# CS581 Theory of Computation: Homework #5

Due on March 2 2016 at 2:00pm

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**Problem 5.3**

Find a match in the following instance of the Post Correspondence Problem.

$$\left\{ \left[\frac{ab}{abab}\right], \left[\frac{b}{a}\right], \left[\frac{aba}{b}\right], \left[\frac{aa}{a}\right] \right\}$$

**Solution**

$$\frac{ab}{abab}, \frac{ab}{abab}, \frac{aba}{b}, \frac{b}{a}, \frac{b}{a}, \frac{aa}{a}, \frac{aa}{a}$$

**Problem 5.4**

If $A \leq_m B$ and B is a regular language, does that imply that A is a regular language? Why or why not?

**Solution**

No it doesn't imply that A is regular, for example: $\{a^n b^n \mid n \geq 0\}$ can be reduced to $\{a^n\} \mid n \geq 0$, by following procedure: Check if input $\in a^n, b^n$, output $a^n$ it is, and $b$ if is not.

Description of the TM form problems 1 and 2.

1. $Q = \{A, B, C, D\}$
2. $\Sigma = \{0, 1\}$
3. $\Gamma = \{0, 1, \_\}$
4. $\delta =$
    1. $\delta(A, 0) = (B, 1, R)$
    2. $\delta(A, 1) = (A, 1, R)$
    3. $\delta(A, \_) = (C, \_, L)$
    4. $\delta(B, 0) = (D, 0, L)$
    5. $\delta(B, 1) = (A, 0, R)$
    6. $\delta(B, \_) = (D, \_, L)$
5. $q_0 =$ A
6. $q_{accept} = C$
7. $q_{reject} = D$

**Problem 1**

Convert this into and instance of the PCP.

**Solution**

Convert the TM into instance of PCP by adding required domino tiles:

Part 1: add first tile

$$\frac{\#}{\#\#Aw_1w_2w_3...}$$

Part 2: Take care of the right transitions

$$\frac{A0}{1B}\ \frac{A1}{1A}\ \frac{B1}{0A}$$

Part 3: Take care of the left transitions

$$\frac{0A\_}{C0\_}\ \frac{1A\_}{C1\_}\ \frac{\_A\_}{C\_\_}\ \frac{0B0}{D00}\ \frac{1B0}{D10}\ \frac{\_B0}{D\_0}\ \frac{0B\_}{D0\_}\ \frac{1B\_}{D1\_}\ \frac{\_B\_}{D\_\_}$$
$$\frac{\#A\_}{\_\#C\_}\ \frac{\#B0}{\_\#D0}\ \frac{\#B\_}{\_\#D\_}$$

Part 4: For every $a \in \Gamma$ put $\frac{a}{a}$

$$\frac{0}{0}\ \frac{1}{1}\ \frac{\_}{\_}$$

Part5

$$\frac{\#}{\#}\ \frac{\#}{\_\#}$$

Part 6: Accept states

$$\frac{0C}{C}\ \frac{1C}{C}\ \frac{\_C}{C}\ \frac{C0}{C}\ \frac{C1}{C}\ \frac{C\_}{C}$$

Part7: Final domino

$$\frac{C\#\#}{\#}$$

**Problem 2**

Show that the string "01" is in the language recognized by this TM by showing a solution to your instance of the PCP.

**Solution**

$$\frac{\#}{\#\#A01}\frac{\#}{\#}\frac{A0}{1B}\frac{1}{1}\frac{\#}{\#}\frac{1}{1}\frac{B1}{0A}\frac{\#}{\#}\frac{1}{1}\frac{0A\_}{C0\_}\frac{\#}{\#}\frac{1}{1}\frac{C0}{C}\frac{\#}{\#}\frac{1C}{C}\frac{\#}{\#}\frac{C\#\#}{\#}$$

$$\frac{\#}{\#\#A01\_}\frac{\#}{\#}\frac{A0}{1B}\frac{1}{1}\frac{\_}{\_}\frac{\#}{\#}\frac{1}{1}\frac{B1}{0A}\frac{\_}{\_}\frac{\#}{\#}\frac{1}{1}\frac{0A\_}{C0\_}\frac{\#}{\#}\frac{1}{1}\frac{C0}{C}\frac{\_}{\_}\frac{\#}{\#}\frac{1C}{C}\frac{\_}{\_}\frac{\#}{\#}\frac{C\_}{C}\frac{\#}{\#}\frac{C\#\#}{\#}$$