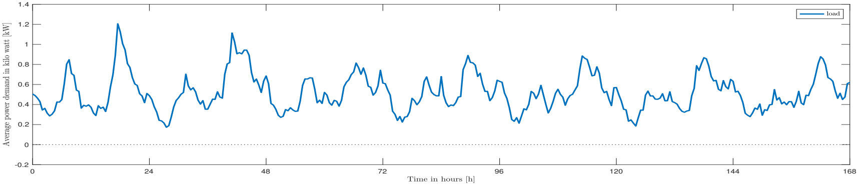
- Optimal Control of Distributed Energy Storage Devices
  - Motivation
  - Modelling Residential Energy Systems
  - Distributed Optimization via ADMM

Outline

- Optimal Control of Distributed Energy Storage Devices
    - Motivation
    - Modelling Residential Energy Systems
    - Distributed Optimization via ADMM
- Current Research
    - Distributed Optimization via ALADIN
    - Multiobjective Optimization
    - Coupled Microgrids
    - Surrogates

Motivation

Aggregated power demand profile (1 week)



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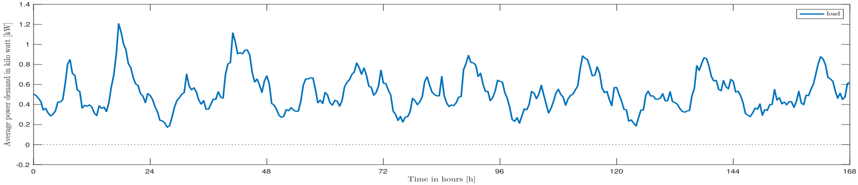
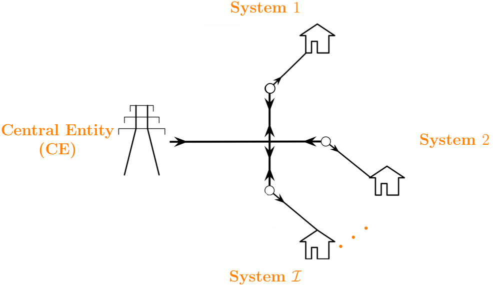


Problem

- volatile load

Smart grid



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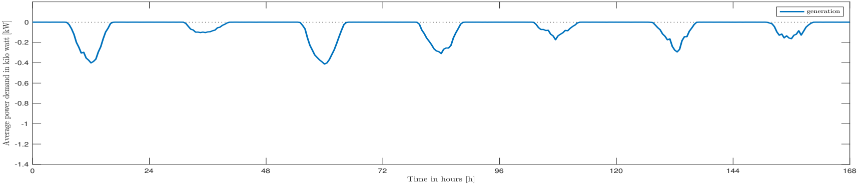

Problem

- volatile load
- generation via photovoltaic

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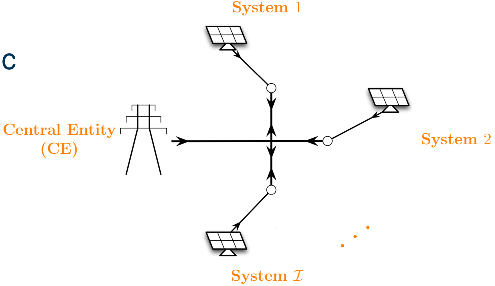

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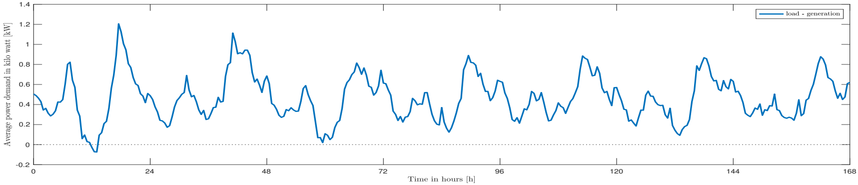

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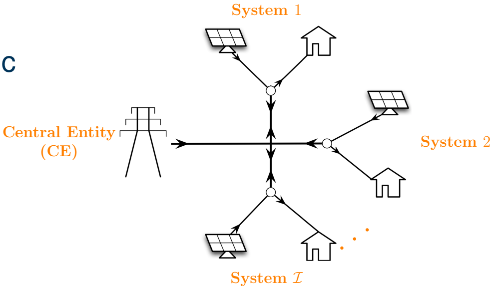

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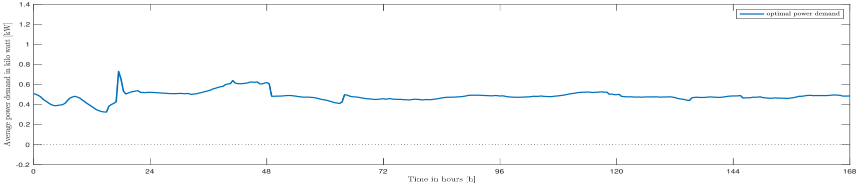

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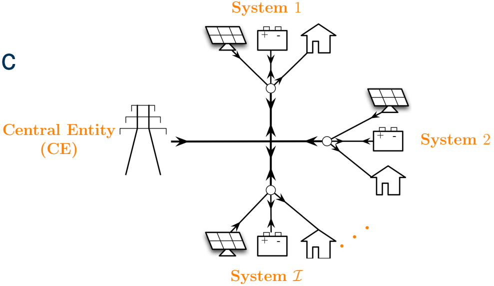
Remedy: exploit flexibilities

- storage devices

Smart grid



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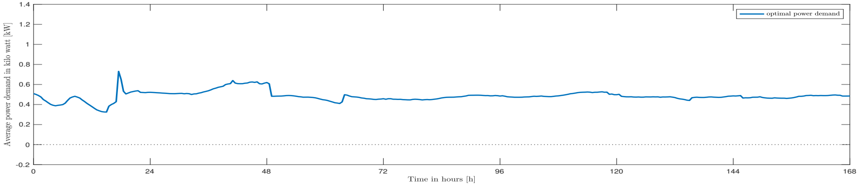

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Remedy: exploit flexibilities

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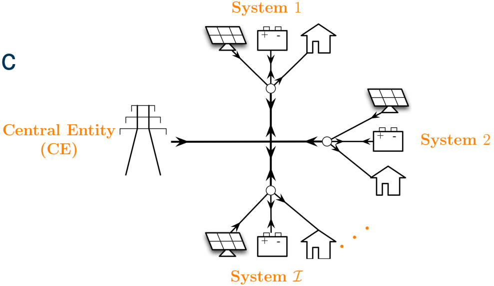
Smart grid



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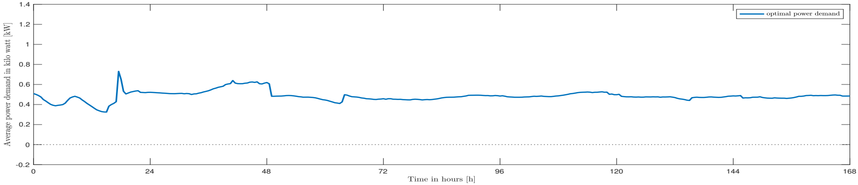

Problem

- volatile load
- generation via photovoltaic

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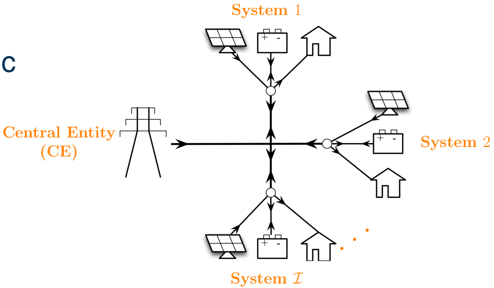
Remedy: exploit flexibilities

- storage devices
- energy exchange
- controllable loads, ...

Smart grid



Dynamics & Constraints

Given: $\mathcal{I} \in \mathbb{N}$ subsystems (smart homes)

System equation of subsystem $i \in [1 : \mathcal{I}] := \{1, 2, \ldots, \mathcal{I}\}$ at time instants $n \in [k : k + N - 1]$, $k \in \mathbb{N}_0$, $x_i(k) = \hat{x}_i$:

$$x_i(n+1)$$
$$z_i(n)$$

Notation

- State of charge $x_i(n)$
- Power demand $z_i(n) \in \mathbb{R}$

Constraints: For all

$i \in [1 : \mathcal{I}]$ and all $n \in \mathbb{N}_0$

$$0 \leq \qquad x_i(n) \qquad \leq C_i$$

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$$x_i(n+1) = x_i(n) + T(\quad u_i^+(n) + u_i^-(n))$$
$$z_i(n)$$

Notation

- State of charge $x_i(n)$
- Power demand $z_i(n) \in \mathbb{R}$
- (Dis-)Charging rate $u_i(n)$
- Time interval length $T > 0$

Constraints: For all $i \in [1:\mathcal{I}]$ and all $n \in \mathbb{N}_0$

$$\begin{aligned} 0 \leq & \quad x_i(n) & \leq C_i \\ \underline{u}_i \leq & \quad u_i^-(n) & \leq 0 \\ 0 \leq & \quad u_i^+(n) & \leq \bar{u}_i \\ 0 \leq & \quad \frac{u_i^-(n)}{\underline{u}_i} + \frac{u_i^+(n)}{\bar{u}_i} & \leq 1 \end{aligned}$$

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$$
\begin{aligned}
x_i(n+1) &= x_i(n) + T\left(u_i^+(n) + u_i^-(n)\right) \\
z_i(n) &= w_i(n) + u_i^+(n) + u_i^-(n)
\end{aligned}
$$

Notation

- State of charge $x_i(n)$
- Power demand $z_i(n) \in \mathbb{R}$
- Net consumption $w_i(n) = \ell_i(n) - g_i(n) \in \mathbb{R}$
- (Dis-)Charging rate $u_i(n)$
- Time interval length $T > 0$

Constraints: For all $i \in [1 : \mathcal{I}]$ and all $n \in \mathbb{N}_0$

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0 \leq\ & x_i(n) && \leq C_i \\
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Philipp Sauerteig
Institute for Mathematics

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$$
\begin{aligned}
x_i(n+1) &= \alpha_i x_i(n) + T(\beta_i u_i^+(n) + u_i^-(n)) \\
z_i(n) &= w_i(n) + u_i^+(n) + \gamma_i u_i^-(n)
\end{aligned}
$$

Notation

- State of charge $x_i(n)$
- Power demand $z_i(n) \in \mathbb{R}$
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- (Dis-)Charging rate $u_i(n)$
- Time interval length $T > 0$
- Efficiencies $\alpha_i, \beta_i, \gamma_i \in (0, 1]$

Constraints: For all $i \in [1:\mathcal{I}]$ and all $n \in \mathbb{N}_0$

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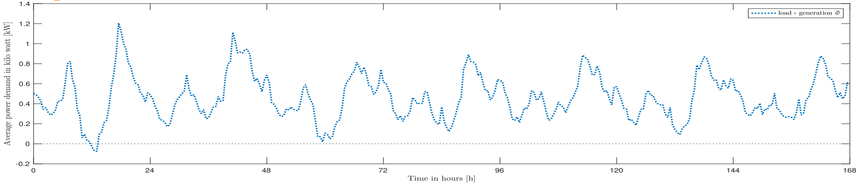
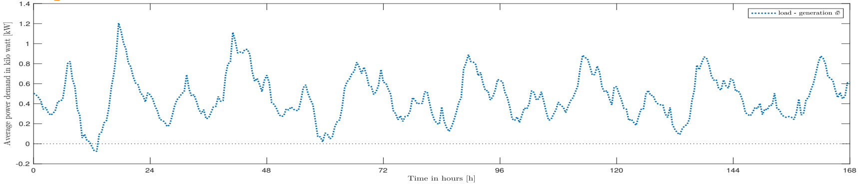
Objective



Goal: Flatten aggregated power demand profile

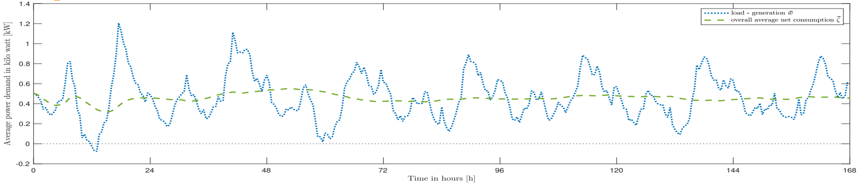
Objective



Goal: Flatten aggregated power demand profile

Idea: Trace some steady reference trajectory

Objective



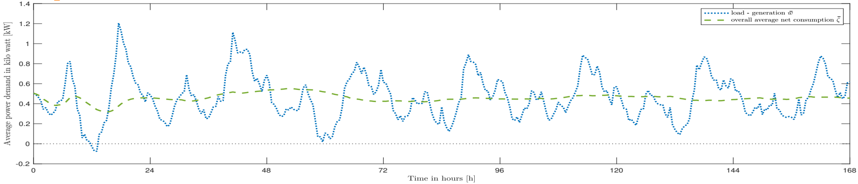
Goal: Flatten aggregated power demand profile

Idea: Trace some steady reference trajectory, e.g. overall average net consumption

$$\bar{\zeta}(n) = \frac{1}{\mathcal{I} \cdot \min\{N_1, n+1\}} \sum_{j=n-\min\{n, N_1-1\}}^{n} \sum_{i=1}^{\mathcal{I}} w_i(j)$$

for some $N_1 \in \mathbb{N}_{\geq 2}$

Objective



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for some $N_1 \in \mathbb{N}_{\geq 2}$

Problem: Future net consumption w_i unknown

$\rightsquigarrow$ prediction of $w_i(j)$, $j \in [k : k + N_2 - 1]$
($N_1 = N_2 = N$ prediction horizon)

Model Predictive Control

Open loop

Closed loop

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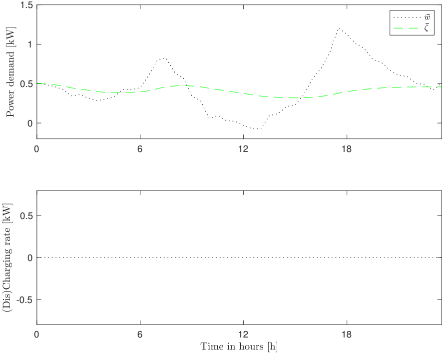

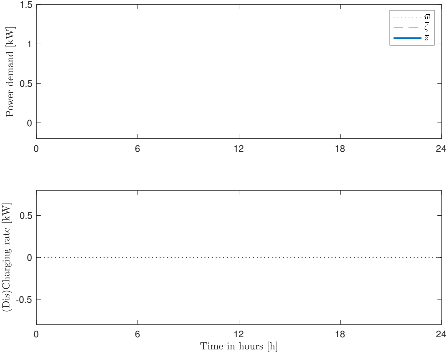

1. Measure current SoC $\hat{x} = x(k)$ and predict (exogenous) input $w = (w(k), \ldots, w(k+N-1))^\top$.

Model Predictive Control

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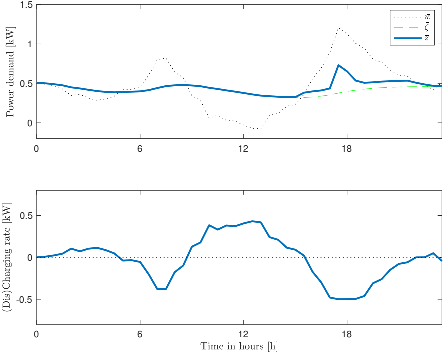

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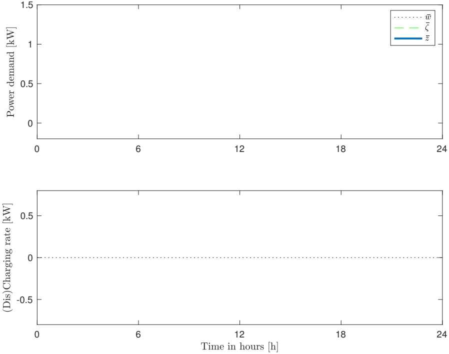

2. Compute optimal input $u^* = (u^*(k), \dots, u^*(k+N-1))^\top$.

Model Predictive Control

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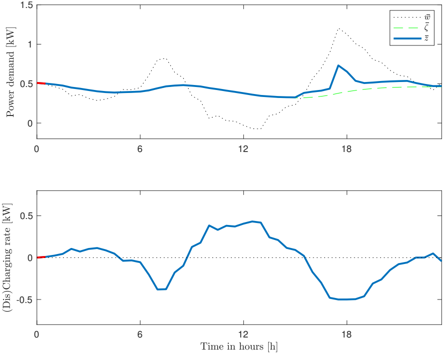

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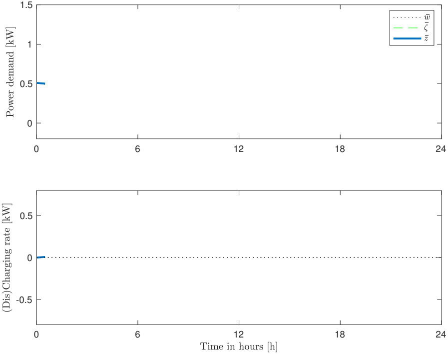

3. Implement $u^*(k)$ and increment k.

Model Predictive Control

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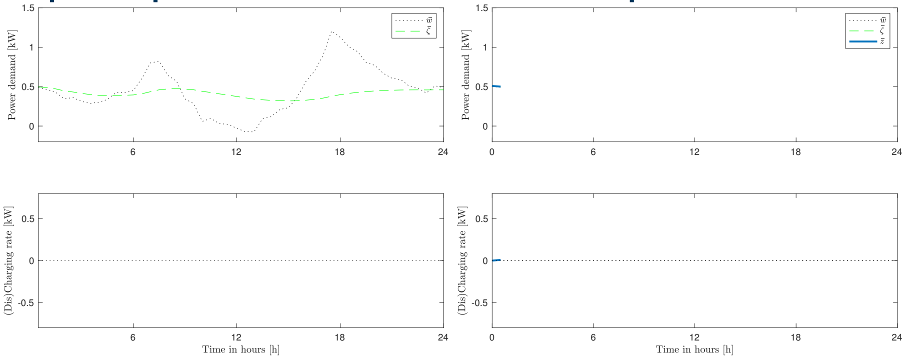

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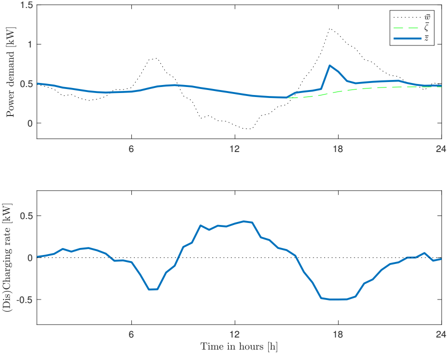

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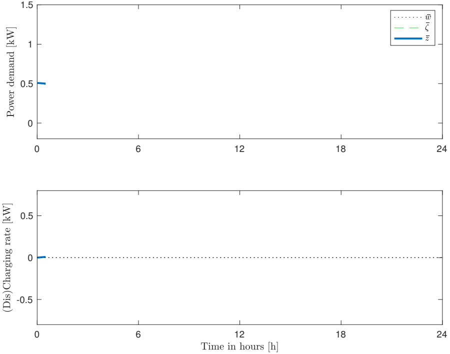

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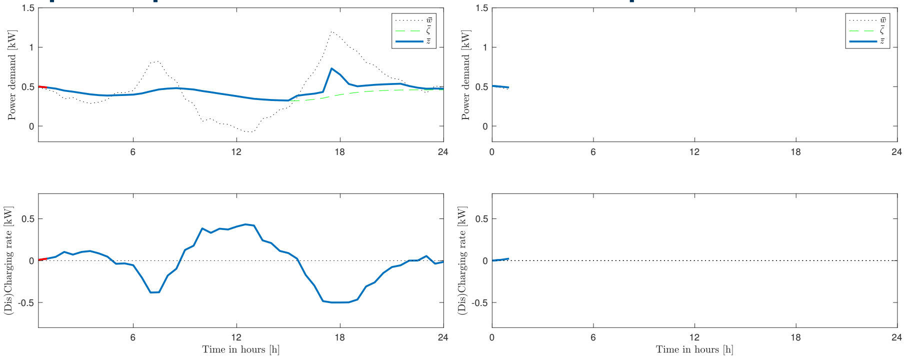

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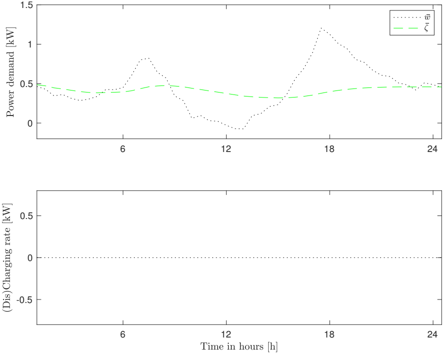

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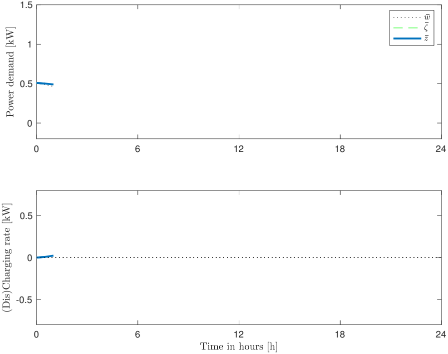

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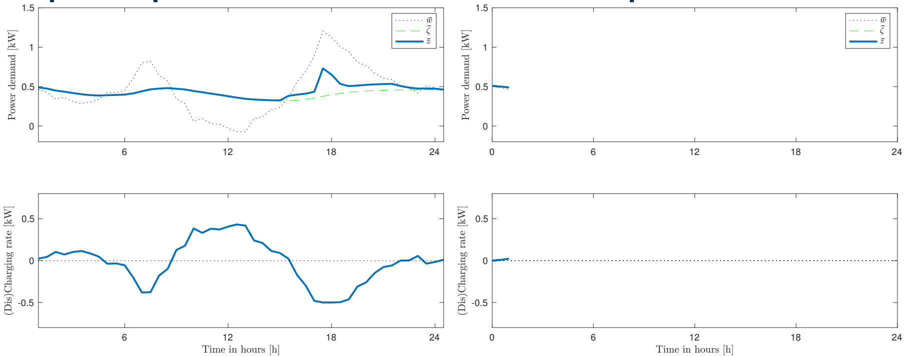

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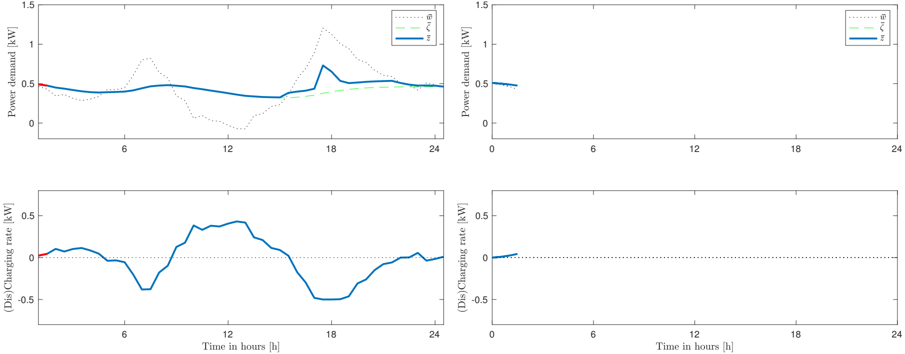

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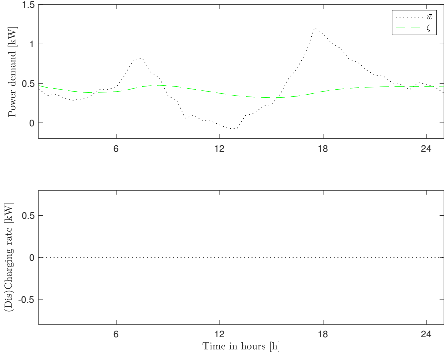

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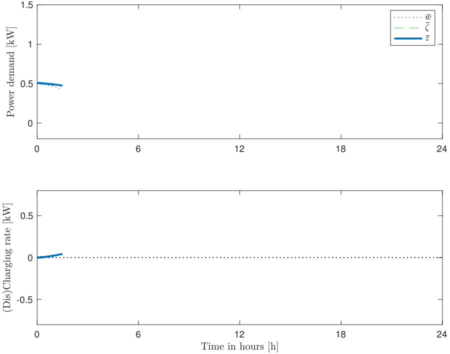

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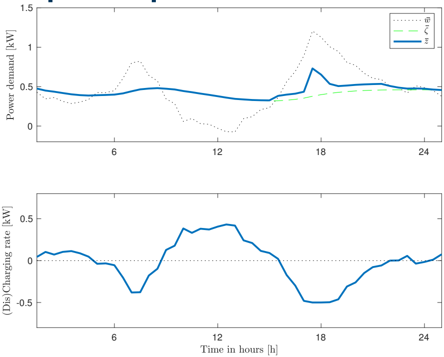

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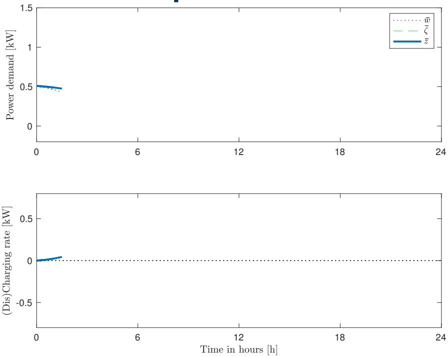

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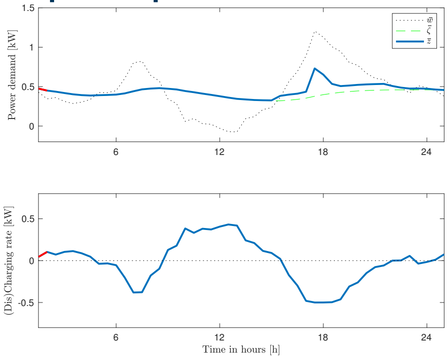

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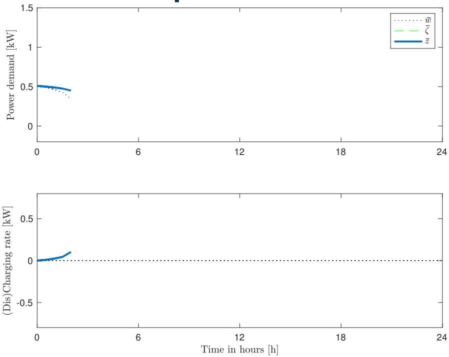

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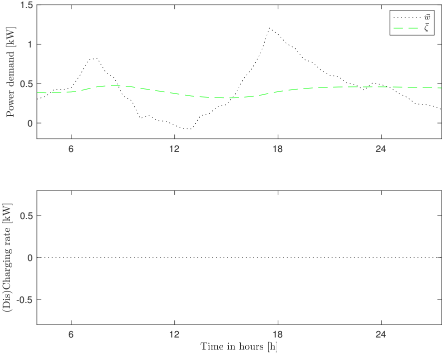

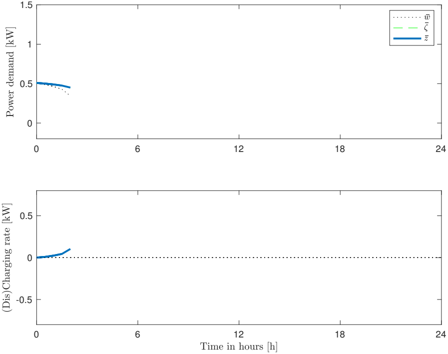

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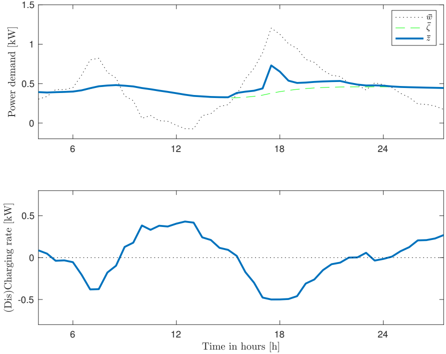
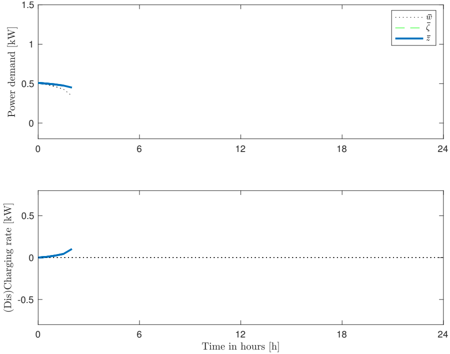
Model Predictive Control

Open loop



Closed loop



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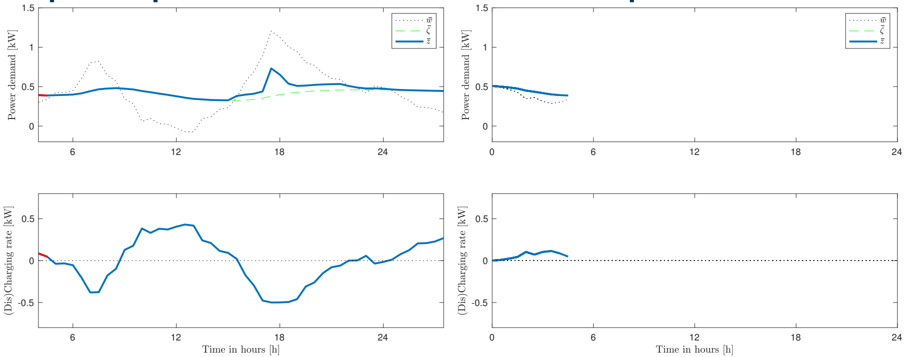

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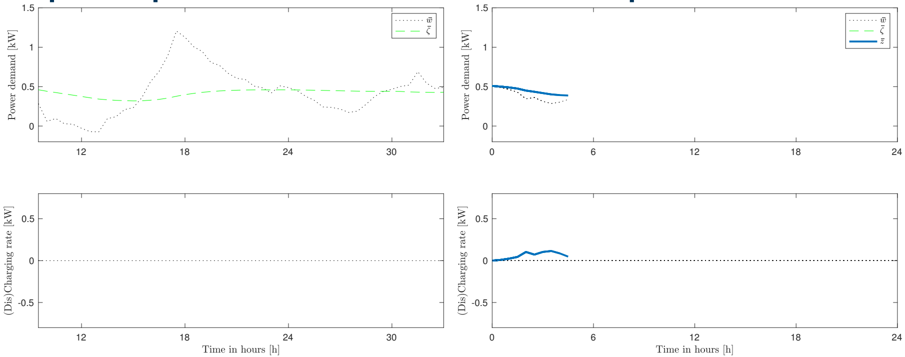

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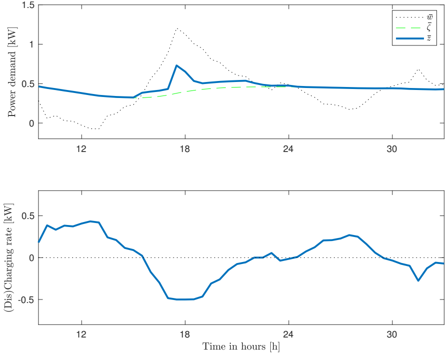

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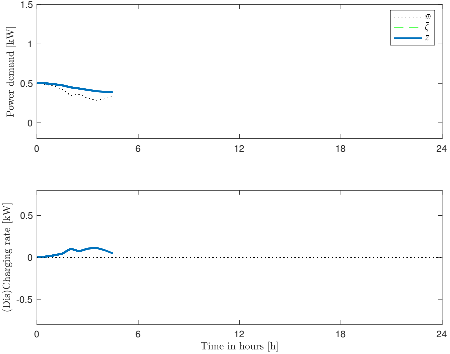

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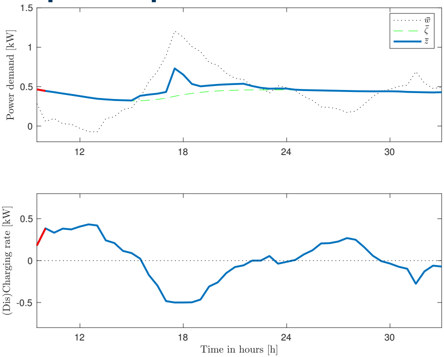

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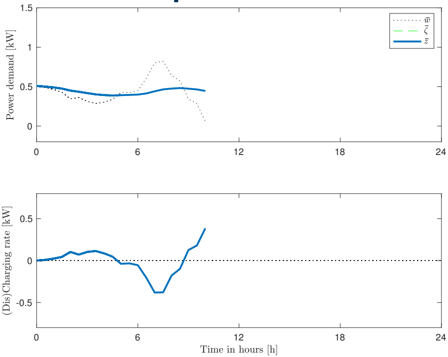

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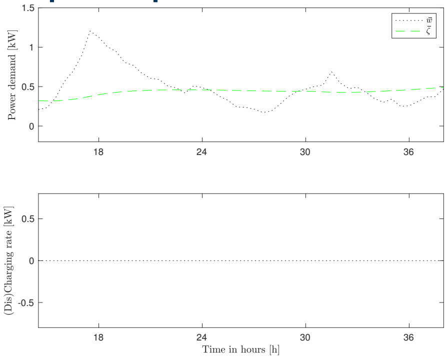

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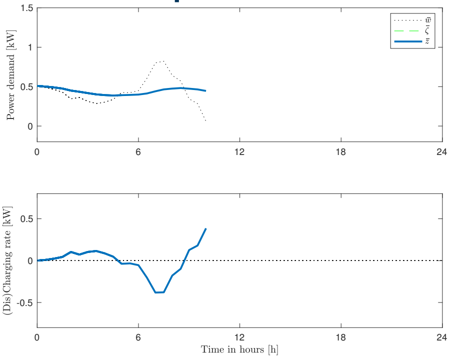

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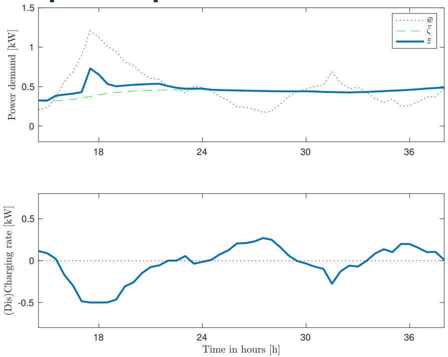

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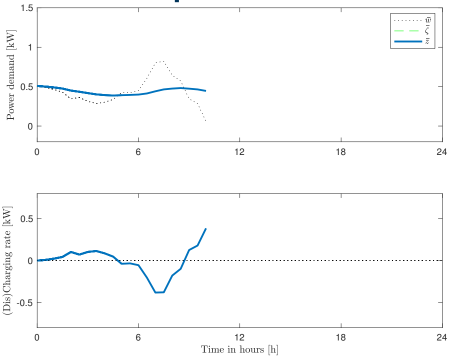

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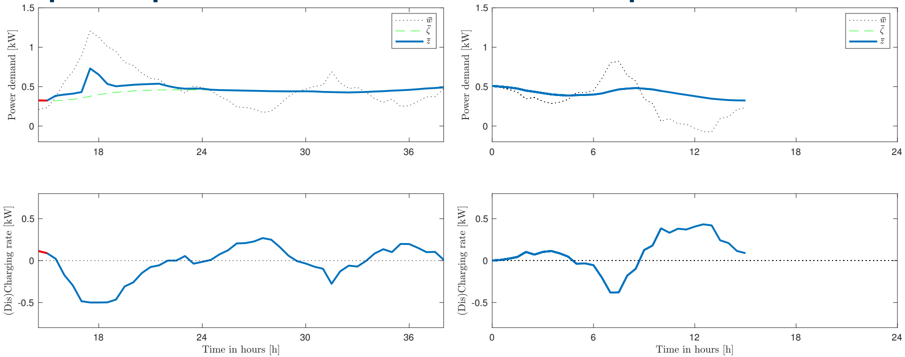

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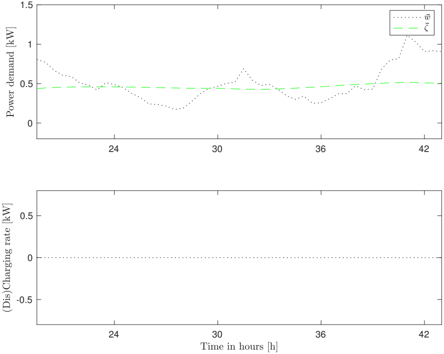

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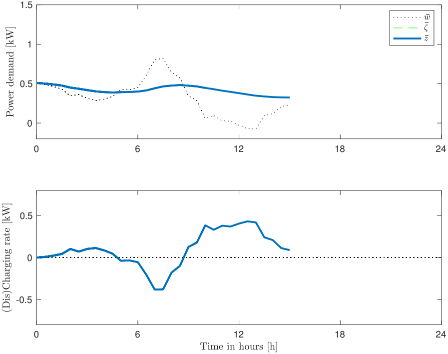

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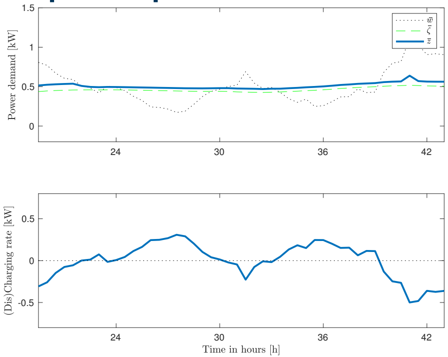

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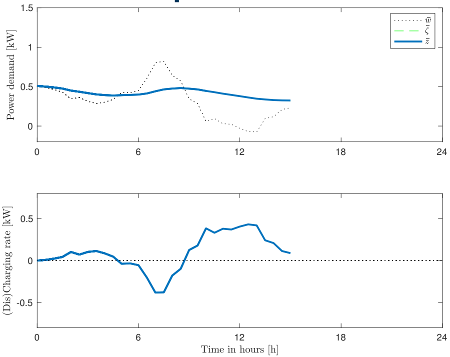

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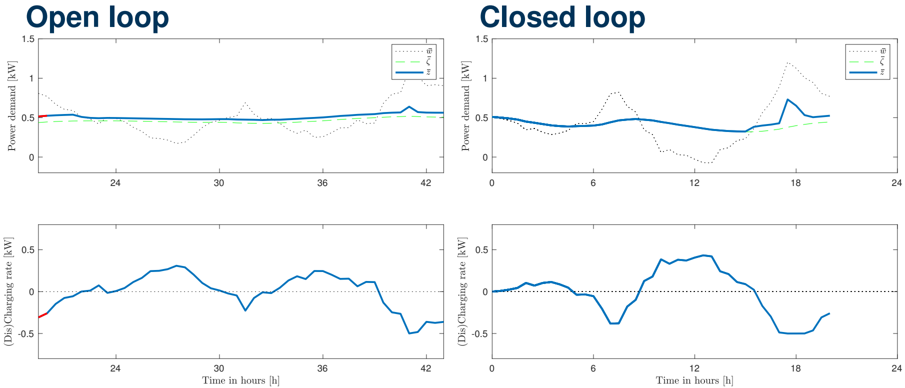
Model Predictive Control

Open loop / Closed loop



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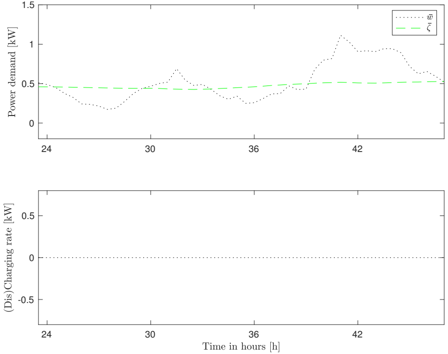

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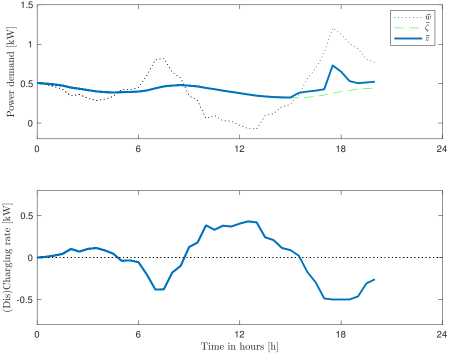

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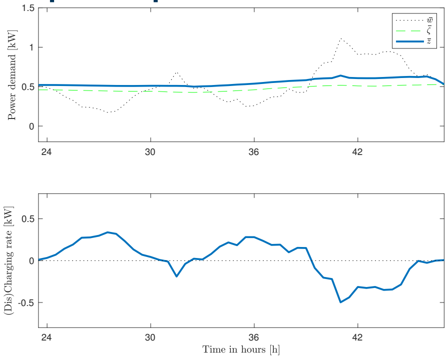

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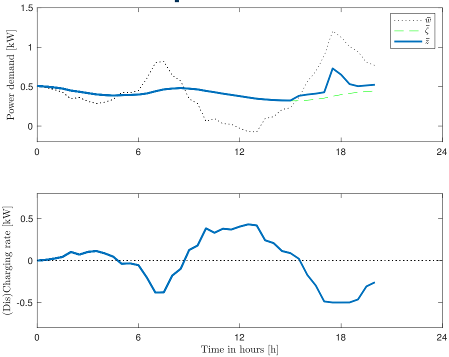

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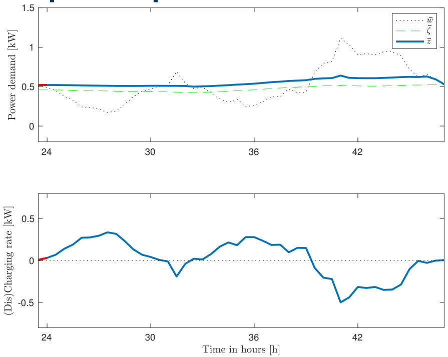

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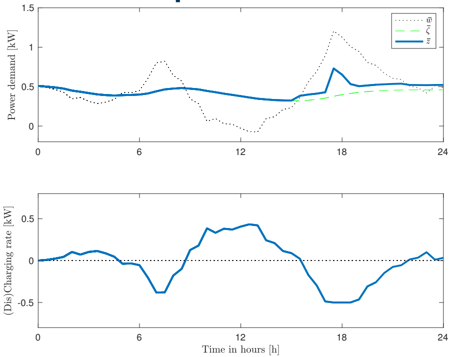

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Optimization Problem

Centralized formulation

$$\min_{u=(u^+,u^-)} \quad \sum_{n=k}^{k+N-1} \left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \left[w_i(n) + u_i^+(n) + \gamma_i u_i^-(n) \right] - \bar{\zeta}(n) \right)^2$$

$$\text{s.t.} \quad \text{system dynamics and constraints}$$

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Distributed formulation

$$\min_{(z,\bar{a})} \sum_{n=k}^{k+N-1} \left(\bar{a}(n) - \bar{\zeta}(n) \right)^2$$

$$\text{s.t.} \quad \text{system dynamics and constraints}$$

$$z_i - a_i = 0, i \in [1:\mathcal{I}], \quad \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} a_i - \bar{a} = 0$$

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Distributed formulation

$$\min_{(z,\bar{a})} \sum_{n=k}^{k+N-1} \left(\bar{a}(n) - \bar{\zeta}(n)\right)^2 = \|\bar{a} - \bar{\zeta}\|_2^2 =: g(\bar{a})$$

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Alternating Direction Method of Multipliers

[Boyd et al., 2011]

Augmented Lagrangian: dual λ and $\rho > 0$

$$\mathcal{L}_\rho(z, a, \lambda) = g(\bar{a}) + \sum_{i=1}^{\mathcal{I}} \left(\lambda_i^\top (z_i - a_i) + \frac{\rho}{2} \|z_i - a_i\|_2^2 \right)$$

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Modified ADMM Scheme [Braun et al., 2018]

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4. Broadcast variable
   $\Pi^{\ell+1} \; = \; \bar{z}^{\ell+1} - \bar{a}^{\ell+1} + \frac{\bar{\lambda}^{\ell+1}}{\rho}$

Current Research

Distributed Optimization using ALADIN

(joint work with Yuning Jiang, ShanghaiTech University)

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Additional Local Costs

Penalize the (dis-)charging of the batteries by introducing $f_i : \mathbb{R}^{2N} \to \mathbb{R}$

$$f_i(u_i) = \|u_i\|_{Q_i}^2$$

with some scaling matrix $Q_i \in \mathbb{R}^{2N}$.

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Optimization Problem

$$\begin{aligned}
\min_{\bar{z},u} \quad & g(\bar{z}) + \sum_{i=1}^{\mathcal{I}} f_i(u_i) \\
\text{s.t.} \quad & \bar{z} = \bar{w} + \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} I_N \otimes \begin{pmatrix} 1 & \gamma_i \end{pmatrix} u_i \\
& u_i \in \mathbb{D}_i = \left\{ u_i \in \mathbb{R}^{2N} \;\middle|\; D_i u_i \leq d_i \right\}, \quad i \in [1 : \mathcal{I}]
\end{aligned}$$

Distributed Optimization using ALADIN

[Houska et al., 2016]

1. Parallel step: Solve

$$\min_{v_i \in \mathbb{D}_i} \quad f_i(v_i) - \lambda^\top A_i v_i + \frac{1}{2} \|v_i - u_i\|_{H_i}^2$$

and compute $g_i = A_i^\top \lambda + H_i(u_i - v_i)$ in parallel.

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2. Terminal condition: Terminate if $\max_{i \in [1:\mathcal{I}]} \|v_i - u_i\|_\infty \leq \varepsilon$.

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2. Terminal condition: Terminate if $\max_{i \in [1:\mathcal{I}]} \left\| v_i - u_i \right\|_\infty \leq \varepsilon$.
3. Consensus step: Update μ and $H_i = 2Q_i + \mu D_i^{\mathrm{act}^\top} D_i^{\mathrm{act}}$ optionally, solve

$$\min_{\bar{z}^+, u^+} \quad g(\bar{z}^+) + \sum_{i=1}^{\mathcal{I}} \frac{1}{2} \left\| u_i^+ - v_i \right\|_{H_i}^2 + g_i^\top u_i^+$$

$$\text{s.t.} \quad \bar{z}^+ = \bar{w} + \sum_{i=1}^{\mathcal{I}} A_i u_i^+ \quad | \lambda^+$$

   and update $\bar{z} = \bar{z}^+$, $u = u^+$, $\lambda = \lambda^+$.

Comparison of Both Algorithms

	ADMM	ALADIN
requirements		
convergence rate		
communication ↑		
communication ↓		

Comparison of Both Algorithms

	ADMM	ALADIN
requirements	convex objective functions	
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Institute for Mathematics

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Comparison of Both Algorithms

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Outlook: ALADIN could be applied to

- more complex (in particular non-convex) objective functions
- nonlinear battery dynamics

Multiobjective Optimization Problem

[S., Worthmann, 2019]

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Objective 1 Flatten aggregated power demand profile:

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Multiobjective Optimization Problem Formulation

$$\min_{\bar{z}, s} \quad \begin{pmatrix} g(\bar{z}) \\ h(s) \end{pmatrix} \quad \text{s.t.} \quad \text{system dynamics and constraints and (1)}$$

Efficiency, Pareto Frontier

Definition

Consider

$$\min_{(\bar{z},s)\in\mathbb{S}} \begin{pmatrix} g(\bar{z}) \\ h(s) \end{pmatrix}, \tag{2}$$

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The set

$$\left\{ (g(\bar{z}^*), h(s^*)) \in \mathbb{R}^2 \;\middle|\; (\bar{z}^*, s^*) \textit{ is efficient for } (2) \right\}$$

is called *Pareto frontier*.

Characterization of Efficiency

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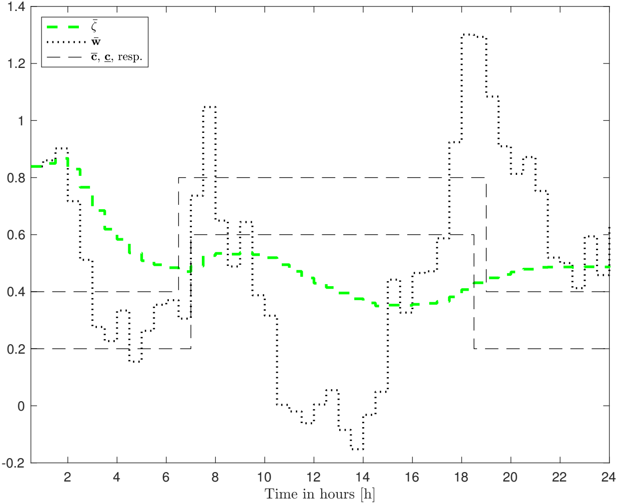
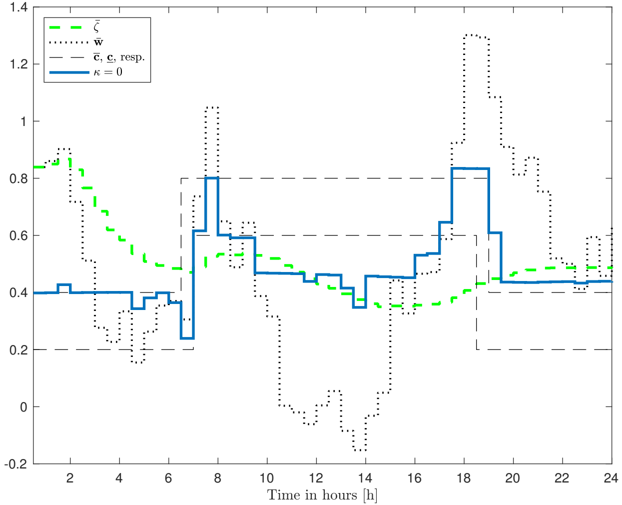
Efficient Control

Setting



Efficient Control

Open Loop Performance



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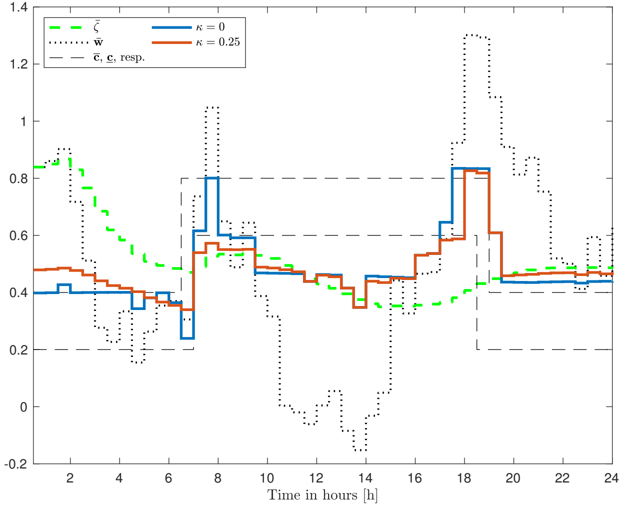

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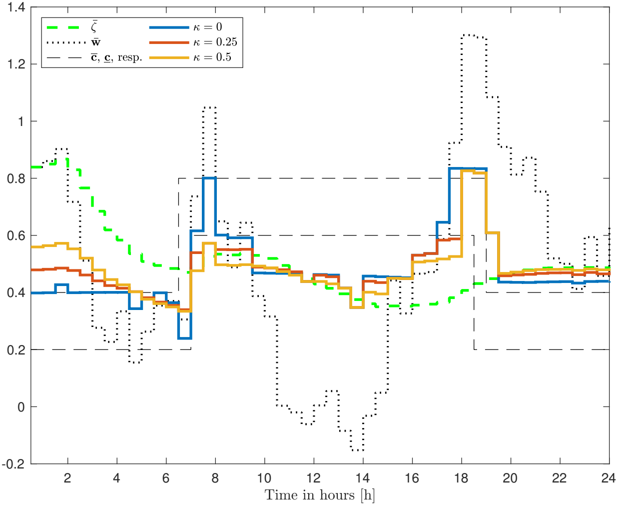

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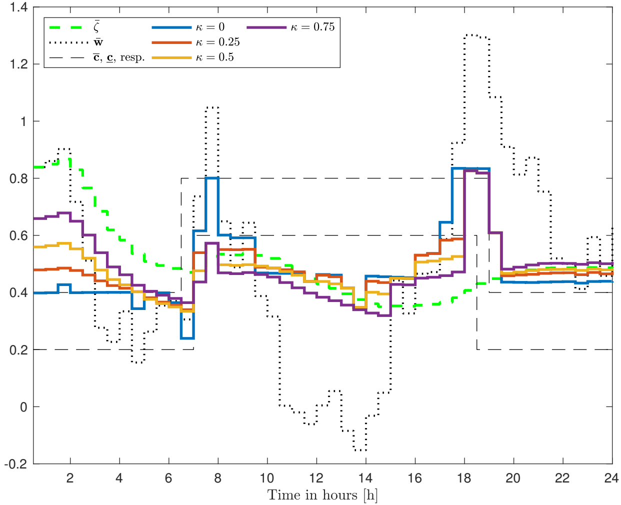

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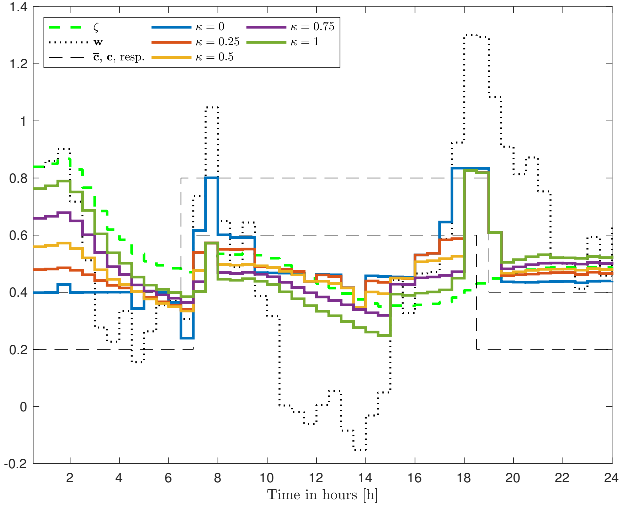

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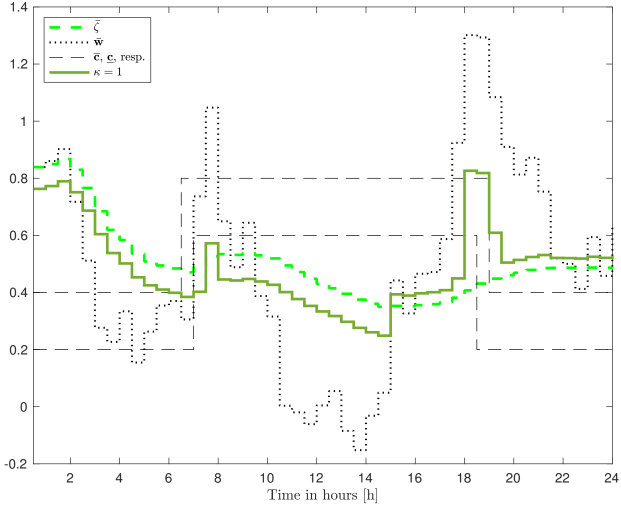
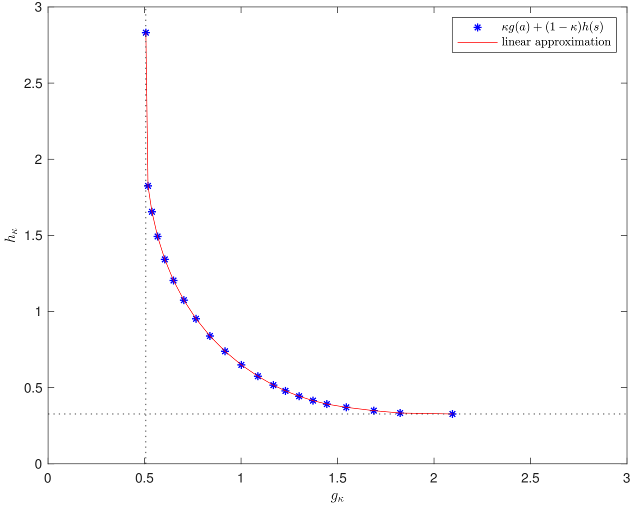


Efficient Control

Pareto Frontier



Proper Efficiency

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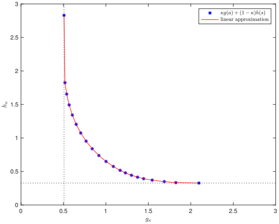

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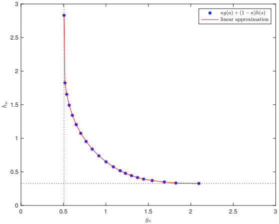

Interpretation

- efficient points = non-dominated points

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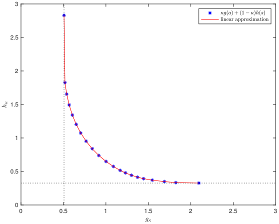

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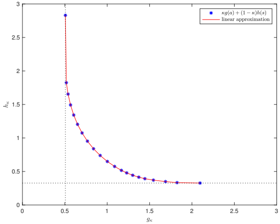

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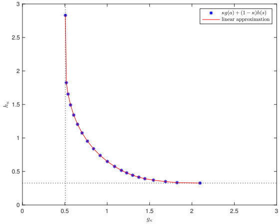

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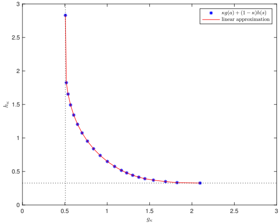

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$\rightsquigarrow$ information on costs of improvement w.r.t. one objective

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(joint work with Philipp Braun (University of Newcastle),
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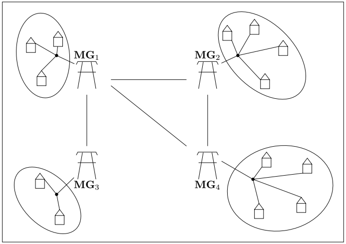

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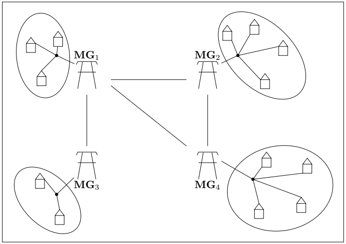

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$$\min_{\delta} \quad J(\delta) = \sum_{n=k}^{k+N-1} \sum_{\kappa=1}^{\Xi} \left(\mathcal{I}_\kappa \bar{\zeta}_\kappa(n) - \sum_{\nu=1}^{\Xi} \delta_{\nu,\kappa}(n) \eta_{\nu,\kappa} \mathcal{I}_\nu \bar{z}_\nu(n) \right)^2$$

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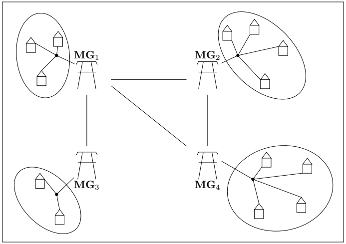

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$$\text{s.t.} \quad \delta \in \Delta = \left\{ \delta \;\middle|\; \sum_{\nu=1}^{\Xi} \delta_{\kappa,\nu}(n) = 1 \text{ and } \delta_{\kappa,\nu}(n) \cdot \delta_{\nu,\kappa}(n) \leq 0 \; (\kappa \neq \nu) \right\}$$

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Results

MG	$\lVert\bar{z} - \bar{\zeta}\rVert_2^2$	$\lVert\bar{z}^+ - \bar{\zeta}\rVert_2^2$
1	920.57	451.44
2	124.85	85.07
3	32.25	79.82
4	34.03	85.07
overall	1111.70	701.39

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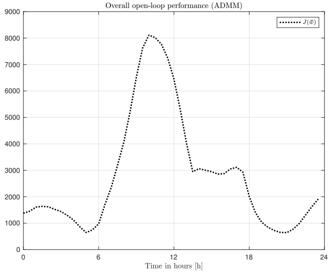
MG	$\lVert \bar{z} - \bar{\zeta} \rVert_2^2$	$\lVert \bar{z}^+ - \bar{\zeta} \rVert_2^2$
1	920.57	451.44
2	124.85	85.07
3	32.25	79.82
4	34.03	85.07
overall	1111.70	701.39



Bilevel Optimization Scheme

1. MG_κ: Solve $\min_{\bar{z}_\kappa \in \mathbb{D}_\kappa} g_\kappa(\bar{z}_\kappa)$ and send $\bar{z}_\kappa$ to CE.
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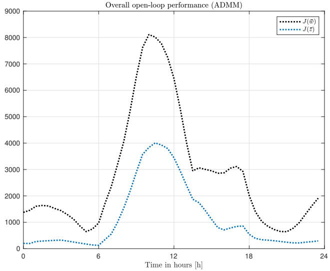

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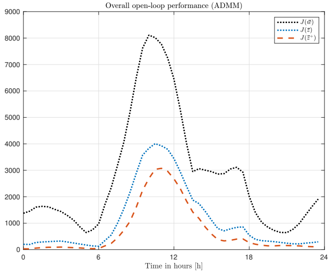

Philipp Sauerteig
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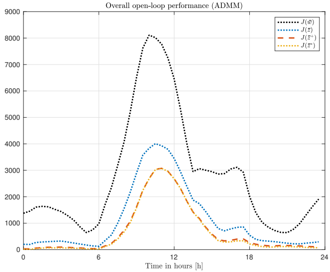

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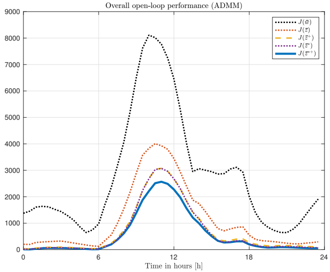

Surrogate Models for Microgrids

(joint work with Sara Grundel and Manuel Baumann, MPI Magdeburg, [Grundel, S., Worthmann, 2019])

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$\rightsquigarrow$ solve optimization problem in each MPC iteration

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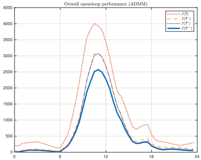

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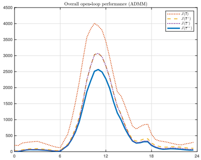

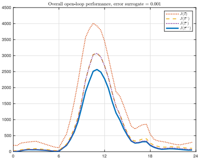

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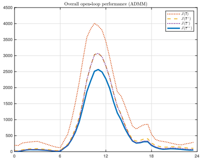

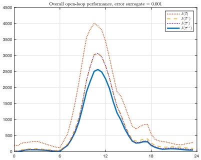

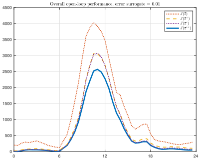

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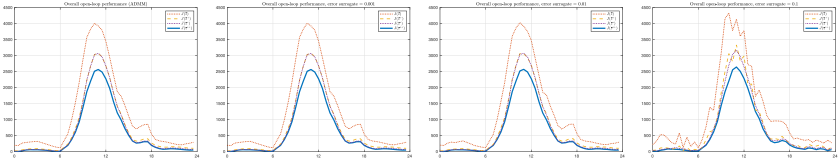
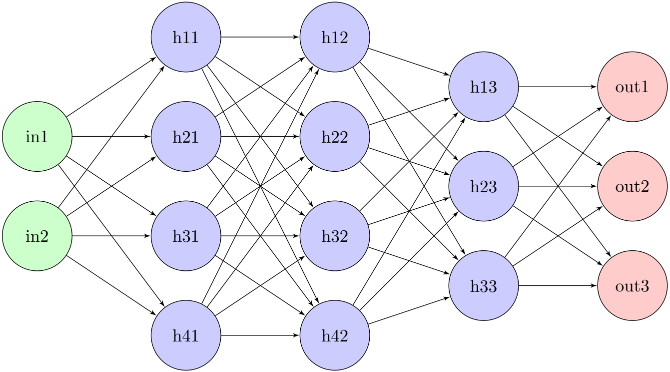


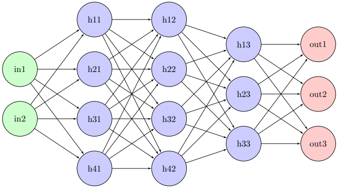
Introduction to Neural Networks

Basic Scheme incorporating 5 layers



- 1 input layer with 2 neurons
- 1 output layer with 3 neurons
- 3 hidden layers with 3 or 4 neurons, resp.

Introduction to Neural Networks



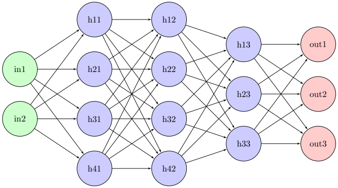
- sigmoid function $\sigma(x) = \frac{1}{1+\mathrm{e}^{-x}}$
- weights $W^{[\ell+1]} \in \mathbb{R}^{m_{\ell+1} \times m_\ell}$
- biases $b^{[\ell+1]} \in \mathbb{R}^{m_{\ell+1}}$

with number m_ℓ of neutrons in layer ℓ.

Output of layer $\ell + 1$ given output y^ℓ of previous layer

$$y^{\ell+1} = \sigma(W^{[\ell+1]} y^\ell + b^{[\ell+1]})$$

Introduction to Neural Networks



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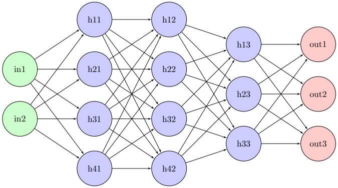
Output of neural network given input x

$$z = F(x) = \sigma(W^{[4]}\sigma(W^{[3]}\sigma(W^{[2]}x + b^{[2]}) + b^{[3]}) + b^{[4]})$$

Introduction to Neural Networks



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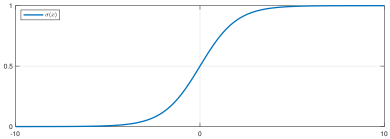
Sigmoid function



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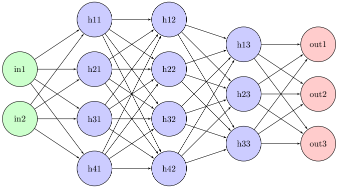

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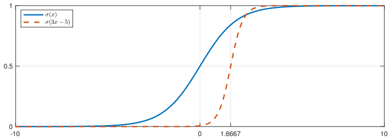

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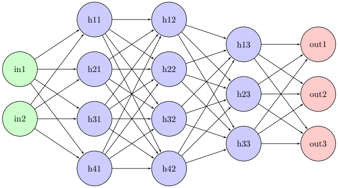

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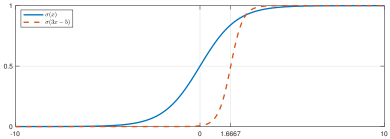
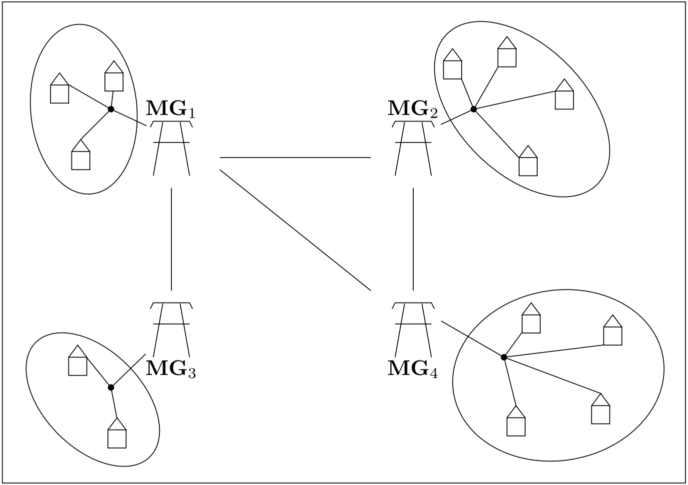


- "smoothed" step function
- imitating neurons in brain:
  - $\sigma = 1 \Leftrightarrow$ neuron firing
  - $\sigma = 0 \Leftrightarrow$ neuron inactive
- [Higham, Higham, 2018]

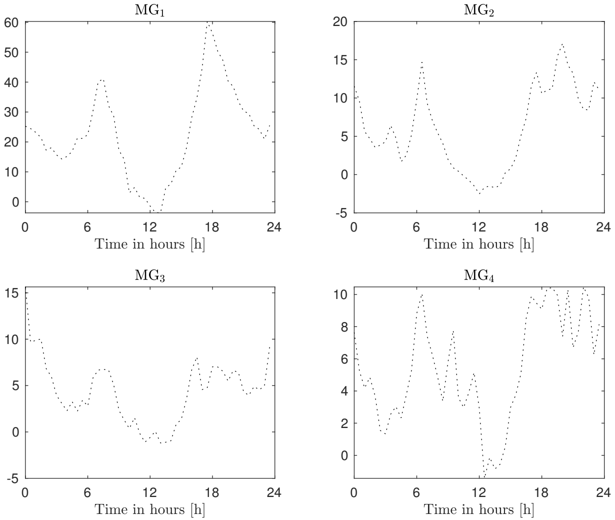
Results

Coupled MGs



Results

Net consumption



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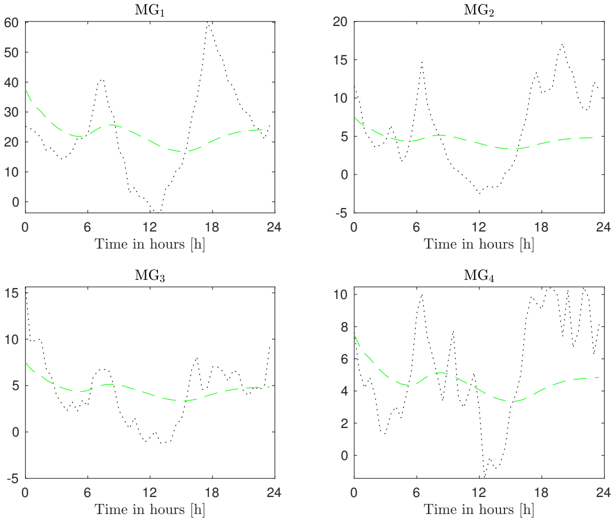
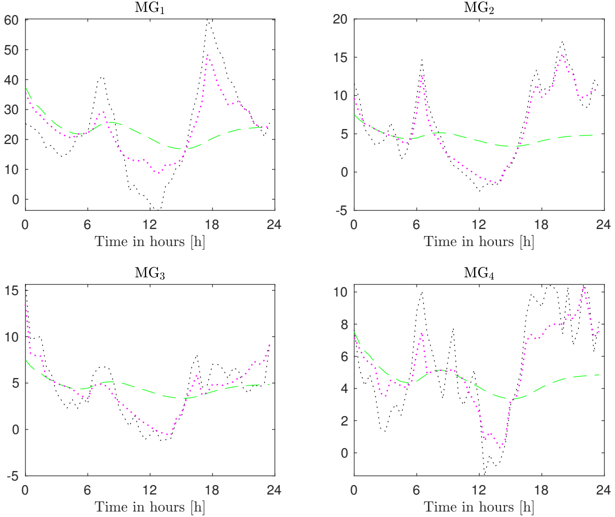
Reference trajectory



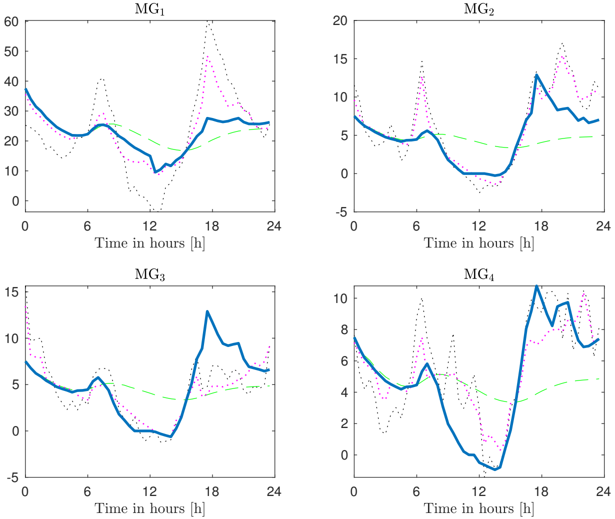
Results

Solution without additional exchange



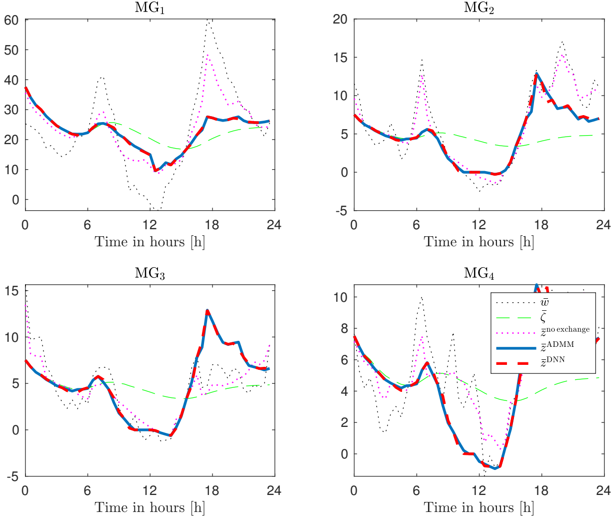
Results

Solution incorporating exchange within MGs



Results

Approximation (using MatLab's Neural Network)



Thank you for your attention!

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Appendix

Extension: Controllable Loads

Assumptions

- Net consumption is split

$$w_i(n) = w_i^s(n) + w_i^c(n)$$

  with static part $w_i^s(n)$ and controllable part $w_i^c(n)$.
- $w_i^c(n)$ is scheduled within time window $[n : n + \bar{N}]$ with $\bar{N} \in \mathbb{N}$
- $w_i^c(n) \in [0, \bar{w}_i^c]$ with $\bar{w}_i^c \geq 0$

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System Dynamics

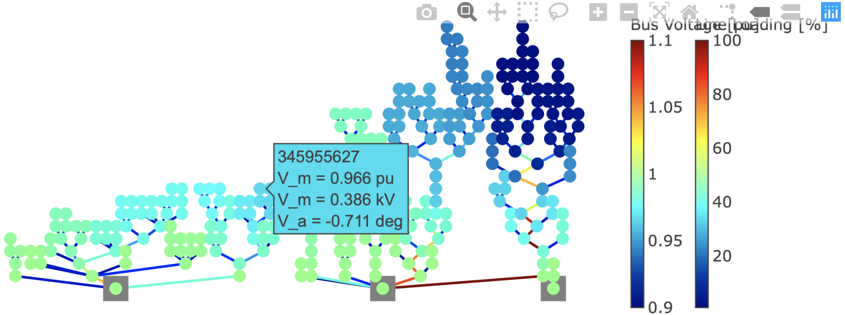
$$\begin{aligned} x_i(n+1) &= \alpha_i x_i(n) + T(\beta_i u_i^+(n) + u_i^-(n)) \\ z_i(n) &= w_i^s(n) + u_i^+(n) + \gamma_i u_i^-(n) + u_i^c(n) \end{aligned}$$

Towards Optimal Line Line Loading

Line Loading of a Test Grid

Towards Optimal Line Line Loading

Line Loading of a Test Grid



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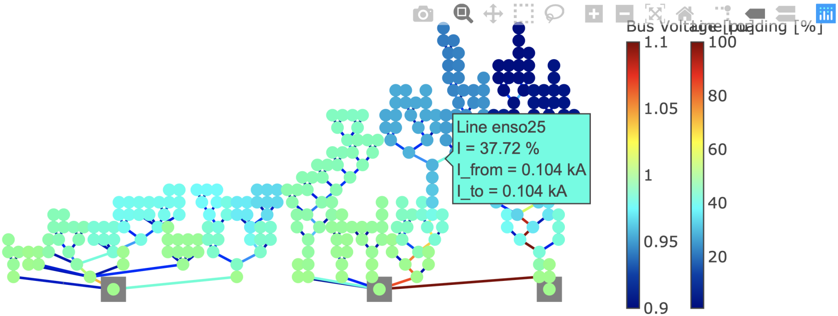

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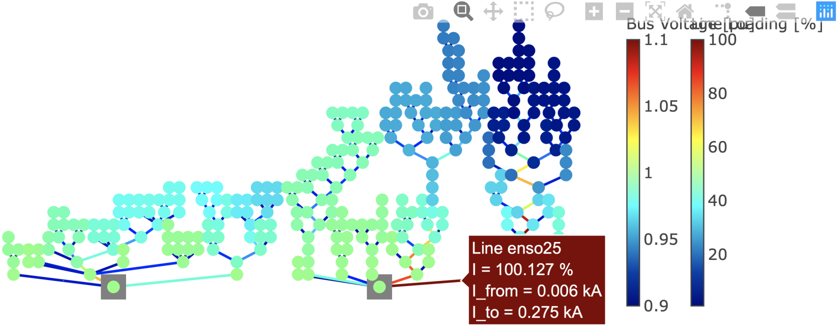

created with `python` toolbox `PandaPower`

Towards Optimal Line Loading

Optimization Problem

$$
\begin{aligned}
\min_{u,Q} \quad & \|s\|_2^2 \\
\text{s.t.} \quad & P_i - |V_i| \sum_{j \neq i} |V_j| (G_{ij} \cos(\delta_{ij}) + B_{ij} \sin(\delta_{ij})) = 0 \\
& Q_i - |V_i| \sum_{j \neq i} |V_j| (G_{ij} \sin(\delta_{ij}) - B_{ij} \cos(\delta_{ij})) = 0 \\
& x_i(k+1) = \alpha_i x_i(k) + T(\beta_i u_i^+(k) + u_i^-(k)) \\
& P_i(k) = z_i(k) = w_i(k) + u_i^+(k) + \gamma_i u_i^-(k) \\
& \underline{c} - \underline{s} \leq \mathcal{J}(|\mathbf{V}|, \delta) \leq \bar{c} + \bar{s} \\
& \text{constraints of each subsystem } i
\end{aligned}
$$

- line loading $\mathcal{J}$: function of voltage $\mathbf{V} = \left(|V_1|\mathrm{e}^{\mathrm{i}\delta_1}, \dots, |V_{\mathcal{I}}|\mathrm{e}^{\mathrm{i}\delta_{\mathcal{I}}}\right)$
- admittance matrix $Y_{ij} = G_{ij} + \mathrm{i}B_{ij}$ encodes grid topology
- tube constraints $\underline{c} < \bar{c}$