The Effects of Monetary Policy Shocks on Risk

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October 26, 2023

Abstract

I study how monetary policy affects the predicted distributions of GDP growth and inflation in the US, in particular whether it can alleviate tail risks. These risks are captured as the time-varying conditional 5^{th} (downside risk) and 95^{th} (upside risk) quantiles of the forecasted distributions of the variables of interest. I find that contractionary shocks shift the expected distributions of both variables to the left, in line with established results, but exacerbate tail risks and create additional modes in the distribution which suggest increased probabilities of very bad growth outcomes and policy ineffectiveness vis-à-vis inflation. The bimodality is not present for expansionary shocks. In addition, in response to the shock, the distributions are skewed and inflation uncertainty actually increases in response to an attempt by the authority to control it. The results recall the idea of multiple equilibria resulting from weak policy responses to increases in inflation, but suggest that multiplicity may be related only to contractions. The findings are derived using a new model setup which combines quantile regression with a dynamic factor model.

Keywords: monetary policy; quantile regression; dynamic factor models; growth-at-risk; inflation-at-risk.

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1 Introduction

Central Banks want to understand how their policies can be expected to affect the real economy. Typically, this is done by tracing out the impulse response functions (IRFs) of key variables such as aggregate prices or gross domestic product (GDP) to a surprise policy implementation. These IRFs are informative about the expected reactions of variables to the shock. To complement these mean reactions, in recent years policymakers have shifted more attention to the assessment of risks of extreme events, that is very poor or very good expected future economic performances and price rises or falls. The rationale for this is a desire to anticipate trends or risk accumulations in specific market segments which signal potential real economic trouble ahead. For example, after the Great Recession, special focus has been dedicated to monitoring financial market conditions to expose potential threats to economic stability coming from the banking and financial services sector. In principle, if such risk accumulations are detected, the policymaker can adjust the relevant tools to prevent them from spiraling into a full-blown crisis. In this paper, I investigate the question whether adjustments in the monetary policy stance are able to reduce the risk of unusually bad economic outcomes by studying the response of the entire distributions of GDP growth and inflation in the US. Using a combination of a structural dynamic factor model (DFM) and quantile regressions I document that contractionary monetary policy shocks slightly increase short run growth risks and can even be ineffective in reducing prices.

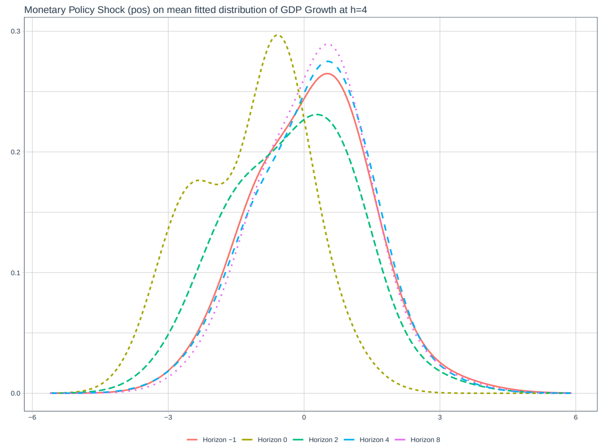
In the financial sector example above, the typical tool used for risk management is macroprudential policy, which aims at strengthening the institutions' positions, for instance, through increased capital requirements. Theoretically, this allows banks to absorb shocks more effectively and thus keeps financing conditions for the real economy steady. However, as has been pointed out in FOMC meetings[1], the effectiveness of such macroprudential policies to mitigate risks to growth and price stability is just beginning to be better understood in the literature. Instead, adjustments in the policy stance have been suggested as a potentially more powerful tool to combat risk accumulation. A change in the policy stance is a less surgical tool than macroprudential policy, but can be expected to have effects beyond financial markets and could therefore be more suitable to address risk accumulations in other sectors. Hence, this paper investigates how risks to growth

[1] Minutes of FOMC meeting on 28/29 January 2020 p.9: link.

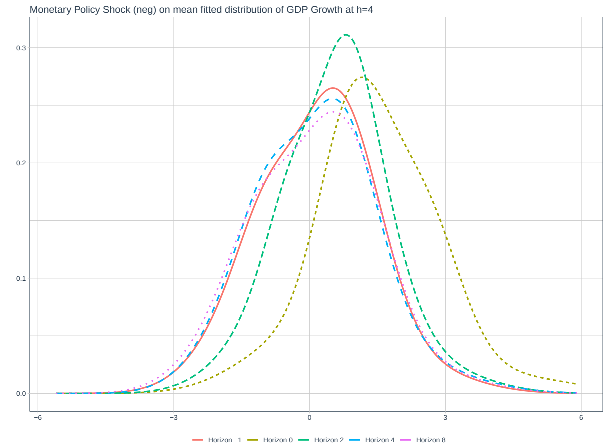
and price stability respond to monetary policy shocks in the US.

Contributions: Figure 1 shows that while both expected distributions of GDP growth and PCE inflation shift to the left in response to a contractionary shock – in line with the established results in the literature – the shape of the distributions is significantly altered in the short run. In particular, the steady state expected distributions become multimodal after the contractionary shock which has important new implications for understanding monetary policy. First, it implies that contractionary monetary policy creates potential future growth scenarios which generate worse than average growth outcomes with high probability. This observation is reminiscent of Lubik & Schorfheide (2004) who show that with multiple equilibria, the real effects of monetary policy can be significantly different. Second, it implies that while on average the contractionary policy contains inflation, there is a significant probability that the policy proves ineffective in reducing inflation. Moreover, in the converse case – in response to an expansionary shock – the multimodality in the expected distributions is less clear cut. It vanishes for GDP growth and is much less pronounced for expected inflation. This points to non-linearities in the transmission of monetary policy shocks due to the sign of the policy. Expansionary shocks shift the mean in line with theory and empirical evidence and in addition to this also reduce the spread and tail weights of the expected distributions. This compression of expected distributions implies that expansionary policy paves the way for more favorable outcomes centered around the mean forecast, i.e. less forecasting uncertainty, but contractions can potentially turn out far more adverse, are accompanied by larger uncertainty, or even prove ineffective.

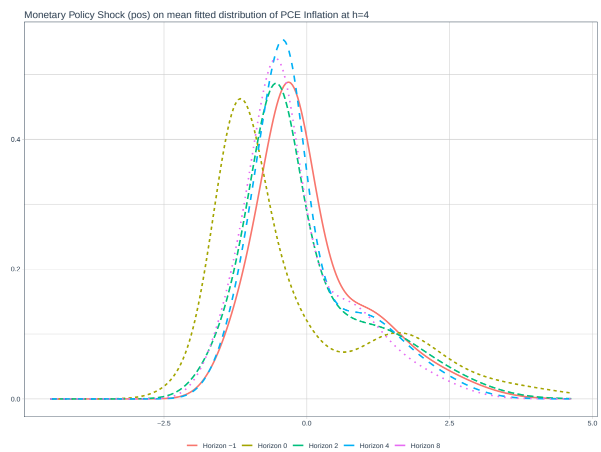
In addition to these main results, this paper adds to the literature on growth-at-risk and inflation-at-risk by providing new estimates of the conditional expected distributions of the two variables using the quantile regression framework of Adrian et al. (2019), where instead of using specific conditioning variables, I condition the predicted distributions on a set of static factors which summarize the information contained in a large macroeconomic data set of the US economy. This method does not allow for a study of how single variables influence the shape of the conditional distributions of GDP growth and inflation, but is less prone to omitted information and collinear regressors since the factors are by construction orthogonal to each other. I find that the main result of Adrian et al. (2019) persists, in that the lower tail of GDP growth is more volatile than the upper tail, albeit not as



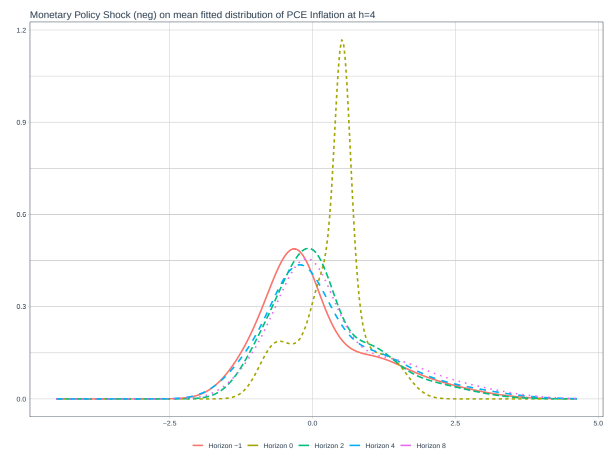
(a) ΔGDP h=4 (contract.)



(b) ΔGDP h=4 (expans.)



(c) PCE Inflation h=4 (contract.)



(d) PCE Inflation h=4 (expans.)

Figure 1: Response of demeaned fitted distribution to monetary policy shocks. Steady state distribution is fitted to the average predicted distribution. All forecast distributions are at h=4. Red line is initial distribution. Mustard line is response at $j = 0$. Green line is response at $j = 2$. Turquoise line is response at $j = 4$. Purple line is response at $j = 8$. 50 bp shock.

strongly as their paper seems to suggest. In the case of inflation I find higher volatility in the upper tail relative to the lower tail.

The methodological contribution of this paper is based on Forni et al. (2021) who combine quantile regressions with SVAR analysis to compute how fluctuations in uncertainty as measured by changes in the left and/or right tail of the predicted distributions affect real and financial variables. A combination of quantile regressions and SVAR techniques is used also in Duprey & Ueberfeldt (2020) who study monetary policy in Canada. Instead of combining the quantile regression framework with an SVAR, I opt for a dynamic factor model. This has the advantage of having been shown to deliver reliable estimates of monetary policy shocks (e.g. Forni & Gambetti, 2010 or Kerssenfischer, 2019), provides high information content for the quantile regression, and allows for the study of the monetary policy shock on any of the more than 240 variables in the macroeconomic data set. Moreover, I use a way of describing the responses of the full conditional distributions of GDP growth and inflation by computing the responses of each quantile to the shock and fitting a kernel density to the shocked quantiles of the distribution. This approach allows for flexible shapes of the distribution and uncovers the existence of multiple modes. Other studies such as Loria et al. (2019) which also study the effect of monetary policy on conditional quantiles use the parametric skew-t distribution (Azzalini & Capitano, 2003) instead, which does not allow more than a single mode. Since the predicted quantiles often have multiple points of inflection, this suggests that multiple modes should not be ruled out when fitting the densities.

Relation to the literature: Multimodality of the conditional distribution of GDP growth has been shown in Adrian et al. (2021) and Forni et al. (2021). It appears to arise during recessionary periods more than in normal times, an observation echoed in this paper as well. Adrian et al. (2021) discuss how poor financial conditions induce the multimodularity and conjecture that bad policy decisions can lead to persistent selection of the worse equilibrium. Their research differs from this paper in the sense that I study the effect of monetary policy shocks on the predicted distributions of GDP growth and inflation, while they study the joint behavior of economic and financial conditions with no particular focus on monetary policy. Forni et al. (2021) on the other hand establish the methodological basis for this paper but focus on how changes in the tails of the predicted distributions affect real and financial outcomes. The multimodality in their paper obtains again in recessionary periods, although the set of predictors and their estimation procedure differ.

In a paper closely related to this one, Boire et al. (2021) use a QR-SVAR to study the response of the GDP growth distribution in six developed countries to monetary and fiscal policy shocks. They find mostly location effects for monetary policy and shape effects for fiscal shocks. In this paper, on the other hand, my approach suggests shape effects to be present in the short run. In another study close to this one, Loria et al. (2019) analyze how different macroeconomic shocks – monetary policy included – affect the conditional quantiles of GDP growth using the quantile regression model of Adrian et al. (2019) as a case study. They find that monetary policy drives the conditional distribution of forecasted GDP growth to the left and that the effect on the 5^{th} quantile is more negative than the effect on the 95^{th} quantile. Their study differs from this paper in the following ways: first, they use the conditional quantiles of Adrian et al. (2019), whereas I construct new forecasted quantiles using a factor approach. Second, they use local projections to gauge the effect of the monetary policy shock, whereas I use a combination of quantile regression and a dynamic factor model. Third, I construct quantile forecasts also for inflation and study their reaction to the monetary policy shock. Such forecasts are constructed also in Adams et al. (2020) using financial conditions and forecast errors from the Survey of Professional Forecasters (SPF) as predictors instead of the factor approach used in this paper. The findings in Loria et al. (2019) are broadly in line with those of this paper, in the sense that the forecasted distribution of GDP growth moves to the left in response to a monetary contraction. This research is also related to the literature on the effects of monetary policy on different measures of uncertainty. For instance, Bekaert et al. (2013) study the effect of monetary policy shocks on a "risk aversion" and an "uncertainty" component of the Chicago Board Options Exchange (CBOE) volatility index (VIX). They find that expansionary monetary policy shocks lead to a decreases in both risk aversion and uncertainty, pointing to a risk-taking channel for monetary policy. By a similar token, I show that the spread of the expected distributions – a measure of uncertainty in Forni et al. (2021) – increases in response to contractionary policy, which could point to the same channel.

In addition, this paper is related to the growing literature on quantile forecasts of macroeconomic variables as in Adrian et al. (2019), Adams et al. (2020), Ghysels et al. (2018) and Plagborg-Møller et al. (2020) to name only a few. Rather than trying to determine the best predictors of different conditional quantiles, as is typical in most of the research to date, I use anonymous factors extracted from a large data set as predictors. Such factors have been shown to capture large portions of the variance of important macroeconomic variables and have been used to some success in mean forecasts (Stock & Watson, 2002). I transfer the idea of using large information to the quantile regression setup similar to Ando & Tsay (2011), who provide an information criterion for factor-augmented quantile regressions. Beyond their paper, there appears to be limited usage of factors in quantile prediction in the literature thus far.

The rest of this paper is organsied as follows: section 2 presents how the DFM is combined with quantile regressions to obtain quantile IRFs. Section 3 describes the data set, transformations, and model specification. Section 4 presents the results. Section 5 offers a discussion of the results and section 6 concludes.

2 Econometric Approach

The main relationship that I use in this paper is an extension of the QR-SVAR of Forni et al. (2021) and is given by

$$Q_{t+h,\tau} = \beta_\tau(L)Wf_t = \beta_\tau(L)WH(L)u_t \tag{1}$$

$Q_{t+h,\tau}$ stands for the h period ahead prediction of the conditional quantile of a variable of interest. In this paper, the variables of interest are GDP growth and inflation. $\beta_\tau(L)$ is a polynomial in the lag-operator L of the coefficients $\beta\tau$ obtained from a quantile regression of the variable of interest on a set of predictors, selected by the matrix W. The set of predictors is made up of the static factors f_t that are extracted from a large macroeconomic data set. The factors are assumed to follow an invertible VAR relationship and therefore, can be cast into the corresponding moving average (MA) representation. The structural MA representation is then given by $H(L)u_t$, where $H(L)$ are the structural impulse responses and u_t are the structural shocks. The monetary policy shock is identified using the dynamic factor model approach of Forni & Gambetti (2010). This relationship exploits the high information content in the static factors as potentially useful predictors for the conditional quantiles of the variables of interest and simultaneously makes use of large information approach to the identification of monetary policy shocks that has been shown to perform similarly to other accepted methods such as high-frequency identification (Kerssenfischer, 2019).

In what follows I explain in more detail the two main relationships – quantile regression and factor model – their specification, treatment and identification of the structural shocks.

2.1 The Dynamic Factor Model

The methodology used to estimate factors and identify the structural shocks is based on the dynamic factor model of Forni & Gambetti (2010). Given a large data set of macroeconomic variables with cross sectional dimension N and time span T the following relationship is assumed to be present in the set:

$$x_{it} = \chi_{it} + \xi_{it} \tag{2}$$

Here, x_{it} is one of the N variables in the data set at time t. χ_{it} is the common component and ξ_{it} is the idiosyncratic component. Common and idiosyncratic components are assumed to be orthogonal. The idiosyncratic components of different variables are allowed to be mildly correlated. The latter implies that the factor model is approximate and note exact (Bai & Ng, 2002). In addition, I assume that the common component χ_{it} is driven by r common factors f_t. The importance each of these r factors has for the common component is captured by a vector of loadings a_i. Hence,

$$\chi_{it} = a_i f_t \tag{3}$$

Next, I assume that the common factors follow a VAR which is assumed to be invertible and given by

$$D(L)f_t = \epsilon_t \tag{4}$$

with $D(L)$ a matrix polynomial of coefficients in the lag operator L and ϵ_t a vector of zero-mean reduced form shocks. As in Forni & Gambetti (2010), the number of structural shocks u_t driving the common factors f_t need not be equal to the number of factors r. Instead, it is possible that the number of deep shocks q is smaller than the number of factors, i.e. $q \leq r$. This is captured by a rotation R such that $\epsilon_t = Ru_t$.

$$D(L)f_t = Ru_t \tag{5}$$

The vector u_t is assumed to be vector white noise and holds the deep shocks in the economy. One of these shocks is assumed to be a monetary policy shock which can be identified in the following way.

2.2 Identification of the Monetary Policy Shock

Beginning from equation 5, invertibility provides that the factors can be written as

$$f_t = D(L)^{-1} R u_t \tag{6}$$

For any orthogonal matrix H equation 6 can be rewritten as

$$f_t = D(L)^{-1} R H' H u_t = D(L)^{-1} S H u_t = S(L) H u_t = H(L) u_t \tag{7}$$

which is the relation used in model 1 with $S = RH'$, $S(L) = D(L)^{-1}S$ and $H(L) = S(L)H$. Similar to the SVAR literature, the model is not identified without further assumptions on the matrix H. The objective is to identify a monetary policy shock in the same way that traditional recursive SVARs, as for example, Christiano et al. (1999) do – by imposing that the monetary policy shock does not contemporaneously affect slow moving variables such as GDP or prices, but can contemporaneously affect financial variables and interest rates. However, it is not possible to impose such restrictions on the factors f_t directly as they lack a concrete interpretation. Hence, I impose the restriction with a view to the common component χ_{it} of the macro panel instead.

Using equations 7 and 3 we obtain the structural IRFs of the common components $b_i(L)$ from

$$\chi_{it} = a_i D(L)^{-1} R H' H u_t = a_i S(L) v_t = c_i(L) v_t = c_i(L) H u_t = b_i(L) u_t \tag{8}$$

where $c_i(L) = a_i S(L)$ and $b_i(L) = c_i(L)H$. Therefore, the IRFs in $b_i(L)$ have to fulfill the identifying assumptions. In the Cholesky case preferred in this paper, this is achieved by selecting exactly $m = q$ variables from x_{it} and ordering them according to the assumed recursive structure. Since,

$$B_m(L) = C_m(L) H \tag{9}$$

the restrictions are imposed contemporaneously by setting

$$H = C_m(0)^{-1} G_m \tag{10}$$

where $C_m(0)^{-1}$ is the inverse of the non-structural IRF on impact and G_m is the lower triangular Cholesky factor of $C_m(0)C_m(0)'$.

2.3 Quantile Regressions

The second step consists in running a predictive regression of the variables of interest – GDP growth and inflation – on the respective sets of predictors, the factors in model 1. While OLS regressions are concerned with the mean response of the dependent variable to changes in different explanatory variables, quantile regression (Koenker & Bassett, 1978) is concerned with the median and other quantiles of the distribution of the dependent variable. In that sense, it is more robust to potential outliers that may distort the mean and, in addition, allows the study of the drivers of more extreme outcomes of GDP growth and inflation, such as $\tau = 0.05, 0.1, 0.9, 0.95$, which are the common quantiles studied in the literature on "growth-at-risk".

Koenker (2005) shows that $\hat{\beta}\tau$ solves the minimization problem of the conditional sample quantile function $Q_{t,\tau}$ as in

$$\min_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} \rho_\tau(y_i - x_i'\beta) \tag{11}$$

where p is the number of regressors, ρ is the loss function $\rho_\tau(u) = u(\tau - \mathbb{1}(u < 0))$ with $u_i = y_i - x_i'\beta$ and y_i being the dependent variable. This is a linear-programming problem that can be solved using simplex methods. Hence, the quantile regression that I run is given by

$$Q_{t+h,\tau} = \beta_\tau(L)Wf_t \tag{12}$$

2.4 Combining Quantile Regression and Dynamic Factor Model

From the estimated coefficients $\widehat{\beta_\tau(L)}$ and given the selection of regressors Wf_t I can pin down the impulse response functions of the conditional quantiles of GDP growth and inflation for a monetary policy shock. This allows an assessment of whether the uncertainty around future economic conditions is reduced, increased or not affected by policy action. Moreover, since I study the three quantiles indivudually it may be that downside risk (as measured by Q05) is affected differently than upside risk (as measured by Q95) by the shock.

The IRFs are given by

$$IRF_{\tau}^{GDP} = \widehat{\boldsymbol{\beta_{\tau}^{GDP}}(L)} W_{GDP} H(L) \tag{13}$$

$$IRF_{\tau}^{\pi} = \widehat{\boldsymbol{\beta_{\tau}^{\pi}}(L)} W_{\pi} H(L) \tag{14}$$

Since I include all factors as regressors, the matrices W are identity matrices in both cases. To repeat, the matrix polynomial $H(L)$ is given by $D(L)^{-1}SH$ as described in equation 7.

3 Data and Model Specification

The data set for the factor model is the collection of US quarterly macroeconomic variables of McCracken & Ng (2020), which is gathered by FRED of the Federal Reserve Bank of St. Louis. The data set contains 248 variables from Q1:1959 to Q1:2021 and is built on top of the data set used in Stock and Watson (2012). I restrict my analysis to the time period between Q1:1959 and Q4:2019 to avoid the recent COVID-19 induced recession. The dependent variables for the prediction exercise whose quantile impulse responses I study are real GDP growth (GDPC1) and Personal Consumer Expenditures Price Index inflation (PCECTPI). Four quarter ahead growth rates ($h = 4$) are computed as the difference of the logs between t and $t - 4$ and multiplied by 100.

The first choice to be made for model specification is the transformation of the variables in the data set. While some authors prefer to keep transformation minimal for comparability to classical SVARs (e.g. Bernanke et al., 2005 or Forni & Gambetti, 2010), McCracken & Ng (2020) examine how using the default transformations provided with the data set compares to using a transformation based on repeated unit root tests. The authors find that the best performing factor prediction models for real GDP exclusively use the original transformation which leaves some variables non-stationary, whereas for prices most, but not all, top performing models use the transformation based on unit roots. The performance differences in the latter are relatively small. Since I estimate the factors using principal components which can be thrown off in small samples by non-stationary data (Alshammri & Pan, 2021), I use a transformation that ensures stationarity of all series as tested using augmented Dickey-Fuller tests, except for the Federal Funds Rate which is kept in levels for better comparability to the existing SVAR and DFM literature on monetary policy. This allows me to interpret the monetary policy shock in terms of basis points. The appendix reports all variables that are included, their transformations and the variation explained by the static factors.

The data set contains missing values and outliers. Outliers are problematic for principal components analysis and are dealt with as suggested in McCracken & Ng (2020) by omitting values whose difference from the median is larger than ten times the interquartile range. These outliers are replaced by missing values. To deal with the missing values I use the expectation maximization algorithm of McCracken & Ng (2020) which comes with the FRED-MD monthly data set. The algorithm imputes missing values by extracting factors and loadings to recompute the implied common component of the data. If it is close to the original common component, the algorithm stops, if not, the newly imputed values are used to extract new factors and loadings until convergence. I tweak the procedure described in McCracken & Ng (2020) by using the selection criterion of Alessi et al. (2010) which is a refined version of the criterion of Bai & Ng (2002) to determine the optimal number of factors at each iteration. The optimal number of static factors to include in f_t is determined to be between $r = 6$ and $r = 13$. Given this relatively wide range I rely on the screeplot in Figure 2 which flattens roughly at $r = 6$. Indeed, after $r = 6$ the increases in overall explained variances are smaller than 3% from adding additional factors. Moreover, I check simple OLS regressions of all variables in the scaled data set on the six factors and the maximum number of factors of thirteen to compare the goodness-of-fit. For the variables used for identification the gains in fit from six to thirteen factors are relatively small, so I opt for the more parsimonious specification. In the robustness section I present all results for all specifications of r between five (to check also fewer factors) and thirteen. Significant differences compared to the $r = 6$ case are only observed at the lower end when $r = 5$ which suggests that too few factors are used. Hence, for the remainder of this paper, $r = 6$, which is also the number of factors found in the comparable quarterly data set by Stock & Watson (2012). Factors are estimated using the principal component estimator.

In a second step, I determine the lag length to be used in the FAVAR using the Hannan-Quinn (HQ) information criterion. This suggests a lag length of $p = 2$. A robustness check using the Akaike information criterion (AIC) delivers $p = 3$, which has no important effect on the results, so I also opt for the parsimonious approach. Finally, the number of dynamic factors or primitive shocks is determined using the criterion of Bai & Ng (2007), which suggests to set $q = 4$. For the purpose of identification $m = q = 4$ identifying variables are required. I use the variables for real GDP (GDPC1), GDP price deflator (GDPCTPI), the Federal Funds Rate (FEDFUNDS), and the USD-CAD exchange rate, in that order. This setup is similar to Forni & Gambetti (2010) and implies that the monetary policy shock is the third one in u_t. It does not drive GDP or prices on impact, but can contemporaneously influence the exchange rate.

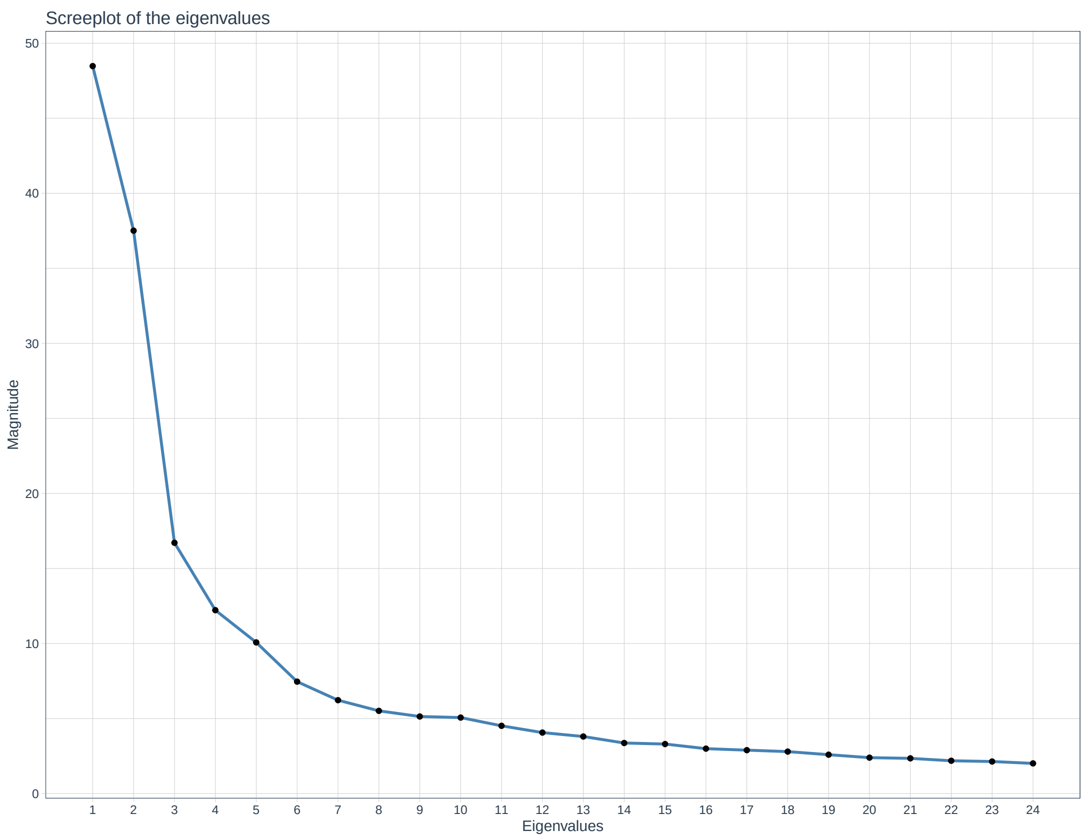


Figure 2: Magnitudes of the estimated eigenvalues of the covariance of the data matrix.

Finally, in the quantile regression exercise I follow Adrian et al. (2019) and set $\tau = 0.05, 0.5, 0.95$ to capture the tail behavior of the conditional distribution of real GDP growth and inflation. The median is useful for comparison to the mean and to compute descriptive statistics such as the Kelley skewness of the conditional distributions implied by the estimates. The forecast horizon is $h = 4$ quarters. For the estimation of the quantile regression part in model 1, there are $r = 6$ static factors available as potential predictors, as well as their lags. To keep the model relatively simple I do not include additional regressors and use only the first, respectively the fourth lag for the predictive regressions. Hence, the matrix W in equation 1 is an identity matrix and $\beta_\tau(L)$ is just β_τ.

4 Results

This section presents the results from the dynamic factor model shock identification, the quantile forecasting exercise and the impulse responses of the conditional quantiles to the monetary policy shock.

4.1 DFM Monetary Policy Shock

Figure 3 shows the impulse responses of the four identifying variables to the Cholesky monetary policy shock, scaled to a 50 basis points increase in the Federal Funds Rate. The shock decreases the levels of GDP and prices and affects the exchange rate between US and Canadian Dollars in line with the overshooting principle of Dornbusch (1976). The classical "price puzzle" and "delayed-overshooting puzzle" do not obtain and the results are in line with Cholesky-DFM results presented in Forni & Gambetti (2010) as well as Kerssenfischer (2019).

4.2 Quantile Regression Predictions

Figure 4 shows the time-varying quantiles as predicted using the factor-augmented quantile approach from equation 12 for real GDP growth and PCE inflation at the four-period horizon. As distinct from other forecasts in the literature, the median and Q95 predictions of GDP growth appear more volatile, while the high volatility of Q05 that is observed in Adrian et al. (2019) and Forni et al. (2021) is also featured as shown in Table 1. These differences may be due to conditioning on a larger set of information. In fact, for every quantile of size 0.05 between 0.05 and 0.95 at least three of the factors are highly significant predictors. Moreover, the factors do not suffer from collinearity by construction which may influence results as well. Especially the downside growth risk increases around the recessionary periods in the mid 1970s, early 1980s, early 1990s and 2000s as well as the

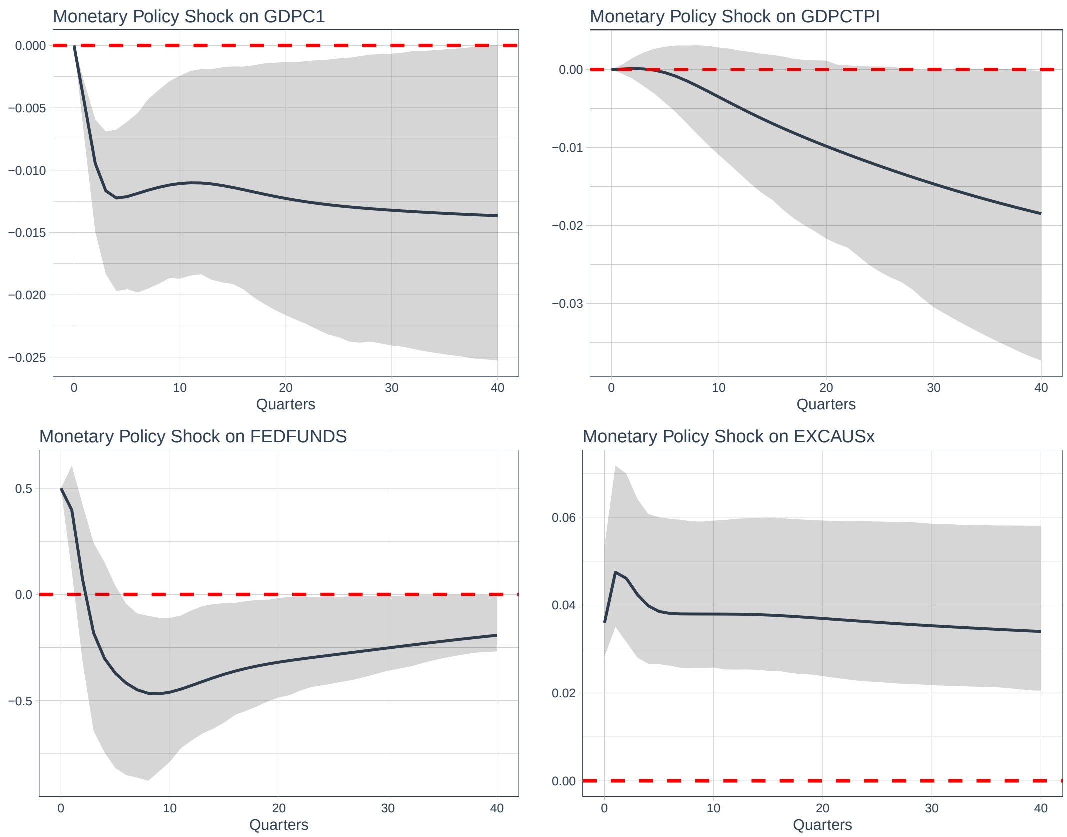


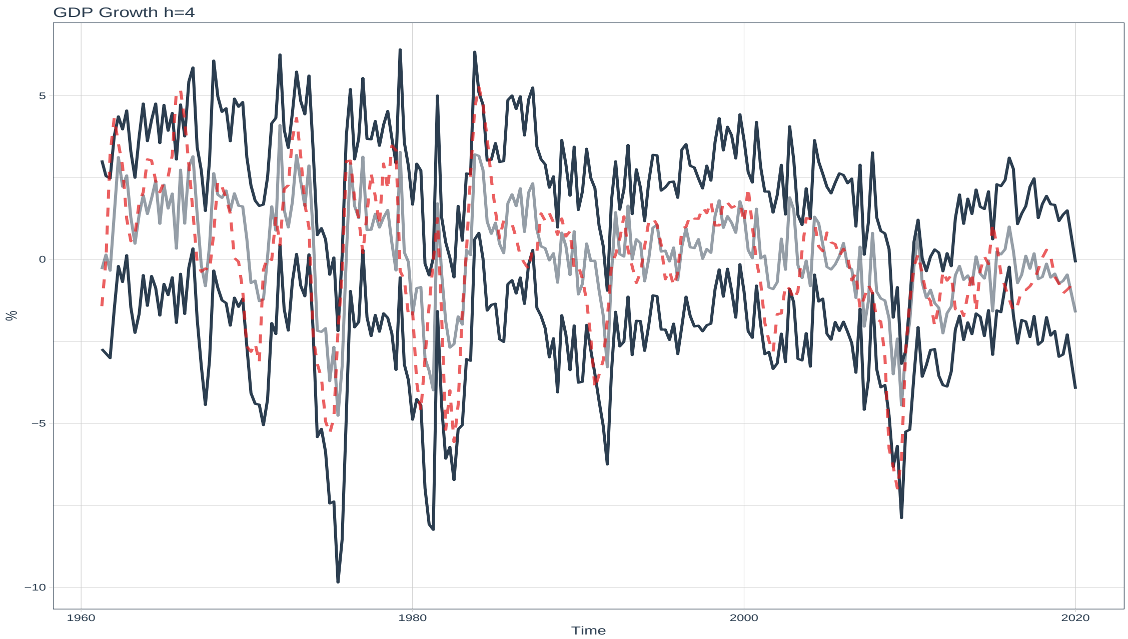
Figure 3: Point estimates of the impulse responses of the identifying variables to a contractionary 50bp monetary policy shock with bootstrapped 90% confidence intervals.

Great Recession are clearly visible. However, the model predicts a very rapid recovery of downside growth risk and median risk after the 2007-2008 recessionary episode. Moreover, The interquantile range decreases markedly with the Great Moderation as shown in Figure 5. During most periods the expected distribution of GDP growth was negatively (left) skewed which suggests a longer left tail relative to the right tail.

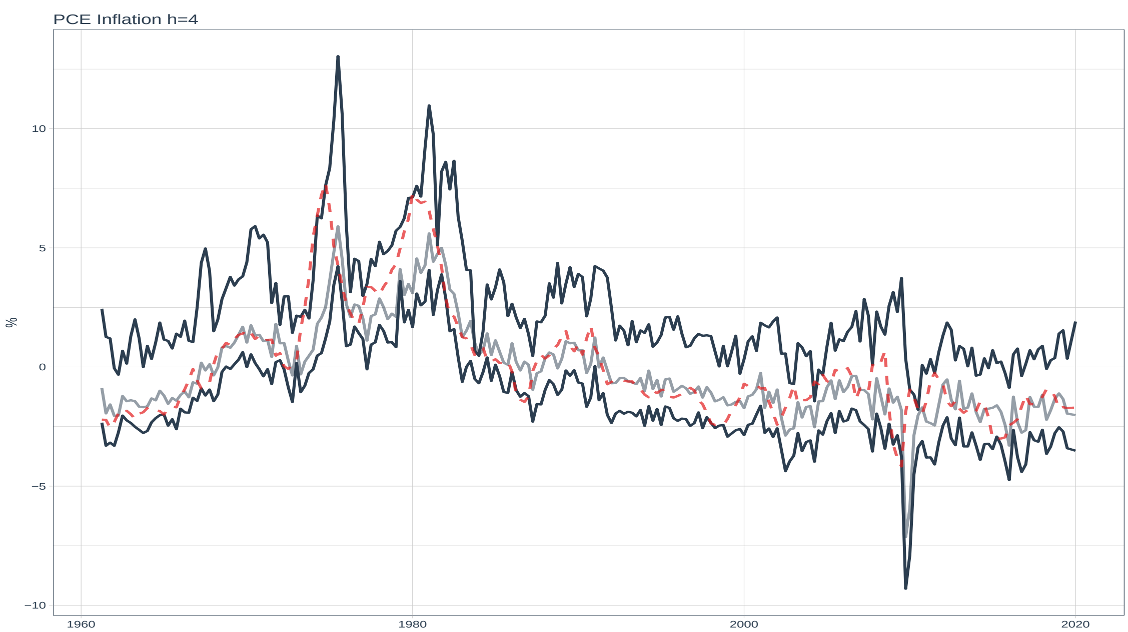
As for the predicted quantiles of PCE inflation, a downward trend is observable with sharp increases in the right tail in the recessions of the 1970s and 1980s but a pronounced drop in the lower tail during the Great Recession. The fall in the interquantile range is not as clear cut as with GDP growth, but during the period of the Great Moderation, the spread measure is less volatile up to the Great Recession. According to the non-normalized Kelley skewness, PCE inflation predictions were almost always positively (right) skewed, which suggests a longer right tail with more extremely high predictions. By a similar token, Table 1 shows that the upper tail was much more volatile than the lower tail over the considered horizon.

Metric	ΔGDP h=4	ΔPCE h=4
σ_{05}	1.78	1.95
σ_{50}	1.53	1.91
σ_{95}	1.70	2.45
$\rho_{5,95}$	0.83	0.87

Table 1: Standard deviations of predicted quantiles and correlation coefficient for the extreme quantiles.



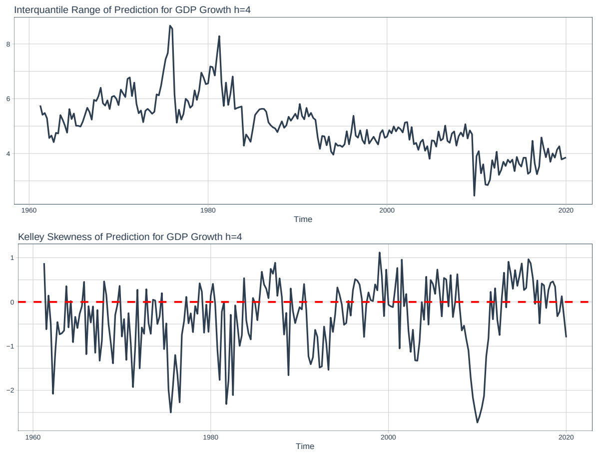
(a) ΔGDP h=4



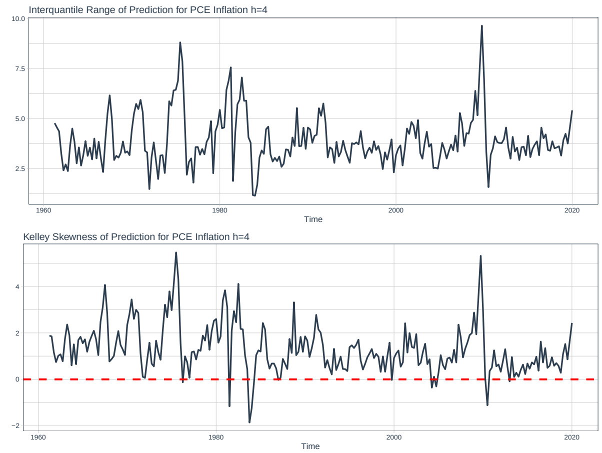
(b) PCE Inflation h=4

Figure 4: h=4 predicted quantiles (Top line: Q95, Light grey line: Q50, Bottom line: Q05) of GDP growth and PCE inflation. Red dashed line is the realized series.

Model evaluation: Evaluating the fit of the model is not straightforward in the case of quantile regressions, as no direct equivalent to the usual R^2 measure and its adjusted



(a) Moments ΔGDP h=4



(b) Moments ΔPCE h=4

Figure 5: Top graph is interquantile range over time (Q95-Q05), bottom graph is non-normalized Kelley skewness (Q95+Q05-2×Q50).

versions exists. Koenker & Machado (1999) propose a metric called R^1. This statistic evaluates the loss implied by the residuals of the quantile regression (V1) at each quantile against the alternative of running the regressions only on a constant (V0).

$$V1_\tau = \sum_{i=1}^{T} \rho_\tau(u_i^1)$$
$$V0_\tau = \sum_{i=1}^{T} \rho_\tau(u_i^0)$$
$$R^1 = 1 - \frac{V1}{V0}$$

The values will be close to one if the loss of the V0 model is large relative to the V1 model. Table 2 reports the R^1 measures for the quantile regressions of GDP growth and the inflation measure at the prediction horizon of four. The performance of the model over the simple model is very good for the extreme quantiles and for the median. Especially for PCE inflation the inclusion of additional information in the predictive regressions is beneficial. The results suggest that the factor approach is suitably information rich.

To complement the in-sample R1 measure I compute predictive scores for the prediction models in a pseudo out-of-sample exercise. Using a rolling window I forecast a total of

Quantile R1	ΔGDP h=4	ΔPCE h=4
Q_5	0.9853	0.9940
Q_{50}	0.9413	0.9634
Q_{95}	0.9859	0.9838

Table 2: Measures of fit computed for the quantiles of interest.

35 periods with a forecast horizon of $h = 4$ using first the factors approach and second a QAR(1) model with only the fourth lag of the forecasted variable as a regressor. Then I fit a kernel estimate to the forecasted quantiles and compute the implied density of the actually realized value. As is shown in Figure 6 the factor approach generates higher densities in most of the out-of-sample period compared to the QAR(1) model for the GDP growth forecasts and in more than half the cases for the traditionally difficult to forecast PCE inflation which suggests that the approach can be useful to generate suitable forecasts.

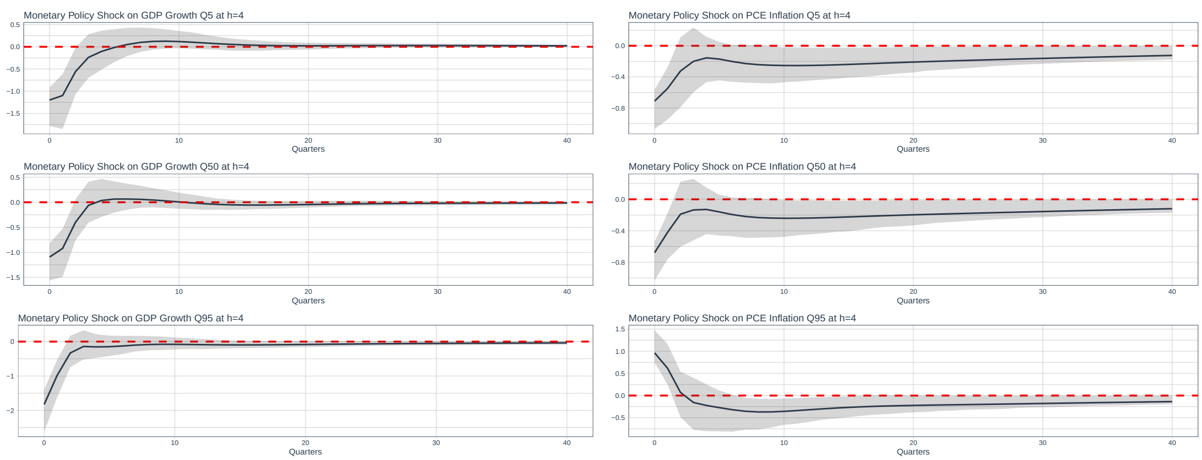
(a) Predictive scores: ΔGDP h=4

(b) Predictive scores: PCE inflation h=4

Figure 6: h=4 predictive scores. Black line is factor model, red line is QAR(1).

4.3 Quantile Impulse Responses

Figure 7 shows the responses of the predicted conditional quantiles to the 50bp contractionary monetary policy shock at horizon $h = 4$. The responses can be interpreted as shifts of the tails (and median) of the predicted distribution after the shock, away from their long-term mean. All forecasted quantiles for GDP growth decrease on impact and return to zero after around five quarters. The initial reaction of downside risk is significantly weaker than for upside risk. The median result is in line with the mean effect depicted in Figure 3. This suggests that the predictions for GDP growth become more pessimistic and the distribution shifts to the left. As distinct from this, we observe a clear spreading of the predicted distribution of PCE inflation, which is a result of the left tail moving further left and the right tail moving further right. Since the median also moves to the left (again in line with the mean results), this implies a skewing of the distribution as well as a spreading – the right tail becomes longer.



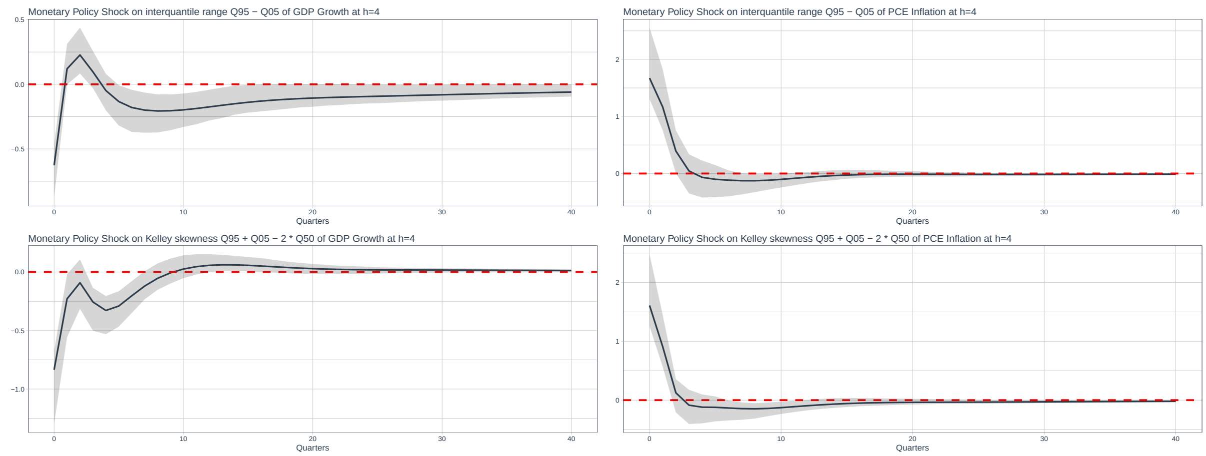
(a) ΔGDP h=4

(b) PCE Inflation h=4

Figure 7: Point estimates of the impulse responses of the quantiles of h=4 GDP growth and PCE inflation to a 50bp monetary policy shock. Bootstrapped 90% confidence intervals. Top graph is Q5 response, middle graph is Q50 response, bottom graph is Q95 response.

From the responses of the conditional quantiles we can compute how the contractionary monetary policy shock affects the interquantile range and the (Kelley) skewness in each case. The predicted GDP growth distribution gets compressed as the right tail moves futher left than the lower tail, whereas the expected PCE inflation distribution spreads out as described above. The skewness of GDP growth decreases relative to the steady state, for PCE inflation it becomes more positive, as shown in Figure 8.

To complement the above results and better visualize the reaction of the *future* conditional distributions of GDP growth and inflation to monetary policy shocks that



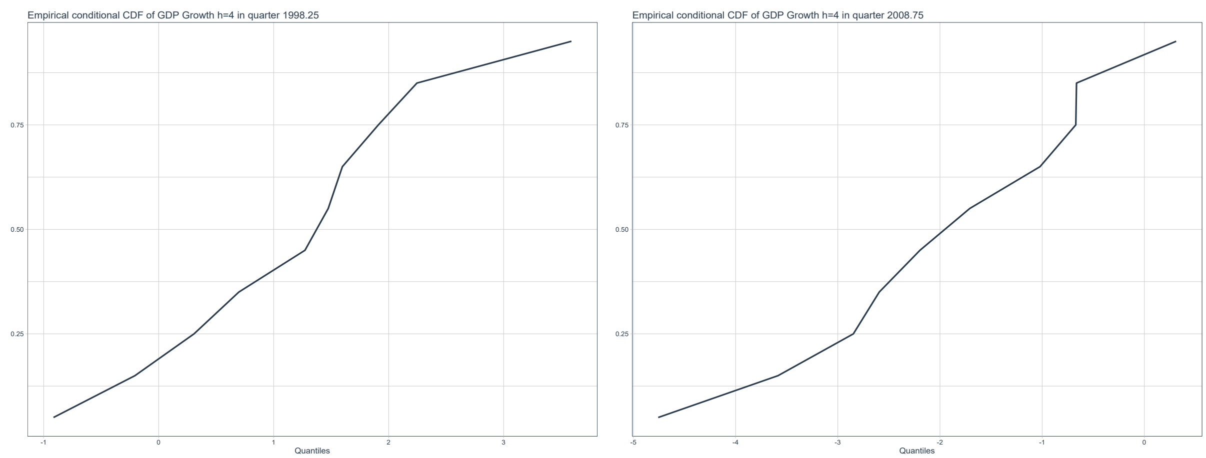
(a) ΔGDP h=4 (b) PCE Inflation h=4

Figure 8: Response of interquantile range (top) and non-normalized Kelley skewness (bottom) to the 50 bp contractionary monetary policy shock. All responses are at $h = 4$. 90% bootstrap intervals shaded in grey.

happen *today*, I use a method which combines the quantile impulse responses with kernel density fitting. First, I compute the average forecasted quantiles for 19 quantiles between 0.05 and 0.95 of each variable to construct a starting point for the impulses. IRFs are usually portrayed as deviations from some steady state or mean and so this is a natural starting point. Then I compute the quantile IRFs for each variable over a horizon of 40 periods. Suppose the IRFs were just the standard IRFs of an SVAR model. Then the reaction of GDP growth would be in percentage points. Hence, I treat the quantile IRFs as the percentage point changes in the location of each quantile. For example, if the initial 5^{th} quantile is at -2% and the response to the shock on impact is -0.2%, then the new 5^{th} quantile is at -2.2%. As the quantile IRFs revert back to zero, the distribution also moves back to its initial location and shape.

Given the initial and shocked quantiles of each variable, the distributions are obtained by fitting adaptive kernel densities to them. The procedure follows Portnoy & Koenker (1989) with a Gaussian kernel and the initial bandwidth computed by the rule of thumb in Silverman (1986, p.48). This initial bandwidth is used to get a sense of regions of low density in the distribution which should receive larger bandwidths. In a second step, the adaptive procedure allows the "bumps" of the fitted densities to have different bandwidths (Silverman, 1986, p. 101). The sensitivity of the adaptive bandwidths is set to $\alpha = 0.5$

as suggested in Silverman (1986, p. 102).[2] This adaptive procedure is favored over fitting parametric distributions to the predicted quantiles since plots of the implied predicted CDFs (Figure 9) associated with GDP growth and inflation suggests the presence of multiple points of inflection. Points of inflection in the CDF occur at the modes of the PDF and so a fitting method should allow for multiple modes. The number of modes that eventually emerge depends on the bandwidth parameters of the kernel densities, which is flexibly handled by the adaptive procedure.



(a) 1998:Q1

(b) 2008:Q3

Figure 9: Implied CDFs of GDP growth h=4. Left panel: CDF is roughly S-shaped. Right panel: CDF has multiple points of inflection.

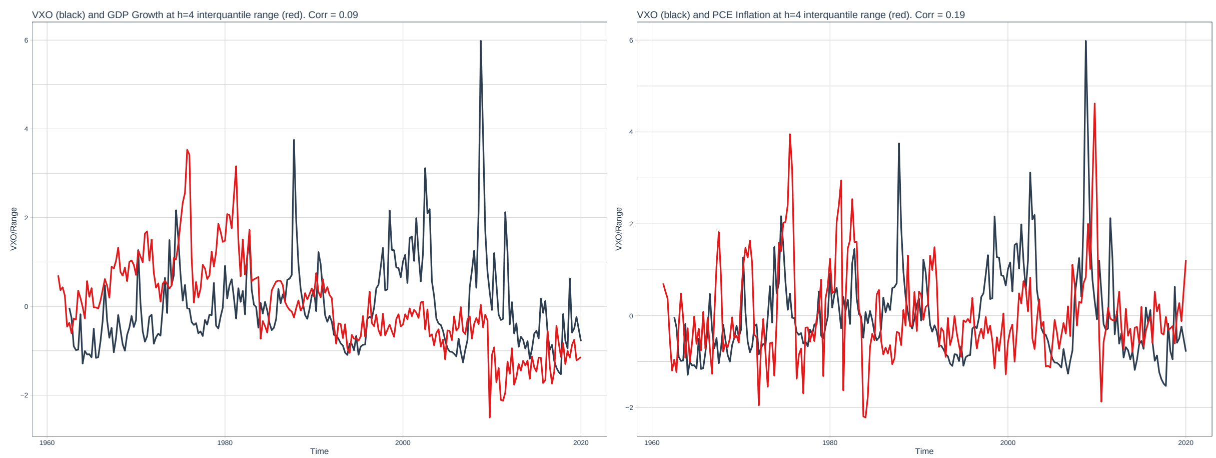
The distributional changes are depicted in Figure 1. In response to the monetary policy shock, the predicted distribution of GDP growth becomes bimodal on impact. A large probability mass is located left of the median distribution. The bimodality recedes after around two quarters and the distribution is nearly back to its initial shape after eight quarters. Interestingly, this result does not obtain for an expansionary shock. In the case of inflation, the initial distribution is slightly right-skewed. In line with the diescription above, this skewness is further increased by the monetary policy contraction, but reversed by an expansion. Moreover, we can see a second mode emerging and clearly observe the spreading of the distribution. On the other hand, the distribution becomes very narrow after the expansionary shock.

[2]The procedure is adapted from the R package quantreg, function *akj*.

5 Discussion

The results presented above suggest that there are important effects of monetary policy on the entire distribution of future GDP growth and prices, especially the extreme quantiles. Firstly, the expected distributions shift location in accordance with the empirical and theoretical literature – an exogenous increase in the Federal Funds Rate leads us to forecast more negative values for both GDP growth and inflation. However, the distributional IRFs show that there are significant effects beyond the location shift. First of all, a second accumulation point for forecasted GDP growth emerges after the shock. In fact, multimodality in forecasted distributions is a feature increasingly documented in the empirical macro literature, for example in Forni et al. (2021) or Adrian et al. (2021). The authors of the latter paper show that around periods of recession the forecasted distribution of GDP growth exhibits multiple modes which can be persistent in the presence of poor financial conditions. The authors conjecture that such multimodality can be resolved by good policy and exacerbated by bad policy. In this paper, I show that if the starting distribution is unimodal, a 50bp increase in the Federal Funds Rate leads to the emergence of a second mode in the conditional distribution of GDP growth, that is, contractionary policy can lead to the emergence of the multimodality observed during recessionary periods. Moreover, there is substantial probability associated to inflation outcomes in the right tail of the distribution which would imply that policy, instead of reducing future inflation, runs a relatively sizable chance of increasing it instead.

The quantile regressions in this paper represent a way of quantifying the expected probabilities of future values of GDP growth and inflation. Since these future values are unknown today, economic agents, professional forecasters or policymakers may want to consider a range of possibilities rather than a single expected value (a point forecast) to adjust their behavior today. As documented in the results section, a decision-maker would assign higher probability masses to extreme inflation events and very poor growth outcomes. It is well known that such an spread in outcome uncertainty is undesirable for risk-averse agents who may in turn resort to more defensive spending behavior. Therefore, given that contractionary monetary policy increases the range of possible outcomes, this may activate an "options" channel (Bernanke, 1983) which induces agents to save instead of invest which contributes to lower growth. In Figure 10 I compute the correlation between the estimated interquantile ranges of the variables of interest and the VXO which is a common uncertainty measure computed from stock price volatility. It is included in the quarterly data set. High volatility is associated with periods of higher uncertainty. The correlations for the GDP growth interquantile range with the VXO is relatively low at 0.09. For the PCE and measure the correlation increases to 0.19 respectively, which is still relatively low. These correlations, however, increase at shorter horizons.



(a) ΔGDP h=4/VXO ($\rho = 0.09$) (b) PCE Inflation h=4/VXO ($\rho = 0.19$)

Figure 10: VXO (black) and interquantile ranges (Q95-Q05, red) implied by the quantile regression predictions at h=f with correlation coefficients.

To shed more light on the importance of monetary policy as a driver of the fluctuations in the individual quantiles, I perform forecast error variance decompositions (FEVD) based on the quantile IRFs for the four primitive shocks. This gives some information whether the movement in the quantiles is largely attributable to monetary policy or rather to the other three unidentified shocks. The results are reported in Table 3. The monetary policy shock explains a sizable share of the variation in the conditional quantiles of GDP growth, expecially of the upper tail. This suggests that policymakers should indeed take the ramifications of their decisions for growth risks into account. Furthermore, the short term importance of monetary policy for the predicted distributions of inflation is around 15% and decreases by a small amount over the horizon of 40 periods. Its contribution is greatest for the lower tail of forecasted inflation. The results for the medians are roughly in line with results obtained for the mean in the literature using DFMs.

The emergence of additional modes in the conditional distributions of GDP growth and

	Horizon	Q05	Q50	Q95		Q05	Q50	Q95
ΔGDP h=4	j=0	0.14	0.15	0.36	ΔPCE h=4	0.15	0.16	0.15
	j=1	0.16	0.17	0.31		0.15	0.13	0.11
	j=8	0.12	0.12	0.23		0.12	0.09	0.07
	j=40	0.12	0.12	0.22		0.14	0.12	0.10

Table 3: Forecast error variance decomposition of GDP growth quantiles and inflation quantiles to a monetary policy shock at different forecast horizons, j periods after the event of the shock.

inflation can be seen as evidence that monetary policy can lead to multiplicity of economic equilibria. Such multiplicity can arise for example if the central bank's reaction to inflation is insufficiently strong in a New-Keynesian framework (Lubik & Schorfheide, 2004). This leads to an indeterminate propagation of monetary policy with potentially many outcomes receiving positive probability. Lubik & Schorfheide (2004) show that depending on the selected equilibrium contractionary shocks can decrease growth by different magnitudes, which is in line with the two modes in the predicted GDP growth distribution representing different equilibria where the contraction has different intensities. Similarly, they show that depending on the selected equilibrium prices can in fact increase shortly after the introduction of the contractionary policy, which is observable in the right mode of Figure 1 in panel c).

Another finding of this paper is the lack of additional modes appearing in the case of expansionary shocks. According to this observation, the emergence of multiple equilibria may not be a result of the intensity of the monetary policy reaction, but rather a result of the direction of policy. To complement this idea, the Appendix shows the reactions when the policy shock is 75 basis points and 25 basis points. In both cases, the emergence of multiple equilibria only happens for the recessionary shocks. In principle, the entire QR-DFM set up is linear, all the way down to the IRFs so that the difference between a positive and a negative shock only lies in the sign of the IRF. Responses to negative shocks are the mirror images about the x-axis of the responses to positive shocks. However, the fitting procedure to the estimated quantiles is highly-nonlinear which explains the emergence of asymmetric distributions in response to the different monetary policy shocks. However, a potential weakness of this approach is quantile crossing, i.e. the potential for, no use an example, the 25th quantile to have a smaller value than the 20th quantile. This crossing problem is not unusual in quantile regressions with realtively small samples or misspecified models. In essence, it provokes the implied CDF to be non-monotone. There are procedures to resolve this To summarize, monetary policy plays a sizable role in driving downside and upside risks around GDP growth and inflation. As the economy contracts in response to the shock, this leads to forecasts which attribute increased probabilities to worse outcomes in the future and could open up important transmission channels for policy through the increase in uncertainty and the emergence of additional equilibria.

6 Conclusion

In this paper I propose a new method to assess how monetary policy shocks influence the conditional distributions of GDP growth and inflation in the US by combining a dynamic factor model with predictive quantile regressions. The methodology is general enough to study the response of any variable included in the data set. I show that the predictive distributions of both variables shift to the left in response to the shock and that downside growth risks are exacerbated by the shock even more than upside risks are reduced. Moreover, the short horizon predicted inflation distribution spreads out. This suggests that monetary policy may open up uncertainty channels which can have real consequences. While the policy achieves a reduction in median expected inflation, it creates potential for more extreme future inflation outcomes as well. Moreover, I show that contractionary shocks can create additional modes in the expected distributions of interest, which lead to significant increase in the probability that growth will decline by much more than on average and even carries the potential for the policy being ineffective in reducing inflation at all. The finding suggests that the multiplicity of equilibria, which theory links to weak policy responses, may only obtain when policy is not expansionary.

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Software

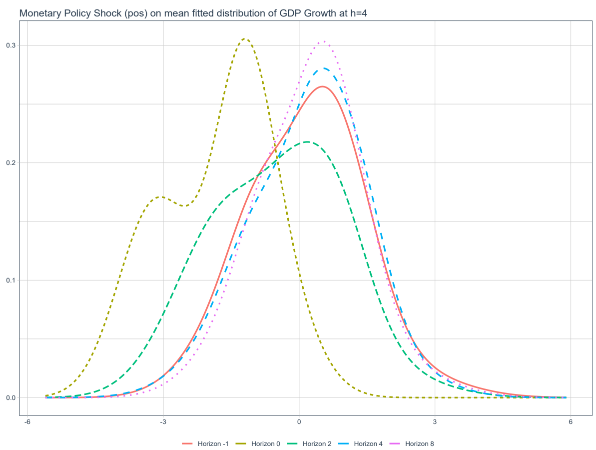
The figures, tables and statistics are calculated using the R language. The packages which are used are reported in Table 4.

Package	Reference
aTSA	https://cran.r-project.org/web/packages/aTSA/index.html
corrplot	https://cran.r-project.org/web/packages/corrplot/index.html
cowplot	https://cran.r-project.org/web/packages/cowplot/index.html
doBy	https://cran.r-project.org/web/packages/doBy/index.html
ggplot2	https://cran.r-project.org/web/packages/ggplot2/index.html
ggpubr	https://cran.r-project.org/web/packages/ggpubr/index.html
gridExtra	https://cran.r-project.org/web/packages/gridExtra/index.html
kableExtra	https://cran.r-project.org/web/packages/kableExtra/index.html
knitr	https://cran.r-project.org/web/packages/knitr/index.html
magic	https://cran.r-project.org/web/packages/magic/index.html
magick	https://cran.r-project.org/web/packages/magick/index.html
maotai	https://cran.r-project.org/web/packages/maotai/index.html
matlib	https://cran.r-project.org/web/packages/matlib/index.html
minpack.lm	https://cran.r-project.org/web/packages/minpack.lm/index.html
MonoInc	https://cran.r-project.org/web/packages/monoinc/index.html
mvtnorm	https://cran.r-project.org/web/packages/mvtnorm/index.html
nowcasting	https://cran.r-project.org/web/packages/nowcasting/index.html
quantreg	https://cran.r-project.org/web/packages/quantreg/index.html
Rcpp	https://cran.r-project.org/web/packages/Rcpp/index.html
readr	https://cran.r-project.org/web/packages/readr/index.html
readr	https://cran.r-project.org/web/packages/readr/index.html
rje	https://cran.r-project.org/web/packages/rje/index.html
sn	https://cran.r-project.org/web/packages/sn/index.html
svMisc	https://cran.r-project.org/web/packages/svMisc/index.html
tidyquant	https://cran.r-project.org/web/packages/tidyquant/index.html
tidyverse	https://cran.r-project.org/web/packages/tidyverse/index.html
tsDyn	https://cran.r-project.org/web/packages/tsDyn/index.html
vars	https://cran.r-project.org/web/packages/vars/index.html

Table 4: R packages used for the paper.

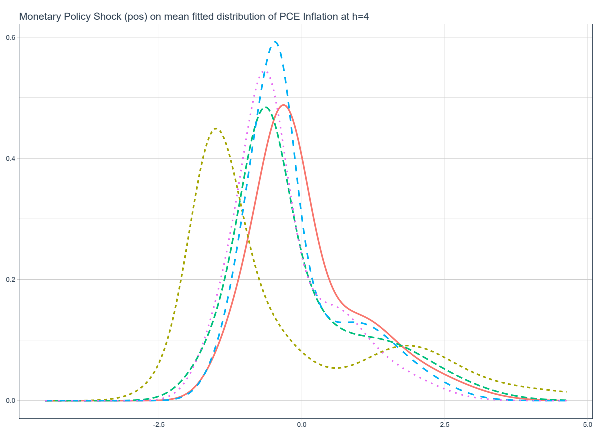
7 Appendix

7.1 Shock sign asymmetry

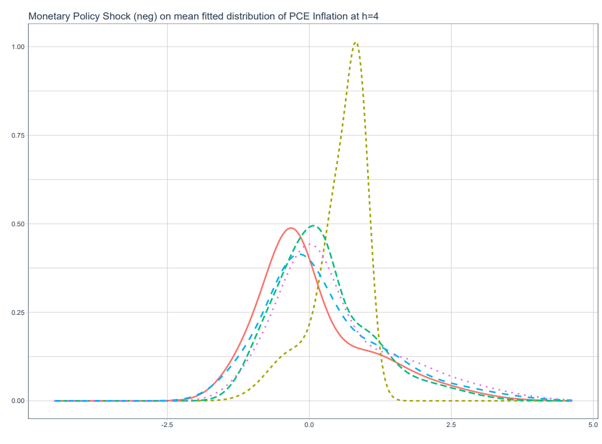


(a) ΔGDP h=4 (contract.)

(b) ΔGDP h=4 (expans.)

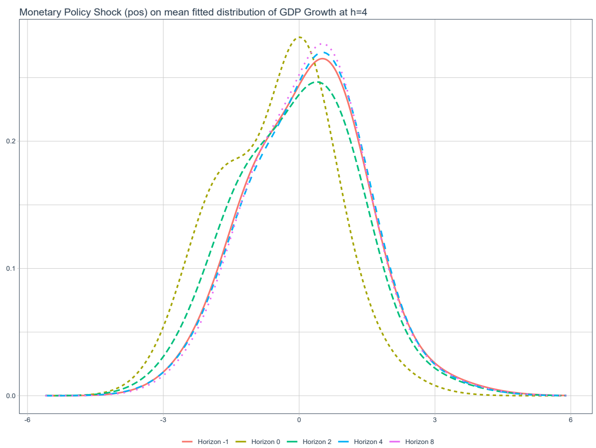


(c) PCE Inflation h=4 (contract.)

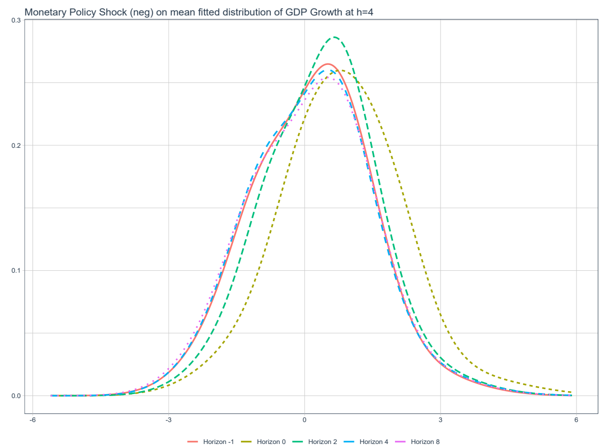


(d) PCE Inflation h=4 (expans.)

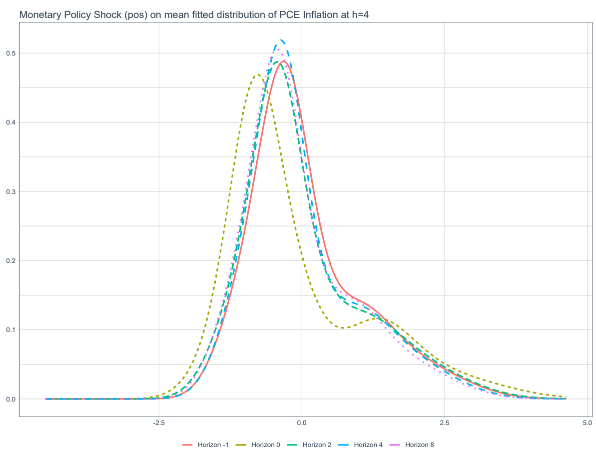
Figure 11: Response of demeaned fitted distribution to monetary policy shocks. Steady state distribution is fitted to the average predicted distribution. All forecast distributions are at h=4. Red line is initial distribution. Mustard line is response at $j = 0$. Green line is response at $j = 2$. Turquoise line is response at $j = 4$. Purple line is response at $j = 8$. 75 bp shock.



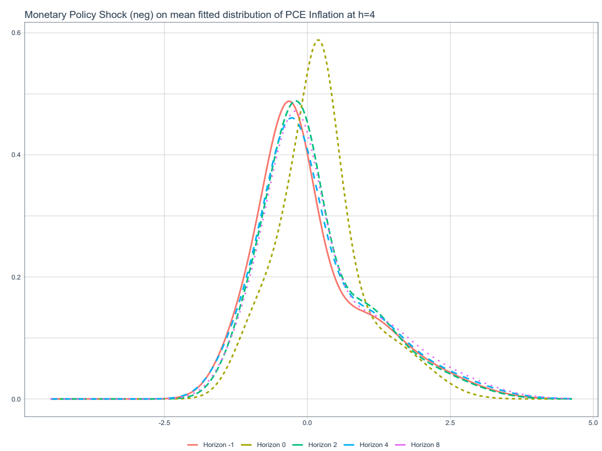
(a) ΔGDP h=4 (contract.)



(b) ΔGDP h=4 (expans.)



(c) PCE Inflation h=4 (contract.)



(d) PCE Inflation h=4 (expans.)

Figure 12: Response of demeaned fitted distribution to monetary policy shocks. Steady state distribution is fitted to the average predicted distribution. All forecast distributions are at h=4. Red line is initial distribution. Mustard line is response at $j = 0$. Green line is response at $j = 2$. Turquoise line is response at $j = 4$. Purple line is response at $j = 8$. 25 bp shock.

7.2 Robustness

I conduct some robustness checks to verify the validity of the approach used in the main specification. First, I identify the monetary policy shock by using between five and thirteen static factors as suggested by the criterion of Alessi et al. (2010). The same approach is used for the computation of the quantile IRFs. Including more factors in the model increases the information content, but may also lead to overfitting the quantile regression and loss of degrees of freedom. Second, I estimate the monetary policy shock with $p = 3$ lags for the VAR in equation 5 as suggested by the AIC.

7.2.1 Changing the number of static factors

Figure 13 shows the responses of the identifying variables to the monetary policy shock for different numbers of static factors $r \in [5, 13]$. Qualitative differences arise only in the case of using $r = 5$ factors. It is well understood that using too few factors leads to poor identification of the shock of interest. For ten and eleven factors the result for the exchange rate IRF changes which may be a result of capturing a lot of irrelevant information in those factors. In sum, results are very similar for intermediate numbers of factors.

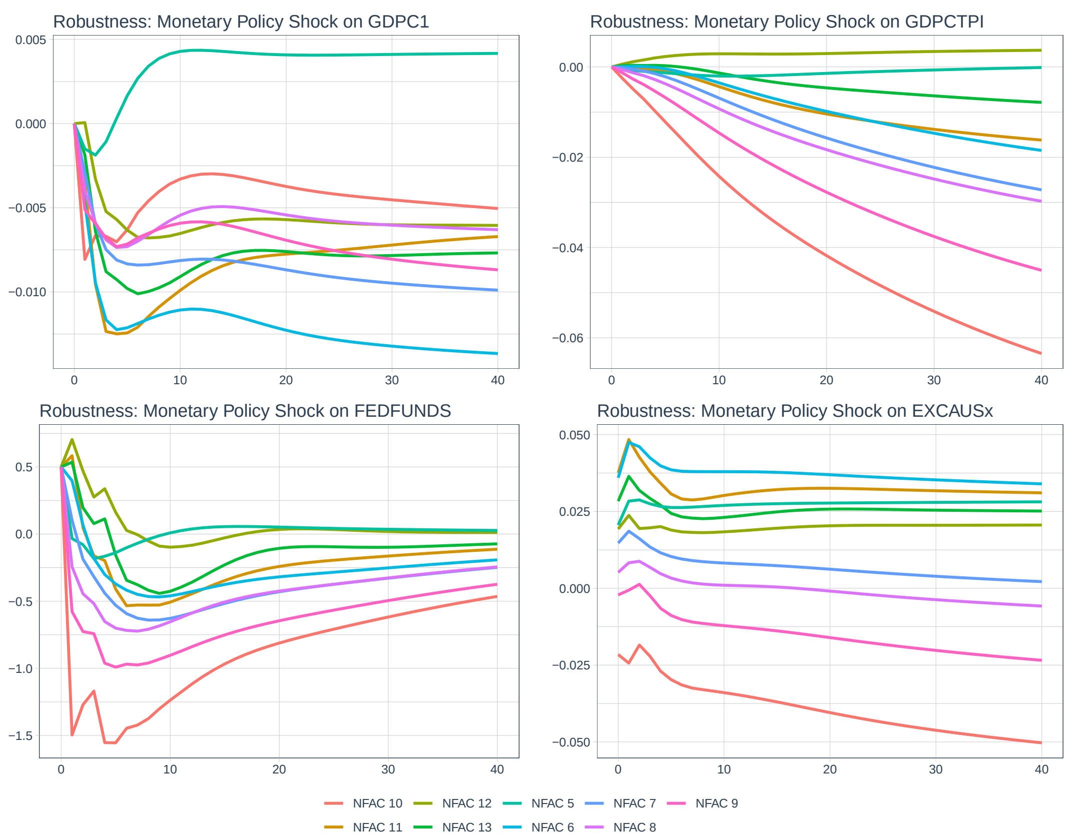
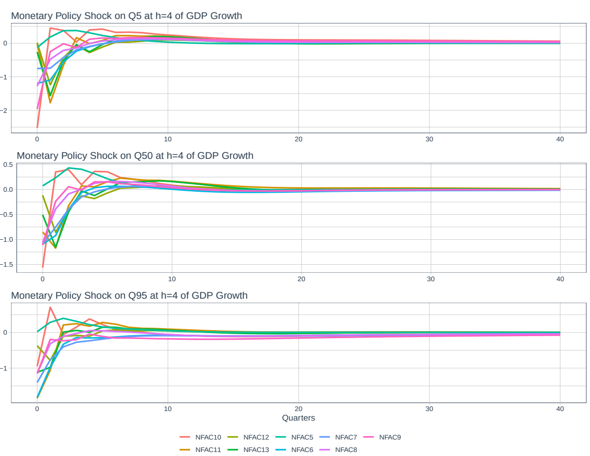


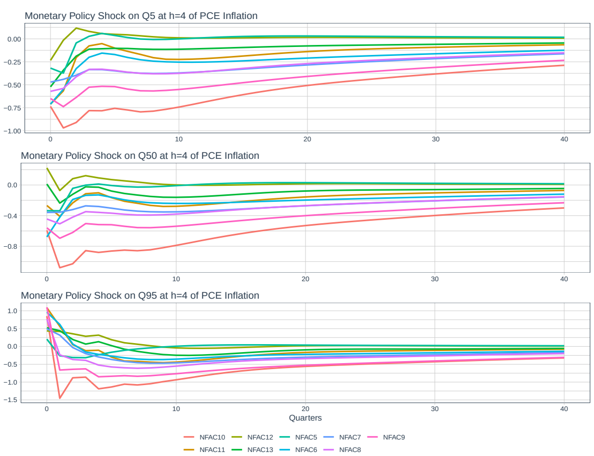
Figure 13: Responses of identifying variables to 50 basis points monetary policy shock with different numbers of static factors.

Figure 14 shows the quantile IRFs that obtain using different numbers of static factors.

Again, the IRFs are similar to the baseline case of $r = 6$, the GDP growth distribution shifts to the left as both tails are pushed in that direction. An exception occurs when $r = 5$, which again suggests that this specification does not cover enough information to yield correct monetary policy shocks. In the case of inflation, for all numbers of factors the left tail and median shift to the left whereas the right tail moves out further to the right. For all specifications, the expected distribution spreads out (the only exception being $r = 12$). Overall, the spread of the expected inflation distribution shows good robustness towards changing the number of static factors used as regressors for the quantile regression and for the identification of the monetary policy shock.



(a) QR-IRFs GDP h=4



(b) QR-IRFs PCE Inflation h=4

Figure 14: Quantile IRFs with differing numbers of static factors.

7.2.2 Changing the number of lags

The AIC criterion suggests $p = 3$ lags for the VAR of the factors given by equation 4. In the main exercise I prefer the more parsimonious specification suggested by the HQ criterion. Figure 15 reports the impulse responses of the identifying variables to the monetary policy shock when $p = 3$. There are no major qualitative differences relative to the baseline case. Therefore, I refrain from reporting the resulting quantile IRFs as the choice of p does not impact the choice of regressors. As the structural IRF is largely unaffected, so are the results for the quantile IRFs.

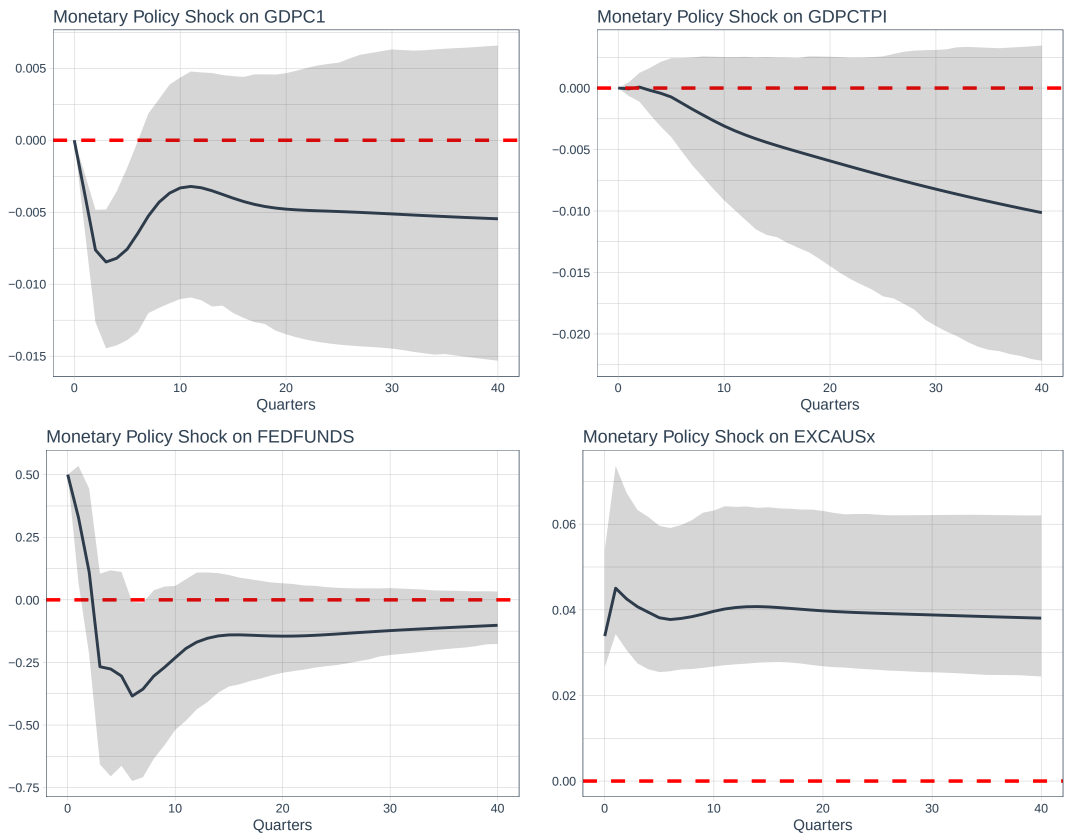


Figure 15: Responses of identifying variables to 50 basis points monetary policy shock with $r = 6$ and $p = 3$.

7.3 Bootstrapping

To compute the confidence intervals for the impulse responses I broadly follow the procedure outlined in Stock & Watson (2016). First, I use a Wild bootstrap with changing sign to bootstrap the factor VAR in equation 5. The bootstrapped factors are then multiplied by the originally estimated matrix of loadings, $\hat{A}$ to obtain the common component χ_b (the subscript b stands for "bootstrapped"). Second, I add the original idiosyncratic component ξ_o to the bootstrapped common component. This step is owed to the fact that the idiosyncratic components are cross-sectionally weakly correlated. The proposed method in Stock & Watson (2016) is to run indivdual AR(p) models on each idiosyncratic series and bootstrap from there. However, this neglects the cross-sectional correlation. Third, I multiply the sum of common component χ_b and ξ_o by the standard deviations of the original data σ_o and add the mean of the original data μ_o. This rescales the bootstrapped data X_b to the scale of the original data.

$$\chi_b = \hat{A}F_b$$
$$X_b = \sigma_o(\chi_b + \xi_o) + \mu_0$$

Once this new data set is constructed I run it through the machinery of estimating the DFM and identifying the structural shocks as described in section 2. To compute the quantile IRFs I use the initial estimates of the β_τ coefficients in combination with the bootstrapped versions of $D(L)^{-1}, S, H$. The procedure is repeated 500 times and I take the 95^{th} and 5^{th} percentiles of the empirical distributions of the IRFs as the confidence bounds.

7.4 Variables, Transformations and Goodness of Fit

The variables are taken from FRED-QD. Transformations are base don repeated ADF tests for unit roots. For more detailed information on the variables names see https://s3.amazonaws.com/files.research.stlouisfed.org/fred-md/FRED-QD_appendix.pdf.

Transformation code 1 means no transformation, 2 means taking first differences, 5 implies difference of the logs. The adjusted R^2 measure is from a linear regression of the transformed variable on the $r = 6$ and $r = 13$ static factors and a constant.

#	Variable	Trans. Code	ADF p-value	Adj. R^2 6 Fac.	Adj. R^2 13 Fac.
1	GDPC1	5	0.01	0.80	0.91
2	PCECC96	5	0.01	0.62	0.67
3	PCDGx	5	0.01	0.40	0.49
4	PCESVx	5	0.01	0.47	0.53
5	PCNDx	5	0.01	0.35	0.44
6	GPDIC1	5	0.01	0.64	0.76
7	FPIx	5	0.01	0.74	0.77
8	Y033RC1Q027SBEAx	5	0.01	0.62	0.64
9	PNFIx	5	0.01	0.67	0.67
10	PRFIx	5	0.01	0.62	0.69
11	A014RE1Q156NBEA	1	0.01	0.61	0.72
12	GCEC1	5	0.01	0.15	0.46
13	A823RL1Q225SBEA	1	0.01	0.04	0.25
14	FGRECPTx	5	0.01	0.31	0.33
15	SLCEx	5	0.01	0.34	0.48
16	EXPGSC1	5	0.01	0.13	0.17
17	IMPGSC1	5	0.01	0.30	0.36
18	DPIC96	5	0.01	0.22	0.34
19	OUTNFB	5	0.01	0.79	0.90
20	OUTBS	5	0.01	0.78	0.90
21	OUTMS	5	0.01	0.96	0.97
22	INDPRO	5	0.01	0.87	0.92
23	IPFINAL	5	0.01	0.79	0.87

#	Variable	Trans. Code	ADF p-value	Adj. R^2 6 Fac.	Adj. R^2 13 Fac.
24	IPCONGD	5	0.01	0.67	0.81
25	IPMAT	5	0.01	0.73	0.79
26	IPDMAT	5	0.01	0.70	0.78
27	IPNMAT	5	0.01	0.61	0.67
28	IPDCONGD	5	0.01	0.70	0.75
29	IPB51110SQ	5	0.01	0.47	0.57
30	IPNCONGD	5	0.01	0.31	0.59
31	IPBUSEQ	5	0.01	0.72	0.76
32	IPB51220SQ	5	0.01	0.04	0.49
33	TCU	2	0.01	0.89	0.93
34	CUMFNS	2	0.01	0.86	0.91
35	PAYEMS	5	0.01	0.92	0.95
36	USPRIV	5	0.01	0.93	0.95
37	MANEMP	5	0.01	0.87	0.92
38	SRVPRD	5	0.01	0.86	0.91
39	USGOOD	5	0.01	0.92	0.94
40	DMANEMP	5	0.01	0.87	0.91
41	NDMANEMP	5	0.01	0.64	0.76
42	USCONS	5	0.01	0.69	0.79
43	USEHS	5	0.01	0.40	0.55
44	USFIRE	5	0.01	0.63	0.64
45	USINFO	5	0.01	0.32	0.35
46	USPBS	5	0.01	0.75	0.79
47	USLAH	5	0.01	0.52	0.55
48	USSERV	5	0.01	0.69	0.75
49	USMINE	5	0.01	0.18	0.30
50	USTPU	5	0.01	0.85	0.88
51	USGOVT	5	0.01	0.56	0.63
52	USTRADE	5	0.01	0.72	0.77
53	USWTRADE	5	0.01	0.81	0.84
54	CES9091000001	5	0.01	0.08	0.15

#	Variable	Trans. Code	ADF p-value	Adj. R^2 6 Fac.	Adj. R^2 13 Fac.
55	CES9092000001	5	0.01	0.54	0.63
56	CES9093000001	5	0.01	0.63	0.69
57	CE16OV	5	0.01	0.69	0.72
58	CIVPART	5	0.01	0.15	0.23
59	UNRATE	2	0.01	0.89	0.90
60	UNRATESTx	2	0.01	0.83	0.85
61	UNRATELTx	2	0.01	0.67	0.73
62	LNS14000012	2	0.01	0.43	0.45
63	LNS14000025	2	0.01	0.85	0.87
64	LNS14000026	2	0.01	0.71	0.73
65	UEMPLT5	5	0.01	0.39	0.44
66	UEMP5TO14	5	0.01	0.64	0.65
67	UEMP15T26	5	0.01	0.58	0.63
68	UEMP27OV	5	0.01	0.68	0.72
69	LNS13023621	5	0.01	0.83	0.84
70	LNS13023557	5	0.01	0.27	0.27
71	LNS13023705	5	0.01	0.03	0.05
72	LNS13023569	5	0.01	0.21	0.26
73	LNS12032194	5	0.01	0.50	0.53
74	HOABS	5	0.01	0.78	0.83
75	HOAMS	5	0.01	0.90	0.91
76	HOANBS	5	0.01	0.82	0.86
77	AWHMAN	2	0.01	0.59	0.60
78	AWHNONAG	2	0.01	0.53	0.56
79	AWOTMAN	2	0.01	0.64	0.65
80	HWIx	5	0.01	0.74	0.75
81	HOUST	5	0.01	0.59	0.84
82	HOUST5F	5	0.01	0.22	0.40
83	PERMIT	5	0.01	0.63	0.86
84	HOUSTMW	5	0.01	0.22	0.46
85	HOUSTNE	5	0.01	0.20	0.32

#	Variable	Trans. Code	ADF p-value	Adj. R^2 6 Fac.	Adj. R^2 13 Fac.
86	HOUSTS	5	0.01	0.44	0.57
87	HOUSTW	5	0.01	0.42	0.58
88	CMRMTSPLx	5	0.01	0.81	0.83
89	RSAFSx	5	0.01	0.54	0.58
90	AMDMNOx	5	0.01	0.61	0.64
91	ACOGNOx	5	0.01	0.88	0.87
92	AMDMUOx	5	0.01	0.44	0.49
93	ANDENOx	5	0.01	0.31	0.34
94	INVCQRMTSPL	5	0.01	0.63	0.73
95	PCECTPI	5	0.01	0.96	0.97
96	PCEPILFE	5	0.03	0.96	0.97
97	GDPCTPI	5	0.01	0.90	0.94
98	GPDICTPI	5	0.01	0.69	0.81
99	IPDBS	5	0.01	0.87	0.91
100	DGDSRG3Q086SBEA	5	0.01	0.92	0.94
101	DDURRG3Q086SBEA	5	0.01	0.81	0.85
102	DSERRG3Q086SBEA	5	0.01	0.90	0.92
103	DNDGRG3Q086SBEA	5	0.01	0.88	0.90
104	DHCERG3Q086SBEA	5	0.01	0.89	0.91
105	DMOTRG3Q086SBEA	5	0.01	0.46	0.51
106	DFDHRG3Q086SBEA	5	0.01	0.74	0.74
107	DREQRG3Q086SBEA	5	0.01	0.68	0.76
108	DODGRG3Q086SBEA	5	0.01	0.52	0.58
109	DFXARG3Q086SBEA	5	0.01	0.41	0.58
110	DCLORG3Q086SBEA	5	0.01	0.37	0.47
111	DGOERG3Q086SBEA	5	0.01	0.80	0.84
112	DONGRG3Q086SBEA	5	0.01	0.70	0.73
113	DHUTRG3Q086SBEA	5	0.01	0.69	0.79
114	DHLCRG3Q086SBEA	5	0.01	0.77	0.81
115	DTRSRG3Q086SBEA	5	0.01	0.59	0.61

#	Variable	Trans. Code	ADF p-value	Adj. R^2 6 Fac.	Adj. R^2 13 Fac.
116	DRCARG3Q086SBEA	5	0.01	0.64	0.66
117	DFSARG3Q086SBEA	5	0.01	0.76	0.79
118	DIFSRG3Q086SBEA	5	0.01	0.17	0.21
119	DOTSRG3Q086SBEA	5	0.01	0.58	0.63
120	CPIAUCSL	5	0.01	0.94	0.95
121	CPILFESL	5	0.01	0.89	0.91
122	WPSFD49207	5	0.01	0.85	0.88
123	PPIACO	5	0.01	0.84	0.87
124	WPSFD49502	5	0.01	0.84	0.87
125	WPSFD4111	5	0.01	0.19	0.36
126	PPIIDC	5	0.01	0.85	0.87
127	WPSID61	5	0.01	0.83	0.86
128	WPU0531	5	0.01	0.18	0.35
129	WPU0561	5	0.01	0.70	0.72
130	OILPRICEx	5	0.01	0.63	0.64
131	AHETPIx	5	0.01	0.40	0.59
132	CES2000000008x	5	0.01	0.23	0.45
133	CES3000000008x	5	0.01	0.23	0.55
134	COMPRMS	5	0.01	0.45	0.61
135	COMPRNFB	5	0.01	0.25	0.75
136	RCPHBS	5	0.01	0.24	0.74
137	OPHMFG	5	0.01	0.79	0.82
138	OPHNFB	5	0.01	0.57	0.74
139	OPHPBS	5	0.01	0.57	0.74
140	ULCBS	5	0.01	0.59	0.88
141	ULCMFG	5	0.01	0.75	0.78
142	ULCNFB	5	0.01	0.61	0.88
143	UNLPNBS	5	0.01	0.33	0.81
144	FEDFUNDS	1	0.41	0.66	0.81
145	TB3MS	2	0.01	0.36	0.71

#	Variable	Trans. Code	ADF p-value	Adj. R^2 6 Fac.	Adj. R^2 13 Fac.
146	TB6MS	2	0.01	0.41	0.81
147	GS1	2	0.01	0.42	0.87
148	GS10	2	0.01	0.35	0.85
149	MORTGAGE30US	2	0.01	0.51	0.68
150	AAA	2	0.01	0.40	0.76
151	BAA	2	0.01	0.44	0.68
152	BAA10YM	1	0.01	0.67	0.76
153	MORTG10YRx	1	0.01	0.45	0.62
154	TB6M3Mx	1	0.01	0.09	0.33
155	GS1TB3Mx	1	0.01	0.26	0.44
156	GS10TB3Mx	1	0.01	0.40	0.69
157	CPF3MTB3Mx	1	0.01	0.61	0.63
158	BOGMBASEREALx	5	0.01	0.07	0.28
159	IMFSLx	5	0.01	0.67	0.68
160	M1REAL	5	0.01	0.51	0.61
161	M2REAL	5	0.01	0.59	0.67
162	MZMREAL	5	0.01	0.46	0.54
163	BUSLOANSx	5	0.01	0.47	0.53
164	CONSUMERx	5	0.01	0.44	0.59
165	NONREVSLx	5	0.01	0.49	0.67
166	REALLNx	5	0.01	0.35	0.49
167	REVOLSLx	5	0.01	0.47	0.54
168	TOTALSLx	5	0.01	0.59	0.74
169	DRIWCIL	1	0.01	0.70	0.75
170	TABSHNOx	5	0.01	0.57	0.86
171	TLBSHNOx	5	0.01	0.33	0.63
172	LIABPIx	5	0.01	0.15	0.51
173	TNWBSHNOx	5	0.01	0.54	0.86
174	NWPIx	5	0.01	0.45	0.73
175	TARESAx	5	0.01	0.49	0.82

#	Variable	Trans. Code	ADF p-value	Adj. R^2 6 Fac.	Adj. R^2 13 Fac.
176	HNOREMQ027Sx	5	0.01	0.39	0.60
177	TFAABSHNOx	5	0.01	0.49	0.82
178	VXOCLSx	1	0.01	0.39	0.55
179	USSTHPI	5	0.01	0.51	0.55
180	SPCS10RSA	6	0.01	0.51	0.54
181	SPCS20RSA	6	0.01	0.82	0.83
182	TWEXAFEGSMTHx	5	0.01	0.26	0.61
183	EXUSEU	5	0.01	0.64	0.66
184	EXSZUSx	5	0.01	0.05	0.47
185	EXJPUSx	5	0.01	-0.01	0.34
186	EXUSUKx	5	0.01	0.18	0.33
187	EXCAUSx	5	0.01	0.41	0.46
188	UMCSENTx	1	0.02	0.66	0.77
189	USEPUINDXM	2	0.01	0.38	0.44
190	B020RE1Q156NBEA	2	0.01	0.40	0.51
191	B021RE1Q156NBEA	2	0.01	0.54	0.66
192	GFDEGDQ188S	2	0.01	0.48	0.65
193	GFDEBTNx	2	0.01	0.49	0.56
194	IPMANSICS	5	0.01	0.88	0.92
195	IPB51222S	5	0.01	0.00	0.51
196	IPFUELS	5	0.01	0.08	0.07
197	UEMPMEAN	2	0.01	0.45	0.53
198	CES0600000007	2	0.01	0.48	0.52
199	TOTRESNS	5	0.01	0.05	0.24
200	NONBORRES	7	0.01	0.01	0.00
201	GS5	2	0.01	0.37	0.90
202	TB3SMFFM	1	0.01	0.62	0.76
203	T5YFFM	1	0.01	0.50	0.73
204	AAAFFM	1	0.02	0.62	0.81
205	WPSID62	5	0.01	0.61	0.69

#	Variable	Trans. Code	ADF p-value	Adj. R^2 6 Fac.	Adj. R^2 13 Fac.
206	PPICMM	5	0.01	0.40	0.44
207	CPIAPPSL	5	0.01	0.49	0.58
208	CPITRNSL	5	0.01	0.81	0.85
209	CPIMEDSL	5	0.01	0.74	0.79
210	CUSR0000SAC	5	0.01	0.91	0.92
211	CUSR0000SAD	5	0.01	0.74	0.77
212	CUSR0000SAS	5	0.01	0.82	0.86
213	CPIULFSL	5	0.01	0.90	0.92
214	CUSR0000SA0L2	5	0.01	0.94	0.95
215	CUSR0000SA0L5	5	0.01	0.94	0.94
216	CES0600000008	5	0.01	0.65	0.80
217	DTCOLNVHFNM	5	0.01	0.07	0.42
218	DTCTHFNM	5	0.01	0.22	0.49
219	INVEST	5	0.01	0.31	0.32
220	HWIURATIOx	2	0.01	0.68	0.69
221	CLAIMSx	5	0.01	0.73	0.74
222	BUSINVx	5	0.01	0.68	0.76
223	ISRATIOx	2	0.01	0.74	0.78
224	CONSPIx	2	0.01	0.30	0.61
225	CP3M	2	0.01	0.49	0.73
226	COMPAPFF	1	0.01	0.38	0.64
227	PERMITNE	5	0.01	0.22	0.32
228	PERMITMW	5	0.01	0.44	0.68
229	PERMITS	5	0.01	0.54	0.72
230	PERMITW	5	0.01	0.43	0.62
231	NIKKEI225	5	0.01	0.19	0.31
232	NASDAQCOM	5	0.01	0.50	0.66
233	CUSR0000SEHC	5	0.01	0.87	0.87
234	TLBSNNCBx	5	0.01	0.20	0.30
235	TLBSNNCBBDIx	1	0.01	0.33	0.51

#	Variable	Trans. Code	ADF p-value	Adj. R^2 6 Fac.	Adj. R^2 13 Fac.
236	TTAABSNNCBx	5	0.01	0.35	0.57
237	TNWMVBSNNCBx	5	0.01	0.15	0.44
238	TNWMVBSNNCBBDIx	2	0.01	0.17	0.46
239	TLBSNNBx	5	0.01	0.23	0.41
240	TLBSNNBBDIx	1	0.01	0.35	0.42
241	TABSNNBx	5	0.01	0.39	0.70
242	TNWBSNNBx	5	0.01	0.34	0.60
243	TNWBSNNBBDIx	2	0.01	0.18	0.46
244	CNCFx	5	0.01	0.14	0.51
245	S.P.500	5	0.01	0.59	0.80
246	S.P..indust	5	0.01	0.55	0.77
247	S.P.div.yield	2	0.01	0.60	0.71
248	S.P.PE.ratio	5	0.01	0.40	0.54