CS581 Theory of Computation: Homework #4

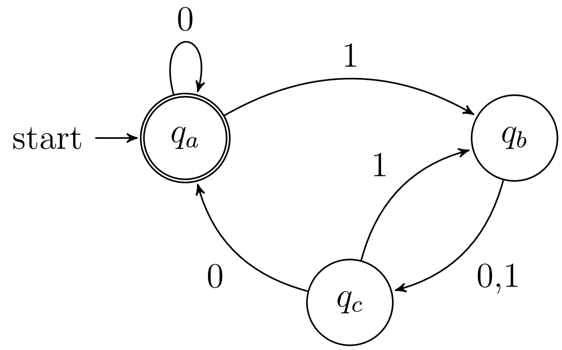
Due on February 22 2016 at 2:00pm

Harry H. Porter Winter 2016

Konstantin Macarenco

Problem 4.1

Answer all parts for the following DFA M and give reasons for your answers.



1. Is $\langle M, 0100\rangle \in A_{DFA}$?
   Yes. The DFA M accepts 0100.
2. Is $\langle M, 011\rangle \in A_{DFA}$?
   No. The DFA M doesn't accept 011.
3. Is $\langle M\rangle \in A_{DFA}$?
   No. This input has only a single component and thus is not of the correct form.
4. Is $\langle M, 0100\rangle \in A_{REX}$?
   No. The first component is not a regular expression and so the input is not of the correct form.
5. Is $\langle M\rangle \in E_{DFA}$?
   No. M's language isn't empty.
6. Is $\langle M, M\rangle \in EQ_{DFA}$?
   Yes. M accepts the same language as itself.

Problem 4.2

Consider the problem of determining whether a DFA and regular expression are equivalent. Express this problem as a language and show that it is decidable.

Solution

Let $EQ_{DFA,REGEX} = \{\langle A, R\rangle \mid A \text{ is a DFA}, R \text{ is a Regular Expression and } L(A) = L(R)\}$. The following TM decides $EQ_{DFA,REGEX}$
On input $\langle A, R\langle$, where A is a DFA, and R is a regular expression do the following:

1. Convert R to equivalent DFA R_D.
2. Construct and run EQ_{DFA} E as a subroutine on $\langle A, R_D\rangle$.
3. *Accept* if E accepts, otherwise *reject*.

Problem 4.3

Let $ALL_{DFA} = \{\langle A\rangle | A \text{ is a } DFA \text{ and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.

Solution

Since class of DFA is closed under complement, we can prove that ALL_{DFA} is decidable by constructing $\overline{ALL_{DFA}}$: and testing if it accepts empty language.

We construct ALL_{DFA} as follows:
On input $\langle A\rangle$, where A is a DFA, do the following:

1. Convert A to $\overline{A}$ (complement of A).
2. Construct and run E_{DFA} E as a subroutine on $\overline{A}$, check if $L(\overline{A}) = \emptyset$ or not.
3. Accept if E return *accept*, otherwise *reject*.

Problem 4.6

Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 19\}$. We describe the functions $f : X \to Y$ and $g : X \to Y$ in the following tables. Answer each part and give a reason for each negative answer.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

n	$g(n)$
1	10
2	9
3	8
4	7
5	6

Solution

1. Is f one-to-one?
   No. f is not one-to-one because $f(1) = f(3)$
2. Is f onto?
   No. f is not onto, because there doesn't exist $x \in X$ such that $f(x) = 8$.
3. Is f a correspondence?
   No. f is not a correspondence because f is not one-to-one and onto.
4. Is g one-to-one?
   Yes. g is one-to-one.
5. Is g onto?
   Yes. g is onto.
6. Is g a correspondence?
   Yes. g is a correspondence because g is one-to-one and onto.

Problem 4.11

Let $INFINITE_{PDA} = \{\langle M \rangle \mid M \text{ is a } PDA \text{ and } L(M) \text{ is an infinite language}\}$. Show that $INFINITE_{PDA}$ is decidable

Solution

Build a Turing machine that will do the following.
On input $\langle M \rangle$, where M is a PDA:

1. Read $\langle M \rangle$ and create an equivalent context-free grammar G.
2. Convert G into Chomsky Normal Form G'
3. Do a breadth-first search of the grammar rules of G' looking for recursion. That is, does there exist a derivation $A \overset{+}{\Rightarrow} uAv$?
4. If yes, then accept $\langle M \rangle$, *reject* otherwise.