Quantitative Macro

Problem Set 8

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Problem 1:

A three period model with a discrete choice on working in the last period.

- Look at the plot of the comparison between the explicit and the numerical solution for the three different solution methods.

Comparing 1 and 2

1 = global solution

2 = discrete-continuous endogeneous grid method

→ we see that solutions largely overlap
→ numerical solution continues for more values of x
→ discontinuity in global solution is around $x \approx 2.5$

Comparing 1 and 3

3 = exogeneous grid method

→ exogeneous grid method gets "discontinuity" (not really discontinuous) at $x \approx 2$, so below the analytical solution
→ after the drop the trend is picked up correctly and as of $x \approx 3$, the two solutions overlap again

2. Describe in words those 3 solution methods:

Solution method 1: "cloud". This is an implementation of the analytical results in the handout.

- using the perturbation from taste shocks we can get an analytical expression for V_2
- we need a threshold condition when the discrete choice in the last period becomes 1 or when it stays zero

- if savings are apparently high, working may be suboptimal for the individual
- using the condition we distinguish two cases and assign choice probabilities using logit
- the expected utility in the last period can be expressed using the log-sum property
- we can solve the maximization problem in the penultimate period for savings and consumption policies, obtain $V_0(x_0)$ and then solve the maximization in the initial period for optimal savings and consumption
- this gives us all policies for consumption and savings and we can check the discrete choice in the last.
- because we have to loop over a grid of shocks for the value function evaluations

solution method 2: endogenous grid method

- only necessary to solve pre-discrete choice problem with discontinuous/continuous choice
  - -> create a grid of savings
  - -> recursively loop over the savings grid and compute the solution for the household problem at each grid point
  - -> gives you a consumption policy, cash-on-hand policy and value function
  - -> this computation is made for all ~~discrete choice~~ state states given by the income shock (i.e. endogenously calculating cash-on-hand)

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following Ishakov, Jorgensen, Rust, Schjerning (2016)

- -> take a current decision d_t and form exogenous, ascending wealth grid (end of period wealth)
- -> compute from this a "grid" of next period wealth
- -> if $u(\cdot)$ is invertible, calculate the discrete choice d_t

from the inverted Euler equation

→ compute current consumption from the inverted Euler equation

→ this gives the endogeneous grid over current period wealth

→ get the choice specific value function by maximizing plugging in the consumption policy into the value function

(→ approximate corresponding functions by linear interpolation)

→ now we have approximations to all solutions of the Euler equation

→ but: some of these may not be optimal choices what are the suboptimal choices we need to kick out?

→ cleanup step: I think this is skipped in the code ???
⇒ it is, line 359 (checking monotonicity)

· how do the preference shocks appear?

→ Type-I EV shocks allow for log-sum representation of continuation value in workers' problem given a discrete choice

→ this is needed when we recursively get the value-functions in the EGM step.

solution method 1 ("traditional VFI") →

· ~~for every point on the~~ savings grid:
· guess a solution for the consumption policy
· evaluate the value function at your guess
· pick the maximum value of the vector
· verify if local or global
· store consumption policy and value function

Method 3: Exogeneous grid root finder

→ make a grid for cash-on-hand

→ use univariate solver to compute optimal consumption from Euler equation

→ solve value function problem of worker (for all possible states given by shocks and decisions) in T-1

→ here we use again the logsum property created by the taste shocks

→ obtain V_0 as $V + \sum P(\eta_j) \cdot V_j(\eta_j)$

~~expected value~~:

~~for every~~

→ do for other periods

3. Compare the closed form solution in period 1 to the numerical solution.

→ solutions: policy functions and value function

$g_1^c(x)$

$g_1^{a'}(x)$

$V_1(x)$

→ we set param. nt = 1 (meaning the agent solves essentially a 2-period model where in the last period they have a discrete choice)

→ we keep the solution method equal to 2

- with 1 period the global solution for c_1 gives larger values for the same grid than for 2 periods before the discrete choice.

=> there is one period with uncertainty fewer than before

- we also get lower value functions (clearly since we have fewer periods to add up) (i.e. lower value function for t=1 perspective relative to t=0 perspective)

- consumption and savings policies change quite significantly
- in fact, they overlap after the kink in $X=2$

4) Measure the running times of the three alternatives
→ intel i7, 8GB RAM

method 1: 25.6787

method 2: 1.2831

method 3: 18.4754

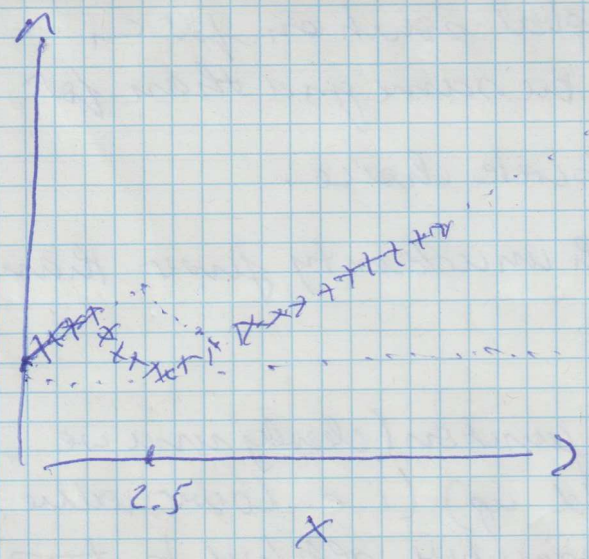
→ endogeneous grid is by far the fastest, method 2 needs to solve non linear problem to find root,

→ method 1 is using a golden search method to check global maximum

→ endogenous grid moves state to one side of Eulr equation

5) Why does method 3 have problems in characterizing the solution?

→ recall, the solution path of method 3 does not coincide well with the path of the global / analytical one



- exogeneous grid method uses Brent's algorithm to find the root of the Euler equation
- the Euler equation ~~terms of c_0 is not continuous~~ and the may have exponential and $1/x$ terms which cangive Brent's algorithm problems to converge
- the method is unable to locate the 1 (analytical) kinks precisely due to linear interpolation.

6. What happens if you increase the variance of the income shocks to

$\sigma_\varepsilon = 0.01$ under the three methods?

method 3: we get a second kink around $x = 1$ for the solution of the problem in terms of c_0

the policies also get a second kink (consumption and savings at $t=0$

method 2: method 2 no longer follows the global solution for higher values of x

also appears to have multiple kink in solution path and policy function

method 1: also two kinks

Why do we get more kinks?

- kinks occur at points where an agent is indifferent between working and not working (i.e. where the value function is not differentiable)
- these so-called primary kinks reverberate back through the system (i.e. through the impact of V_t on V_{t-1} and of the discontinuity in the policies) and can manifest themselves as additional kinks (secondary kinks)
- the number of kinks depends on the wealth threshold attained per period and so depend on the magnitude of the wage shock
- ~~if the precautionary saving (in wealth) of optimal consumption policy carries for $\tau = 3$ periods~~ which ~~calculated are such that choices jump at different~~ realizations of the wage shock ~~we can get~~
- the optimal consumption policy is piecewise linear and may therefore exhibit many kinks depending on the wealth thresholds (and hence the shocks to income).

7.) Elsing param. sigma = 1

method 2: analytical and numerical solution are indistinguishable

smoothing is strong so that kinks appear to be gone

method 3: corresponds closely to analytical and global solution

slight kink in consumption and in savings policies, but close to none

=> numerical solution get closer to analytical solution due to smoothing

→ for $\sigma = 1$ (I assume they mean the scale parameter for the Gumbel shocks) the log sums become very simple and the expected utility can be evaluated well

Problem B

$$V(x, \eta, j) = \max_{c, a'} \left\{ u(c) + \beta \underbrace{E_j}_{\text{survival risk}} \sum_{\eta'} \pi(\eta'|\eta) V(x', \eta', j+1) \right\}$$

s.t.

$$a' = s = \begin{cases} a(1+r) + \eta w - c & \text{for } j < j_r \\ a(1+r) + p - c & \text{for } j \geq j_r \end{cases}$$

$$x = a(1+r) + \eta w$$

$$a' \geq 0$$

$$u(c) = \frac{c^{1-\theta}}{1-\theta} \qquad \theta \neq 1$$

$$\log c \qquad \theta = 1$$

Problem 1: a) Explain all sections of the code

1. lines 1–16 : Intro

→ we have a couple of options to set, whether to include survival risk, stochastic element and we need to set the number of states for the income shock, in this case 2

2. lines 17–33: Solution

→ uses function that are created later to

a) calibrate the parameters of the model to typical values

get invariant distribution starting from 50%/50%

b) solve the household model

c) aggregate onto

d) create the trajectories over the lifecycle of our variables of interest

3. lines 34 - 108: Plots

- Plots paths for variables of interest over the life-cycle + growth rate of consumption, the asset distribution and the value function

4. line 109 - 210: Calibration

- based on input from intro (essentially whether stochastic or deterministic) calibrate model parameters, long run distribution, survival rates, Markov process shocks and transitions, population and pension income

5. line 211 - 343: solve the household problem

- uses backward iteration and an endogeneous grid method to recuperate consumption policy and value function
- first set up grids for cash-on-hand, savings and assets at all times and states
- for the two states compute final period wealth to get final period value, policies
- from there iterate backwards in time using a grid on savings
- for all income shocks (i.e. high or low), for all grid points in the c-o-h grid do the following
  a) compute the value tomorrow under each shock by interpolating
  b) since we have ~~consumption~~ savings only on RHS of Euler equation → evaluate
  c) solve for

grid

e) calculate value function and update

f) go back one period and proceed as before

6. line 344 - 377: function interpolation

- implement linear interpolation of a function between points

7. line 378 - ~~445~~ 511: Aggregation

- this section controls for the measure of individuals at points in the cash-on-hand grid
- for this we need a transfer function which determines the new shape of the distribution
- then we aggregate over the new distribution and store this at each period
- then we check if the measure in total adds to 1
- for the actual aggregation of assets we need these distributions (we also compute total consumption)
- the last bit of this section is a function we use to generate the new distribution, i.e. for the transfer function, from the old distribution

8. line 512 - 544: Life-Cycle Profile

- the function here calculates the asset holdings, consumptions, incomes, value functions at all periods of the population

9. rest: these are auxiliary functions needed in the solution above

b) Compare the solution procedure in func_hh to a standard implementation using the exogeneous grid method

→ What are the differences:

the grid is used for savings instead of cash-on-hand

→ Advantages / disadvantage?

- the method is faster and reduces the number of continuous state variables
- we do not need to use a non-linear root finder but can solve for consumption "analytically" via the Euler equation
- disadvantage: the exogeneous savings grid can lead to an endogeneous assets grid with infeasible grid points

↳ Hence need to figure out a way how to deal with this in interpolation

c) Implement exogeneous grid method

→ running time egm: 6.7431 s

→ running time exogm (flawed): 27.7066 s

d) Have a look at func_aggr.

i) what is the role of the objects TT and Phi?

lecture notes: Ψ_t = cross sectional measure of households

Q_t = Markov transition function

→ TT: how do individuals move within the distribution over the ages

→ Phi: determines the measure over ages, savings and income shocks

- in the lecture notes the survival risk is multiplied onto the transition function rather than the law of motion of the measure
- the integral is not applied in the code, rather we sum over the entire state space (for the law of motion of the measure)

- merging reduces time from 6.0929 to 5.9223

- breaking reduces time to 5.8635 but delivers strange results

e) solve the model under 3 alternative settings

1. opt_ret = true , opt_nosr = true
   - non-stochastic income and the transition function has $\pi = 1$
   - perfect survival until the last period
   - problem: value function is convex
   - consumption constant except 1 period around retirement, policy kinked
2. opt_ret = false , opt_nosr = true
   - income is determined by shocks that follow a Markov process, there is no risk from survival chance
   - consumption becomes flat after retirement
   - consumption policy at age 20 and 40 overlap
   - asset decumulation is more concave
3. (I assume there is a typo in the PS)
   opt_ret = false , opt_nosr = false
   → agent save to smooth consumption and face survival risk

Stock

- simulate the model with varying theta
→ I put both previous options to "false"

θ=2:

- value function from −38 to −2 (relatively straight)
- savings from 0.4 to 0.15 (ages 20 − 55)
  at retirement sharp drop, before slight increase
  after retirement steady around −0.45, then increase to 0
- consumption hump shaped, peak at 6.5 (1.25)
- asset "triangular": peaked at 68 (11)
- income: sharp drop at retirement (from 1.4)
- asset tomorrow dist.: mass at zero, hump at (10, 0.05)
  relatively high variance
- policies for young agents almost overlapping (consumption)

θ=4

- value function far lower at early ages
- savings are similar to above setting
- slightly higher consumption, but also hump-shaped
- higher asset accumulation towards retirement
- income very similar (a bit higher)
- asset distribution (tomorrow) even higher variance
- policies for consumption roughly the same

ρ=0.2, r=0.04 (leads to lower β) → more impatience

- consumption policies overlap completely at young age
- value function decreasing after retirement
- savings similar but somewhat lower
- consumption not really hump shaped but skewed towards age 0
- asset accumulation decelerates before retirement
- lower income, asset distribution less variance, higher mass at zero