Quantitative Macro – Homework 3

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October 10, 2019

Computing Transitions in a RA Economy

Part 1a

The exercise requires normalizing output (y) to one in order to exploit the given ratios which are $\frac{k_t}{y_t} = 4$ and $\frac{i_t}{y_t} = 0.25$ for all periods steady states. Since we set y equal to 1 we know that k must be equal to 4 in the first steady state. From the second ratio we obtain $\delta = \frac{1}{16}$. The other parameters of the model are given as $\theta = 0.67$ and $h_t = 0.31$. I calculate the FOC with respect to k_{t+1} as the following:

$$\beta u'(c_{t+1})[(1-\theta)k_{t+1}^{-\theta}(zh_{t+1})^{\theta} + (1-\delta)] = u'(c_t) \tag{1}$$

Rearranging and plugging in the known terms yields a rounded discount factor $\beta = 0.98$. From the production function of the economy I compute the level of z which delivers our required ratios as roughly 1.63. From this we obtain steady state levels of output, consumption and savings as follows:

Table 1: Initial steady state equilibrium

Productivity z	1.63
Capital k	4
Output y	1
Consumption c	0.75
Investment i	0.25

Part 1b

Doubling the productivity level permanently and going through the algebra leads to a new new steady state capital of 8 which could be expected due to the CRS production function. The new steady state is defined by the following values:

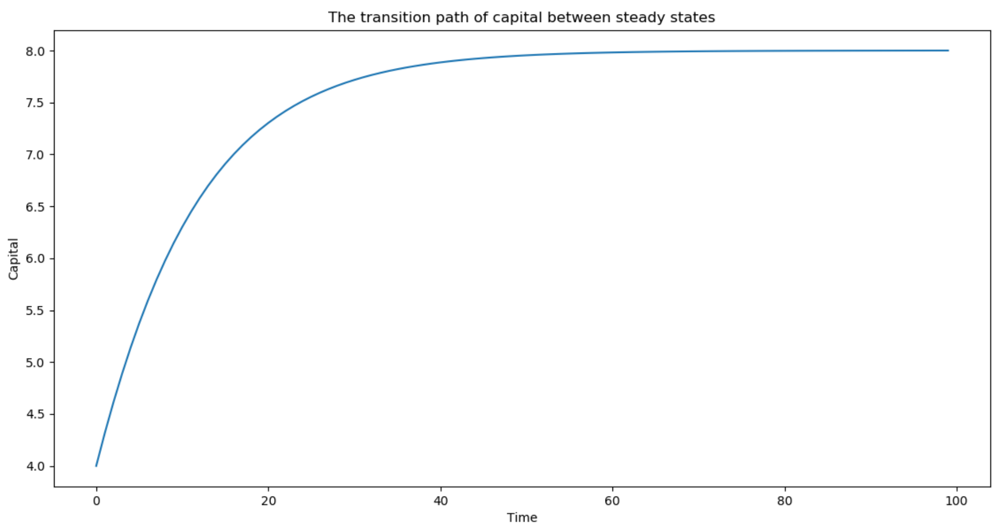
Table 2: Second steady state equilibrium

Productivity z	3.26
Capital k	8
Output y	2
Consumption c	1.5
Investment i	0.5

Part 1c

I compute the transition of capital from the first to the second steady state by essentially solving a large system of Euler equations for an arbitrarily chosen horizon. For this, I constrain the first observation to be equal to 4 and the last being equal to 8 as we need a starting value and an "out of sample value" to terminate the system. The transition path for capital and those for consumption savings and output are plotted below:

Figure 1: Transition path of capital



Part 1d

The shock that occurs at period t=10 drives our variables of interest off the transition path to the foreseen steady state. This is marked by a sharp drop in the level of all variables of interest which are plotted here. It can be seen that upon receiving the shock,agents react strongly and then converge more or less smoothly to the initial steady state. It is somewhat noticeable

Figure 2: Transition paths of consumption, output and savings

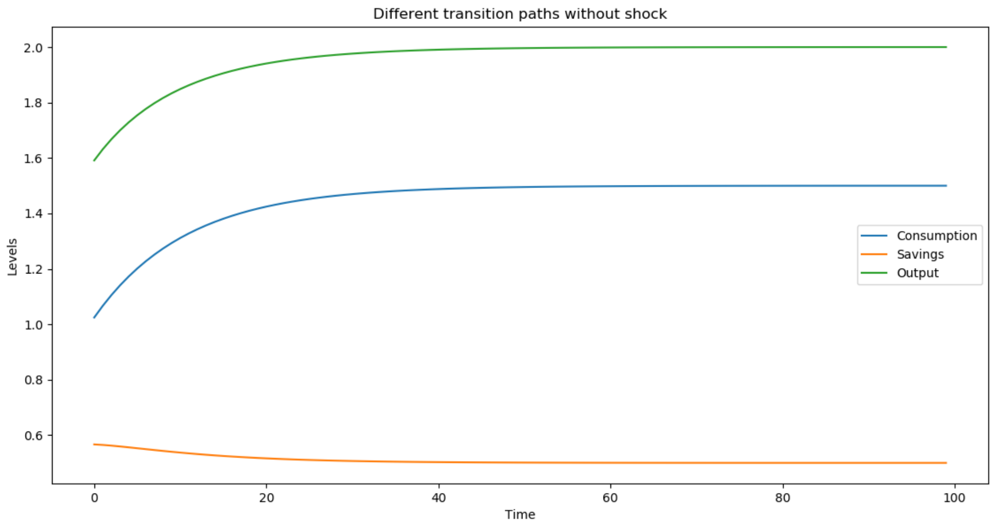
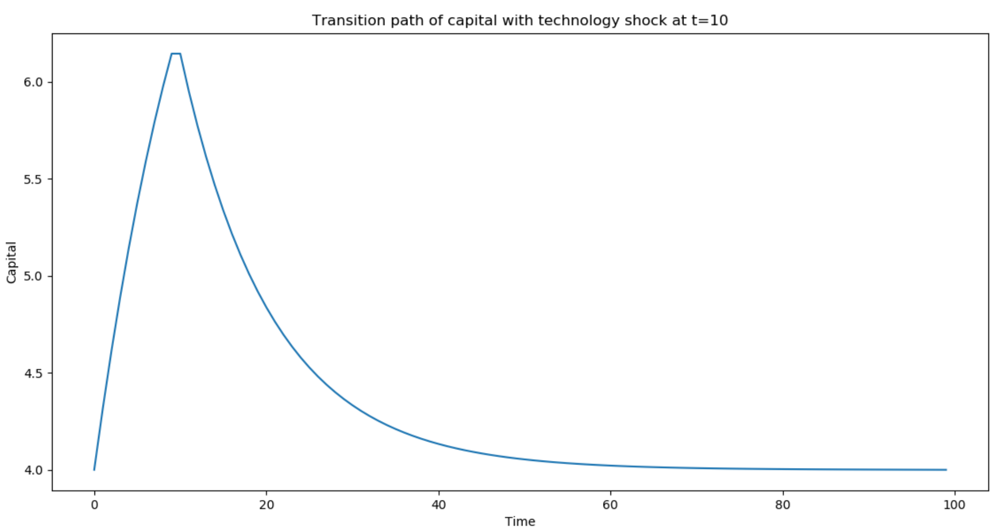


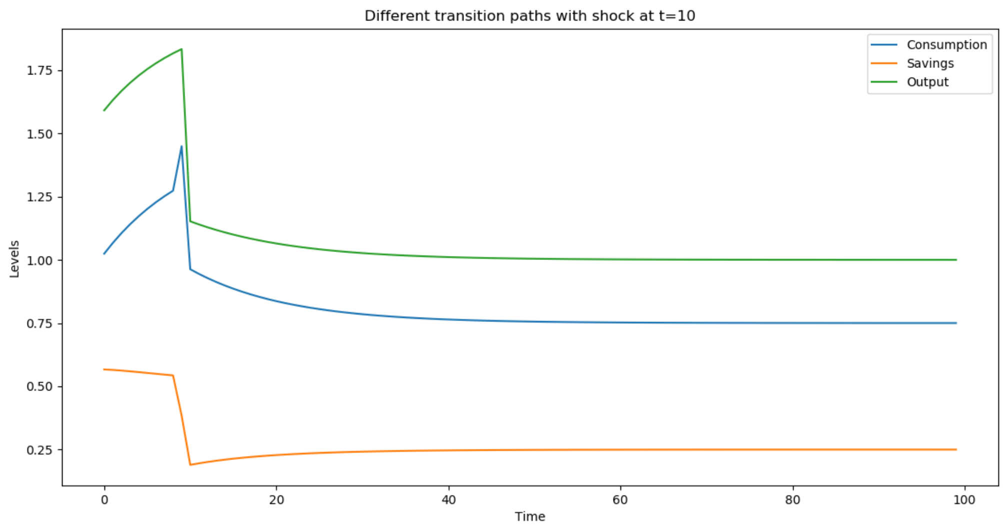
Figure 3: Transition path of capital after shock



A model for a capital union with labor taxes

For the entire exercise I will assume that in each country, there exists a representative household with low productivity and a representative household with high productivity. Changing the population composition - using a unit mass of households - would be

Figure 4: Transition paths of consumption, output and savings after shock



straightforward and would come into play at the aggregation level for the most part. Furthermore, I assume that agents are endowed with a capital level of 1 such that total capital stock in a given country is equal to 2 at the beginning. I consider capital not consumable, so that all of it is plugged into production.

Part 2a

My approach to solving the closed economy is to set up a system of non-linear equations which include Euler equations for high and low types, the two types' budget constraints and the firm's first order conditions which pin down prices. That gives me a system of six equations for the six unknowns consumption (high and low), labor supply (high and low), wage and rate of return on capital. The Euler equations are defined as follows:

$$c_c^{-\sigma}\lambda_c(1-\phi_c)(w_c\eta_c^i)(w_c h_c^i \eta_c^i)^{-\phi_c} = \kappa_c(h_c^i)^{1/\nu} \tag{2}$$

where the subindex c refers to country A or B and the supindex i refers to type. The budget constraints are of the following form:

$$c_c^i = \lambda_c(w_c h_c^i \eta_c^i)^{1-\phi_c} + r_c k_c^{\eta_c^i} \tag{3}$$

The firm first order conditions are given by the following:

$$Z_c(1-\theta_c)K_c^{-\theta}H_c^{\theta} = r_c \tag{4}$$

$$Z_c\theta_c K_c^{1-\theta}H_c^{\theta-1} = w_c \tag{5}$$

The solution to this system of equations requires aggregating capital and labor. For the closed case, as explained above, I set $K_c = 2$ and for labor I use efficient units $H_c = \eta_c^h h_c^h + \eta_c^l h_c^l$

Using the given parameter calibration, the equilibrium allocation is summarized in the following table:

Table 3: Closed economy static equilibria

Variable	*Country A*	*Country B*
Consumption type h	1.48	1.3
Consumption type l	0.48	0.83
Labor supply type h	0.35	0.33
Labor supply type l	0.15	0.26
Wage	0.6	0.55
Return on capital	0.4	0.45

For both countries under the closed economy scenario I get qualitatively similar results. Consumption is higher for the productive type, less productive types enjoy the return on leisure more than using their relatively unproductive labor. Wages are slightly higher than the return on capital because the marginal product of labor is larger than that of capital.

Part 2b

I run the same operation as in part a) for the union with one major difference: I modify the variable for capital supply in the budget constraint to mean domestically supplied capital and add foreign capital investment as a source of income. This also has an effect on the aggregation of capital. Budget constraints are now

$$c_c^i = \lambda_c (w_c h_c^i \eta_c^i)^{1-\phi_c} + r_c (k_c^{i,s})^{\eta_c^i} + r_{-c}(k_c^{i,e} - k_c^{i,s}) \tag{6}$$

where I take $k_c^{i,s}$ to mean domestically supplied capital and $k_c^{i,e}$ to be the initial capital endowment of type i in country c. This leads to additional optimality conditions

$$r_c \eta_c^i (k_c^{i,s})^{\eta_c^i - 1} = r_{-c} \tag{7}$$

and a redefined aggregation for capital as exemplified for country A

$$K_A = k_A^{l,s} + k_A^{h,s} + (k_B^{l,e} - k_B^{l,s}) + (k_B^{h,e} - k_B^{h,s}) \tag{8}$$

Solving this new system of equations leads the following equilibrium allocation:

It is noticeable that factor prices equalize across countries, as we should expect to avoid arbitrage. Moreover, compared to the no-trade equilibrium we see that low types appear to benefit in terms of higher consumption levels. In both countries we observe some foreign capital investment as agents do not domestically supply their entire capital unit. Labor supplies remain fairly stable, as there is no option to trade, but some extra inputs come through foreign capital now.

Table 4: Union static equilibria

Variable	*Country A*	*Country B*
Consumption type h	1.25	1.12
Consumption type l	0.62	0.72
Labor supply type h	0.38	0.37
Labor supply type l	0.12	0.29
Domestic capital supply type h	0.68	0.69
Domestic capital supply type l	0.26	0.55
Wage	0.55	0.56
Return on capital	0.46	0.45

Part 2c

For this part of the exercise, since the environment is static, I would propose a simple Utilitarian social welfare function of the following form:

$$SWF = \psi_1 v_A^l(c,h) + \psi_2 v_A^h(c,h) + \psi_3 v_B^l(c,h) + \psi_4 v_B^h(c,h) \tag{9}$$

where ψ is the Pareto weight and the utilities are indirect and result from the agents and firms' maximization problem. In that sense, I would perhaps follow *Conesa, Kitao and Krüger (2009)* who propose to maximize the SWF by selecting parameters - in our special case they would be λ and ϕ (i.e. to balance the government budget and select the appropriate degree of progressivity)- taken from the *Feldstein (1969)* idea:

$$T(inc) = inc(1 - \lambda inc^{-\phi}) \tag{10}$$

with *inc* referring to taxable labor income. In terms of solving this problem, I would proceed as above with the system of non-linear equations and then solving the maximization problem of the planner.