CS581 Theory of Computation: Homework #5

Due on March 2 2016 at 2:00pm

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Problem 5.3

Find a match in the following instance of the Post Correspondence Problem.

$$\left\{ \left[\frac{ab}{abab}\right], \left[\frac{b}{a}\right], \left[\frac{aba}{b}\right], \left[\frac{aa}{a}\right] \right\}$$

Solution

$$\frac{ab}{abab}, \frac{ab}{abab}, \frac{aba}{b}, \frac{b}{a}, \frac{b}{a}, \frac{aa}{a}, \frac{aa}{a} \text{ or } \frac{aa}{a}, \frac{aa}{a}, \frac{b}{a}, \frac{ab}{abab}$$

Problem 5.4

If $A \leq_m B$ and B is a regular language, does that imply that A is a regular language? Why or why not?

Solution

No it doesn't imply that A is regular, for example: CFL $\{a^n b^n \mid n \geq 0\}$ can be reduced to regular language $\{a^n \mid n \geq 0\}$, by following procedure: check if input $\in a^n b^n$, output a^n if it is, and b if it is not.

Description of the TM form problems 1 and 2.

1. $Q = \{A, B, C, D\}$
2. $\Sigma = \{0, 1\}$
3. $\Gamma = \{0, 1, \_\}$
4. $\delta =$
   1. $\delta(A, 0) = (B, 1, R)$
   2. $\delta(A, 1) = (A, 1, R)$
   3. $\delta(A, \_) = (C, \_, L)$
   4. $\delta(B, 0) = (D, 0, L)$
   5. $\delta(B, 1) = (A, 0, R)$
   6. $\delta(B, \_) = (D, \_, L)$
5. $q_0 =$ A
6. $q_{accept} = C$
7. $q_{reject} = D$

Problem 1

Convert this into and instance of the PCP.

Solution

Convert the TM into instance of PCP by adding required domino tiles:

Part 1: add first tile

$$\left[\frac{\#}{\#\#Aw_1w_2w_3...}\right]$$

Part 2: Take care of the right transitions

$$\left[\frac{A0}{1B}\right] \left[\frac{A1}{1A}\right] \left[\frac{B1}{0A}\right]$$

Part 3: Take care of the left transitions

$$\left[\frac{0A\_}{C0\_}\right] \left[\frac{1A\_}{C1\_}\right] \left[\frac{\_A\_}{C\_\_}\right] \left[\frac{0B0}{D00}\right] \left[\frac{1B0}{D10}\right] \left[\frac{\_B0}{D\_0}\right] \left[\frac{0B\_}{D0\_}\right] \left[\frac{1B\_}{D1\_}\right] \left[\frac{\_B\_}{D\_\_}\right]$$
$$\left[\frac{\#A\_}{\_\#C\_}\right] \left[\frac{\#B0}{\_\#D0}\right] \left[\frac{\#B\_}{\_\#D\_}\right]$$

Part 4: For every $a \in \Gamma$ put $\left[\frac{a}{a}\right]$

$$\left[\frac{0}{0}\right] \left[\frac{1}{1}\right] \left[\frac{\_}{\_}\right]$$

Part5

$$\left[\frac{\#}{\#}\right] \left[\frac{\#}{\_\#}\right]$$

Part 6: Accept states

$$\left[\frac{0C}{C}\right] \left[\frac{1C}{C}\right] \left[\frac{\_C}{C}\right] \left[\frac{C0}{C}\right] \left[\frac{C1}{C}\right] \left[\frac{C\_}{C}\right]$$

Part7: Final domino

$$\left[\frac{C\#\#}{\#}\right]$$

So far we converted the TM into MPCP, usually this this would require further conversion into instance of PCP, by addition of $\frac{\star\, t_1}{\star b_1 \star}$ to the first title, and $\frac{\star\, t_1}{b_1 \star}$ to all the rest to enforce the order of computation, however this procedure was omitted for briefness.

Problem 2

Show that the string "01" is in the language recognized by this TM by showing a solution to your instance of the PCP.

Solution
We find a match in PCP instance.

$$\frac{\#}{\#\#A01\_\#}\frac{\#}{\#}\frac{A0}{1B}\frac{1}{1}\frac{\_}{\_}\frac{\#}{\#}\frac{1}{1}\frac{B1}{0A}\frac{\_}{\_}\frac{\#}{\#}\frac{1}{1}\frac{0A\_}{C0\_}\frac{\#}{\#}\frac{1}{1}\frac{C0}{C}\frac{\_}{\_}\frac{\#}{\#}\frac{1C}{C}\frac{\_}{\_}\frac{\#}{\#}\frac{C\_}{C}\frac{\#}{\#}\frac{C\#\#}{\#}$$

Resulting PCP:

$$\frac{\#\#A01\_\#1B1\_\#10A\_\#1C0\_\#1C\_\#C\_\#C\#\#}{\#\#A01\_\#1B1\_\#10A\_\#1C0\_\#1C\_\#C\_\#C\#\#}$$