

How to Coordinate Countermeasures against COVID-19

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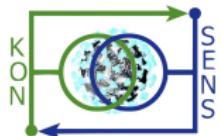
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Project: KONSENS (grants: 05M18SIA, 05M18EVA)

ECMI, 14 April 2021



Outline

1. Epidemiological modelling

1.1 Basics: S(E)IR model

1.2 Extensions tailored to COVID-19: SEIPHR model

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- 2.1 Social distancing
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- 3.1 Model predictive control
- 3.2 Results

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4. Conclusions & outlook

Basic compartmental model: SIR



3 compartments

- susceptible S
- infectious I
- removed R (recovered/deceased)

Basic compartmental model: SIR

β : $\frac{\text{average number of contacts}}{\text{person} \times \text{time}} \cdot \text{Prob(transmission)}$



3 compartments

- susceptible S
- infectious I
- removed R (recovered/deceased)

$$\frac{d}{dt}S(t) = -\beta S(t)I(t)$$

$$\frac{d}{dt}I(t) = \beta S(t)I(t)$$

Basic compartmental model: SIR

η^{-1} : average recovery time



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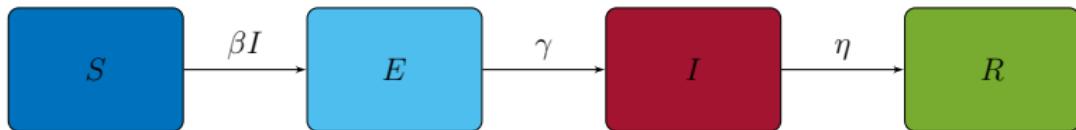
$$\frac{d}{dt}S(t) = -\beta S(t)I(t)$$

$$\frac{d}{dt}I(t) = \beta S(t)I(t) - \eta I(t)$$

$$\frac{d}{dt}R(t) = \eta I(t)$$

Basic compartmental model: SEIR

γ^{-1} : average incubation time



4 compartments

- susceptible S
- exposed E
- infectious I
- removed R (recovered/deceased)

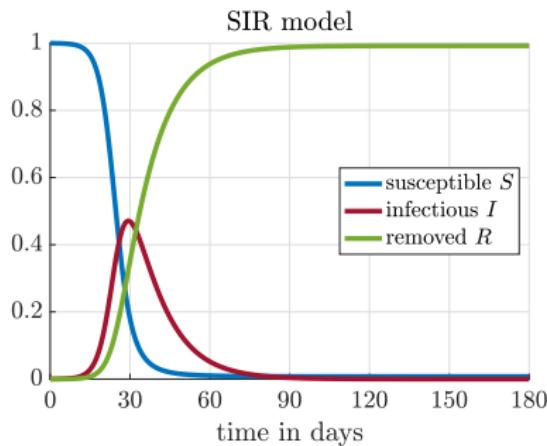
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$$\frac{d}{dt}E(t) = \beta S(t)I(t) - \gamma E(t)$$

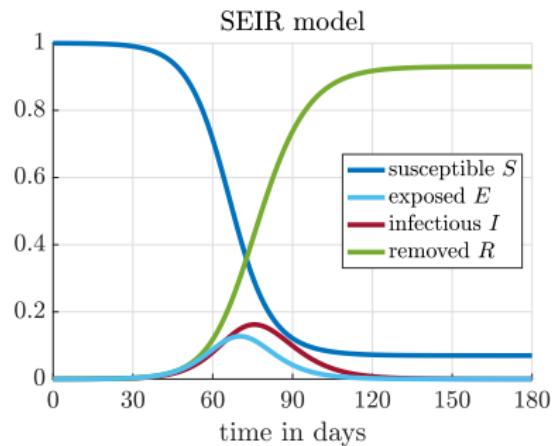
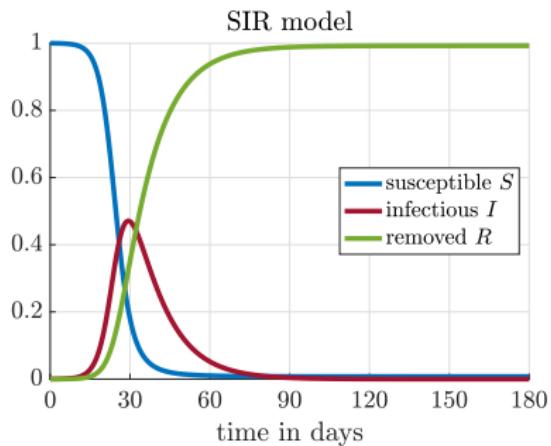
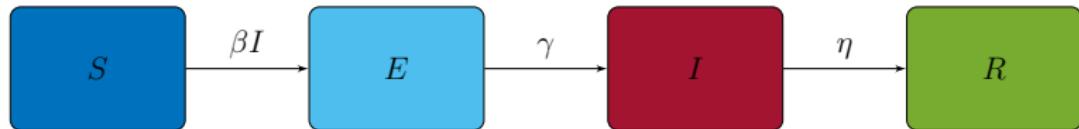
$$\frac{d}{dt}I(t) = \gamma E(t) - \eta I(t)$$

$$\frac{d}{dt}R(t) = \eta I(t)$$

Basic compartmental model: SEIR



Basic compartmental model: SEIR



Drawbacks of the SEIR model

Model does not account for

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- symptom severity (ICU occupancy)

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 - on contacts
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 - mass testing

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Model does not account for

- symptom severity (ICU occupancy)
- demographic influences
 - on contacts
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- counter measures
 - social distancing/quarantine
 - vaccination
 - mass testing
- births and (natural) deaths
- re-infections

Extension: SEIPHR model

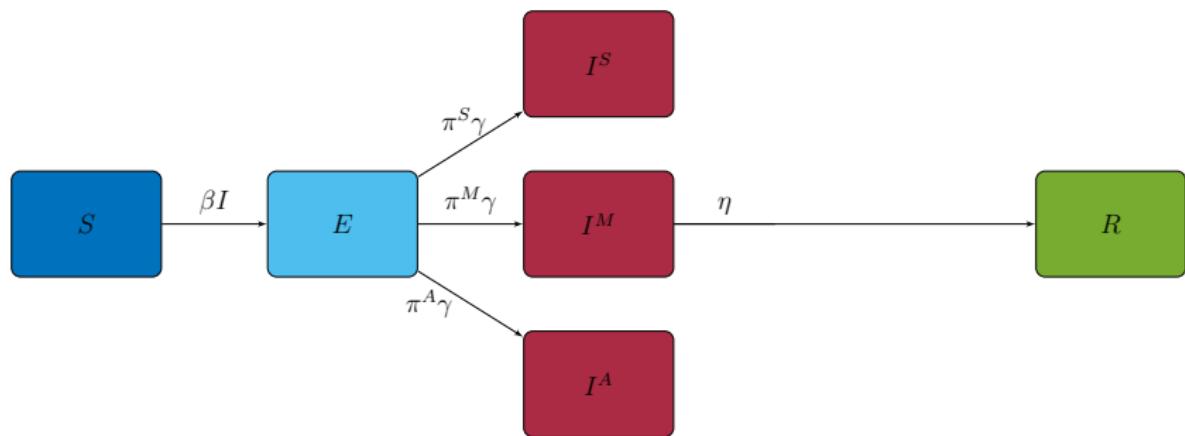
SEIR model



Extension: SEIPHR model

Symptom severity

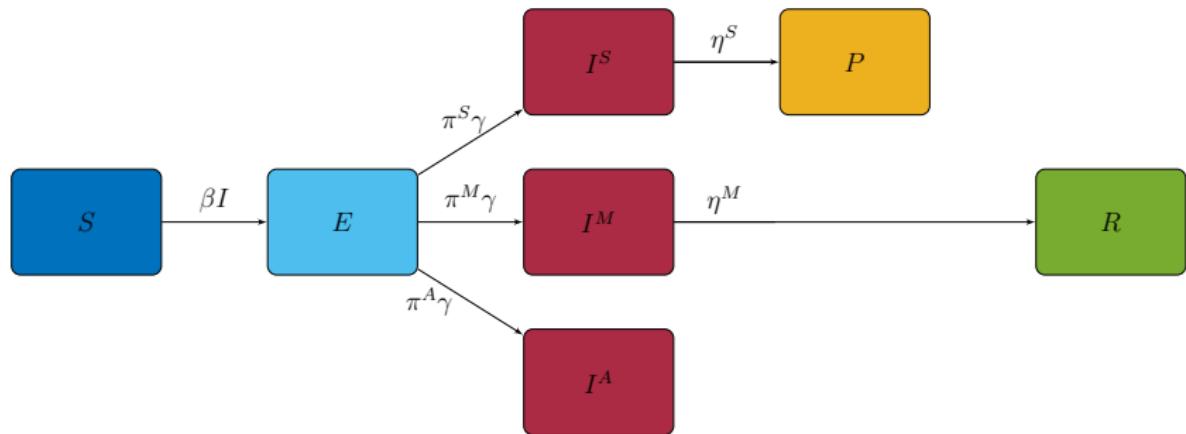
transmission probabilities $\pi^S + \pi^M + \pi^A = 1$



Extension: SEIPHR model

Pre-ICU compartment

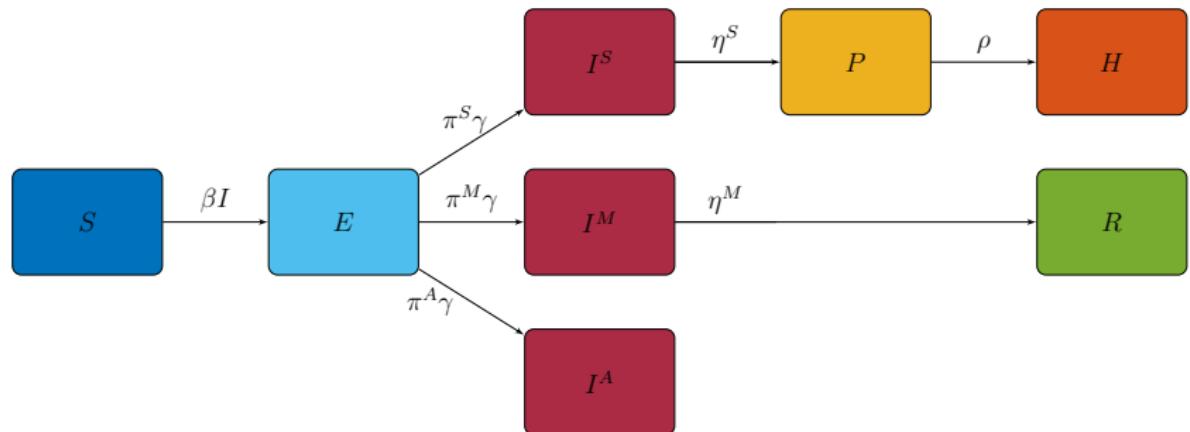
quarantine



Extension: SEIPHR model

ICU compartment

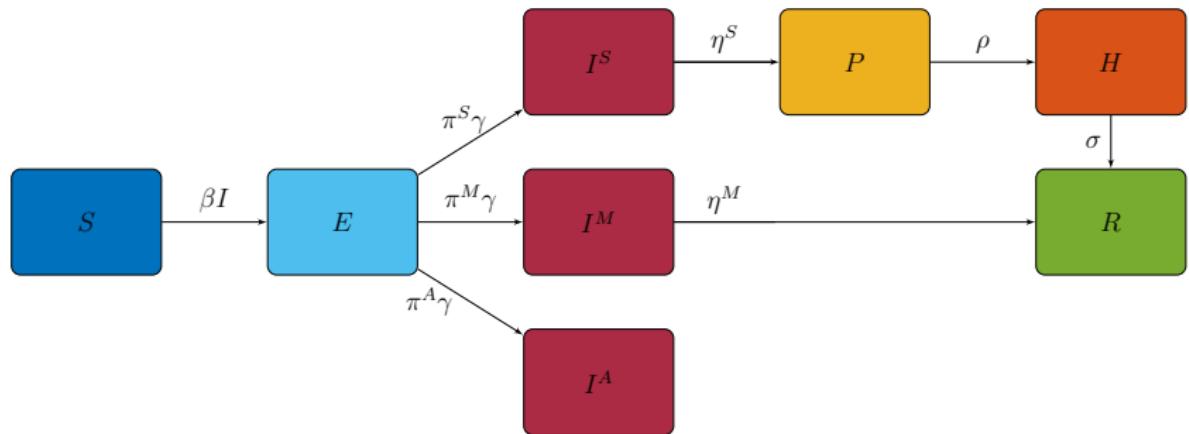
ρ : ICU admittance rate



Extension: SEIPHR model

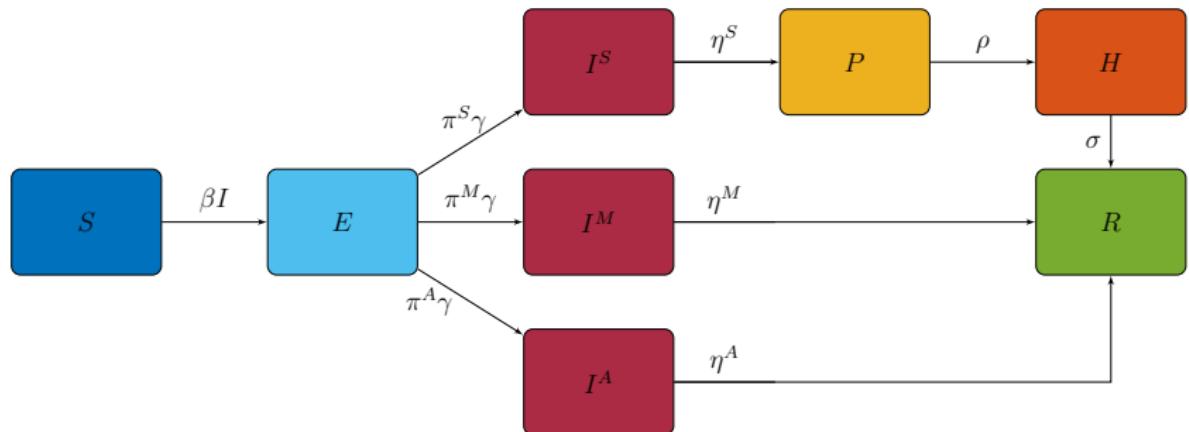
ICU compartment

σ : ICU discharge rate



Extension: SEIPHR model

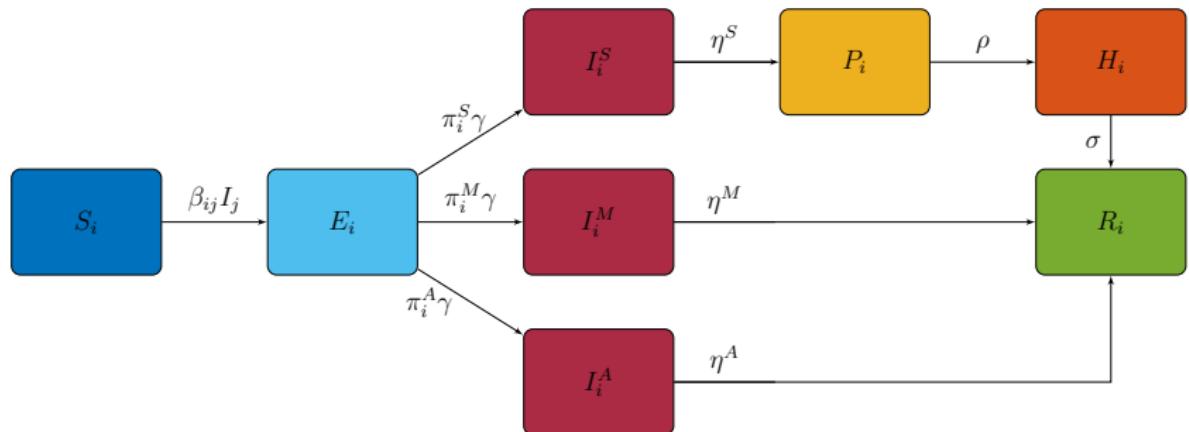
Undetected recovery



Extension: SEIPHR model

Distinguish age groups

$i \in \{1, 2, \dots, n_g\}$ (in particular β_{ij} and π_i)

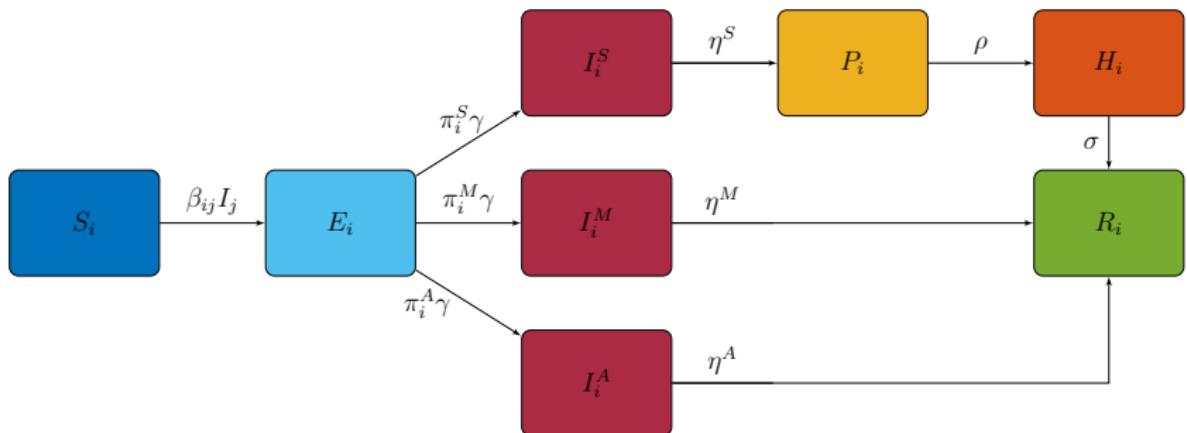


Extension: SEIPHR model

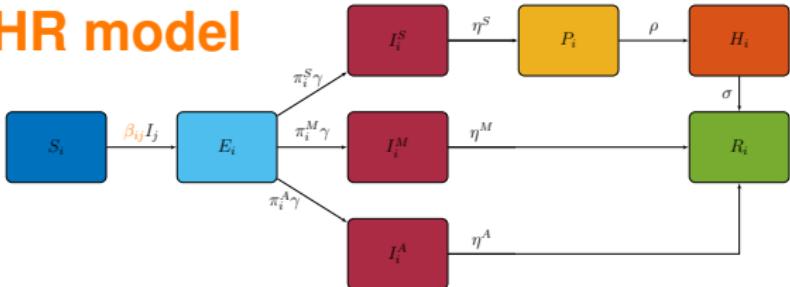
Distinguish age groups

$i \in \{1, 2, \dots, n_g\}$ (in particular β_{ij} and π_i)

$n_g = 3$: children, adults (most contacts), elderly (high-risk)



Extension: SEIPHR model



System dynamics

$$\frac{d}{dt} S_i(t) = - \sum_{j=1}^{n_g} \beta_{ij} S_i(t) I_j(t) \quad (I_i = I_i^S + I_i^M + I_i^A)$$

$$\frac{d}{dt} E_i(t) = \sum_{j=1}^{n_g} \beta_{ij} S_i(t) I_j(t) - \gamma E_i(t)$$

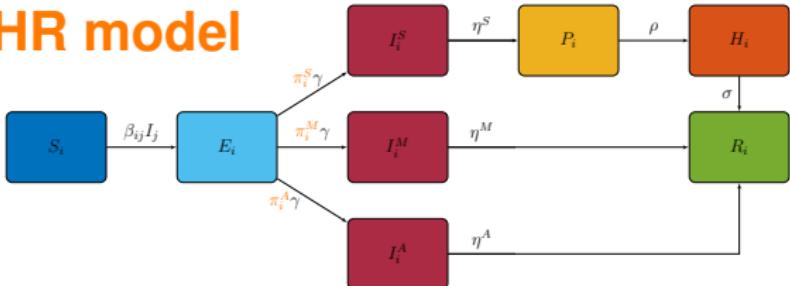
$$\frac{d}{dt} I_i^\#(t) = \pi_i^\# \gamma E_i(t) - \eta^\# I_i^\#(t) \quad \# \in \{S, M, A\}$$

$$\frac{d}{dt} P_i(t) = \eta^S I_i^S(t) - \rho P_i(t)$$

$$\frac{d}{dt} H_i(t) = \rho P_i(t) - \sigma H_i(t)$$

$$\frac{d}{dt} R_i(t) = \eta^M I_i^M(t) + \eta^A I_i^A(t)$$

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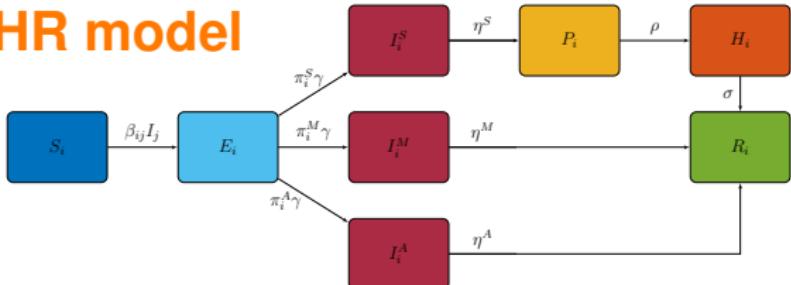
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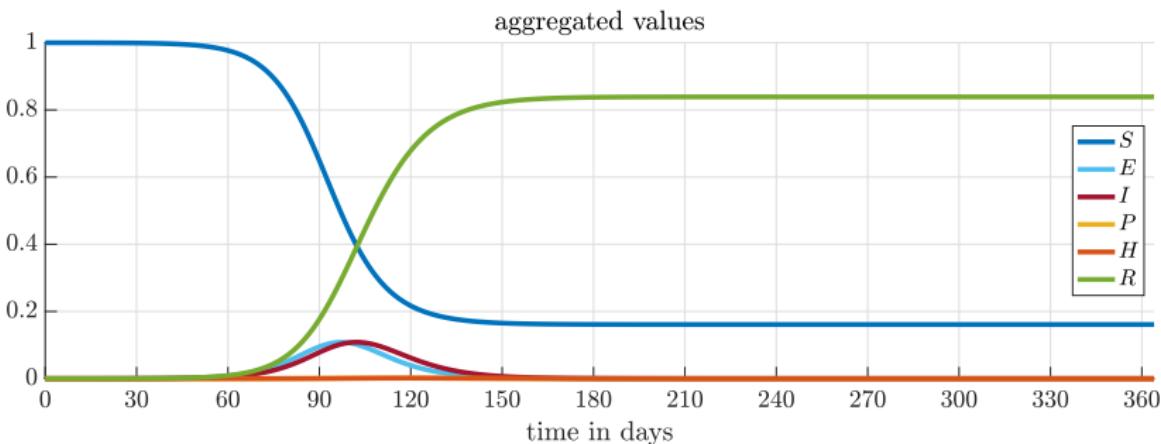
$$\frac{d}{dt} H_i(t) = \rho P_i(t) - \sigma H_i(t)$$

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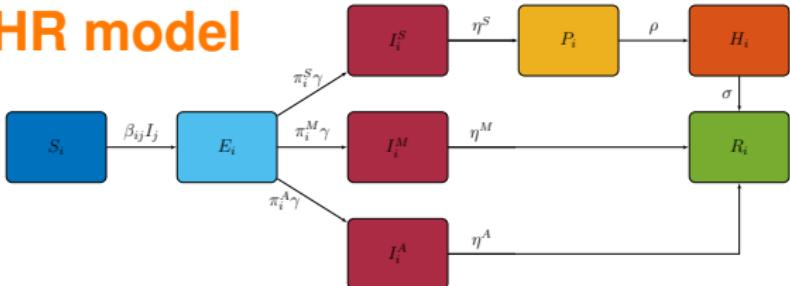
Extension: SEIPHR model



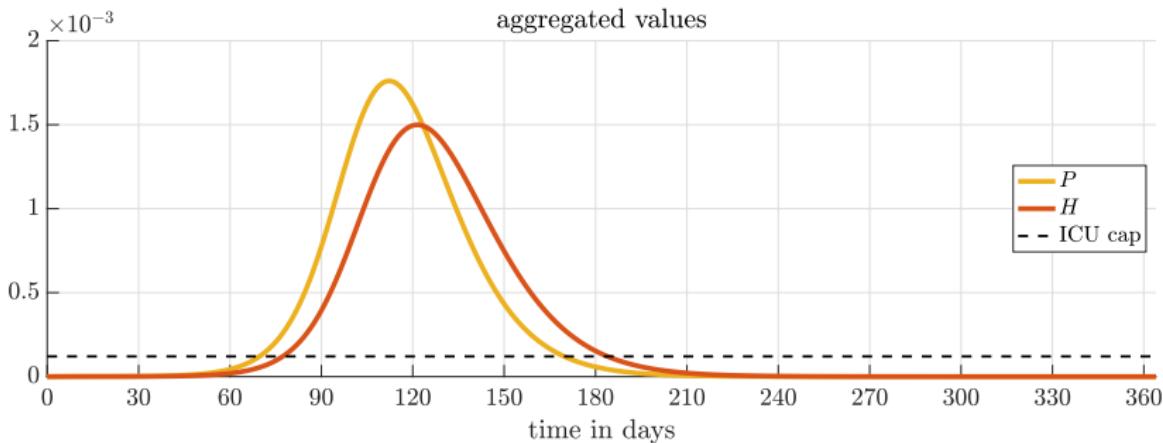
Simulation results



Extension: SEIPHR model

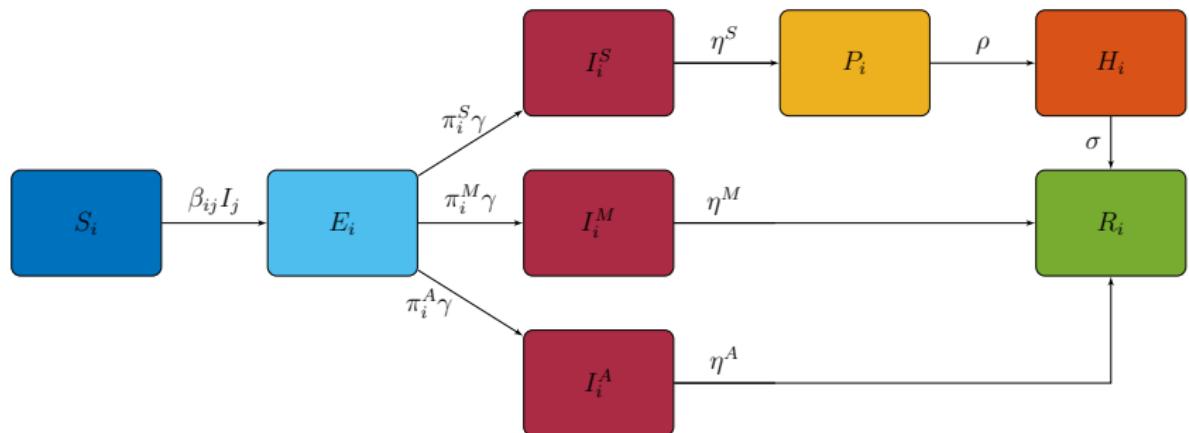


Simulation results



Social distancing

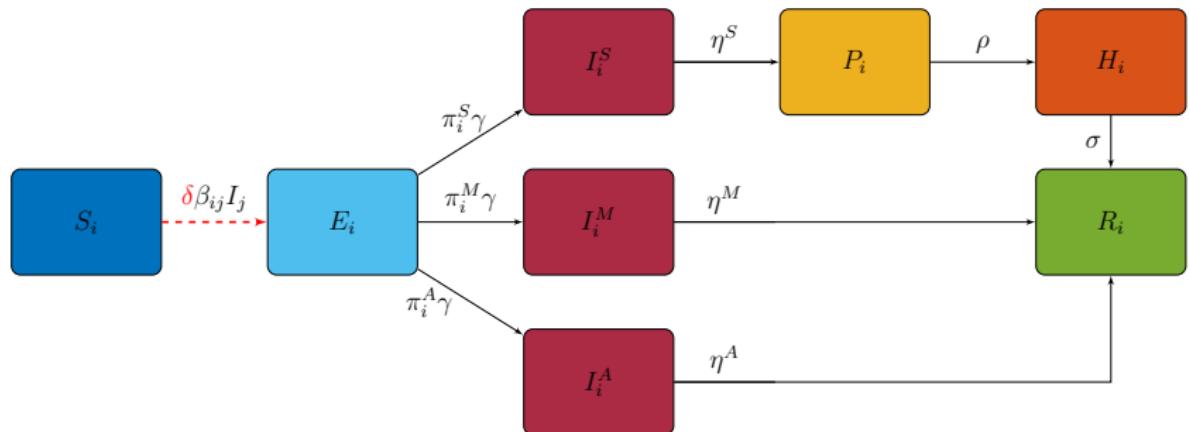
SEIPHR model



Social distancing

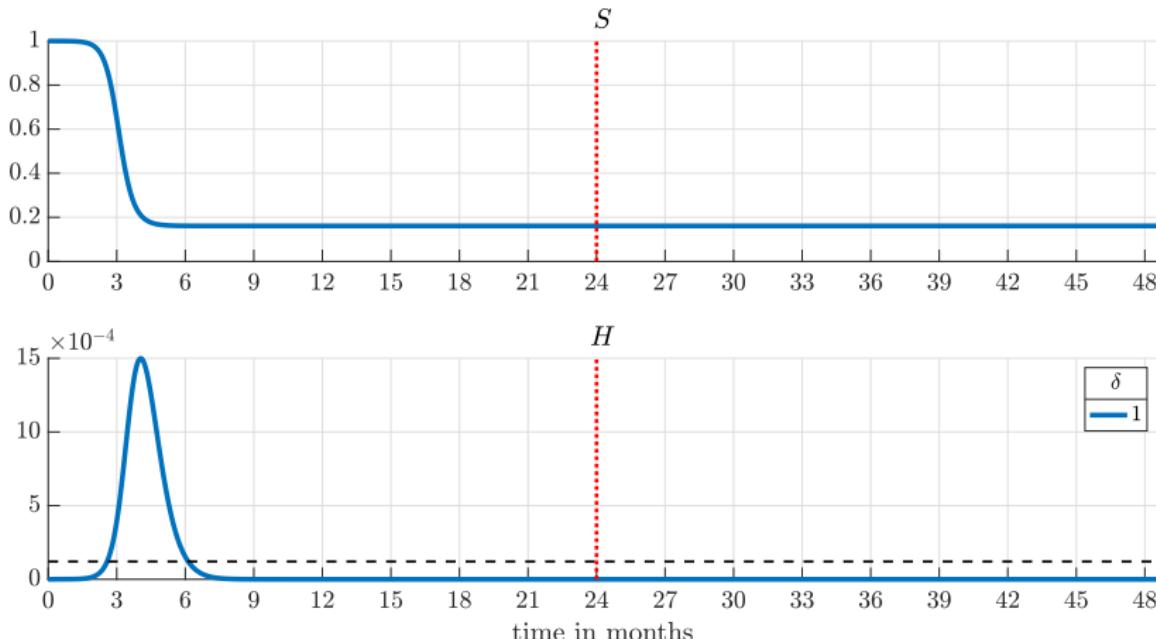
Contact restrictions

control input $\delta : [0, \infty) \rightarrow [0, 1]$



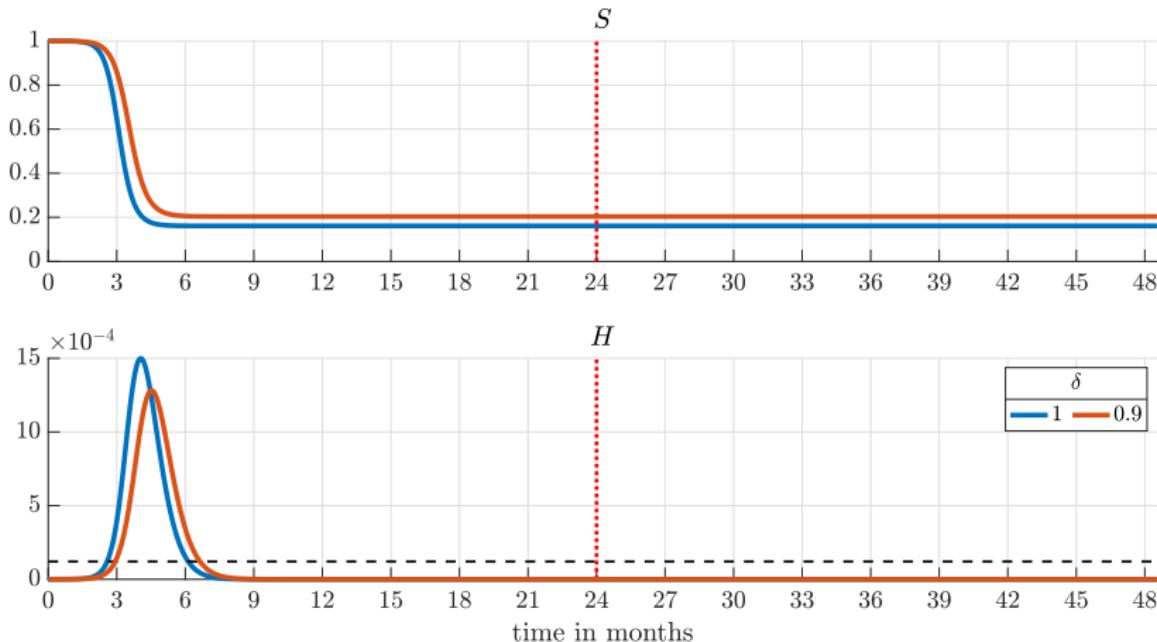
Impact of constant restrictions

Simulation results over 4 years with lift of restrictions after 2 years



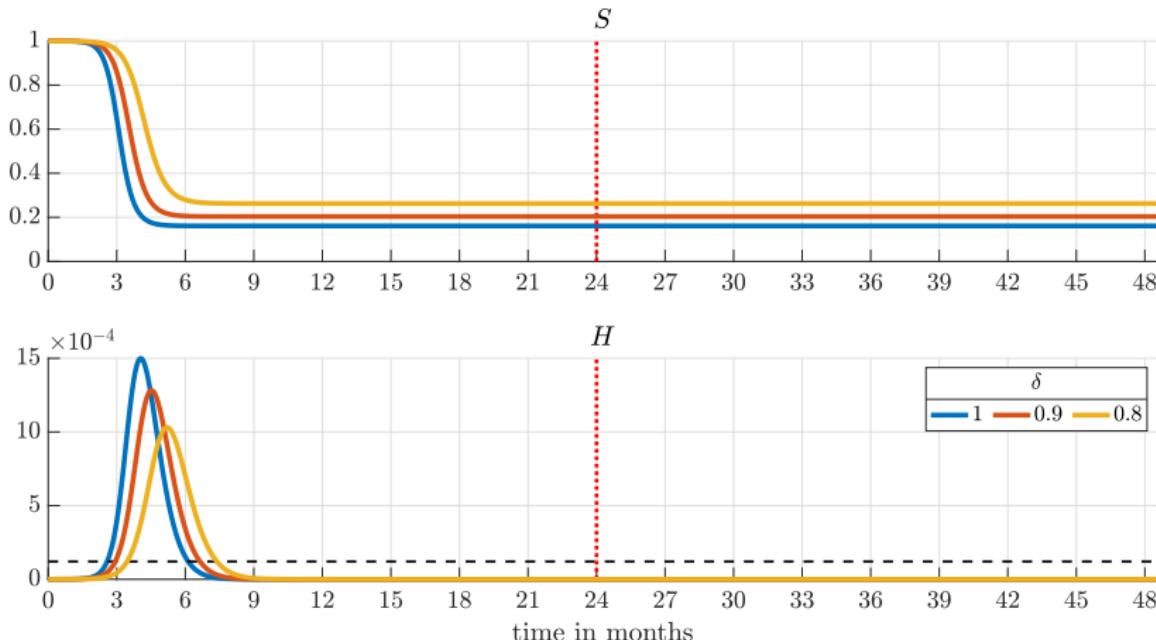
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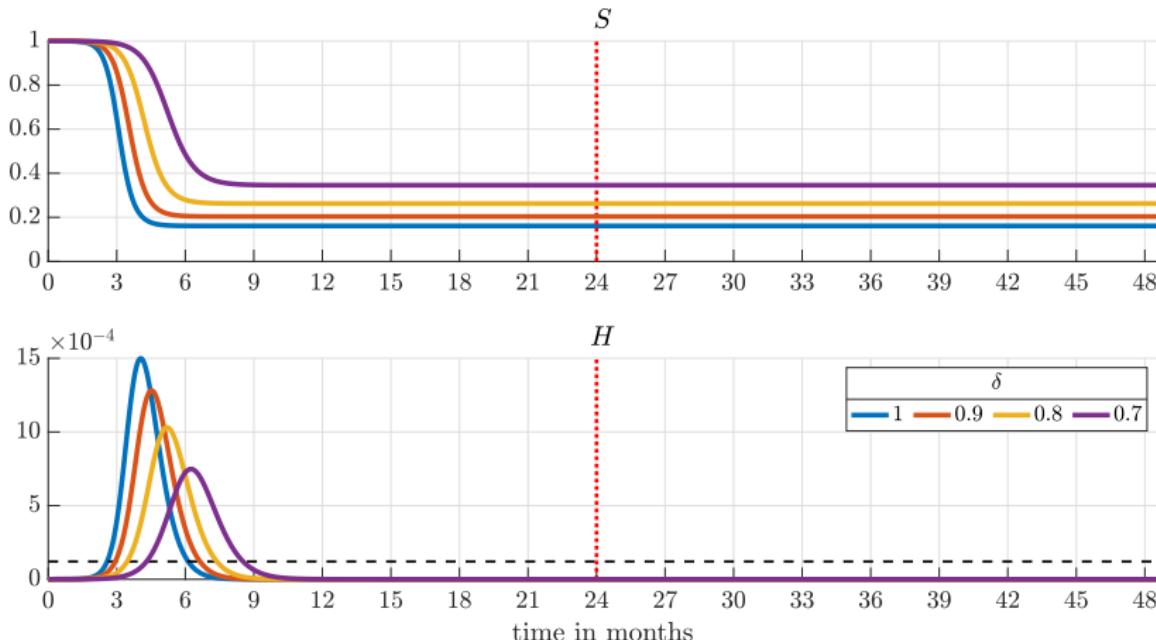
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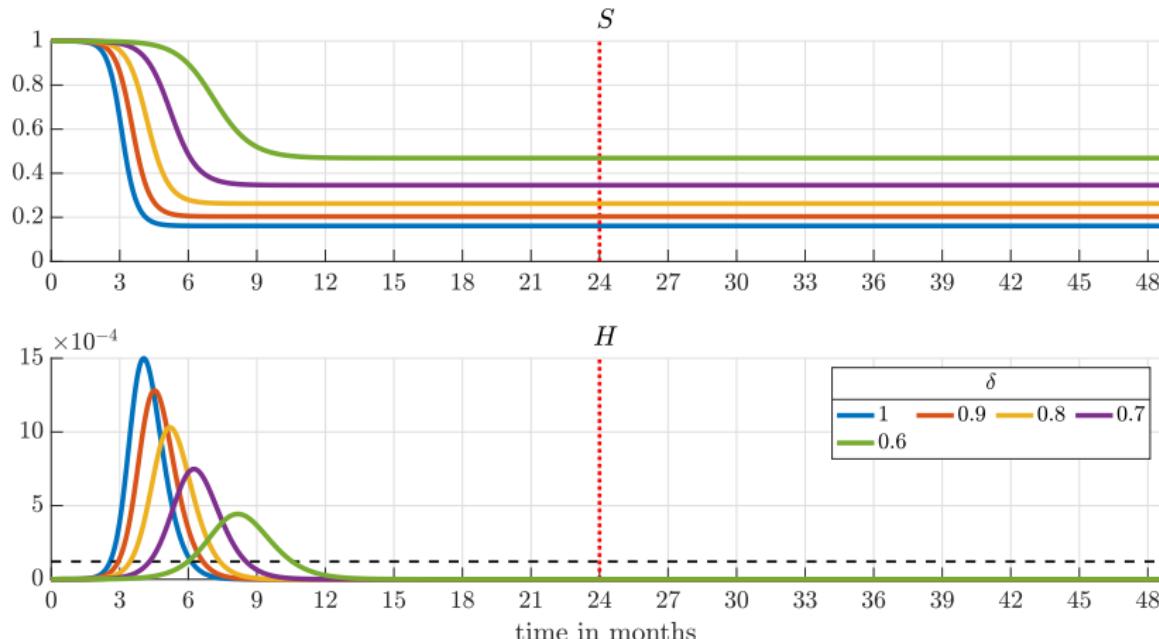
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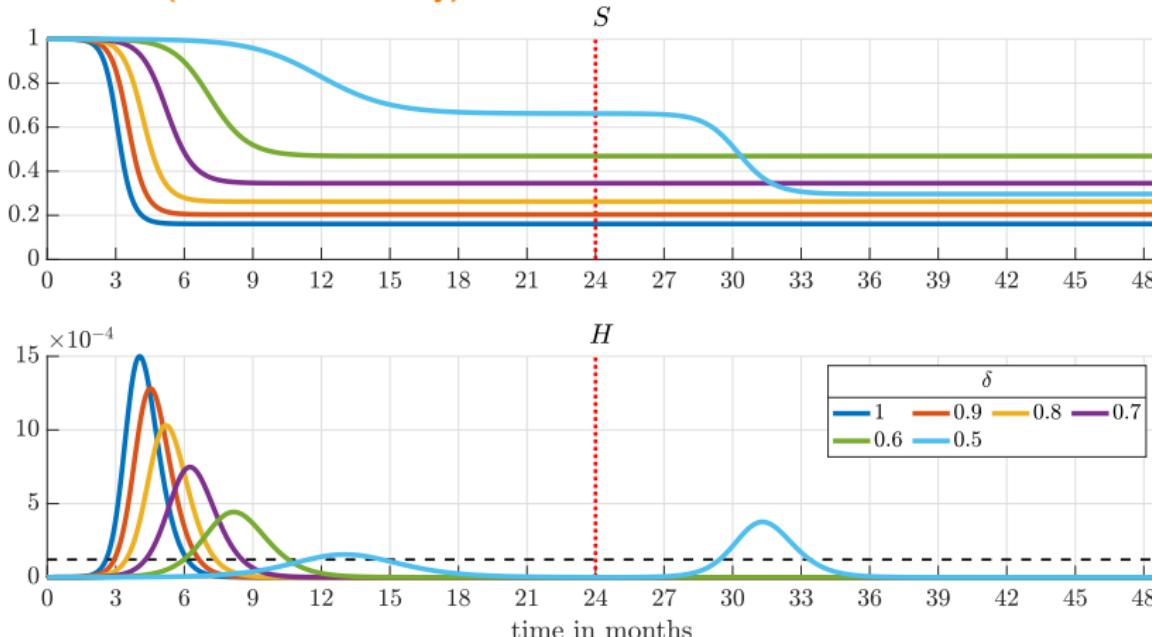
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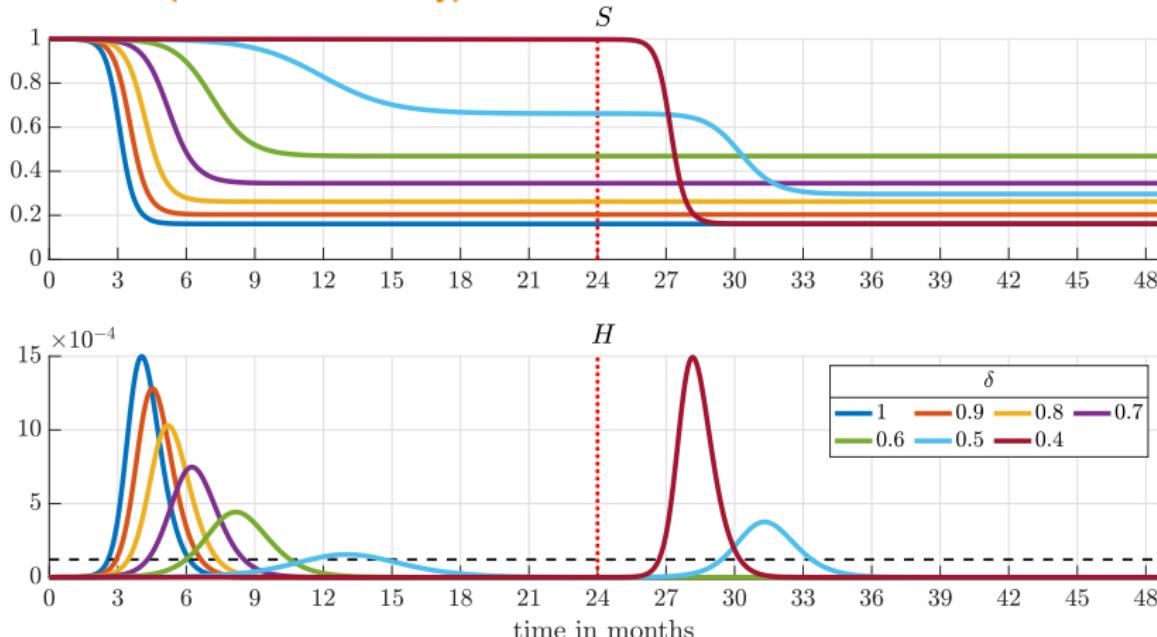
2nd wave (no herd immunity)



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Optimal social distancing

Goal: maintain hard ICU cap with as few contact restrictions as possible

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Optimal control problem

$$\min_{\delta} \quad \int_0^{t_f} (1 - \delta(t))^2 \, dt$$

subject to $\dot{x}(t) = f(x(t), \delta(t)), \quad x(0) = x^0$

$$\sum_{i=1}^{n_g} H_i(t) \leq H^{\max} \quad \forall t \geq 0$$

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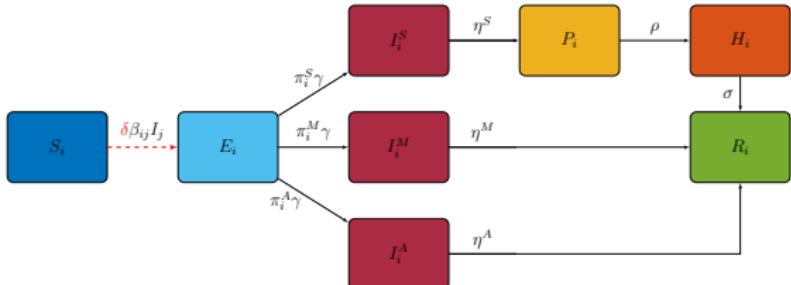
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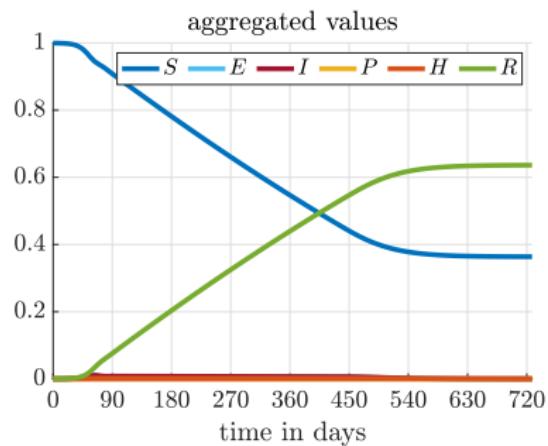
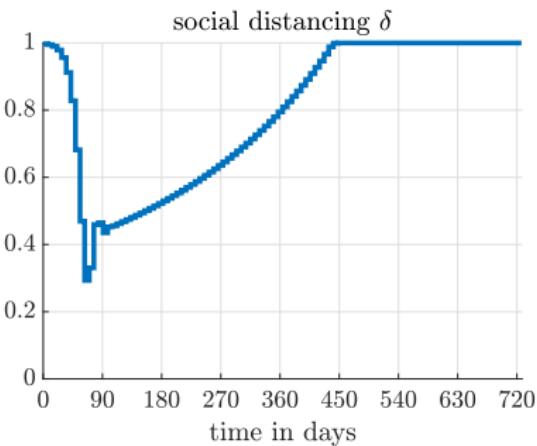
$$\delta(t) = \delta(k\Delta t), \quad t \in [k\Delta t, (k+1)\Delta t), \quad k = 0, 1, \dots$$

with $\Delta t = 1$ week

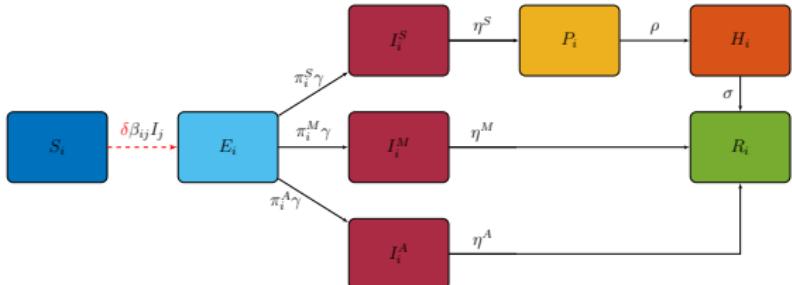
Results



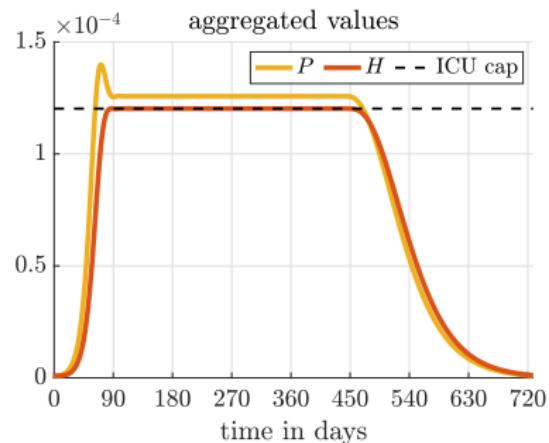
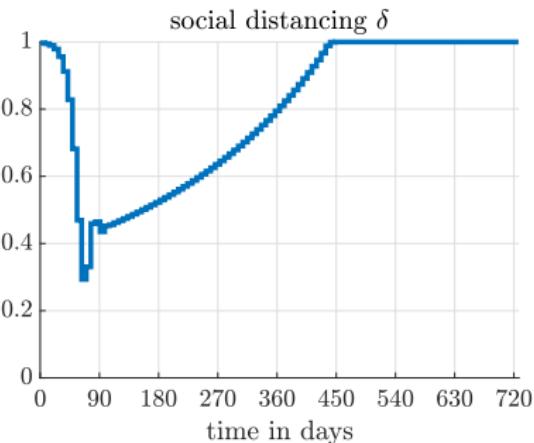
Simulation results



Results



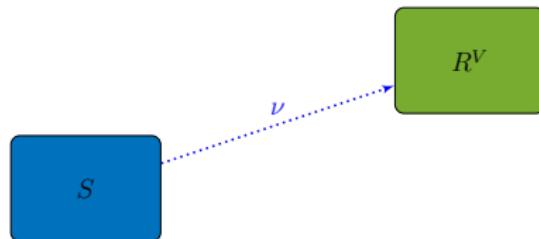
Simulation results



Vaccination

Vaccination of susceptible people

vaccination rate $\nu : [0, \infty) \rightarrow \mathbb{R}_{\geq 0}$

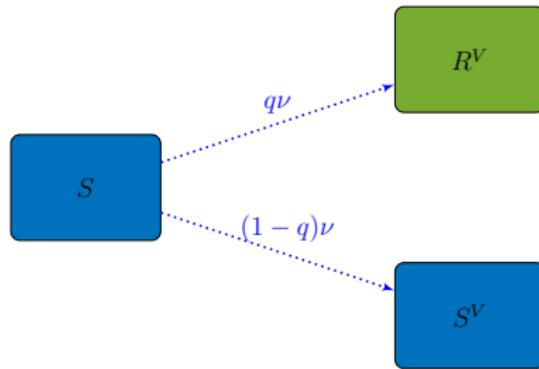


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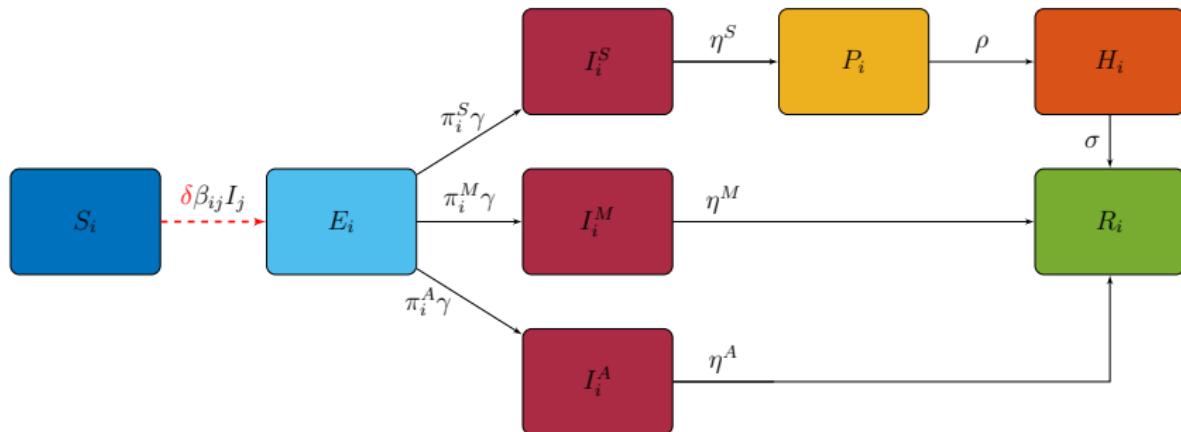
vaccination rate $\nu : [0, \infty) \rightarrow \mathbb{R}_{\geq 0}$

success rate $q \in [0, 1]$



Vaccination

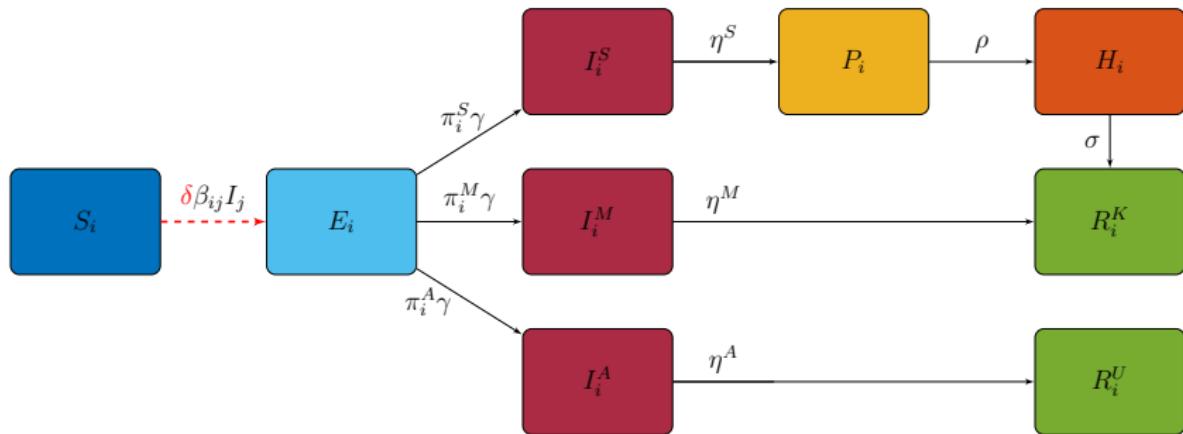
SEIPHR model



Vaccination

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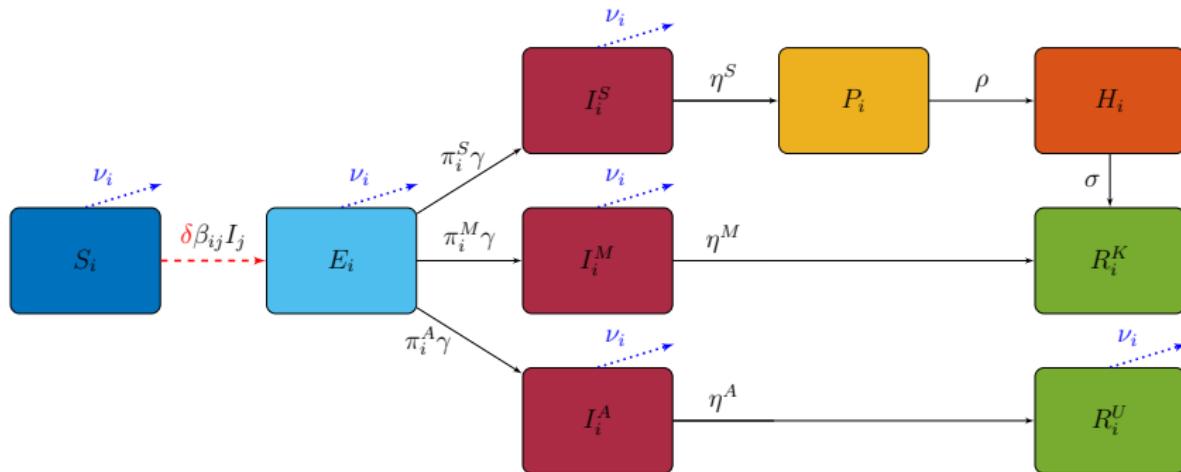
separate undetected recovery



Vaccination

Non-vaccinated part of population

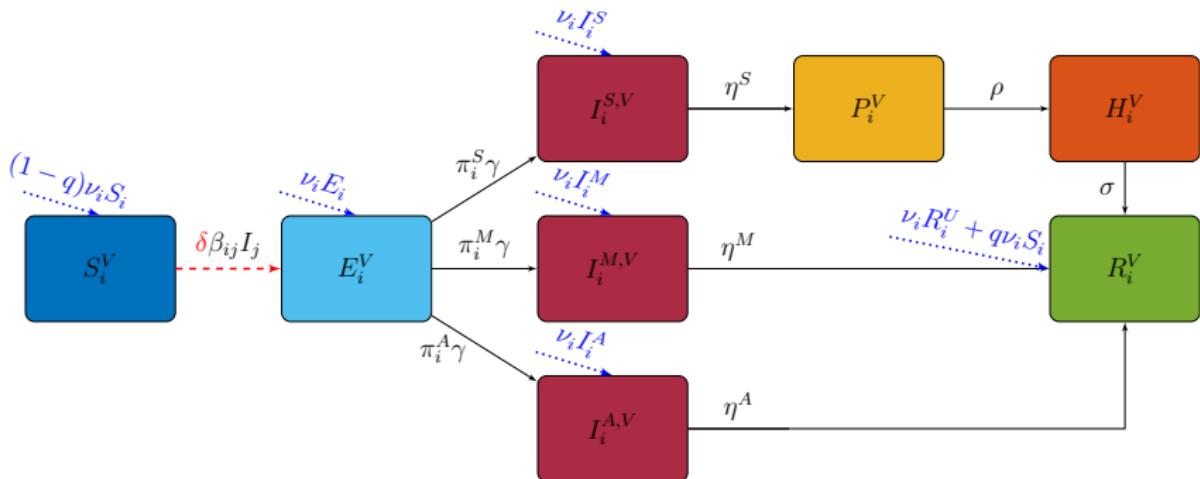
vaccination rate $\nu_i : [0, \infty) \rightarrow \mathbb{R}_{\geq 0}$



Vaccination

Vaccinated part of population

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Coordination of social distancing & vaccination

Goal: reduce social distancing

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$$\min_{\delta} \quad \int_0^{t_f} (1 - \delta(t))^2 \, dt$$

$$\text{subject to } \dot{x}(t) = f(x(t), \delta(t)), \quad x(0) = x^0$$

$$\sum_{i=1}^{n_g} H_i(t) \leq H^{\max} \quad \forall t \geq 0$$

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Optimal control problem

$$\min_{\delta, \nu} \quad \int_0^{t_f} (1 - \delta(t))^2 dt + \kappa \|\nu\|_2^2$$

subject to $\dot{x}(t) = f(x(t), \delta(t), \nu(t)), \quad x(0) = x^0$

$$\sum_{i=1}^{n_g} H_i(t) + H_i^V(t) \leq H^{\max} \quad \forall t \geq 0$$

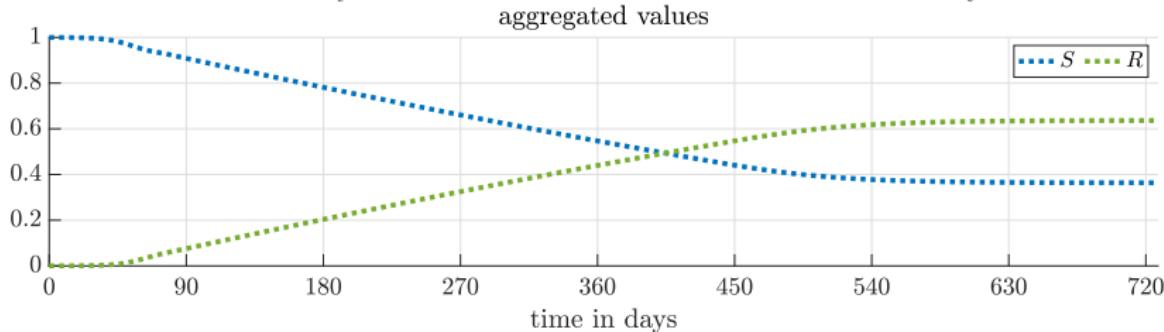
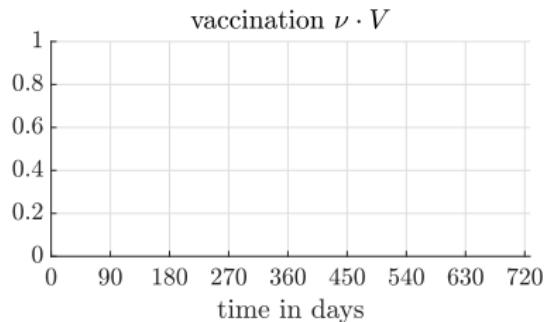
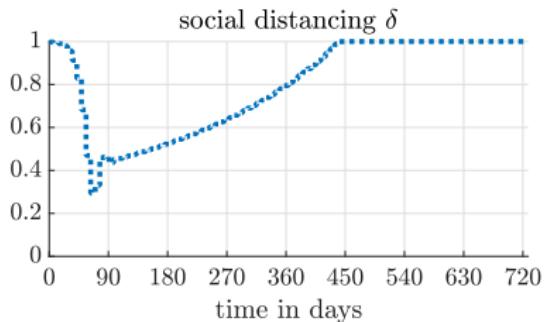
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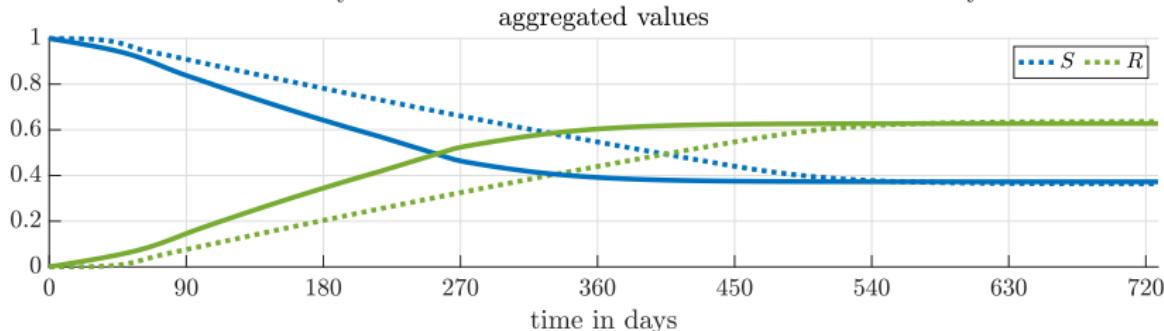
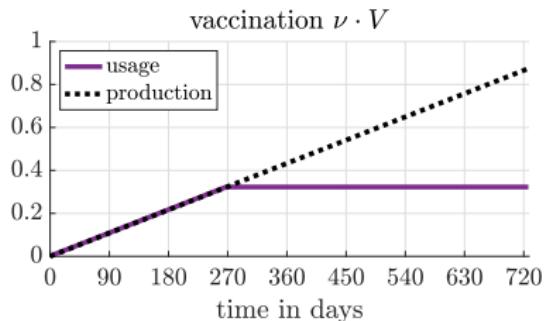
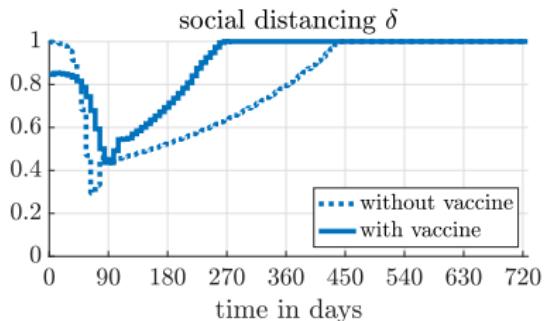
$$\int_0^t \sum_{i=1}^{n_g} \nu_i(s) V_i(s) ds \leq V^{\max} \cdot t \quad \forall t \geq 0$$

$$\nu(t) = \nu(k\Delta t), \quad t \in [k\Delta t, (k+1)\Delta t), \quad k = 0, 1, \dots$$

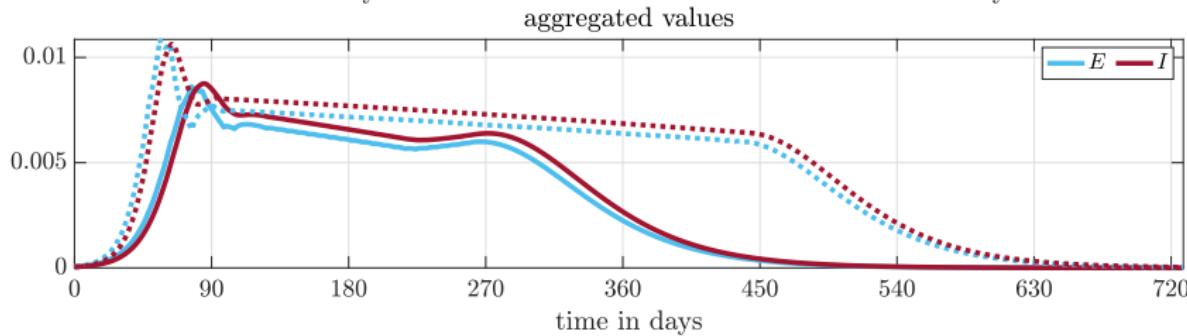
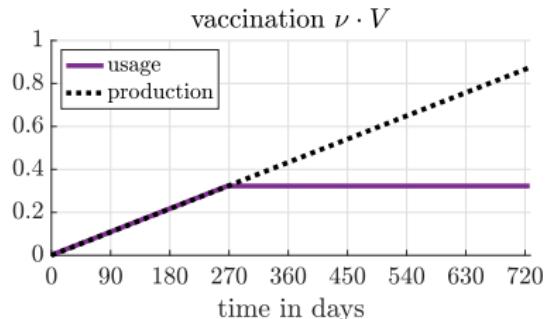
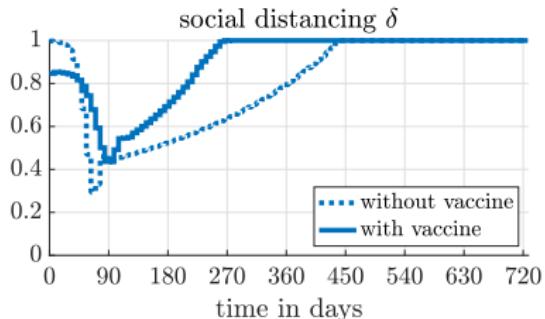
Results



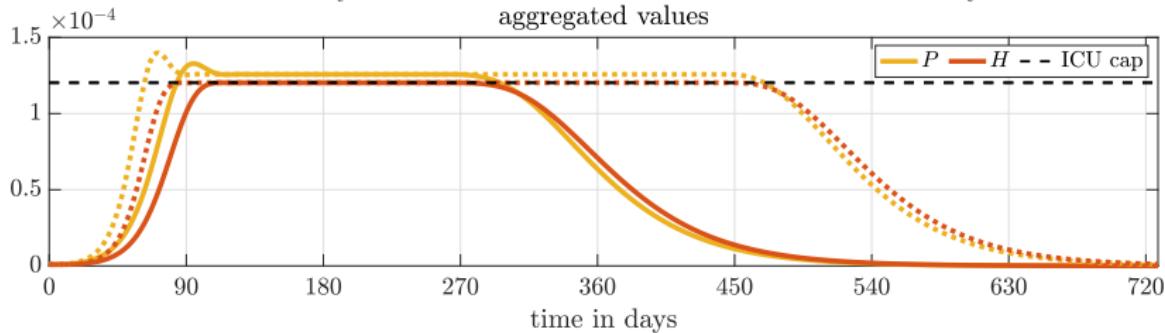
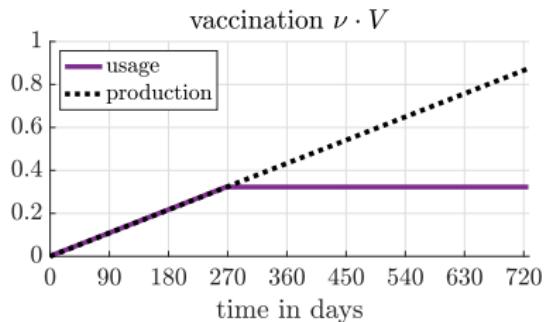
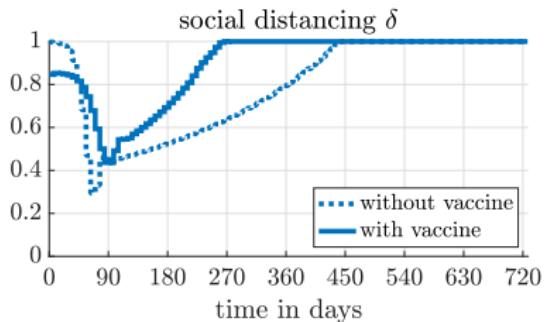
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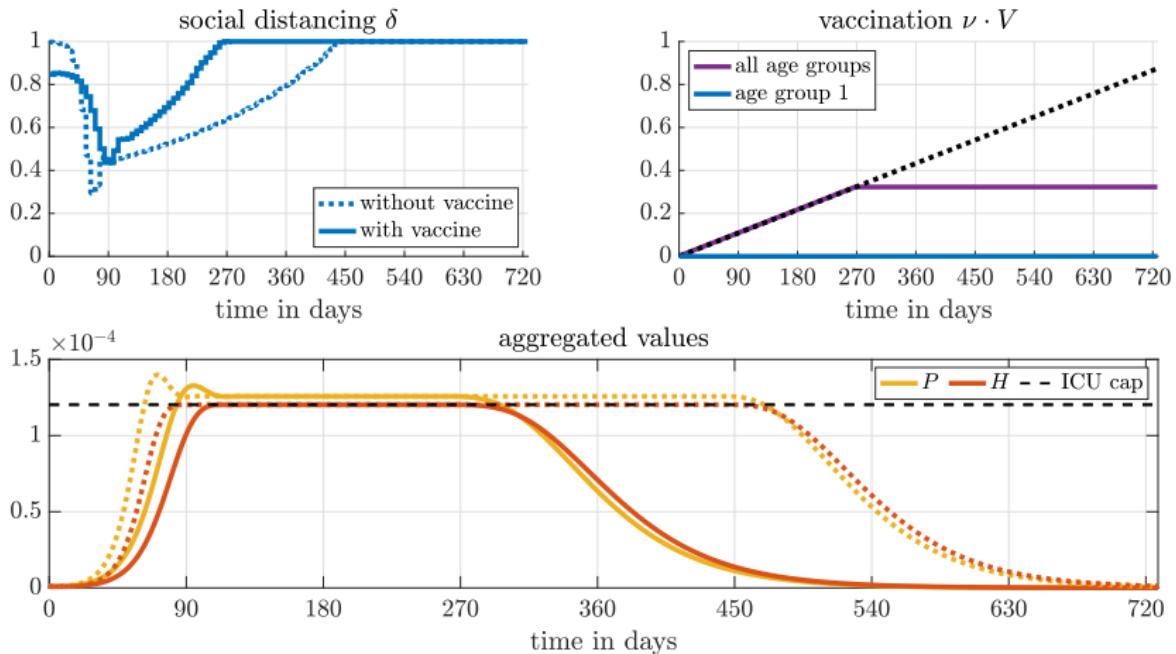
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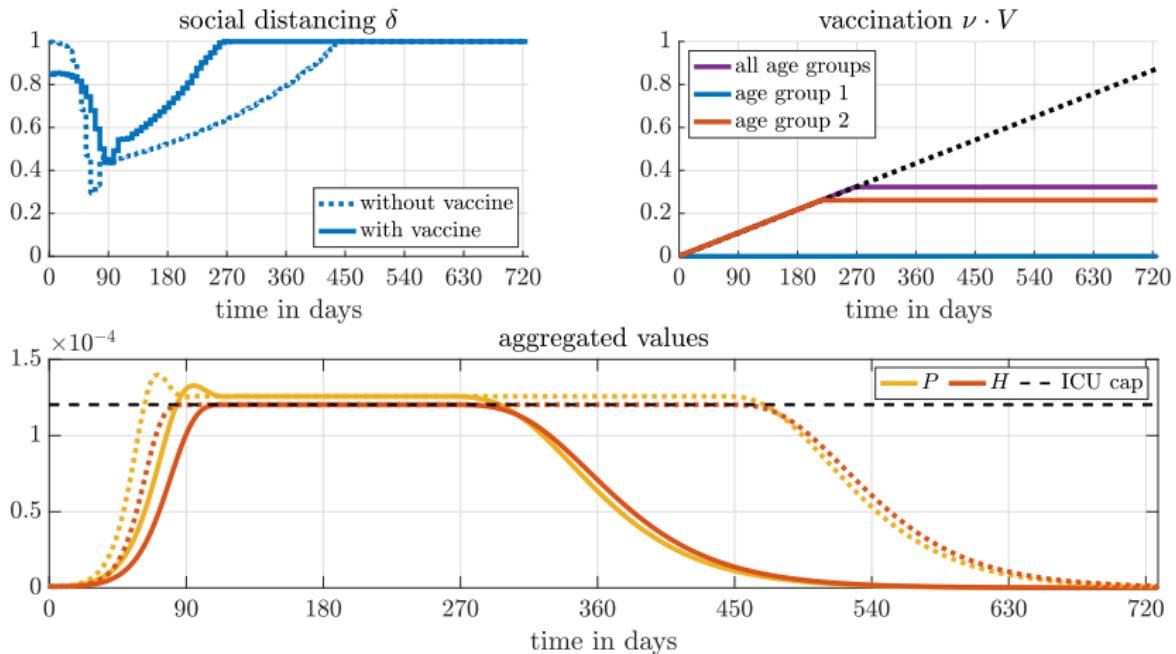
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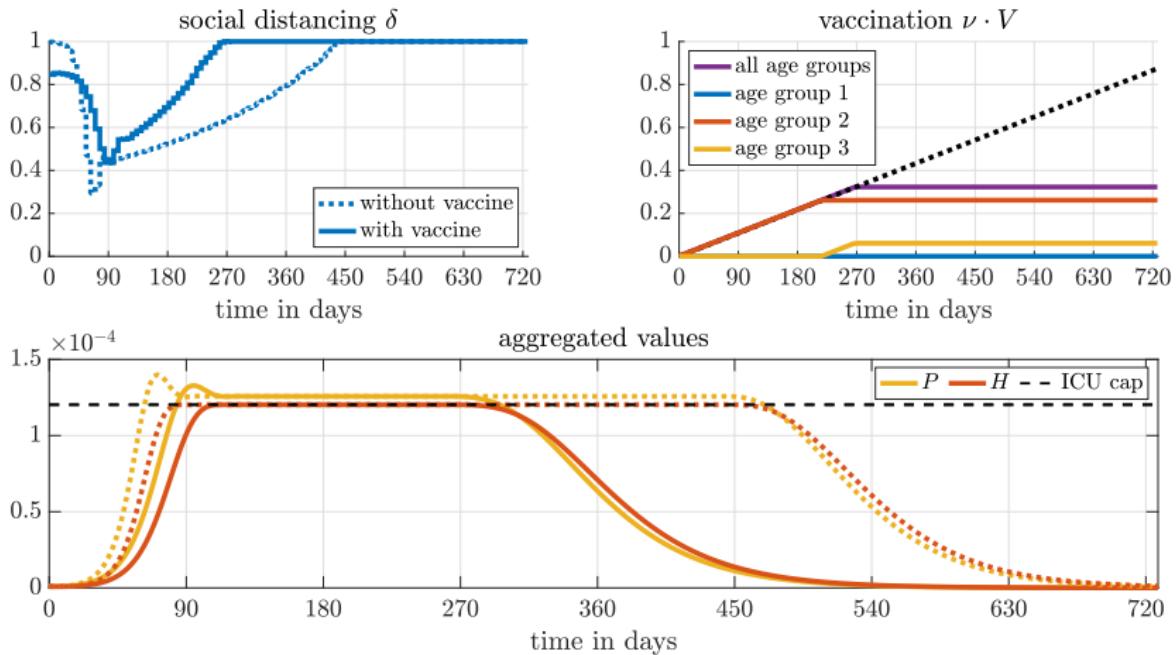
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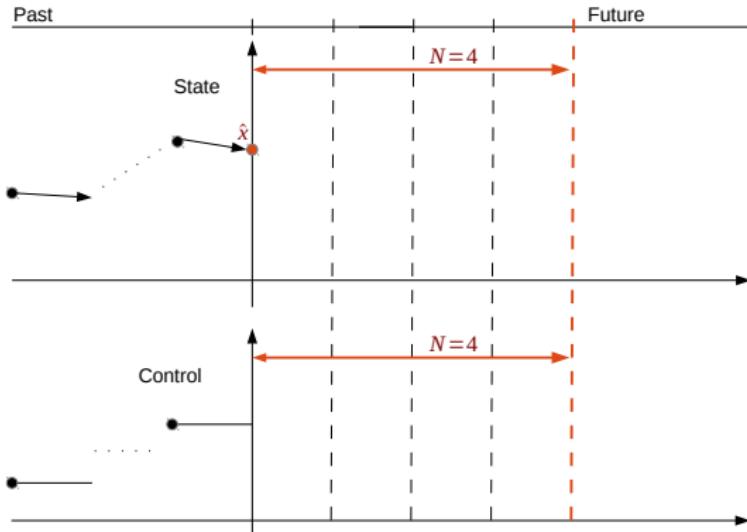
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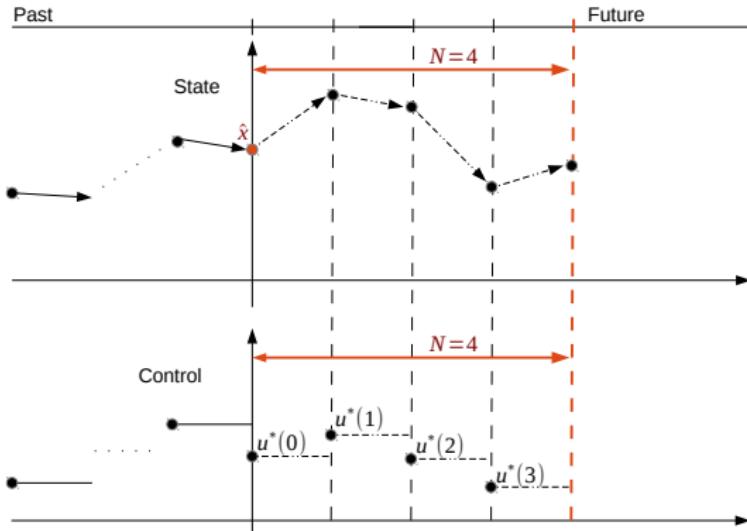


Model Predictive Control



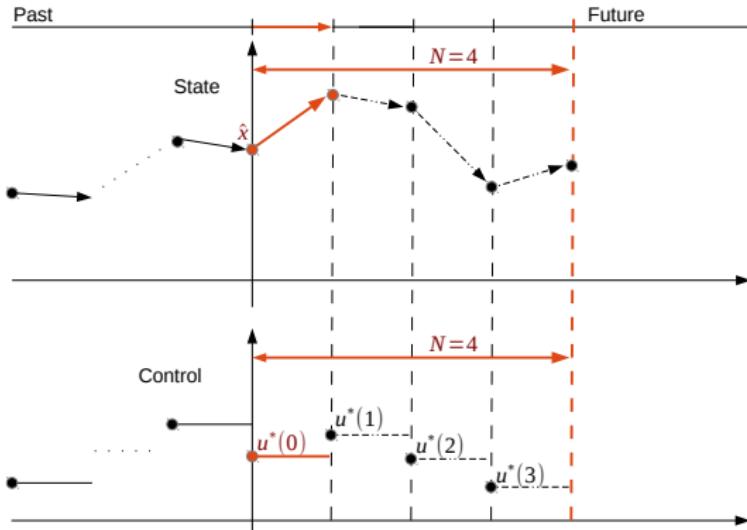
1. Measure/Estimate current state and update parameters.

Model Predictive Control



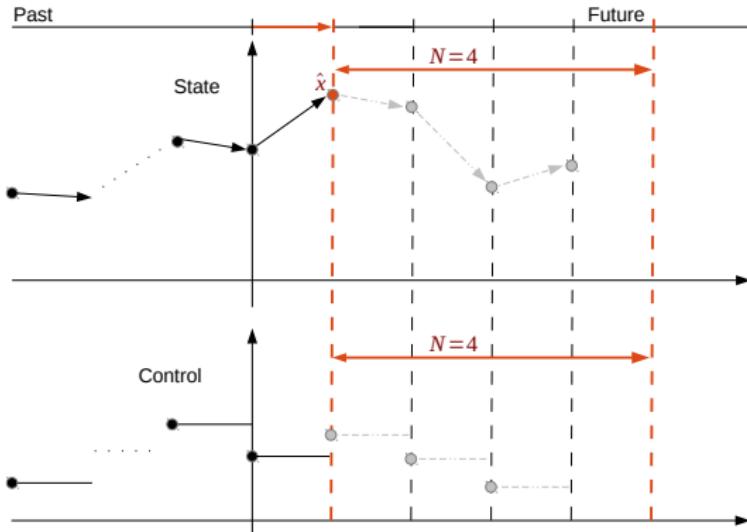
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2. Solve optimal control problem *on small time window*.

Model Predictive Control



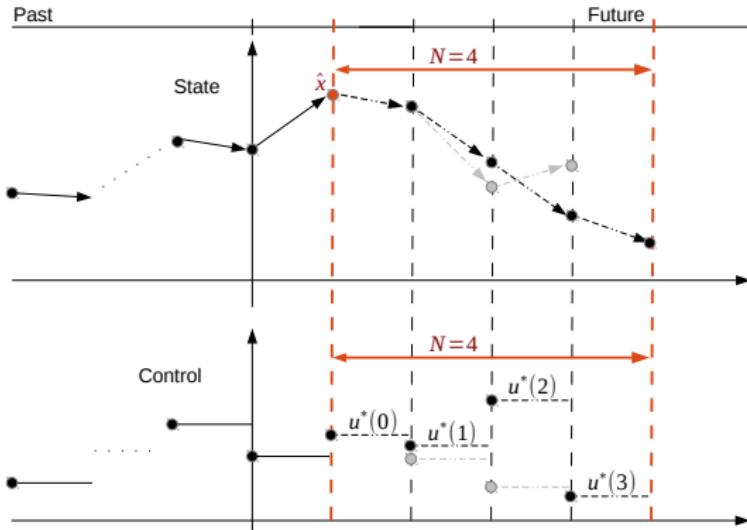
1. Measure/Estimate current state and update parameters.
2. Solve optimal control problem *on small time window*.
3. Implement first control instance.

Model Predictive Control



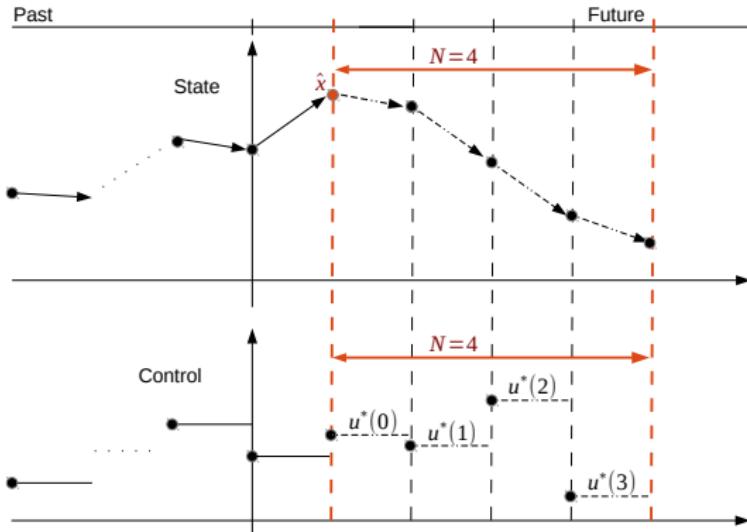
1. Shift time step, measure/estimate current state, and update parameters.
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3. Implement first control instance.

Model Predictive Control



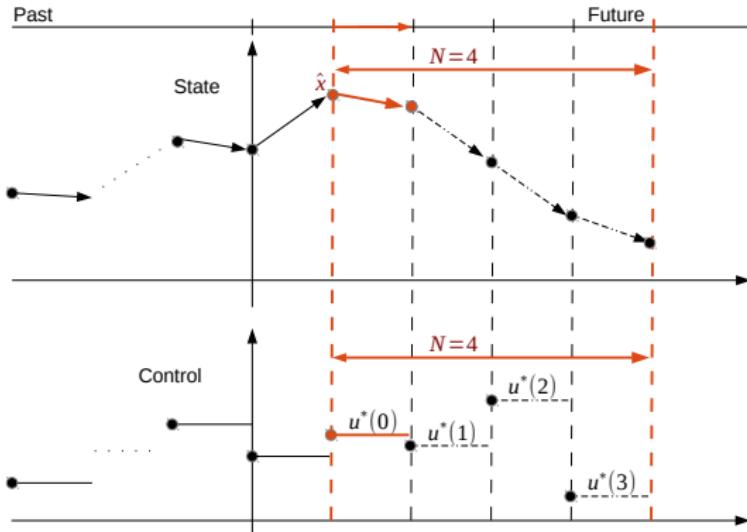
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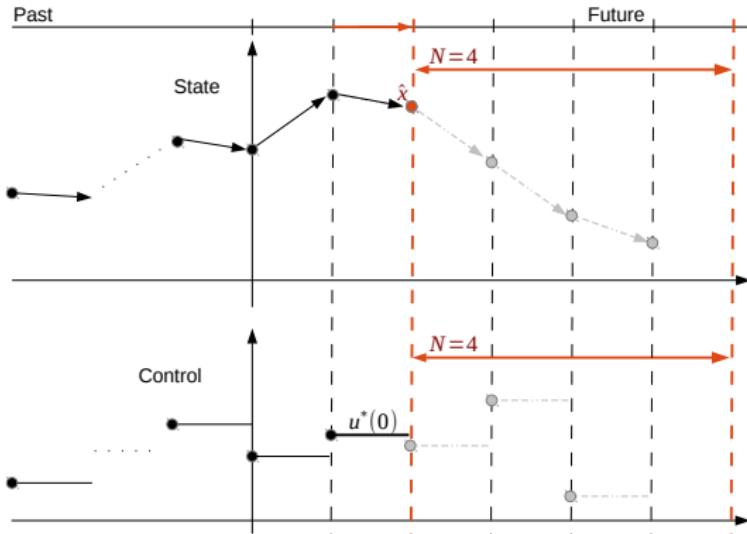
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Model Predictive Control



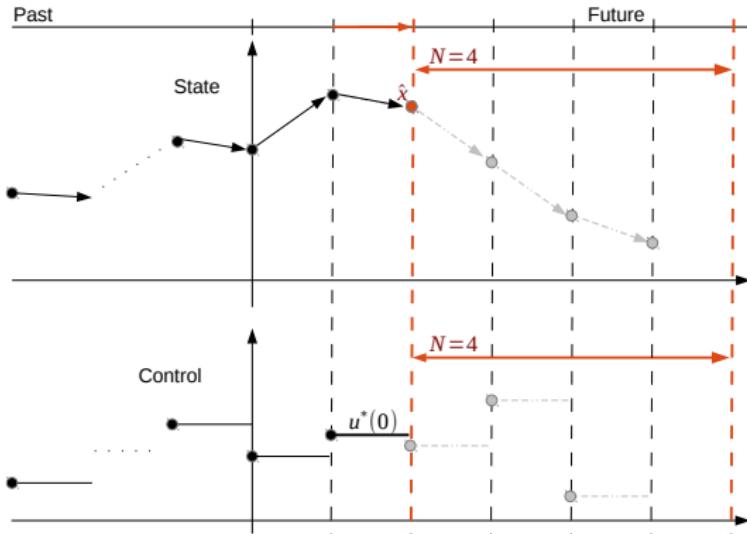
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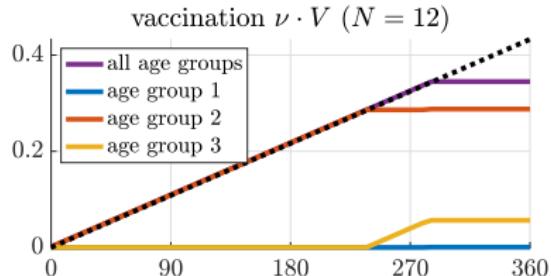
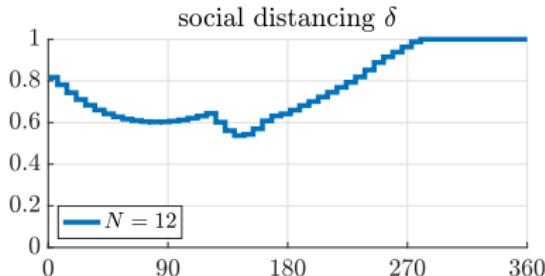
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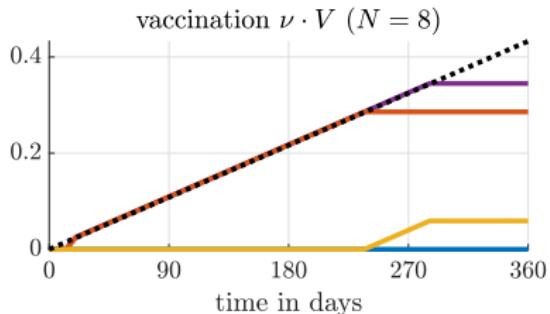
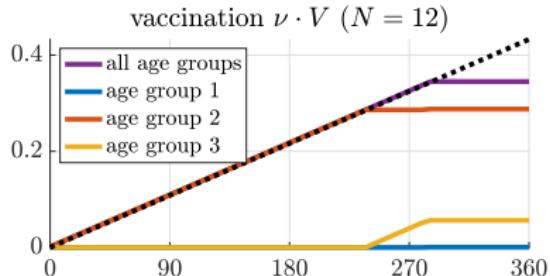
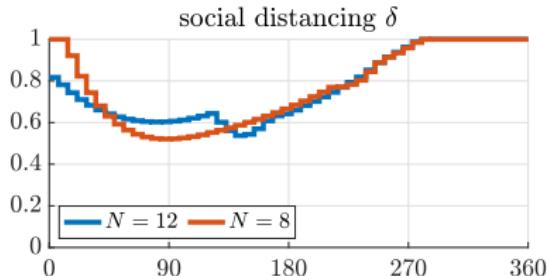
Closed-loop simulations

Impact of prediction horizon N (in weeks) on controls



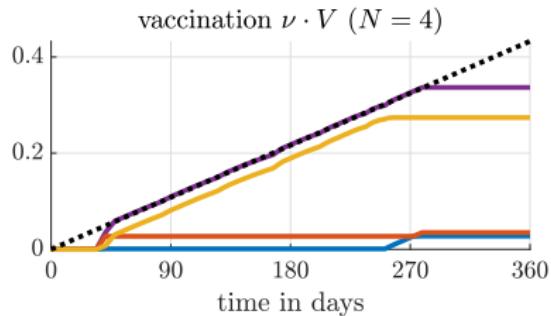
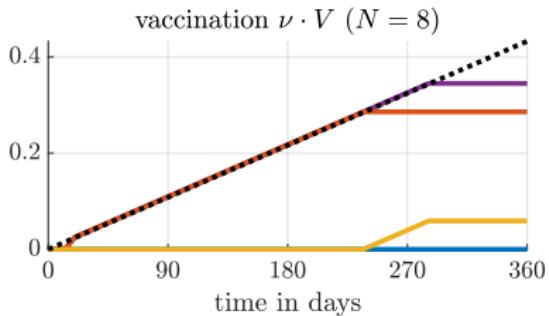
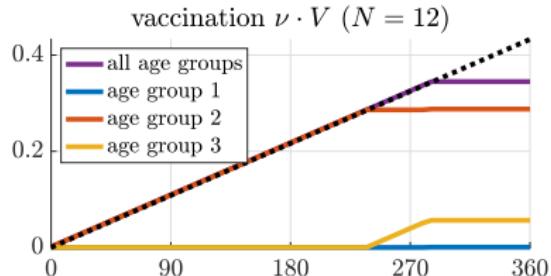
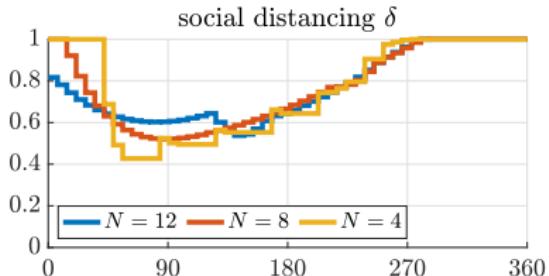
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Conclusions & outlook

Recap

- extension of the SEIR model to account for age-dependent symptom severity and countermeasures

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- minimizing fatalities

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- re-infections
- uncertainty quantification of parameters
- breakpoints/bifurcation

References

The presented results are based on

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Submitted. (Preprint available at <https://arxiv.org/abs/2011.01282>)

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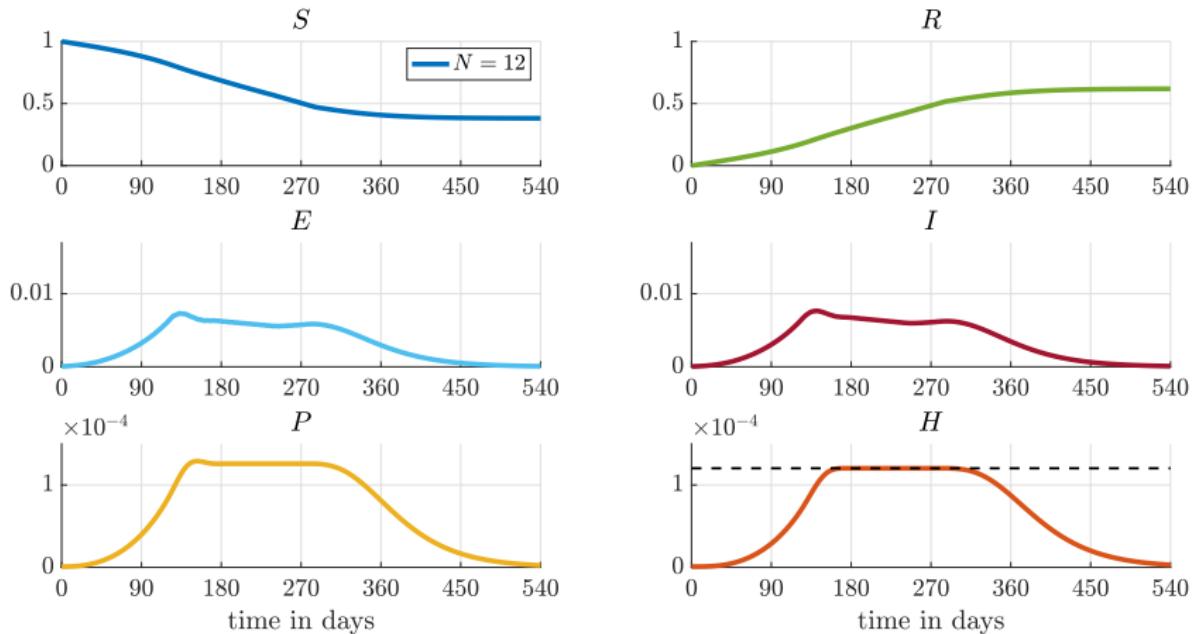
Thank you for your attention!

Parameters

Description	Symbol	Value		
Number of age groups	n_g	3		
Regularization parameter	κ	10^{-3}		
Removal rate (severe)	η^S	0.2500		
Removal rate (mild)	η^M	0.2500		
Removal rate (asymptomatic)	η^A	0.1667		
Rate of becoming infectious	γ	0.1923		
ICU admittance rate	ρ	0.0910		
ICU discharge rate	σ	0.0952		
Vaccine production limit	V^{\max}	100,000		
Success rate	q	0.9		
Age-differentiated parameters				
Age group	i	1	2	3
Age range (in years)	–	< 15	15 – 59	> 60
Relative age group size	N_i	0.1370	0.5776	0.2854
Probability of severe symptoms	π_i^S	0.0053	0.0031	0.0302
Probability of mild symptoms	π_i^M	0.1211	0.2201	0.2512
Probability of no symptoms	π_i^A	0.8737	0.7768	0.7186
Transmission rate (age group 1)	β_{1i}	0.4612		
Transmission rate (age group 2)	β_{2i}	0.4819	0.6304	
Transmission rate (age group 3)	β_{3i}	0.1243	0.2944	0.1802

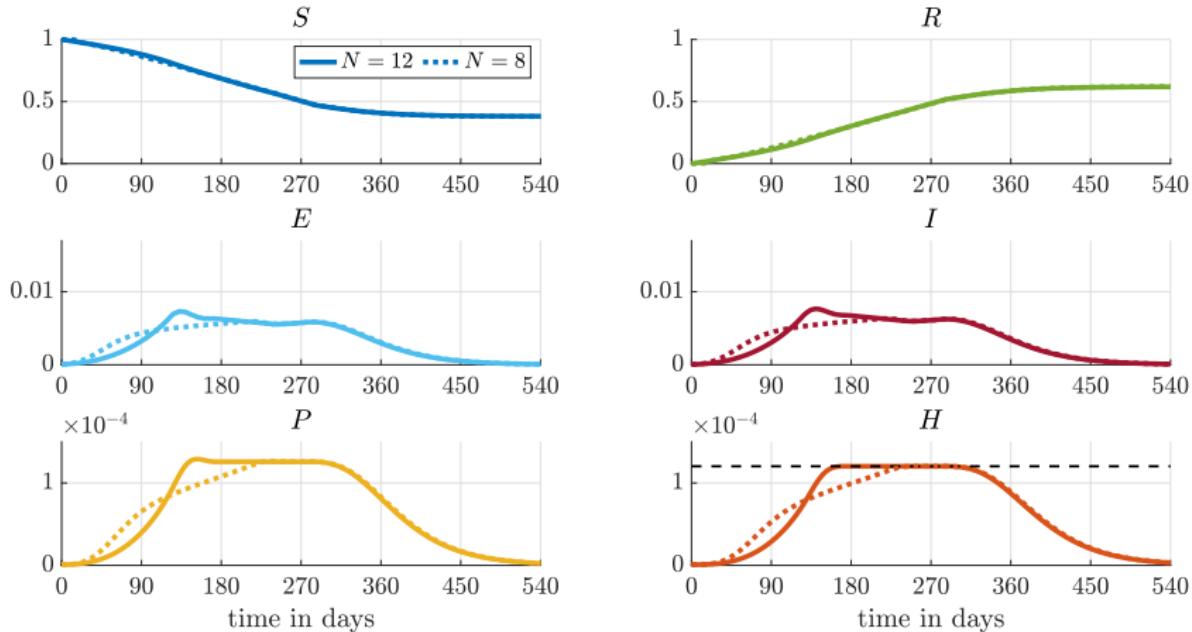
Closed-loop simulations II

Impact of prediction horizon N (in weeks) on states



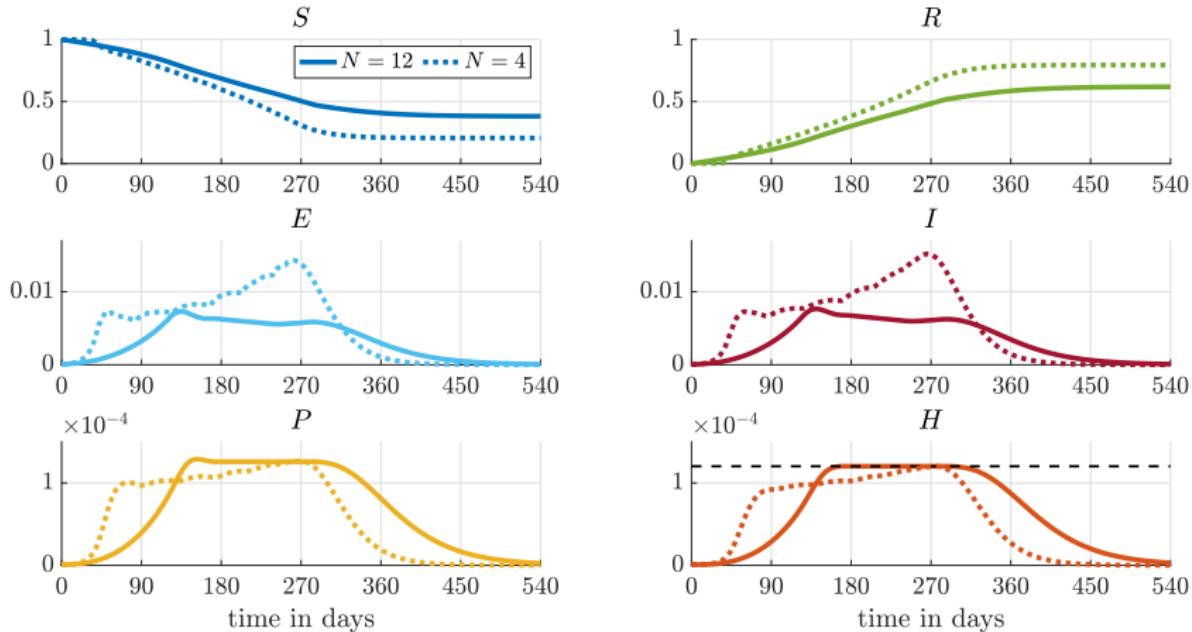
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Minimizing fatalities

Assumption: total number of fatalities \propto total number of people treated on ICU

For given $\delta^c \in [0, 1]$ solve

$$\min_{\nu} H^C(t_f) + \kappa \|\nu\|_2^2$$

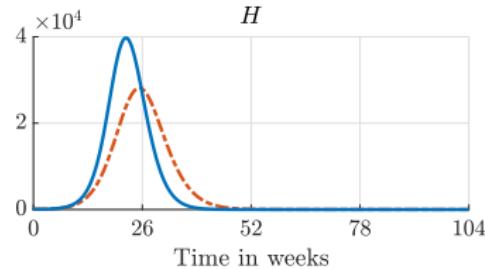
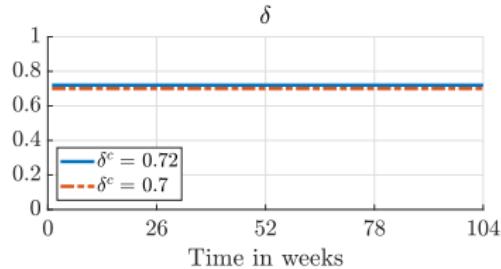
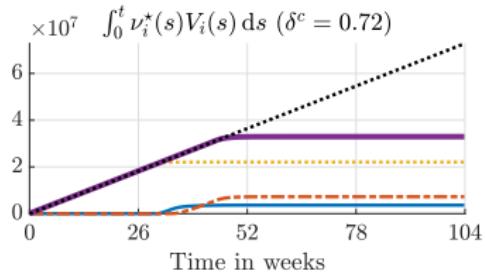
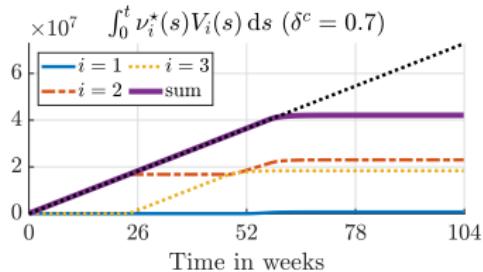
subject to $\dot{H}^C(t) = \sum_{i=1}^{n_g} \sigma(H_i(t) + H_i^V(t)), \quad H^C(0) = 0$

$$\dot{x}(t) = f(x(t), \delta^c, \nu(t)), \quad x(0) = x^0$$

$$\int_0^t \sum_{i=1}^{n_g} \nu_i(s) V_i(s) \, ds \leq V^{\max} \cdot t \quad \forall t \geq 0$$

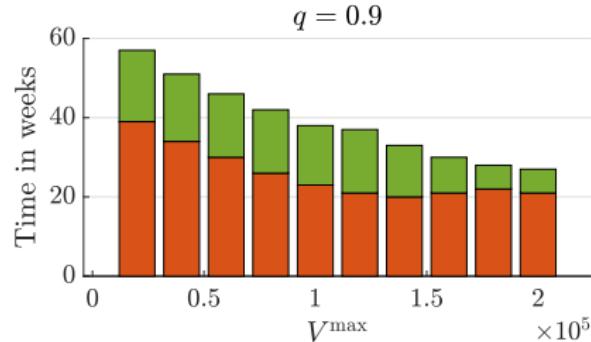
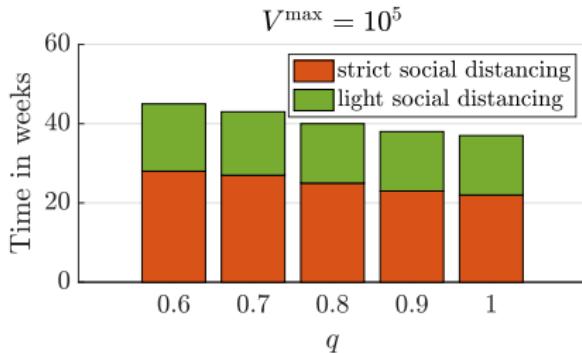
$$\nu(t) = \nu(k\Delta t), \quad t \in [k\Delta t, (k+1)\Delta t], \quad k = 0, \dots, N-1$$

Minimizing fatalities



- contact restrictions sufficiently strict \rightsquigarrow vaccinate group with most contacts first
- otherwise \rightsquigarrow vaccinate high-risk group first ("damage control")

Impact of V^{\max} and q



for convenience:

- light contact restrictions: $0.8 \leq \delta$
- strict contact restrictions: $0.6 \leq \delta < 0.8$
- lockdown: $\delta < 0.6$