CS581 Theory of Computation: Homework #4

Due on February 22 2016 at 2:00pm

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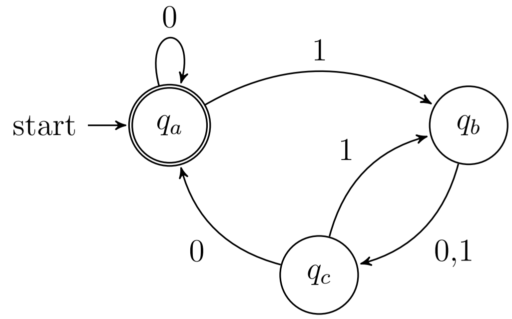
Problem 4.1

Answer all parts for the following DFA M and give reasons for your answers.



1. Is $\langle M, 0100 \rangle \in A_{DFA}$?
   Yes. The DFA M accepts 0100.
2. Is $\langle M, 011 \rangle \in A_{DFA}$? No. The DFA M doesn't accept 011.
3. Is $\langle M \rangle \in A_{DFA}$? No. This input has only a single component and thus is not of the correct form.
4. Is $\langle M, 0100 \rangle \in A_{REX}$? No. The first component is not a regular expression and so the input is not of the correct form.
5. Is $\langle M \rangle \in E_{DFA}$? No. M's language isn't empty.
6. Is $\langle M, M \rangle \in EQ_{DFA}$? Yes. M accepts the same language as itself.

Problem 4.2

Consider the problem of determining whether a DFA and regular expression are equivalent. Express this problem as a language and show that it is decidable.

Solution

Let $EQ_{DFA,REGEX} = \{\langle A, R \rangle | A \text{ is a } DFA, R \text{ is a Regular Expression and } L(A) = L(R)\}$.
The following TM decides $EQ_{DFA,REGEX}$
On input $\langle A, R \langle$, where A is a DFA, and R is a regular expression do the following:

1. Convert R to equivalent DFA R_D.
2. Run EQ_{DFA} as a subroutine on $\langle A, R_D \rangle$.
3. If EQ_{DFA} accepts, *accept*, otherwise *reject*.

Problem 4.3

Let $ALL_{DFA} = \{\langle A\rangle | A \text{ is a } DFA \text{ and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.

Since class of DFA is closed under complement, we can prove that ALL_{DFA} is decidable by constructing it:

On input $\langle A\langle$, where A is a DFA, do the following:

1. Convert A to $\overline{A}$ (complement of A).
2. Run E_{DFA} as a subroutine on $\overline{A}$, check if $L(\overline{A}) = \emptyset$ or not.
3. $L(\overline{A}) = \emptyset$ return *accept*, otherwise return $reject$

Problem 4.6

Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 19\}$. We describe the functions $f : X \to Y$ and $g : X \to Y$ in the following tables. Answer each part and give a reason for each negative answer.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

n	$g(b)$
1	10
2	9
3	8
4	7
5	6

1. Is f one-to-one? No. f is not one-to-one because $f(1) = f(3)$
2. Is f onto? No. f is not onto, because there doesn't exist $x \in X$ such that $f(x) = 8$.
3. Is f a correspondence? No. f is not a correspondence because f is not one-to-one and onto.
4. Is g one-to-one? Yes. g is one-to-one.
5. Is g onto? Yes. g is onto.
6. Is g a correspondence? Yes. g is a correspondence because g is one-to-one and onto.

Problem 4.11

Let $INFINITE_{PDA} = \{\langle M \rangle | M$ is a PDA and $L(M)$ is an infinite language$\}$. Show that $INFINITE_{PFA} is decidable$

Build a Turing machine that will do the following. Read hMi and create an equivalent context-free grammar G. Convert G to Chomsky Normal Form. Call it G 0 . Do a breadth-first search of the grammar rules of G 0 looking for recursion. That is, does there exist a derivation A += uAv? If so, then accept hMi. If not, then reject hMi.
http://people.hsc.edu/faculty-staff/robbk/Coms461/Lectures