CS581 Theory of Computation: Homework #2

Due on February 1 2016 at 2:00pm

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Problem 2.6
Give context-free grammars generating the following languages.

Problem 2.6 b
The complement of the language $\{a^n b^n | n \geq 0\}$

$$\begin{aligned} &S \to aSb \mid bY \mid Ya \\ &Y \to bY \mid aY \mid \epsilon \end{aligned}$$

Problem 2.6 d
$\{x_1 \# x_2 \# \cdots \# x_k | k \geq 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j, x_i = {x_j}^R\}$

$$\begin{aligned} &S \to ABC \\ &A \to D\#A \mid \epsilon \\ &B \to 0B0 \mid 1B1 \mid E \\ &C \to \#DC \mid \epsilon \\ &D \to 1D \mid 0D \mid \epsilon \\ &E \to \#DE \mid \# \end{aligned}$$

Problem 2.7
Give informal English description of PDAs for the languages in Exercise 2.6

Problem 2.7 b
The complement of the language $\{a^n b^n | n \geq 0\}$

Problem 2.7 d
$\{x_1 \# x_2 \# \cdots \# x_k | k \geq 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j, x_i = {x_j}^R\}$

Problem 2.9
Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k | i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$$

Is your grammar ambiguous? Why or why not?

$$\begin{aligned} &S \to AB \mid CD \\ &A \to aAb \mid \epsilon \\ &B \to cB \mid \epsilon \\ &C \to aC \mid \epsilon \\ &D \to bDc \mid \epsilon \end{aligned}$$

The language is not ambiguous...

Problem 2.13
Let $G = (V, \sum, R, S)$ be the following grammar. $V = \{S < TU\}; \sum = \{0, \#\}$; and R is the set of rules:

$$\begin{aligned} &\mathrm{S} \rightarrow TT \mid U \\ &\mathrm{T} \rightarrow 0T \mid T0 \mid \# \\ &\mathrm{U} \rightarrow 0U00 \mid \# \end{aligned}$$

Problem 2.13 a
Describe $L(G)$ in English.

Informally $L(G)$ is either two or more # separated by arbitrary number of 0's (zero or more) or zero or more zero followed by # and by twice as many zeros as in before #. More formally it is $\{0_1^{i_1}\#0_2^{i_2}\#0_3^{i_3}\#0_4^{i_4}\#\cdots\#0_k^{i_n} \mid \text{where } i_j \geq 0 \text{ and } k \geq 3\}$ or $\{0^n\#0^{2n}|n \geq 1\}$

Problem 2.13 b
Prove that $L(G)$ is not regular.

1. Assume that $L(G)$ is regular.
2. Consider the word $0^p\#0^{2p} \in L(G)$. By pumping lemma there eixsts a word $xyz \in L(G)$, and puping lenght p, such that $|xy| \leq p$ and for all $i \geq 0$, $xy^iz \in L$. In case of word $=$ $0^p\#0^{2p}$, xy can only be in 0^p, however for $i \geq 0$ xy^iz is $0^{p+i}\#0^{2p} \notin L(G)$ is a contradiction. Therefore $L(G)$ is no regular.

Problem 2.15
Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by th CFG $G = (V, \sum, R, S)$. Add the new rule $S \rightarrow SS$ and call the resulting grammar G'A. This grammar is supposed to generate $A*$.

This rule applied to grammar $G = (\{S\}, \{a, b\}, \{S \rightarrow aSb|\epsilon\}, S) + S \rightarrow SS$ we get grammar G' that can produce a word: $aababb \notin A^*$

Problem 2.19
Let CFG G be the following grammar.

$$\begin{aligned} &S \rightarrow aSb \mid bY \mid Ya \\ &Y \rightarrow bY \mid aY \mid \epsilon \end{aligned}$$

Give a simple description of L(G) in English. Use that description to give a CFG for $\overline{L(G)}$, the compliment of $L(G)$.

L(G) is the language that produces all strings not in $a^n b^n$, i.e. compliment of of $a^n b^n$.

$$S \to aSb \mid \epsilon$$

Problem 2.28

Give unambiguous CFGs for the following languages.

Problem 2.28 a
$\{w|$ in every prefix of w the number of a's is at least the number of b's $\}$

$$\begin{aligned} S &\to aS \mid BS \mid \epsilon \\ B &\to aBBb \mid \epsilon \end{aligned}$$

Problem 2.28 b
$\{w|$ the number of a's and the number of b's in w are equal $\}$

$$\begin{aligned} S &\to aA \mid bB \mid \epsilon \\ A &\to bS \mid aAA \\ B &\to aS \mid bBB \end{aligned}$$

Problem 2.28 c
$\{w|$ the number of a's is at least the number of b's in $w\}$

$$\begin{aligned} S &\to aA \mid bB \mid a \\ A &\to bS \mid aAA \\ B &\to aS \mid bBB \end{aligned}$$

$$\begin{aligned} S &\to aS \mid bB \mid a \\ B &\to aS \mid bBB \end{aligned}$$

Problem 2.30
Use the pumping lemma to show that the following languages are not context free.

Problem 2.30 a
$\{0^n 1^n 0^n 1^n | n \geq 0\}$

Problem 2.30 d
$\{t_1 \# t_2 \# \cdots \# t_k | k \geq 2, \text{ each } t_i \in \{a,b\}^*, \text{ and } t_i = t_j \text{ for some } i \neq j\}$

Problem 2.31
Let B be the language of all palindromes over $\{0,1\}$ containing equal numbers of 0s and 1s. Show that B is not context free.

Problem 2.33
Show that $F = \{a^i, b^j | i = kj \text{ for some positive integer } k\}$ is not context free.

Problem 2.35
Let G be a CFG in Chomsky normal form that contains b variables. Show that if G generates some string with a derivation having at least 2^b steps, $L(G)$ is infinite.

Problem 2.46
Consider the following CFG G: Describe $L(G)$ and show that G is ambiguous. Give an

$$S \to SS \mid T$$
$$T \to aTb \mid ab$$

unambiguous grammar H where $L(H) = L(G)$ and sketch a proof that H is unambiguous.