MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS MAGDEBURG
CSC COMPUTATIONAL METHODS IN SYSTEMS AND CONTROL THEORY
KONSENS
Federal Ministry of Education and Research

Complexity Reduction for Power Flow Simulations

Team *Simulation of Energy Systems* (SES 1)

Motivation

The *energy transition* in Germany comes with new challenges for the mathematical modeling of power grids. Most prominently, this includes renewable energy resources that are modeled as time-dependent generators. Newly developed storage units can be added to improve stability of the grid.

The simulation of dynamical systems is improved by means of Model Order Reduction (MOR),

$$\dot{x} = f(x,t) \quad \rightsquigarrow \quad \dot{\hat{x}} = W_r^T f(V_r \hat{x}, t), \tag{1}$$

where bi-orthogonal projection matrices V_r and W_r are used.

Optimal Power Flow Simulation

Power flow equations An electrical network is modeled as a mathematical graph. The power flow at node i is described by the complex power $S_i = P_i + jQ_i$ and the complex voltage $V_i = |V_i|e^{j\delta_i}$ governed by the (stationary) *power flow equations*,

$$c_E(x,t) = \begin{bmatrix} P_i - |V_i|\sum_{k=1}^N |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \\ Q_i - |V_i|\sum_{k=1}^N |V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \end{bmatrix} \overset{!}{=} 0,$$

for $x_i := [|V_i|, \delta_i, P_i, Q_i]^T$, $\delta_{ik} := \delta_i - \delta_k$, and at all times t.

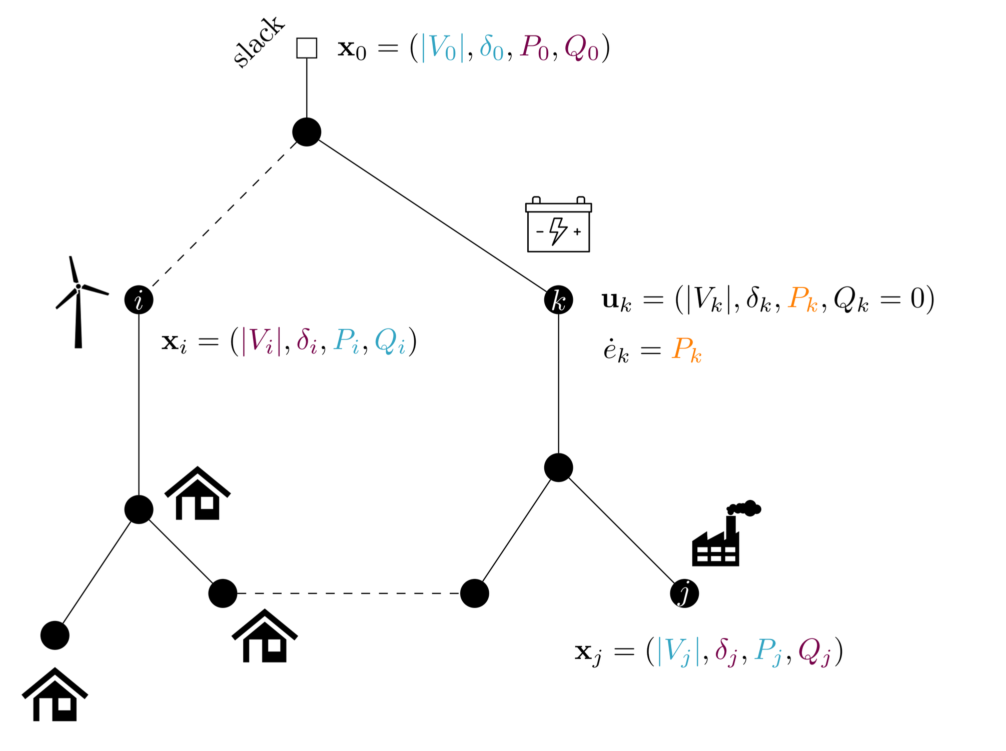

Figure 1: Overview of different components in a power grid. The colors indicate if a nodal variable is known, unknown, or controllable.

Battery Controls We introduce storage units at some nodes of the power grid in order to improve the line loading in the network. The reactive power of these batteries are the control quantities u. The following optimal control problem is derived,

$$\begin{array}{lll} \underset{u}{\text{minimize}} & \int_{t_0}^{t_e} \ell(x(u),t)\,dt & \text{(line loading)} \\ \text{subject to} & c_E(x,t) + Bu(t) = 0 & \text{(network state)} \\ & \dot{e}_k = u_k,\ e_k(0) = e_k^0 & \text{(battery state-of-charge)} \\ & x_{\min} \le x_i(t) \le x_{\max} & \text{(network constraints)} \\ & u_{\min} \le u_k(t) \le u_{\max} & \text{(control constraints)} \\ & e_{\min} \le e_k(t) \le e_{\max} & \text{(battery constraints)} \end{array}$$

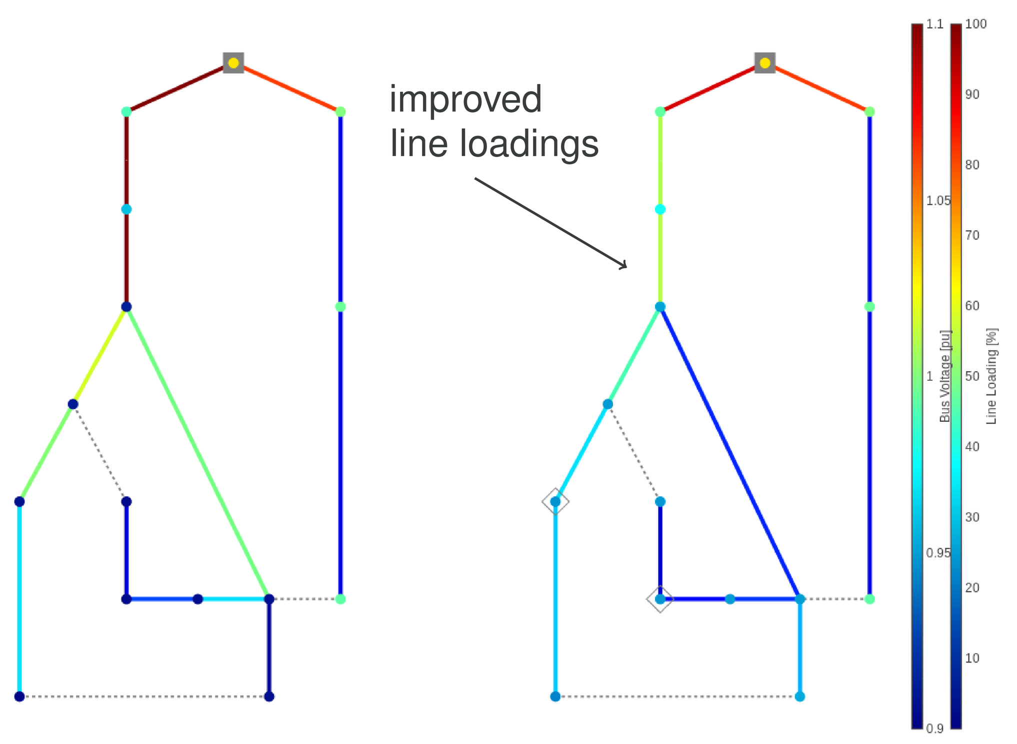

Figure 2: Improved line loadings in the CIGRE network, with battery locations marked by '⋄'. Visualization ©pandapower v1.2.2.

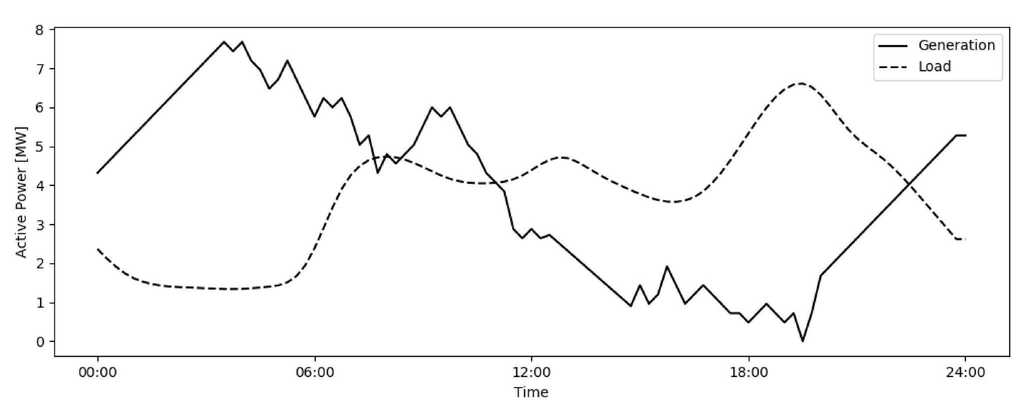

Figure 3: A typical (dis-)charging behavior of the battery controls.

Model Order Reduction by Clustering

Clustering-based MOR can be realized using Galerkin projection with, i.e.,

$$V_r = W_r = P(\pi) \quad \text{in (1)},$$

where $P(\pi)$ is a characteristic matrix of a partition π, e.g. for $\pi = \{\{1,2,3,4\},\{5,6\},\{7\},\{8\},\{9,10\}\}$ we have

$$P(\pi) = \begin{bmatrix} 1&1&1&1&0&0&0&0&0&0 \\ 0&0&0&0&1&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&1 \end{bmatrix}^T.$$

Then, if $\mathcal{A} = [a_{ij}]$ is the adjacency matrix of the graph over which the dynamics is evolving, the adjacency matrix of the reduced graph is $\hat{\mathcal{A}} = P(\pi)^T \mathcal{A} P(\pi)$.

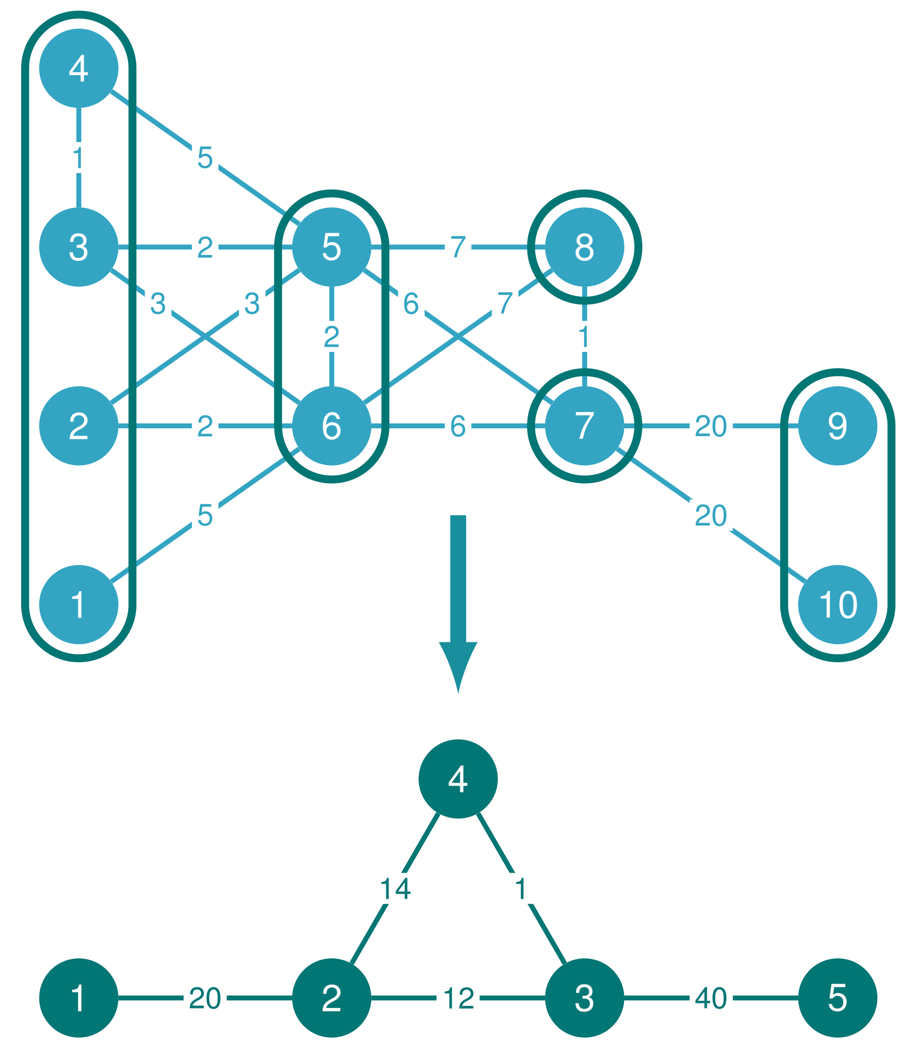

Figure 4: Reduction of a graph using clustering.

Dynamic Power Flow

A network of generators is modeled using swing equations [2]

$$M_i\ddot{\delta}_i(t) + D_i\dot{\delta}_i(t) = P_i - \sum_{j=1}^N a_{ij}\sin(\delta_i(t) - \delta_j(t)), \tag{2}$$

with $N = 10$ in the following. The reduced order model (ROM) we present is derived from a two-step clustering algorithm:

1. Collect the leading POD modes of δ_i and $\dot{\delta}_i$ from (2).
2. Apply k-means clustering with respect to the leading POD modes using PYTHON's `sklearn.cluster.KMeans`.

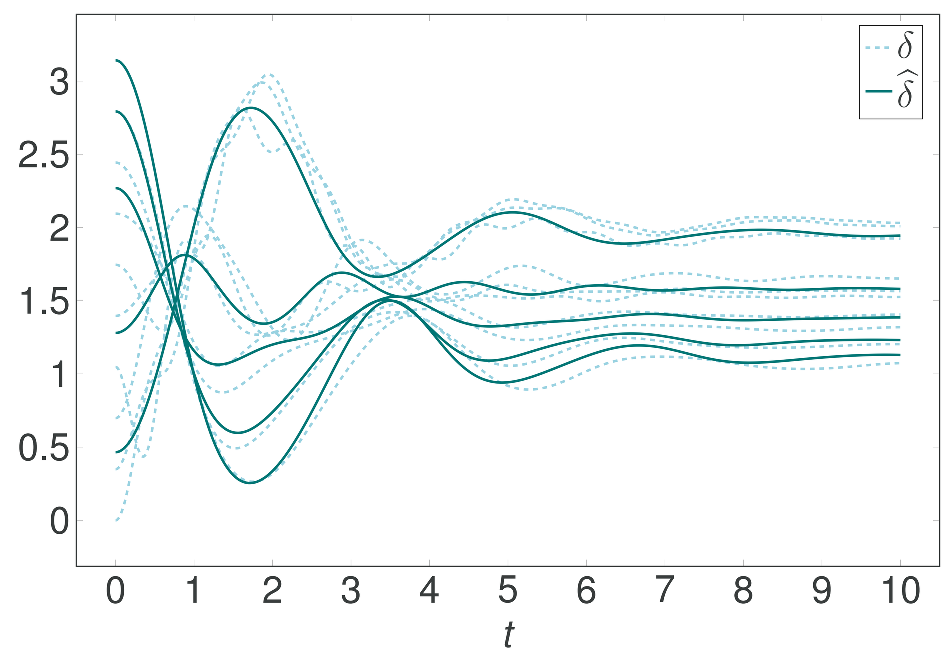

Figure 5: Simulation of the original swing equation (dashed lines) and its clustering-based ROM (solid lines).

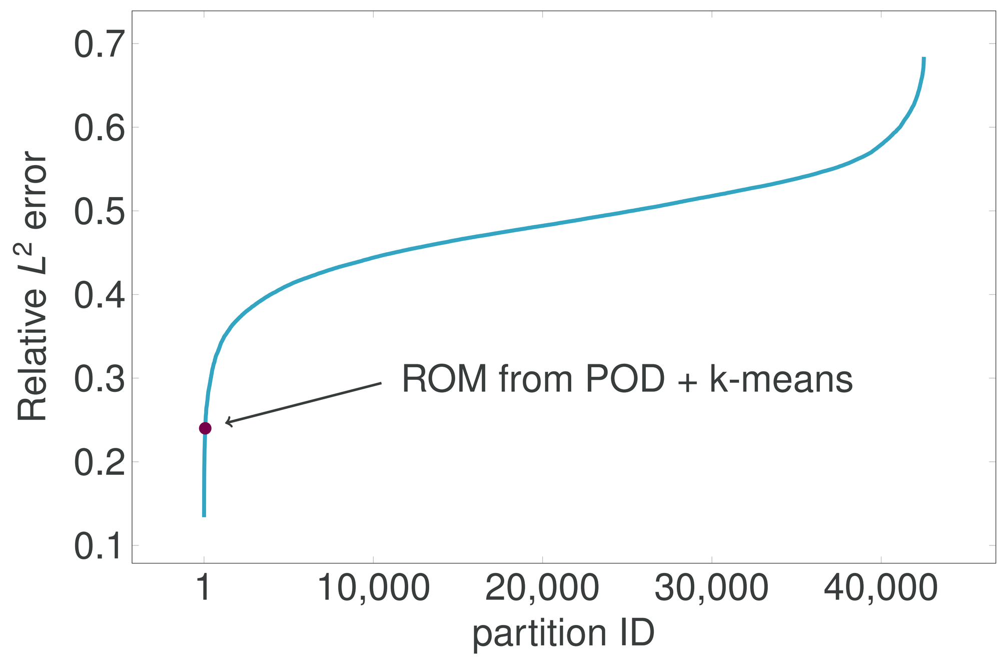

Figure 6: Relative error of all possible clusterings with partition size $r = 5$.

MOR in Quadratic-Bilinear Form

The swing equation (2) can be written in Quadratic-Bilinear (QB) form when introducing the elongated state variable $x_i = [\delta_i, \dot{\delta}_i, \sin(\delta_i), \cos(\delta_i)]^T$,

$$\dot{x}(t) = Ax(t) + H\, x(t) \otimes x(t) + Bu(t),$$
$$y(t) = Cx(t), \quad x(0) = 0.$$

In [1], the balanced truncation method is extended to derive matrices $\{\hat{A}, \hat{B}, \hat{C}, \hat{H}\}$ of a reduced-order QB-system, cf. Figure 7.

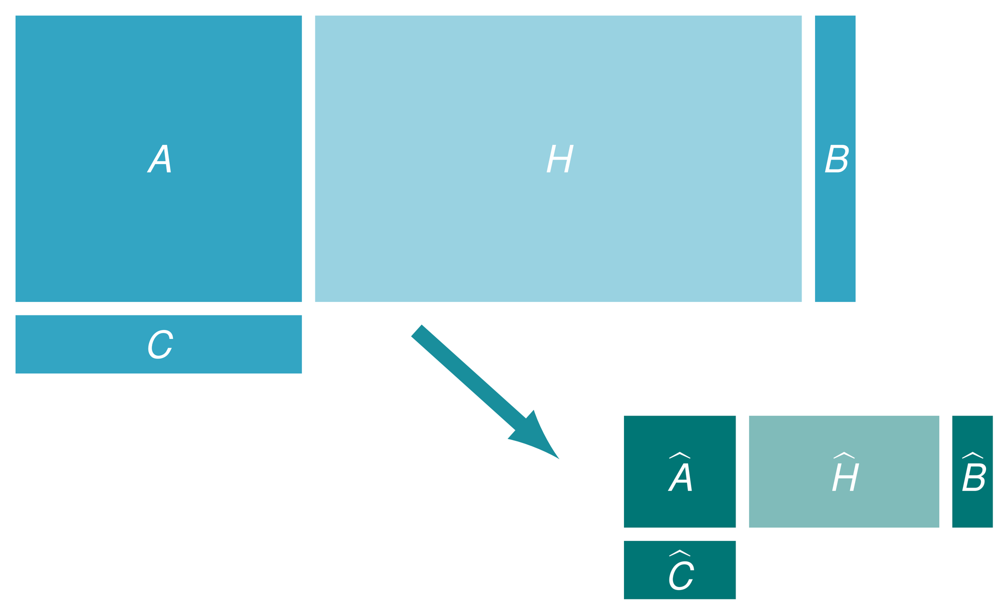

Figure 7: Illustration of the Petrov-Galerkin projections $\hat{A} = W_r^T A V_r$, $\hat{B} = W_r^T B$, $\hat{C} = C V_r$, and $\hat{H} = W_r^T H (V_r \otimes V_r)$.

Requires fixpoint iteration for non-linear Lyapunov equations:

$$A\mathbf{P}_{k+1} + \mathbf{P}_{k+1}A^T + H(\mathbf{P}_k \otimes \mathbf{P}_k)H^T = -BB^T, \quad \mathbf{P} =: RR^T,$$
$$A^T\mathbf{Q}_{k+1} + \mathbf{Q}_{k+1}A + H(P \otimes \mathbf{Q}_k)H^T = -C^TC, \quad \mathbf{Q} =: SS^T.$$

Obtain projection spaces based on truncated SVD of S^TR.

Future Work

Switched QB-systems In [3] linear switched systems are considered, i.e. when $H_{q(t)} \equiv 0\ \forall t$ in,

$$\dot{x}(t) = A_{q(t)}x(t) + H_{q(t)}\, x(t) \otimes x(t) + B_{q(t)}u(t),$$
$$y(t) = C_{q(t)}x(t), \quad x(0) = 0.$$

Switched system are relevant for power grid simulations when, for instance, line failure occurs.

Reduced-Order Model for Power Flow Simulation Apply MOR algorithms to the efficient numerical solution of optimal power flow.

- Apply clustering to a power grid with varying node types (multi-agent clustering).
- First-order optimization algorithms required the efficient solution of sequences of linear systems with system matrix $\nabla_x c_E(x,t)$ for which fast solvers are, hence, crucial.
- Identify preferred battery locations for grid stabilization.

References

[1] P. Benner and P. Goyal, *Balanced truncation model order reduction for quadratic-bilinear systems*, e-prints 1705.00160, arXiv, 2017. math.OC.
[2] P. Mlinarić, T. Ishizaki, A. Chakrabortty, S. Grundel, P. Benner, and J.-i. Imura, *Synchronization and aggregation of nonlinear power systems with consideration of bus network structures*, e-prints 1803.00439, arXiv, 2018. cs.SY.
[3] I. Pontes Duff, S. Grundel, and P. Benner, *New Gramians for linear switched systems: Reachability, observability, and model reduction*, e-print 1806.00406, arXiv, 2018. math.OC.

The KONSENS Project

As part of the initiative *Mathematics for Innovation* by the Federal Ministry of Education and Research, the project "Konsistente Optimierung und Stabilisierung elektrischer Netzwerksysteme" (KONSENS) addresses mathematical challenges arising from the energy transition in Germany.

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