Differentiable Neural Computers

Hybrid Computing using a neural network with dynamic external memory (Graves et al. 2016)

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Logic and Computation

Overview: Probabilistic Programming

Cross-domain

- Data Flow Programming
- Bayesian Reasoning
- Machine Learning
- Functional Programming

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Program	Model
Discrete	Continuous
Deterministic	Stochastic
Static	Adaptive

Overview: DNC

Differentiable Neural Computer

A recurrent neural network coupled with an external memory.

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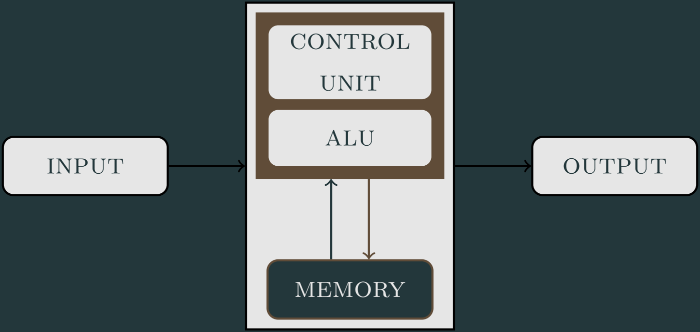
Differentiable Neural Computer

A recurrent neural network coupled with an external memory.

- Inspired by biological memory
- Reimaging of classical concepts of computation
- Extension of NTMs
  - End-to-end differentiable
  - Auto-associative memory
  - Turing complete
  - \+ Stronger memory management

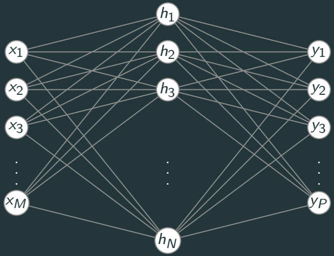
Introduction: Classic Computation

Von Neumann architecture



Introduction: Neural Networks

Simple Neural Net



$$NN : \boldsymbol{x}_{t_i} \mapsto \boldsymbol{y}_{t_i}$$

No memory

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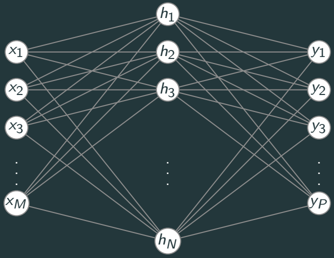

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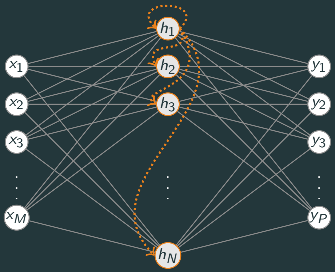
Simple Recurrent Net



$$RNN : \boldsymbol{x}_{t_0} \otimes \boldsymbol{x}_{t_1} \otimes \cdots \otimes \boldsymbol{x}_{t_i} \mapsto \boldsymbol{y}_{t_i}$$

Finite memory

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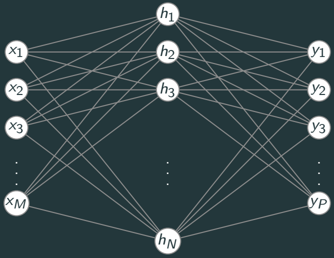

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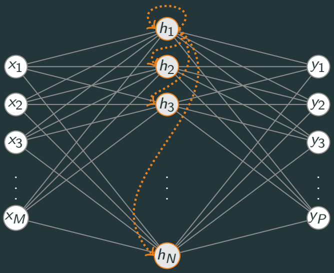

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Finite memory

"If training vanilla neural nets is optimization over functions, training recurrent nets is optimization over programs."

Approach

Train a RNN to act as the controller of a memory matrix M of N addresses through R read heads and one write head.

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1. Content Lookup
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   - Shift operations defined by $Lw,\ L^\top w$
3. Dynamic Allocation
   - Mark memory locations with $\{0,1\}$ to signal usage
   - Manipulate signals during R/W operations to enable reallocation
   - Generalization to unbounded memory

Controller: Overview

A deep long short-term memory network receiving input:

$$\boldsymbol{\mathcal{X}}_t = [\boldsymbol{x}_t; \boldsymbol{r}_{t-1}^1; \ldots; \boldsymbol{r}_{t-1}^R] \qquad \text{(timestep t)}$$

and producing output:

$$(\boldsymbol{\upsilon}_T, \boldsymbol{\xi}_T) = \mathcal{N}([\boldsymbol{\mathcal{X}}_1; \ldots; \boldsymbol{\mathcal{X}}_T]; \vartheta) \qquad \text{(entire sequence)}$$

where $\mathcal{N}$ a set of state equations and ϑ their trainable parameters.

Controller: State Equations

A more detailed look into $\mathcal{N}$.

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LSTM layer equations

Input: $[\mathcal{X}_t; \boldsymbol{h}_{t-1}^{l}; \boldsymbol{h}_{t}^{l-1}]$
Output: $\boldsymbol{h}_t^l$

Controller: State Equations

A more detailed look into $\mathcal{N}$.

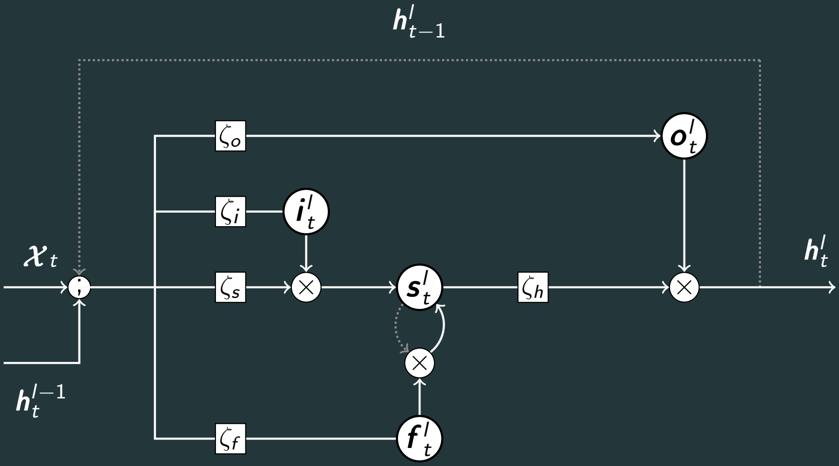
LSTM layer equations

Input: $[\mathcal{X}_t; \boldsymbol{h}^l_{t-1}; \boldsymbol{h}^{l-1}_t]$
Output: $\boldsymbol{h}^l_t$

$$
\begin{aligned}
\boldsymbol{i}^l_t &= \zeta_i([\mathcal{X}_t; \boldsymbol{h}^l_{t-1}; \boldsymbol{h}^{l-1}_t]) && \text{(input gate)}\\
\boldsymbol{f}^l_t &= \zeta_f([\mathcal{X}_t; \boldsymbol{h}^l_{t-1}; \boldsymbol{h}^{l-1}_t]) && \text{(forget gate)}\\
\boldsymbol{s}^l_t &= \boldsymbol{f}^l_t \boldsymbol{s}^l_{t-1} + \boldsymbol{i}^l_t \zeta_s([\mathcal{X}_t; \boldsymbol{h}^l_{t_1}; \boldsymbol{h}^{l-1}_t]) && \text{(state update)}\\
\boldsymbol{o}^l_t &= \zeta_o([\mathcal{X}_t; \boldsymbol{h}^l_{t-1}; \boldsymbol{h}^{l-1}_t]) && \text{(output gate)}\\
\boldsymbol{h}^l_t &= \boldsymbol{o}^l_t \zeta_h(\boldsymbol{s}^l_t) && \text{(hidden layer)}
\end{aligned}
$$

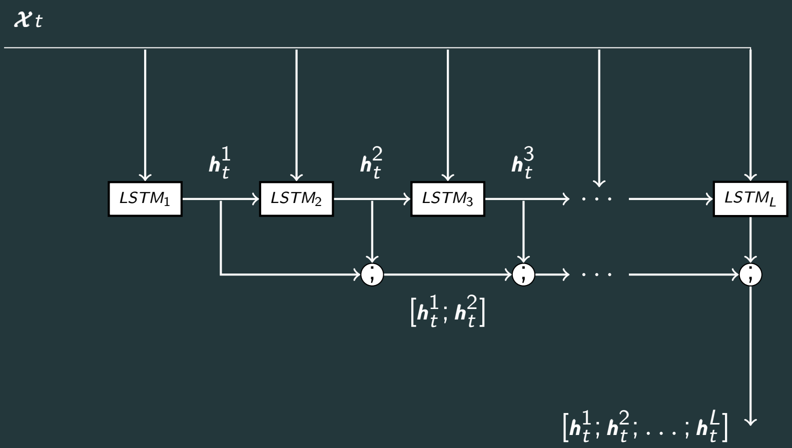
Controller: Signal-Flow (1/2)

Single LSTM layer



Controller: Signal-Flow (2/2)

LSTM Network (multiple layers)



Controller: Outputs

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Intermediate output $\boldsymbol{\upsilon}_t = W_y[\boldsymbol{h}_t^1; \ldots; \boldsymbol{h}_t^L]$

$$\boldsymbol{y}_t = \boldsymbol{\upsilon}_t + W_R[\boldsymbol{r}_t^1; \ldots; \boldsymbol{r}_t^R] \qquad \text{(Memory-conditioning)}$$

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Interface vector $\boldsymbol{\xi}_t = W_\xi[\boldsymbol{h}_t^1; \ldots; \boldsymbol{h}_t^L]$

- Read keys: $\boldsymbol{k}_t^{r,i}$
- Read strengths: $\beta_t^{r,i}$
- Write key: $\boldsymbol{k}_t^w$
- Write strength: β_t^w
- Erase vector: $\boldsymbol{e}_t$
- Write vector: $\boldsymbol{v}_t$
- Free gates: $\boldsymbol{\phi}_t^i$
- Allocation gate: g_t^a
- Write gate: g_t^w
- Read modes: $\boldsymbol{\pi}_t^i$

Memory Adressing: Content-Lookup

R read keys $\boldsymbol{k}^{r,i} \in \mathbb{R}^W, \ i = 1 \ldots R$

R read strengths $\beta^{r,i} \in [1, \infty), \ i = 1 \ldots R$

Write key $\boldsymbol{k}^w \in \mathbb{R}^W$

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Matching function $\mathcal{D}$ comparing memory contents

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Matching function $\mathcal{D}$ comparing memory contents

Weighting function $\mathcal{C}$ normalizing and sharpening matches

$$\mathcal{C}(M, \boldsymbol{k}, \beta)[i] = \frac{exp\{\beta \mathcal{D}(\boldsymbol{k}, M[i, :])\}}{\sum_j exp\{\beta \mathcal{D}(\boldsymbol{k}, M[j, :])\}}$$

Memory Adressing: R/W

Attention dictated by weightings $\boldsymbol{w} \in \mathcal{S}^N$

Erase vector $\boldsymbol{e}_t \in [0,1]^W$

Write vector $\boldsymbol{v}_t \in \mathbb{R}^W$

Read operations

$$\boldsymbol{r}_t^i = M_t^\top \boldsymbol{w}_t^{r,i}$$

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Write operations

$$M_t = \underbrace{M_{t-1} \circ (\mathbf{1} - \boldsymbol{w}_t^w \boldsymbol{e}_t^\top)}_{\text{erased memory}} + \underbrace{\boldsymbol{w}_t^w \boldsymbol{v}_t^\top}_{\text{new write}}$$

Memory Adressing: Dynamic Allocation

Free gates $\boldsymbol{\phi}_t^i \in [0,1]^W$

Allocation gate $g_t^a \in [0,1]$

Write gate $g_t^w \in [0,1]$

"Free list" scheme

$$\boldsymbol{\psi}_t = \prod_{i=1}^{R}(\mathbf{1} - \boldsymbol{\phi}_t^i w_{t-1}^{r,i}) \qquad \text{(memory retention)}$$

$$u_t = (\boldsymbol{u}_{t-1} + \boldsymbol{w}_{t-1}^w + \boldsymbol{u}_{t-1} \circ \boldsymbol{w}_{t-1}^w) \circ \boldsymbol{\psi}_t \qquad \text{(usage tracking)}$$

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Attention shift

- Obtain the allocation vector $\boldsymbol{a}_t$ by normalizing $\boldsymbol{u}_t$
- Shift $\boldsymbol{w}_t$ by $g_t^a \boldsymbol{a}_t$ and scale by g_t^w

Memory Adressing: Temporal Linking

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Allocation vector $\boldsymbol{a}_t$ given by normalizing and sorting $\boldsymbol{u}_t$

Further Reading

- Neural Turing Machines (Graves, Wayne, Danihelka)
- Entity Networks (Henaff, Weston, Szlam, Bordes, LeCun)
- End-to-End Memory Networks (Sukhbaatar, Szlam, Weston, Fergus)
- Jointly Learning to Align and Translate (Bahdanau, Cho, Bengio)
- Principles of Probabilistic Programming Languages (Goodman)
- Backprop as a Functor (Fong, Spivak, Tuyras)
- Formal Methods for Probabilistic Programming (Selsam, Liang, Dill)

Conclusion

Takeaway

We can automatically infer simple functions over complex data structures, in the form of probability distributions just by using examples.

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Thank you!