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# SIMPLE LINEAR-TIME ALGORITHMS TO TEST CHORDALITY OF GRAPHS, TEST ACYCLICITY OF HYPERGRAPHS, AND SELECTIVELY REDUCE ACYCLIC HYPERGRAPHS*

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**Abstract.** Chordal graphs arise naturally in the study of Gaussian elimination on sparse symmetric matrices; acyclic hypergraphs arise in the study of relational data bases. Rose, Tarjan and Lueker [SIAM J. Comput., 5 (1976), pp. 266–283] have given a linear-time algorithm to test whether a graph is chordal, which Yannakakis has modified to test whether a hypergraph is acyclic. Here we develop a simplified linear-time test for graph chordality and hypergraph acyclicity. The test uses a new kind of graph (and hypergraph) search, which we call *maximum cardinality search.* A variant of the method gives a way to selectively reduce acyclic hypergraphs, which is needed for evaluating queries in acyclic relational data bases.

**Key words.** graph algorithm, acyclic data base scheme, sparse Gaussian elimination, graph search, hypergraph

**1. Introduction.** We shall use more-or-less standard terminology from the theory of graphs and hypergraphs [3], some of which we review here. A *hypergraph* $H=(V, E)$ consists of a set of *vertices* $V$ and a set of *edges* $E$; each edge is a subset of $V$. A *graph* is a hypergraph all of whose edges have size two. The *graph* $G(H)$ of a hypergraph $H$ is the graph whose vertices are those of $H$ and whose edges are the vertex pairs $\{v, w\}$ such that $v$ and $w$ are in a common edge of $H$. Two vertices of a graph $G$ are *adjacent* if they are contained in an edge. A *path* in $G$ is a sequence of distinct vertices $v_0, v_1, \cdots, v_k$ such that $v_i$ and $v_{i+1}$ are adjacent for $0 \leqq i < k$. A *cycle* is a path $v_0, v_1, \cdots, v_k$ such that $k \geqq 2$ and $v_0$ and $v_k$ are adjacent. Vertices $v_i$ and $v_{(i+1) \bmod (k+1)}$ for $0 \leqq i \leqq k$ are *consecutive* on the cycle. A *clique* of $G$ is a set of pairwise adjacent vertices. A hypergraph $H$ is *conformal* if every clique of $G(H)$ is contained in an edge of $H$. A graph $G$ is *chordal* if every cycle of length at least four has a chord, i.e., an edge joining two nonconsecutive vertices on the cycle. A hypergraph $H$ is *acyclic* if $H$ is conformal and $G(H)$ is chordal.

Chordal graphs arise in the study of Gaussian elmination on sparse symmetric matrices [12]. Acyclic hypergraphs arise in the study of relational data base schemes [1], [7], [21]; they are powerful enough to capture most real-world situations but simple enough to have many desirable properties [1], [2], [9], [18]. Rose, Tarjan and Lueker [15] have given an $O(n+m)$-time[1] algorithm (henceforth called the RTL algorithm) to test whether a graph is chordal. Yannakakis [19] has extended the algorithm to the problem of testing whether a hypergraph is acyclic. In this paper we propose a simplified version of the RTL algorithm that can be used for testing both chordality of graphs and acyclicity of hypergraphs. In § 2 we develop the algorithm as it applies to graph chordality testing. In § 3 we modify the algorithm for hypergraph acyclicity testing. Besides leading to a method simpler than the RTL test, our analysis provides additional insight into the structure of chordal graphs and acyclic hypergraphs. In § 4 we use this insight to develop a simple linear-time algorithm for selectively reducing acyclic hypergraphs, a problem that arises in evaluating queries in acyclic relational data bases.

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[1] We shall use $n$ to denote the number of vertices and $m$ to denote the total size of the edges in a hypergraph.