Solution to CS243 Assignment2

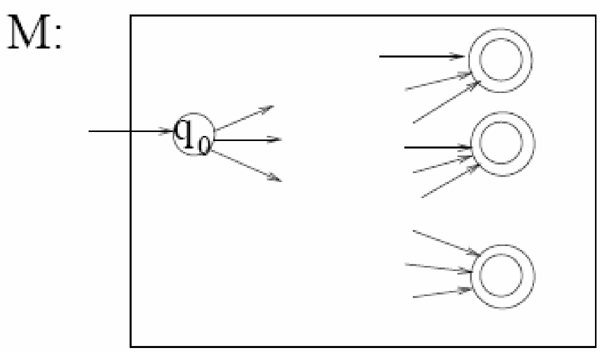
1. Text (Sipser, second edition) Chapter 1 (p.88) 1.29b [14%] $A_2 = \{\omega\omega\omega \mid \omega \text{ is in } \{a,b\}^*\}$

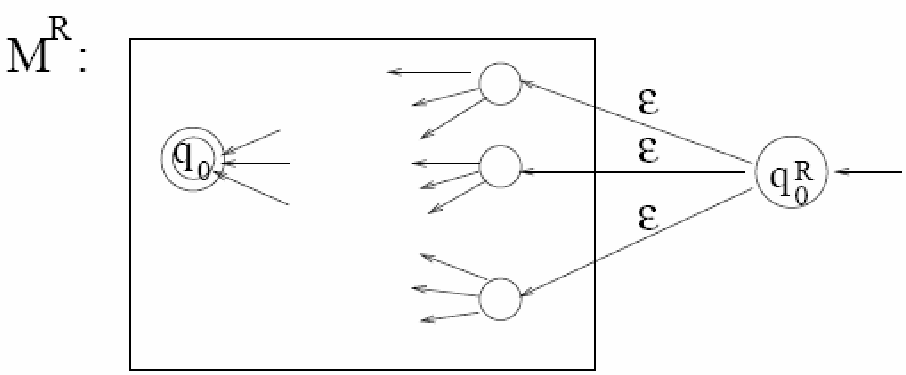
Assume to the contrary that A_2 is regular, and let p be the pumping length given by pumping lemma. Choose $s = a^p b a^p b a^p b$, which can be divided into three pieces $s = xyz$, where $|xy| \le p$. This means xy contains only a's. Since $|y| > 0$, let $y = a^k$, $k>0$. However, $xy^2z = a^{p+k} b a^p b a^p b$, where $p+k>p$, is not in A_2. That is s cannot be pumped. This is a contradiction. Thus, A_2 is not regular.□

2. Text (Sipser, second edition) Chapter 1 (p.89) 1.31 [14%]

Let $M = (Q, \Sigma, \delta, q_0, F$ be the DFA that recognizes A. We can build an NFA $M^R = (Q^R, \Sigma^R, \delta^R, q_0^R, F^R)$ that recognizes the reverse language A^R as follows: We pick $q_0^R \notin Q$ as the start state, $\Sigma^R = \Sigma$, $Q^R = Q \cup \{q_0^R\}$, $F^R = \{q_0\}$, and

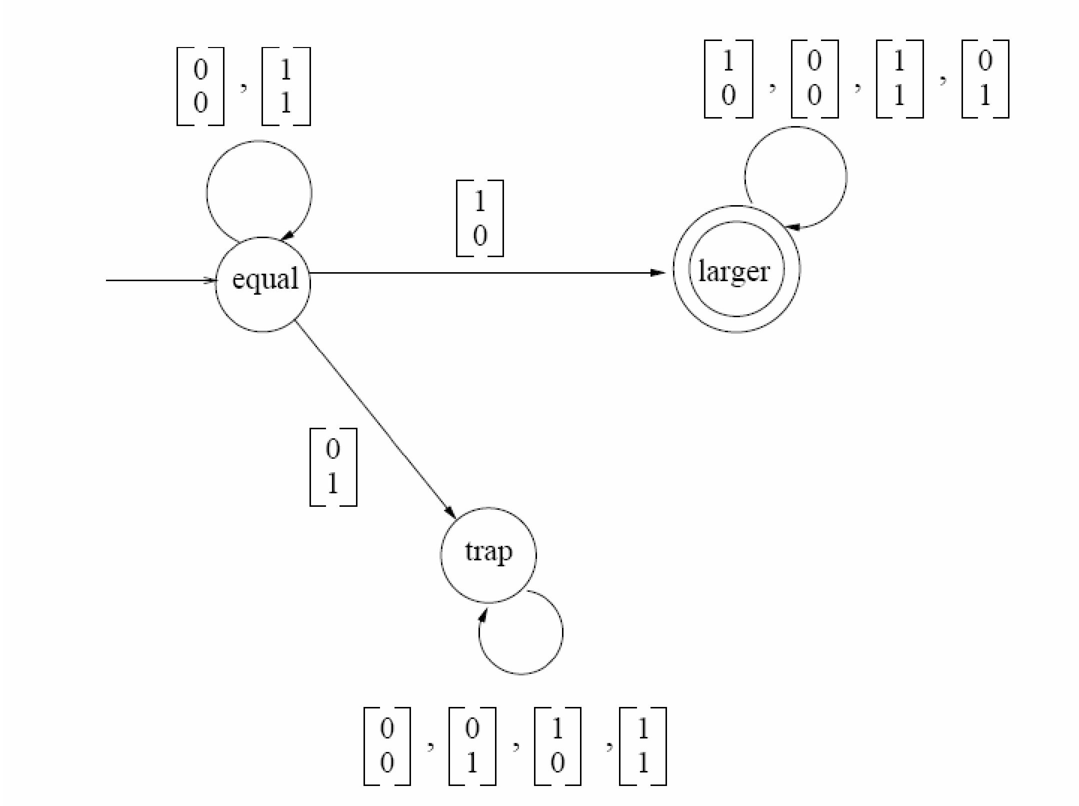
$$\delta^R(q, a) = \bigcup_{\delta(p,a)=q} \{p\}$$





3. Text (Sipser, second edition) Chapter 1 (p.89) 1.34 [14%]

The following DFA recognizes D. Thus, D is regular.



4. Text (Sipser, second edition) Chapter 1 (p.89) 1.37 [14%]

We only need to construct a DFA to keep track of the remainder of the input seen so far (from left to right) divided by n. If it ends up with remainder zero, accept; otherwise reject. Notice the following relations:

If (ω mod n) = k, where $0 \le k \le n-1$, we have $\omega = n\cdot q + k$.

For case $\omega 0$: the remainder is $[2(n\cdot q + k) \bmod n] = [2k \bmod n]$

For case $\omega 1$: the remainder is $[2(n\cdot q + k)+1 \bmod n] = [2k+1 \bmod n]$

Construct DFA M=($\{q_0, q_1, \ldots, q_n\}$, $\{0,1\}$, δ, q_0, $\{q_0\}$),

$$\delta(q_k, a) = q_j, \text{ where } j = \begin{cases} 2k \bmod n & \text{if } a = 0 \\ 2k+1 \bmod n & \text{if } a = 1 \end{cases}$$

M recognizes C_n, thus C_n is regular.

5. Text (Sipser, second edition) Chapter 1 (p.90) 1.46a, 14.6c, 1.46d [10% each]

1.46a $\{0^n 1^m 0^n \mid m, n \ge 0\}$

Use pumping lemma, and choose $s = 0^p 1 0^p = xyz$. Here xy contains only 0's. Let, $y = 0^k$, $k>0$. Thus $xy^0z = 0^{p-k}10^p$ is not in the language. Thus, it's not regular.

14.6c $\{w \mid w \text{ in } \{0,1\}^*$ is not a palindrome$\}$
Let its compliment language L = $\{w \mid w$ in $\{0,1\}^*$ is a palindrome$\}$, we can prove the original language is not regular by showing that L is not regular.
Choose s = 0^p10^p = xyz, a palindrome in L. Here xy contains only 0's. Let, y = 0^k, k>0. Thus $xy^0z = 0^{p-k}10^p$ is not a palindrome, thus it's not in the language L. Thus, L is not regular.

1.46d $\{wtw \mid w, t$ are in $\{0,1\}^+\}$
Use pumping lemma, and choose s = 0^p10^p = xyz. Here xy contains only 0's. Let, y = 0^k, k>0. Thus $xy^0z = 0^{p-k}10^p$ is not in the language. Thus, it's not regular.

6. Text (Sipser, second edition) Chapter 1 (p.90) 1.48 [14%]
The following DFA recognize D, thus D is regular.