Method of moments for the Poisson distr.

$$p(x=k|\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}; \quad k=0,1,2,3\ldots$$

$$E[X]=\lambda$$

$$\bar{X} = \frac{1}{N}\sum_{i}^{N} x_i$$

$$E[X]=\bar{X}$$

$$\lambda=\bar{X}$$

MLE for the Poisson distr:

$$p(X=k|\lambda) = \frac{\lambda^k e^{-k}}{k!}, \quad k=0,1,2\ldots$$

$$L(\lambda|X) = \prod_{i=1}^{N} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}$$

$$l(\lambda|X) = \sum_{i=1}^{N} \ln \frac{\lambda^{x_i} e^{-\lambda}}{x!} =$$

$$= \sum_{i=1}^{N} x_i \ln\lambda - \lambda \cancel{\ln e} - \ln x_i!$$

$$= \ln\lambda\left(\sum_{i=1}^{N} x_i\right) - N\lambda - \sum_{i}^{N} \ln x_i!$$

$$\frac{\partial l(\lambda|X)}{\partial\lambda} = 0 = \frac{\sum_{i=1}^{N} x_i}{\lambda} - N$$

$$= 0 \Rightarrow \sum_{i=1}^{N} x_i = N\lambda \Rightarrow \lambda = \frac{\sum_{i=1}^{N} x_i}{N}$$

Entropy of Gaussian distr

$$N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$H(P) = -\int_{-\infty}^{+\infty} p(x) \log_2 p(x)\, dx$$

Будем использовать натуральный логарифм (ln), так что энтропия будет в натах (nats).

$$\log(N(\mu, \sigma^2)) = -\ln\sqrt{2\pi\sigma^2} - \frac{(x-\mu)^2}{2\sigma^2}$$

$$= -\frac{1}{2}\ln 2\pi\sigma^2 - \frac{(x-\mu)^2}{\sigma^2}$$

$$H(p) = -\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} \cdot \left(-\frac{1}{2}\ln 2\pi\sigma^2 - \frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} \cdot \left(\frac{1}{2}\ln 2\pi\sigma^2 + \frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$= \underbrace{\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} \cdot \frac{1}{2}\ln 2\pi\sigma^2\, dx}_{1} + \underbrace{\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} \cdot \frac{(x-\mu)^2}{2\sigma^2}\, dx}_{2}$$

(1): $\int_{-\infty}^{+\infty} \underbrace{\frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}}_{p(x)} \cdot \underbrace{\frac{1}{2}\ln 2\pi\sigma^2}_{\text{const}}\, dx =$

$$= \frac{1}{2}\ln 2\pi\sigma^2 \underbrace{\int_{-\infty}^{+\infty} p(x)\, dx}_{=1}$$

$$= \frac{1}{2}\ln 2\pi\sigma^2$$

(2) $\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} \cdot \frac{(x-\mu)^2}{2\sigma^2}\, dx$

$$= \int_{-\infty}^{+\infty} p(x) \frac{(x-\mu)^2}{2\sigma^2}\, dx =$$

$$\frac{1}{2\sigma^2} \underbrace{\int_{-\infty}^{+\infty} p(x)(x-\mu)^2\, dx}$$

$$Var(X) = E[(x-\mu)^2] =$$

$$= \int_{-\infty}^{+\infty} (x-\mu)^2\, p(x)\, dx$$

$$= \frac{1}{2\sigma^2} \cdot \sigma^2 \leftarrow Var(X)$$

$$= \frac{1}{2}$$

Summary: $H(N(\mu, \sigma^2)) = \frac{1}{2}\ln 2\pi\sigma^2 + \frac{1}{2}$

$$= \frac{1}{2}(\ln 2\pi\sigma^2 + 1) = \frac{1}{2}\ln 2\pi\sigma^2 e$$