CS581 Theory of Computation: Homework #2

Due on February 1 2016 at 2:00pm

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Problem 2.6
Give context-free grammars generating the following languages.

Problem 2.6 b
The complement of the language $\{a^n b^n | n \geq 0\}$

Problem 2.6 d
$\{x_1 \# x_2 \# \cdots \# x_k | k \geq 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j, x_i = {x_j}^R\}$

Problem 2.7
Give informal English description of PDAs for the languages in Exercise 2.6

Problem 2.7 b
The complement of the language $\{a^n b^n | n \geq 0\}$

Problem 2.7 d
$\{x_1 \# x_2 \# \cdots \# x_k | k \geq 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j, x_i = {x_j}^R\}$

Problem 2.9
Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k | i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$$

Is your grammar ambiguous? Why or why not?

Problem 2.13
Let $G = (V, \sum, R, S)$ be the following grammsr. $V = \{S < TU\}; \sum = \{0, \#\}$; and R is the set of rules: **Problem 2.13 a**

$$\begin{array}{l} \text{S} \rightarrow TT \mid U \\ \text{T} \rightarrow 0T \mid T0 \mid \# \\ \text{U} \rightarrow 0U00 \mid \# \end{array}$$

Describe $L(G)$ in English.

Problem 2.13 b
Prove that $L(G)$ is not regular.

Problem 2.15
Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by th CFG

$G = (V, \sum, R, S)$. Add the new rule $S \to SS$ and call the resulting grammar G'A. This grammar is supposed to generate $A*$.

Problem 2.19
Let CFG G be the following grammar.

$$S \to aSb \mid bY \mid Ya$$
$$Y \to bY \mid aY \mid \epsilon$$

Give a simple description of L(G) in English. Use that description to give a CFG for $\overline{L(G)}$, the compliment of $L(G)$.

Problem 2.28

Give unambiguous CFGs for the following languages.

Problem 2.28 a
$\{w|$ in every prefix of w the number of a's is at least the number of b's $\}$

Problem 2.28 b
$\{w|$ the number of a's and the number of b's in w are equal $\}$

Problem 2.28 c
$\{w|$ the number of a's is at least the number of b's in $w\}$

Problem 2.30
Use the pumping lemma to show that the following languages are not context free.

Problem 2.30 a
$\{0^n 1^n 0^n 1^n | n \geq 0\}$

Problem 2.30 d
$\{t_1 \# t_2 \# \cdots \# t_k | k \geq 2,\ \text{each } t_i \in \{a, b\}^*,\ \text{and } t_i = t_j \text{ for some } i \neq j\}$

Problem 2.31
Let B be the language of all palindromes over $\{0, 1\}$ containing equal numbers of 0s and 1s. Show that B is not context free.

Problem 2.33
Show that $F = \{a^i, b^j | i = kj$ for some positive integer $k\}$ is not context free.

Problem 2.35
Let G be a CFG in Chomsky normal form that contains b variables. Show that if G generates some string with a derivation having at least 2^b steps, $L(G)$ is infinite.

Problem 2.46
Consider the following CFG G: Describe $L(G)$ and show that G is ambiguous. Give an

$$S \to SS \mid T$$
$$T \to aTb \mid ab$$

unambiguous grammar H where $L(H) = L(G)$ and sketch a proof that H is unambiguous.