MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

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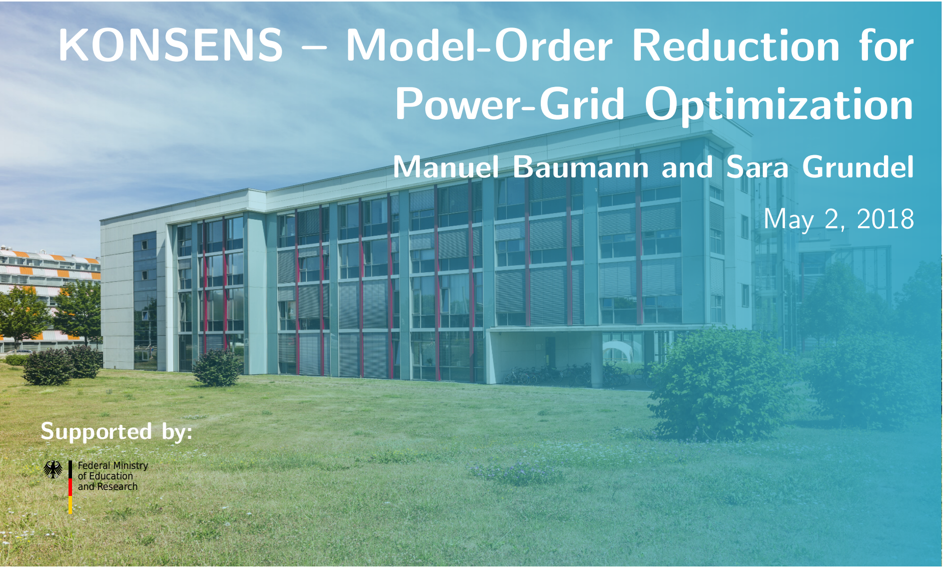
COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

20 YEARS
1998-2018

KONSENS – Model-Order Reduction for Power-Grid Optimization

Manuel Baumann and Sara Grundel

May 2, 2018



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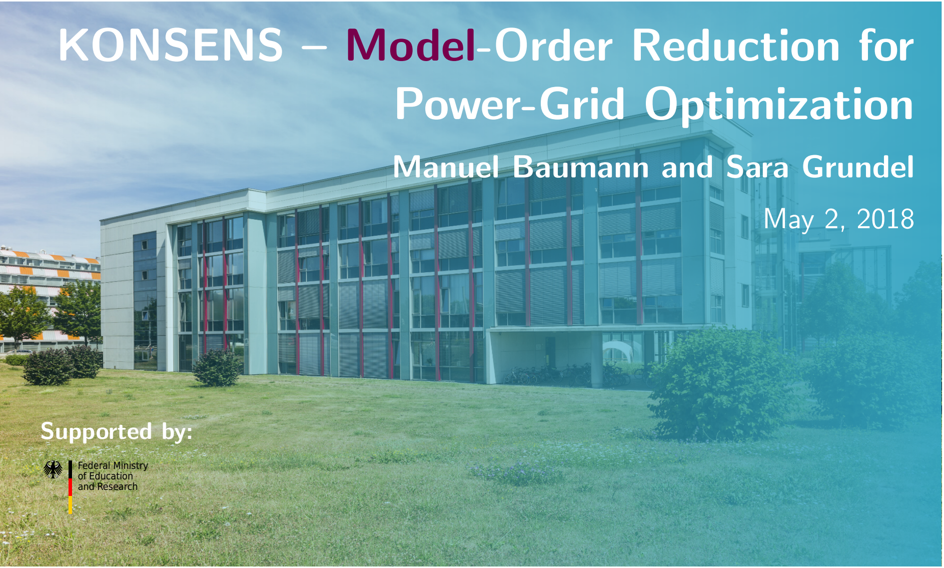

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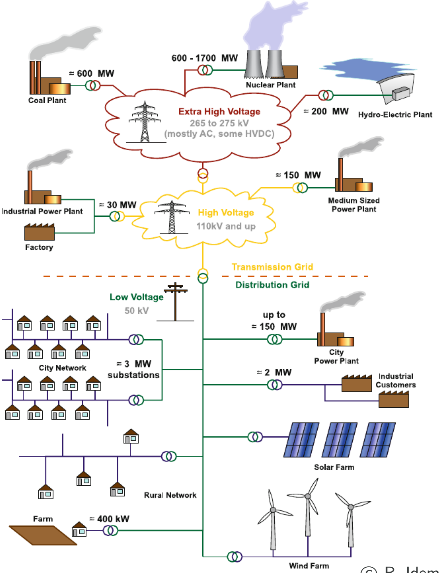
Who are we?

Sara Grundel

- team leader *Simulation of Energy Systems*
- areas of expertise: modeling and simulation of large networks, optimization, model-order reduction

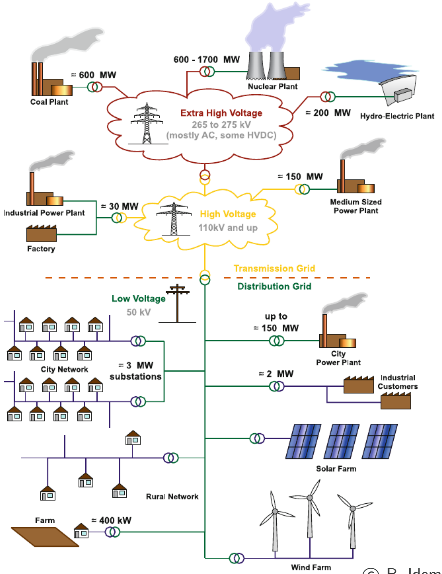
Manuel Baumann

- PostDoc at MPI since April 2018
- PhD from TU Delft in Numerical Linear Algebra
- past projects in: computational geophysics, optimal control, model-order reduction



Recent developments:

- renewables
- E-car
- batteries
- prosumers



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Recent developments:

- renewables
- E-car
- batteries
- prosumers

Gives rise to new mathematical challenges!

Which model?

The network is an undirected graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ with generators $\mathcal{V}_G$ and loads $\mathcal{V}_L$ ($\mathcal{V} = \mathcal{V}_G \cup \mathcal{V}_L$), and battaries. At node $i \in \mathcal{V}$,

$$V_i = |V_i| e^{i\delta_i(t)} \quad \text{or} \quad S_i = P_i + jQ_i.$$

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Classical network (conventional generators):

$$\begin{aligned} &\text{If } i \in \mathcal{V}_G: \quad P_i \text{ and } |V_i| \text{ are known.} \quad \rightsquigarrow \delta_i(t) \\ &\text{If } i \in \mathcal{V}_L: \quad P_i \text{ and } Q_i \text{ are known.} \end{aligned}$$

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Q1: For renewables $|V_i|$ becomes $|V_i(t)|$???

Q2: How to model batteries ???

Which model?

Three leading models

Governing equations (*swing equations*) at network node i,

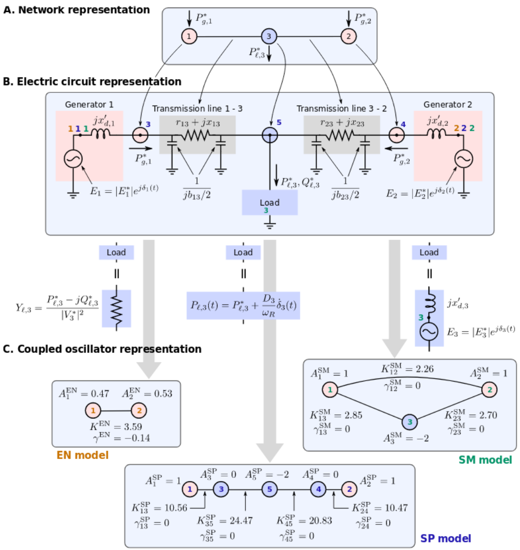
$$\frac{2H_i}{\omega_R}\ddot{\delta}_i + \frac{D_i}{\omega_R}\dot{\delta}_i = A_i - \sum_{j\neq i} K_{ij}\sin(\delta_i - \delta_j - \gamma_{ij}), \quad i \in \{1, ..., N\},$$

yield for a specific choice of parameters the three models

- **EN** effective network model ($N = |\mathcal{V}_G|$),
- **SM** synchronous motor model ($N = |\mathcal{V}|$),
- **SP** structure-preserving model ($N > |\mathcal{V}|$).

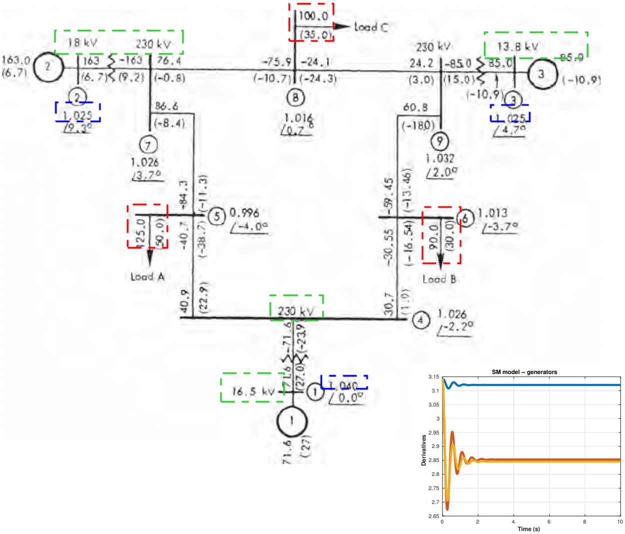
T. Nishikawa and A. E. Motter (2015). *Comparative analysis of existing models for power-grid synchronization.* New Journal of Physics 17:1.

Which model?


A. Network representation
B. Electric circuit representation
Generator 1
Transmission line 1 - 3
Transmission line 3 - 2
Generator 2
$r_{13} + jx_{13}$
$r_{23} + jx_{23}$
$jx'_{d,1}$
$jx'_{d,2}$
$P^*_{g,1}$
$P^*_{g,2}$
$P^*_{\ell,3}$
$P^*_{\ell,3}, Q^*_{\ell,3}$
$E_1 = |E_1^*| e^{j\delta_1(t)}$
$E_2 = |E_2^*| e^{j\delta_2(t)}$
$\frac{1}{jb_{13}/2}$
$\frac{1}{jb_{23}/2}$
Load
$Y_{\ell,3} = \frac{P^*_{\ell,3} - jQ^*_{\ell,3}}{|V_3^*|^2}$
$P_{\ell,3}(t) = P^*_{\ell,3} + \frac{D_3}{\omega_R} \dot{\delta}_3(t)$
$jx'_{d,3}$
$E_3 = |E_3^*| e^{j\delta_3(t)}$
C. Coupled oscillator representation
$A_1^{EN} = 0.47$ $A_2^{EN} = 0.53$
$K^{EN} = 3.59$
$\gamma^{EN} = -0.14$
EN model
$A_1^{SM} = 1$ $K_{12}^{SM} = 2.26$ $A_2^{SM} = 1$
$\gamma_{12}^{SM} = 0$
$K_{13}^{SM} = 2.85$ $\gamma_{13}^{SM} = 0$
$K_{23}^{SM} = 2.70$ $\gamma_{23}^{SM} = 0$
$A_3^{SM} = -2$
SM model
$A_1^{SP} = 1$ $A_3^{SP} = 0$ $A_5^{SP} = -2$ $A_4^{SP} = 0$ $A_2^{SP} = 1$
$K_{13}^{SP} = 10.56$ $\gamma_{13}^{SP} = 0$
$K_{35}^{SP} = 24.47$ $\gamma_{35}^{SP} = 0$
$K_{45}^{SP} = 20.83$ $\gamma_{45}^{SP} = 0$
$K_{24}^{SP} = 10.47$ $\gamma_{24}^{SP} = 0$
SP model

Comparison

Current MSc project (ongoing).



Other models

The Kuramoto model,

$$\dot{\delta}_i = \omega_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\delta_i - \delta_j), \quad i = 1, ..., N,$$

fits the same framework.

- F.A. Rodrigues, T. K. Peron, P.Ji, J. Kurths (2016). *The Kuramoto model in complex networks.* Physics Reports 610:1-98.
- V. Mehrmann, R. Morandi, S. Olmi, and E. Schöll (2017). *Qualitative Stability and Synchronicity Analysis of Power Network Models in Port-Hamiltonian Form*. arXiv:1712.03160v2.

Software tools ?

MATLAB

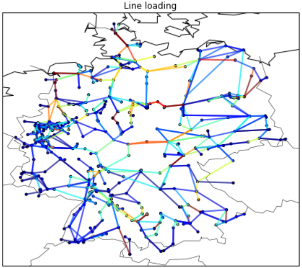
- MATPOWER
- pg_sync_models

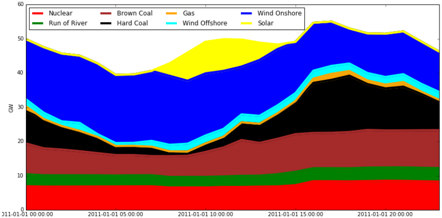
Python

- PyPSA with NetworkX

Others

- ???





Model-order reduction

With $\xi_i := \dot{\delta}_i$, the model equations yield a nonlinear dynamical system,

$$\begin{bmatrix} \dot{\delta}_i \\ \dot{\xi}_i \end{bmatrix} = \begin{bmatrix} \xi_i \\ \frac{\omega_R}{2H_i}\left(A_i - \sum_{j \neq i} K_{ij} \sin(\delta_i - \delta_j - \gamma_{ij}) - \frac{D_i}{\omega_R}\xi_i\right) \end{bmatrix},$$

of the form $\dot{x}_i = f_i(x_i^{(1)}, x_i^{(2)})$, for $i = 1, ..., N$.

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of the form $\dot{x}_i = f_i(x_i^{(1)}, x_i^{(2)})$, for $i = 1, ..., N$.

Nonlinear model-order reduction

The state vector $\mathbf{x} = [x_1, ..., x_N]$ grows with the network size. Therefore, derive a reduced-order model (ROM):

$$\text{FOM}\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t, \mathbf{u}) \\ y = \mathbf{g}(\mathbf{x}, t, \mathbf{u}) \end{cases} \quad \leftrightsquigarrow \quad \text{ROM}\begin{cases} \dot{\hat{\mathbf{x}}} = W^T\mathbf{f}(V\hat{\mathbf{x}}, t, \mathbf{u}) \\ \hat{y} = \mathbf{g}(V\hat{\mathbf{x}}, t, \mathbf{u}) \end{cases}$$

such that for the outputs: $\|y - \hat{y}\|$ is small.

Model-order reduction

Nonlinear model-order reduction

- Proper Orthogonal decomposition
- Reduced basis method
- (Discrete) empirical interpolation method
- Empirical Gramians
- Quadratic/bilinearization method

Model-order reduction

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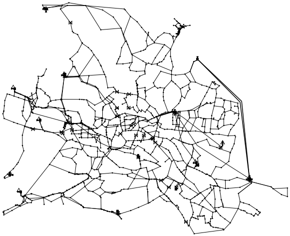
Challenges for power-grids

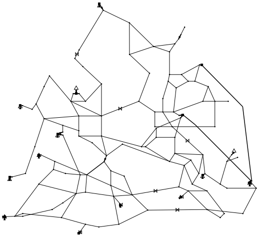
- Preserve certain state variables, and structure
- Low-dimensional projection and transformation spaces

Project goals

Nonlinear model-order reduction for power grids

- uncertainty quantification (UQ)
- error estimates ROM vs. FOM
- structure-preserving clustering





Discussion points

Questions from our side are:

- Which model?

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Discussion points

Questions from our side are:

- Which model?
- Which test cases?
- What is a control and which constraints?
- What to preserve in a reduced-order model?
- Usage of dictionaries (learned steady states)?