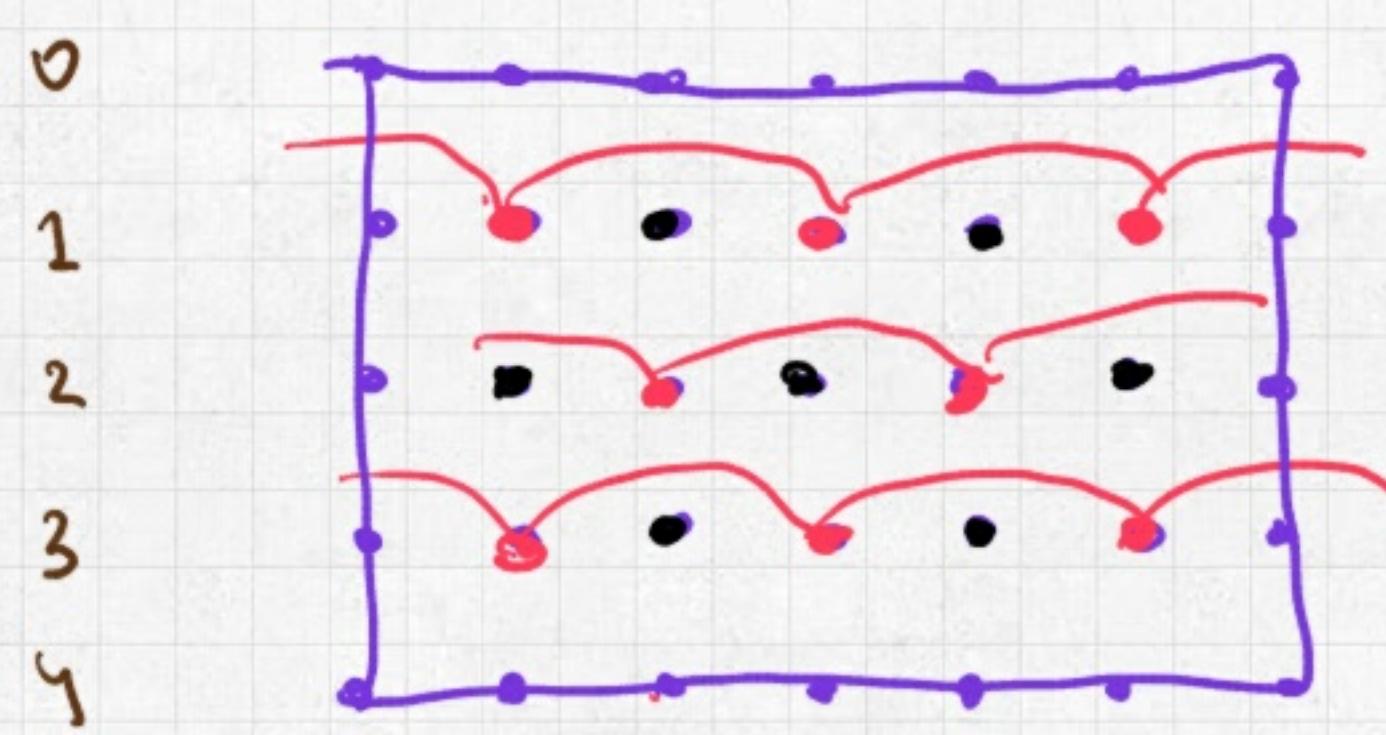


Treating border elements is always, how to focus on



$(i+j)/2 = 0 \rightarrow \text{Red}$   
otherwise  $\rightarrow \text{black}$

$$\begin{array}{|c|} \hline 1+2 \\ \hline 1+3 \\ \hline 1+5 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 2+2 \\ \hline 2+4 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline 3+1 \\ \hline 3+3 \\ \hline 3+5 \\ \hline \end{array}$$

$$\begin{array}{ccc} 1+2 & 2+1 & 3+2 \\ 1+4 & 2+3 & 3+4 \\ 2+5 & & \end{array}$$

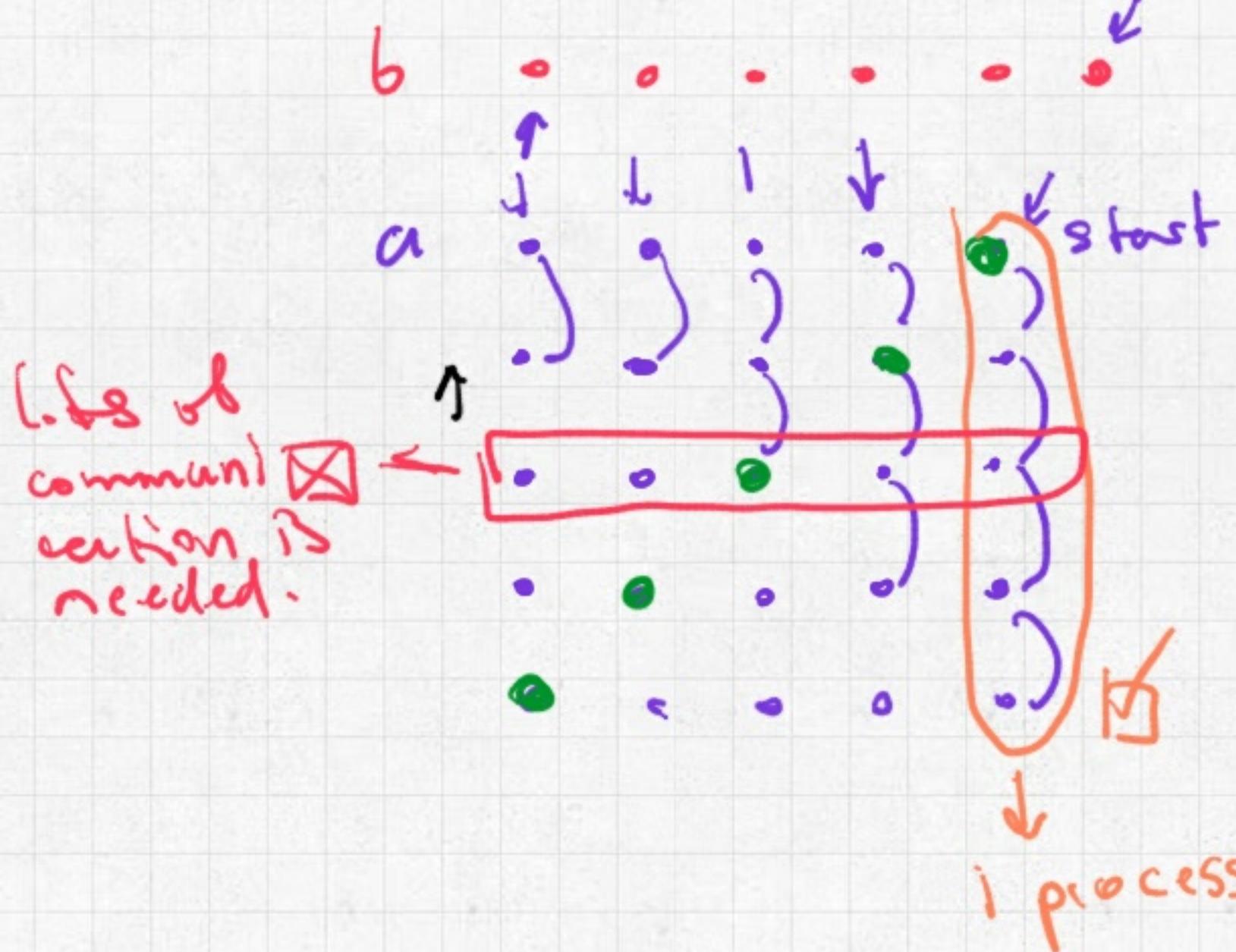
1  
2  
3

Can split it into 4 loops

$$x \cdot \dots \cdot \cdot \cdot \leftarrow x[i] = b[i] \cdot a[i][j]$$

$b$      $\circ \circ \circ \circ \circ \circ$      $\leftarrow$   
 $a$      $\uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$     start

lots of  
communication  
is  
needed.



Column dependent  
Each  $b$   
So it partitioned by first dimension

Each process gets a column of  $a$ .  
almost no communication is needed

upper loop can be

If partitioned by row (second dimension) Then  
lots of communication is needed!

read + write = pass (from/to disk)

Are two passes always sufficient for sorting?

When not enough memory  $\Rightarrow$  more than two passes are required!

Answer depends on the available memory!

$$T_S = 12$$

$$T_P = 2 + \frac{10}{2} = 2 + 5 = 7$$

$$\begin{array}{|c|} \hline 12 \\ \hline 7 \\ \hline \end{array}$$

$$\frac{12}{2 + \frac{10}{2}} = \frac{12}{3} \rightarrow 4x$$

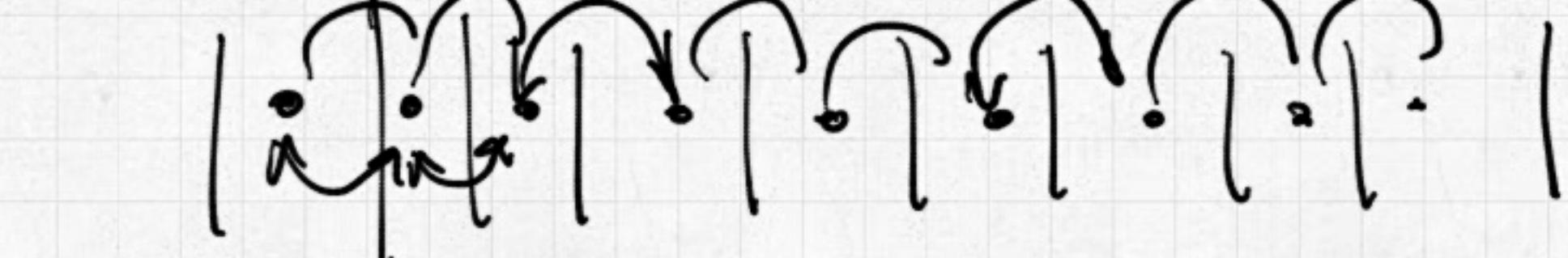
Smallest number  $\rightarrow$  10 or 10+2?

$m \times n \cdot 2$  Computations  
 $2m n$  Reads  
 $m n$  Writes

~~5mn~~  $5mn$  - Overall

$$m \cdot \frac{n}{p} + (p-1)50$$

Parallel Execution.



$(p-1)$  Communications  
 $\rightarrow$  50 units of time

$$\frac{5n}{p} + \frac{n}{p} + (p-1)50$$

$$\frac{10000}{p} + \frac{10000 + (p-1)50}{p}$$

$$\frac{10000}{5000 + 100} + \frac{100 + (p-1)50}{5000 + 100}$$

$$m \cdot \frac{n}{p} + (p-1) \cdot 50 \quad vs \quad 5mn$$

Fixed size speedup.

1. - p processors.

$$S_{FS} = \frac{T_S(N_0)}{T_p(N_0)}$$

$$T_S(N) = 10^6 + 10^3 N + 24N^2$$

$$T_p(N) = 15 \cdot 10^5 + 1050N/p + 24N^2/p$$

$$\frac{1000000 + 1000N + 24N^2}{1500000 + 1050\frac{N}{p} + 24\frac{N^2}{p}}$$

1)  $\frac{N}{p} = N_0 \quad \frac{N}{p} = 1000 \quad N =$

$$T_S(p1000) = \frac{1000000 + 1000000p + 24000000p^2}{1500000 + 1050000 + 24000000p}$$

$$\frac{1 + p + 24p^2}{1.5 + 1.05 + 24p} \approx \frac{1 + p + 24p^2}{2.55 + 24p}$$



Unbounded when  $p \rightarrow \infty$

2)  $T_p = T_S(N_0) \quad ?$

$$S_{FT} = \frac{T_S(N_p)}{T_p(N_p)} \quad T_p(N_p) = T_S(N_0) = 26 \cdot 10^6$$

$$26 \cdot 10^6 = 1500000 + 1050 \frac{N_p}{p} + 24 \frac{N_p^2}{p}$$

$$1500000 - 26 \cdot 10^6 + 1050 \frac{N_p}{p} + 24 \frac{N_p^2}{p}$$

Now just need to find roots of this equation huh.

$$E = \frac{T_s}{T_p \cdot N} \cdot 100\% \quad \frac{5}{2 \cdot 10} \cdot 100\% \quad \frac{5}{20} \approx \text{very low!}$$

Program total Complexity  $t$ ,  $n$  total operations

Processors  $p = t$   
then  $t + \frac{(m-t)}{p}$  theorem! Global sum

Example:  $p = \lfloor \frac{n}{\log n} \rfloor$   $m = n-1$

$$\lceil \log n \rceil + \frac{n-1 - \lceil \log n \rceil}{\lfloor n/\log n \rfloor} = \Theta(\log n)$$

$$\text{Cost} = \lfloor n/\log n \rfloor \times \Theta(\log n) = \Theta(n) - \text{optimal}$$

Same but for prefix sum: