MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC

COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

20 YEARS
1998-2018

Model-Order Reduction for Power Grid Simulations with Renewables and Batteries

Manuel Baumann

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Who are we?

Sara Grundel

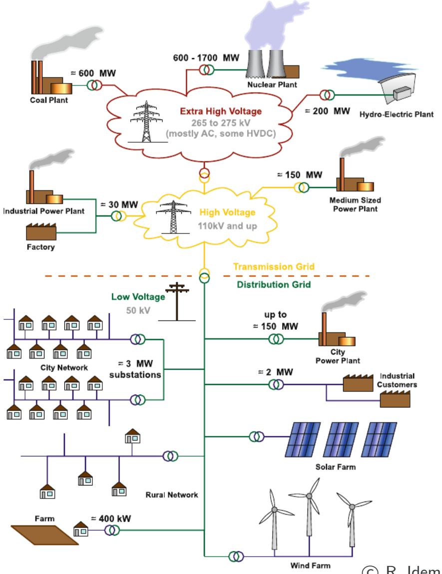
- team leader *Simulation of Energy Systems*
- areas of expertise: modeling and simulation of large networks, optimization, model-order reduction

Manuel Baumann

- PostDoc at MPI since April 2018
- PhD from TU Delft in Numerical Linear Algebra
- past projects in: computational geophysics, optimal control, model-order reduction

https://konsens.github.io/

Power grid – new challenges



Recent developments:

- renewables
- E-car
- batteries
- prosumers

Gives rise to new mathematical challenges!

Classical mathematical model

The network is an undirected graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ with generators $\mathcal{V}_G$ and loads $\mathcal{V}_L$ ($\mathcal{V} = \mathcal{V}_G \cup \mathcal{V}_L$), and batteries. Four variables at node $i \in \mathcal{V}$,

$$V_i = |V_i|e^{j\delta_i} \text{ (voltage)} \quad \text{and} \quad S_i = P_i + jQ_i \text{ (power)}.$$

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Classical network (conventional generators):

$$\text{If } i \in \mathcal{V}_G: \quad P_i \text{ and } |V_i| \text{ are known.}$$
$$\text{If } i \in \mathcal{V}_L: \quad -P_i \text{ and } -Q_i \text{ are known.}$$

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Closed system via $2N_L + N_G$ non-linear power flow equations,

$$\begin{pmatrix} P_i - |V_i| \sum_{k=1}^{N} |V_k| (G_{ik} \cos(\delta_{ik}) + B_{ik} \sin(\delta_{ik})) \\ Q_i - |V_i| \sum_{k=1}^{N} |V_k| (G_{ik} \sin(\delta_{ik}) - B_{ik} \cos(\delta_{ik})) \end{pmatrix} \overset{!}{=} 0 \quad \forall i \in \mathcal{V}.$$

Renewables & batteries

Mathematical model for ...

- renewables: as a time-dependent PQ-bus $\quad \rightsquigarrow \quad +P_i(t), +Q_i(t)$
- batteries: as a control $u_j(t) = \{\pm P_j(t), Q_j(t) \equiv 0\}$ for a few $j \in \mathcal{V}$ with bounds for active power

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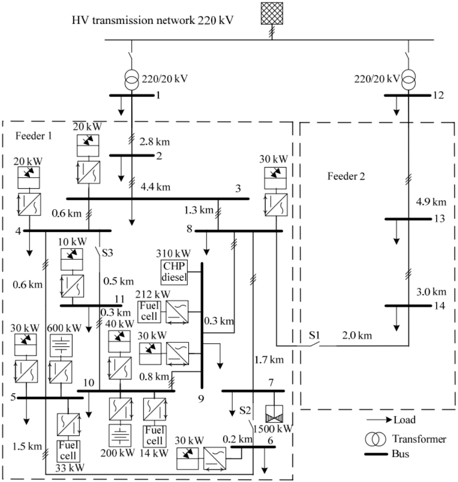
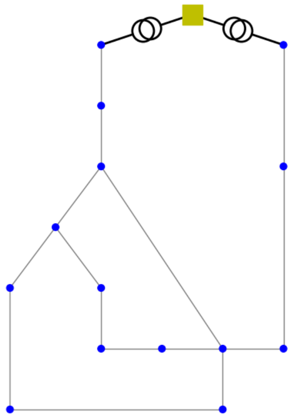
Questions:

- Is this reasonable?
- What is the control goal (objective function)?
- Is solving the power flow equation (via Newton-Raphson ?) a computational bottleneck for you?

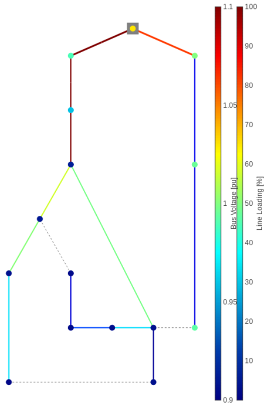
The CIGRE test case (1/2)



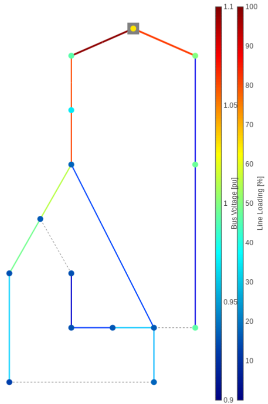


The CIGRE test case (2/2)

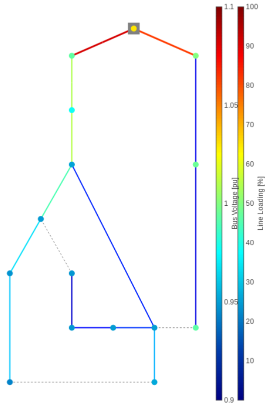
no renewables



8 PVs + 1 wind



9 DERs + batteries



Mathematical model

The abstract optimal control problem

Optimal control problem with (in-)equality constraints:

$$
\begin{aligned}
&\min_{u} \mathcal{J}(x(u), t) \\
\text{s.t.} \quad & f(x,t) + Bu(t) = 0 \\
\text{s.t.} \quad & x_{\min} \leq x_i(t) \leq x_{\max} \\
\text{s.t.} \quad & u_{\min} \leq u_i(t) \leq u_{\max}
\end{aligned}
$$

Here, the state at node i is $x_i = [|V_i|, \delta_i, P_i, Q_i]^T$ and the control u_i are the active power of the batteries (at fewer nodes).

Non-linear Power Flow eqn's $f_i(x,t) = \begin{bmatrix} P_i(t) - \Re[V_i(t) \sum_{k=1}^{N} Y_{ik}^* V_k^*(t)] \\ Q_i(t) - \Im[V_i(t) \sum_{k=1}^{N} Y_{ik}^* V_k^*(t)] \end{bmatrix}$.

Model-order reduction – Basics

Input-state-output systems:

system

$$\xrightarrow{u} \boxed{\begin{aligned} \dot{x} &= f(x,u) \\ y &= g(x) \end{aligned}} \xrightarrow{y}$$

- $x(t) \in \mathbb{R}^{n_x}$ – state, i.e. all quantities $\{V_i, S_i\}\ \forall i \in \mathcal{V}$
- $u(t) \in \mathbb{R}^{n_u}$ – input, e.g. battery configurations
- $y(t) \in \mathbb{R}^{n_y}$ – output, e.g. maximum voltage $\|V\|_\infty$, line loading, ...

Model-order reduction: Reduce the state dimension $n_x \gg 1$ while preserving an accurate input-output relation $u \mapsto y$.

Model-order reduction – Basics

Mathematical systems theory:

Definition 1: Reachability

The state x is **reachable** from the zero state $x_0 = 0$ if there exist an input function $u(t)$ of finite energy such that x can be obtain from the zero state and within a finite period of time $t < \infty$.

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A non-zero state x is **controllable** if there exist an input $u(t)$ with finite energy such that the state of the system goes to zero from x within a finite time $t < \infty$.

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Definition 3: Observability

Given any input $u(t)$, a state x of the system is **observable**, if starting with the state x ($x(0) = x$), and after a finite period of time $t < \infty$, x can be uniquely determined by the output $y(t)$.

Linear model-order reduction

In linear control theory, the full-order model reads,

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t),$$

with $A \in \mathbb{R}^{n_x \times n_x}, B \in \mathbb{R}^{n_x \times n_u}, C \in \mathbb{R}^{n_y \times n_x}$, and, particularly, $x(t) \in \mathbb{R}^{n_x}$.

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Choose (bi-linear) matrices $V_r, W_r \in \mathbb{R}^{n_x \times r}$ and define corresponding reduced-order model,

$$\dot{\hat{x}}(t) = (W_r^T A V_r)\hat{x}(t) + (W_r^T B)u(t),$$
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In the linear case, the transfer function maps input to output,

$$G(s) = C(sI_{n_x} - A)^{-1}B,$$
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- Extensions: Bi-linear MOR, quadratic MOR, ...

Non-linear model-order reduction

The full-order model is coverned by nonlinear equations,

$$\dot{x} = f(x,t) + Bu,$$
$$y = Cx.$$

Algorithm (Proper Orthogonal Decomposition):

- Collect snapshots $X := [x(t_1), ..., x(t_s)]$
- Computer SVD: $X = V\Sigma U^T$
- Define projection space via truncation $V_r := V[:, 1\!:\!r]$

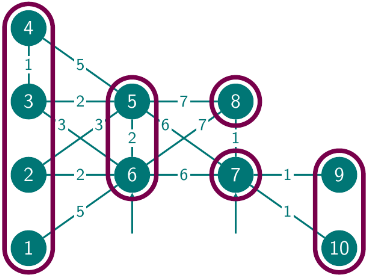
The projection then yields the term $W_r^T f(V_r\hat{x}, t)$ which can be further reduced using hyper-reduction methods (e.g. DEIM).

Model-order reduction for networks

Take V_r from IRKA, and find partition $P(\pi)$ such that,

$$\text{Range } V_r = \text{Range } P(\pi),$$

i.e. find Z such that $V_r = P(\pi)Z$.



Advantages:

- The reduced variable $x \approx P(\pi)\hat{x}$ corresponds to a network.
- Existing code can be re-used.

P. Mlinarić, S. Grundel, P. Benner (2015). *Efficient model order reduction for multi-agent systems using QR decomposition-based clustering.* 54th IEEE Conference on Decision and Control, 4794–4799.

Some open problems

On the mathematical side:

- MOR with state-inequalities

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On the modelling side:

- *dynamic* power flow simulation: $\delta_i \rightarrow \delta_i(t)$
- new players: DER, battery, E-car $\rightsquigarrow$ 3-phases simulation ?

Dynamic power flow simulation?

Three leading models

Governing equations (*swing equations*) at network node i,

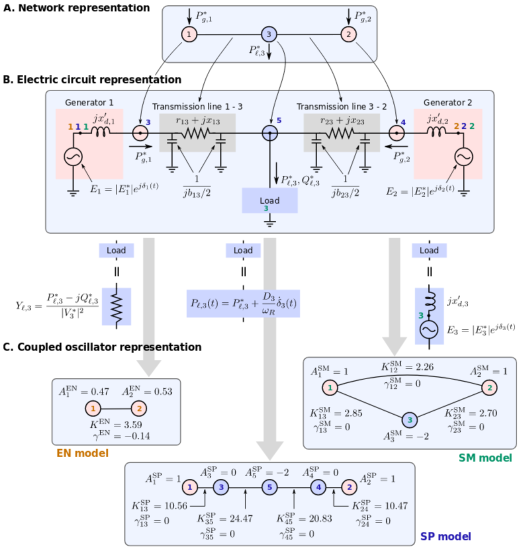
$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta}_i = A_i - \sum_{j\neq i} K_{ij}\sin(\delta_i - \delta_j - \gamma_{ij}), \quad i \in \{1, ..., N\},$$

yield for a specific choice of parameters the three models

- **EN** effective network model ($N = |\mathcal{V}_G|$),
- **SM** synchronous motor model ($N = |\mathcal{V}|$),
- **SP** structure-preserving model ($N > |\mathcal{V}|$).

T. Nishikawa and A. E. Motter (2015). *Comparative analysis of existing models for power-grid synchronization.* New Journal of Physics 17:1.

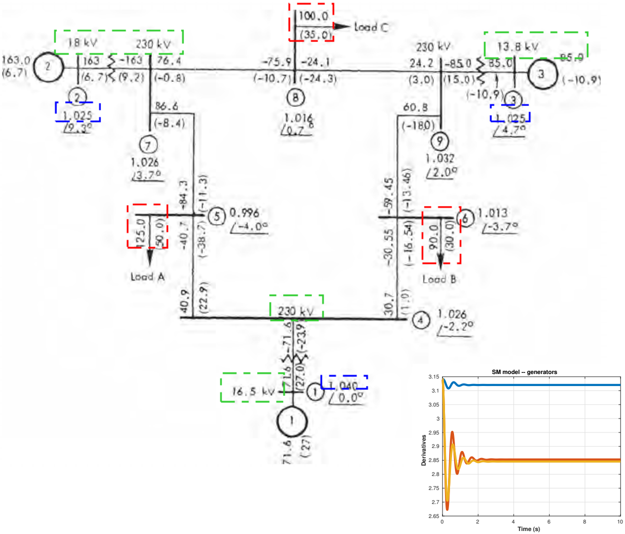
Which model?



A. Network representation
B. Electric circuit representation
Generator 1
Transmission line 1 - 3
Transmission line 3 - 2
Generator 2
Load
C. Coupled oscillator representation
EN model
SM model
SP model

Comparison

Current MSc project (ongoing).



Swing equations,

$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta}_i = A_i - \sum_{j\neq i} K_{ij}\sin(\delta_i - \delta_j - \gamma_{ij}), \quad i \in \{1, ..., N\}.$$

Linearization,

$$\begin{bmatrix}\dot{\delta}_i \\ \dot{\xi}_i\end{bmatrix} = \begin{bmatrix} \xi_i \\ \frac{\omega_R}{2H_i}\left(A_i - \sum_{j\neq i} K_{ij}\sin(\delta_i - \delta_j - \gamma_{ij}) - \frac{D_i}{\omega_R}\xi_i\right)\end{bmatrix},$$

Bi-linear formulation,

$$\begin{bmatrix}\dot{\delta}_i \\ \dot{\xi}_i \\ \dot{s}_i \\ \dot{c}_i\end{bmatrix} = \begin{bmatrix} \xi_i \\ \frac{\omega_R}{2H_i}\left(A_i - \sum_{j\neq i} K_{ij}(s_i c_j \gamma_{ij}^c - c_i s_j \gamma_{ij}^c - c_i c_j \gamma_{ij}^s - s_i s_j \gamma_{ij}^s) - \frac{D_i}{\omega_R}\xi_i\right) \\ c_i \xi_i \\ -s_i \xi_i \end{bmatrix}$$