

CS581 Theory of Computation: Homework #2

Due on February 1 2016 at 2:00pm

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Problem 2.6

Give context-free grammars generating the following languages.

Problem 2.6 b

The complement of the language $\{a^n b^n | n \geq 0\}$

$$\begin{aligned} S &\rightarrow aSb \mid bY \mid Ya \\ Y &\rightarrow bY \mid aY \mid \epsilon \end{aligned}$$

Problem 2.6 d

$\{x_1 \# x_2 \# \cdots \# x_k | k \geq 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j, x_i = x_j^R\}$

$$\begin{aligned} S &\rightarrow ABC \\ A &\rightarrow D \# A \mid \epsilon \\ B &\rightarrow 0B0 \mid 1B1 \mid E \\ C &\rightarrow \#DC \mid \epsilon \\ D &\rightarrow 1D \mid 0D \mid \epsilon \\ E &\rightarrow \#DE \mid \# \end{aligned}$$

Problem 2.7

Give informal English description of PDAs for the languages in Exercise 2.6

Problem 2.7 b

The complement of the language $\{a^n b^n | n \geq 0\}$

Problem 2.7 d

$\{x_1 \# x_2 \# \cdots \# x_k | k \geq 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j, x_i = x_j^R\}$

Problem 2.9

Give a context-free grammar that generates the language

$$A = \{a^i b^j c^k | i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$$

Is your grammar ambiguous? Why or why not?

$$\begin{aligned} S &\rightarrow AB \mid CD \\ A &\rightarrow aAb \mid \epsilon \\ B &\rightarrow cB \mid \epsilon \\ C &\rightarrow aC \mid \epsilon \\ D &\rightarrow bDc \mid \epsilon \end{aligned}$$

The language is not ambiguous...

Problem 2.13

Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S < TU\}$; $\Sigma = \{0, \#\}$; and R is the set of rules:

$$\begin{aligned} S &\rightarrow TT \mid U \\ T &\rightarrow 0T \mid T0 \mid \# \\ U &\rightarrow 0U00 \mid \# \end{aligned}$$

Problem 2.13 a

Describe $L(G)$ in English.

Informally $L(G)$ is either two or more $\#$ separated by arbitrary number of 0's (zero or more) or zero or more zero followed by $\#$ and by twice as many zeros as in before $\#$. More formally it is $\{0_1^{i_1} \# 0_2^{i_2} \# 0_3^{i_3} \# 0_4^{i_4} \# \cdots \# 0_k^{i_k} \mid \text{where } i_j \geq 0 \text{ and } k \geq 3\}$ or $\{0^n \# 0^{2n} \mid n \geq 1\}$

Problem 2.13 b

Prove that $L(G)$ is not regular.

1. Assume that $L(G)$ is regular.
2. Consider the word $0^p \# 0^{2p} \in L(G)$. By pumping lemma there exists a word $xyz \in L(G)$, and pumping length p , such that $|xy| \leq p$ and for all $i \geq 0$, $xy^i z \in L$. In case of word $= 0^p \# 0^{2p}$, xy can only be in 0^p , however for $i \geq 0$ $xy^i z$ is $0^{p+i} \# 0^{2p} \notin L(G)$ is a contradiction. Therefore $L(G)$ is not regular.

Problem 2.15

Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by the CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \rightarrow SS$ and call the resulting grammar G' . This grammar is supposed to generate A^* .

This rule applied to grammar $G = (\{S\}, \{a, b\}, \{S \rightarrow aSb \mid \epsilon\}, S) + S \rightarrow SS$ we get grammar G' that can produce a word: $aababb \notin A^*$

Problem 2.19

Let CFG G be the following grammar.

$$\begin{aligned} S &\rightarrow aSb \mid bY \mid Ya \\ Y &\rightarrow bY \mid aY \mid \epsilon \end{aligned}$$

Give a simple description of $L(G)$ in English. Use that description to give a CFG for $\overline{L(G)}$, the complement of $L(G)$.

$L(G)$ is the language that produces all strings not in $a^n b^n$, i.e. complement of $a^n b^n$.

$$S \rightarrow aSb \mid \epsilon$$

Problem 2.28

Give unambiguous CFGs for the following languages.

Problem 2.28 a

$\{w \mid \text{in every prefix of } w \text{ the number of a's is at least the number of b's} \}$

$$\begin{aligned} S &\rightarrow aS \mid BS \mid \epsilon \\ B &\rightarrow aBBb \mid \epsilon \end{aligned}$$

Problem 2.28 b

$\{w \mid \text{the number of a's and the number of b's in } w \text{ are equal} \}$

$$\begin{aligned} S &\rightarrow aA \mid bB \mid \epsilon \\ A &\rightarrow bS \mid aAA \\ B &\rightarrow aS \mid bBB \end{aligned}$$

Problem 2.28 c

$\{w \mid \text{the number of a's is at least the number of b's in } w\}$

$$\begin{aligned} S &\rightarrow aA \mid bB \mid a \\ A &\rightarrow bS \mid aAA \\ B &\rightarrow aS \mid bBB \end{aligned}$$

$$\begin{aligned} S &\rightarrow aS \mid bB \mid a \\ B &\rightarrow aS \mid bBB \end{aligned}$$

Problem 2.30

Use the pumping lemma to show that the following languages are not context free.

Problem 2.30 a

$\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

Let p be the pumping length, $s = 0^p 1^p 0^p 1^p$. By pumping lemma $s = uvxyz$, where $|vxy| \leq p$ and $|vy| > 1$.

Problem 2.30 d

$\{t_1 \# t_2 \# \cdots \# t_k \mid k \geq 2, \text{ each } t_i \in \{a, b\}^*, \text{ and } t_i = t_j \text{ for some } i \neq j\}$

Problem 2.31

Let B be the language of all palindromes over $\{0, 1\}$ containing equal numbers of 0s and 1s. Show that B is not context free.

Problem 2.33

Show that $F = \{a^i b^j \mid i = kj \text{ for some positive integer } k\}$ is not context free.

Problem 2.35

Let G be a CFG in Chomsky normal form that contains b variables. Show that if G generates some string with a derivation having at least 2^b steps, $L(G)$ is infinite.

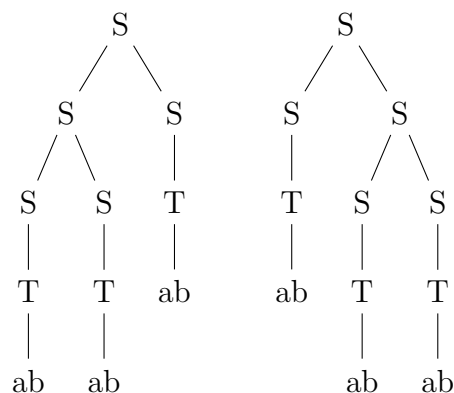
Problem 2.46

Consider the following CFG G : Describe $L(G)$ and show that G is ambiguous. Give an

$$\begin{aligned} S &\rightarrow SS \mid T \\ T &\rightarrow aTb \mid ab \end{aligned}$$

unambiguous grammar H where $L(H) = L(G)$ and sketch a proof that H is unambiguous.

$L(G) = \{(a^n b^n)^i \mid \text{where } i, n > 0\}$. $ababab \in L(G)$ and has two derivation trees



Therefore $L(G)$ is ambiguous.

Unambiguous grammar for this language is:

$$\begin{aligned} S &\rightarrow TS \mid \epsilon \\ T &\rightarrow aTb \mid ab \end{aligned}$$