# Differentiable Neural Computers

## Hybrid Computing using a neural network with dynamic external memory (Graves et al. 2016)

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May 28, 2018

Logic and Computation

# Overview: Probabilistic Programming

## Cross-domain

- Data Flow Programming
- Bayesian Reasoning
- Machine Learning
- Functional Programming

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| PROGRAM | MODEL |
|---:|---|
| Discrete | Continuous |
| Deterministic | Stochastic |
| Static | Adaptive |

# Overview: DNC

Differentiable Neural Computer

A recurrent neural network coupled with an external memory.

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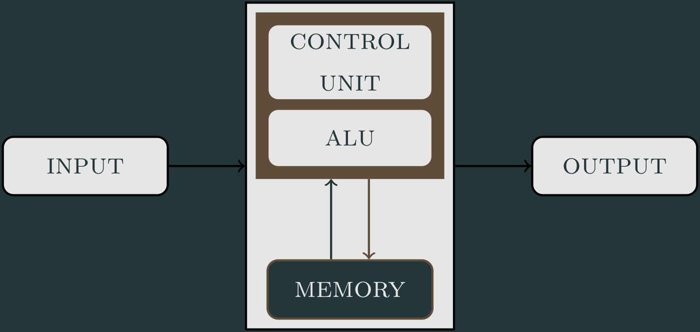
**Differentiable Neural Computer**

A recurrent neural network coupled with an external memory.

- Functional replication of biological memory
- Reimaging of classical concepts of computation
- Extension of NTMs
  - End-to-end differentiable
  - Auto-associative memory
  - Turing complete
  + Computationally efficient memory management

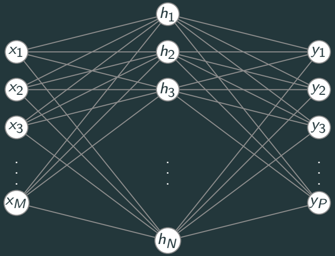
# Introduction: Classic Computation

## Von Neumann architecture



# Introduction: Neural Networks

## Simple Neural Net

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No memory

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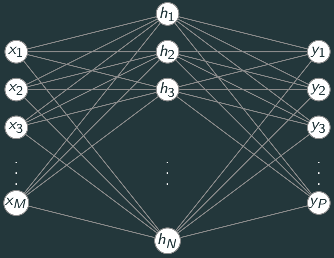


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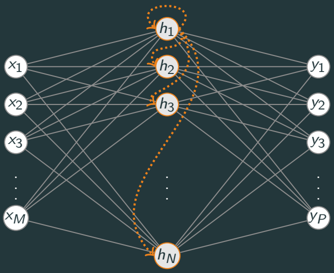


$$RNN : \boldsymbol{x}_{t_0} \otimes \boldsymbol{x}_{t_1} \otimes \cdots \otimes \boldsymbol{x}_{t_i} \mapsto \boldsymbol{y}_{t_i}$$

Finite memory

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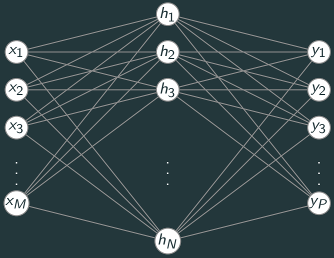


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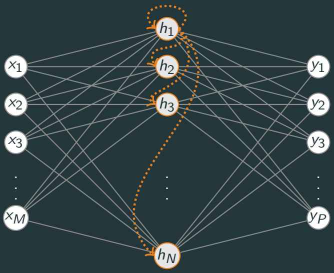


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Finite memory

*"If training vanilla neural nets is optimization over functions, training recurrent nets is optimization over programs."*

# Approach

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3. Dynamic Allocation
   - Mark memory locations with $\boldsymbol{u} \in [0,1]^N$ to signal usage
   - Manipulate signals during R/W operations to enable reallocation
   - Generalization to unbounded memory

# Controller: Overview

A deep long short-term memory network receiving input:

$$\boldsymbol{\mathcal{X}}_t = [\boldsymbol{x}_t; \boldsymbol{r}^1_{t-1}; \ldots; \boldsymbol{r}^R_{t-1}] \qquad \text{(timestep t)}$$

and producing output:

$$(\boldsymbol{\upsilon}_t, \boldsymbol{\xi}_t) = \mathcal{N}([\boldsymbol{\mathcal{X}}_1; \ldots; \boldsymbol{\mathcal{X}}_t]; \vartheta) \qquad \text{(entire sequence)}$$

where $\mathcal{N}$ a set of state equations and $\vartheta$ their trainable parameters.

# LSTM: Overview

### LSTM Network

Multiple stacked LSTM units.

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1. Input gate $\boldsymbol{i}_t^l$
2. Forget gate $\boldsymbol{f}_t^l$
3. Output gate $\boldsymbol{o}_t^l$

# LSTM: Signal-Flow (1/2)

LSTM Unit (single layer)

- Input: $[\mathcal{X}_t; \boldsymbol{h}_{t-1}^l; \boldsymbol{h}_t^{l-1}]$
- Output: $\boldsymbol{h}_t^l$

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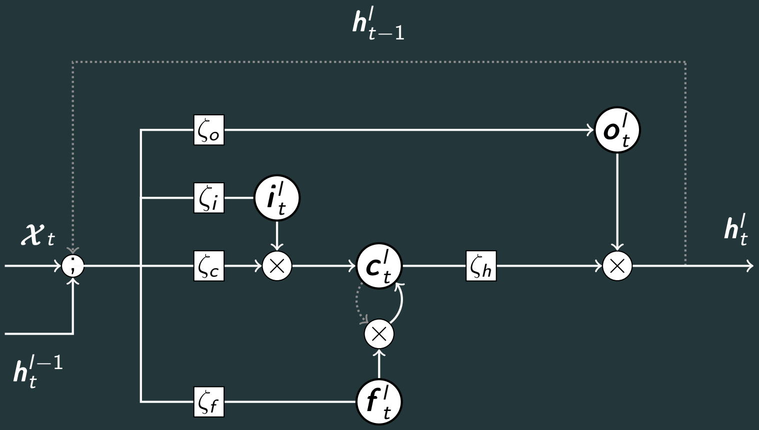

# Controller: Signal-Flow (2/2)

LSTM Network (multiple layers)

- Input: $\boldsymbol{\mathcal{X}}_t$
- Output: $[\boldsymbol{h}_t^1; \boldsymbol{h}_t^2; \ldots \boldsymbol{h}_t^L]$

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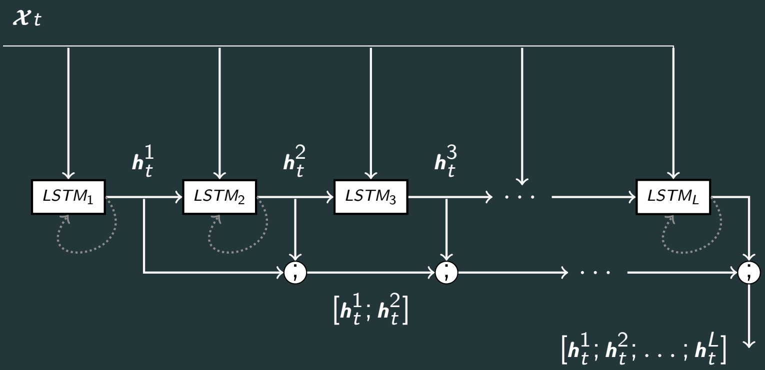

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Intermediate output $\boldsymbol{\upsilon}_t = W_y[\boldsymbol{h}_t^1; \ldots; \boldsymbol{h}_t^L]$

$$\boldsymbol{y}_t = \boldsymbol{\upsilon}_t + W_R[\boldsymbol{r}_t^1; \ldots; \boldsymbol{r}_t^R] \qquad \text{(Memory-conditioning)}$$

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Interface vector $\boldsymbol{\xi}_t = W_\xi[\boldsymbol{h}_t^1; \ldots; \boldsymbol{h}_t^L]$

- Read keys: $\boldsymbol{k}_t^{r,i}$
- Read strengths: $\beta_t^{r,i}$
- Write key: $\boldsymbol{k}_t^w$
- Write strength: $\beta_t^w$
- Erase vector: $\boldsymbol{e}_t$
- Write vector: $\boldsymbol{v}_t$
- Free gates: $\boldsymbol{\phi}_t^i$
- Allocation gate: $g_t^a$
- Write gate: $g_t^w$
- Read modes: $\boldsymbol{\pi}_t^i$

# Memory Adressing: Content-Lookup

R read keys $\boldsymbol{k}^{r,i} \in \mathbb{R}^W,\ i = 1 \dots R$

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Write key $\boldsymbol{k}^w \in \mathbb{R}^W$

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Matching function $\mathcal{D}$ comparing memory contents

Weighting function $\mathcal{C}$ normalizing and sharpening matches

$$\mathcal{C}(M, \boldsymbol{k}, \beta)[i] = \frac{exp\{\beta\mathcal{D}(\boldsymbol{k}, M[i, :])\}}{\sum_j exp\{\beta\mathcal{D}(\boldsymbol{k}, M[j, :])\}}$$

# Memory Adressing: R/W

Attention dictated by weightings $\boldsymbol{w} \in \mathcal{S}^N$

Erase vector $\boldsymbol{e}_t \in [0, 1]^W$

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Read operations

$$\boldsymbol{r}_t^i = M_t^\top \boldsymbol{w}_t^{r,i}$$

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Write operations

$$M_t = \underbrace{M_{t-1} \circ (\mathbf{1} - \boldsymbol{w}_t^w \boldsymbol{e}_t^\top)}_{\text{erased memory}} + \underbrace{\boldsymbol{w}_t^w \boldsymbol{v}_t^\top}_{\text{new write}}$$

# Memory Adressing: Dynamic Allocation

Free gates $\boldsymbol{\phi}_t^i \in [0,1]^W$

Allocation gate $g_t^a \in [0,1]$

Write gate $g_t^w \in [0,1]$

"Free list" scheme

$$\boldsymbol{\psi}_t = \prod_{i=1}^{R} (\mathbf{1} - \boldsymbol{\phi}_t^i w_{t-1}^{r,i}) \qquad \text{(memory retention)}$$

$$u_t = (\boldsymbol{u}_{t-1} + \boldsymbol{w}_{t-1}^w - \boldsymbol{u}_{t-1} \circ \boldsymbol{w}_{t-1}^w) \circ \boldsymbol{\psi}_t \qquad \text{(usage tracking)}$$

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Attention shift

- Obtain the allocation vector $\boldsymbol{a}_t$ by normalizing $\boldsymbol{u}_t$
- Shift $\boldsymbol{w}_t$ by $g_t^a \boldsymbol{a}_t$ and scale by $g_t^w$

# Memory Adressing: Temporal Linking

Read modes: $\boldsymbol{\pi}_t^i \in \mathcal{S}_3$

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Temporal Transition $\boldsymbol{L}_t \in [0,1]^{N \times N}$

$$\boldsymbol{L}[i,j] = \underbrace{(1 - w_t^W[i] - w_t^W[j])\boldsymbol{L}_{t-1}[i,j]}_{\text{Part of last transition}} + \underbrace{w_t^w[i] w_{t-1}^w[j]}_{\text{Current transition}}$$

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Mode Interpolation

$$\boldsymbol{w}_t^{r,i} = \underbrace{\boldsymbol{\pi}_t^i[1]L\boldsymbol{w}_t^{r,i}}_{\text{Forward shift}} + \underbrace{\boldsymbol{\pi}_t^i[2]\boldsymbol{w}_t^{r,i}}_{\text{No shift}} + \underbrace{\boldsymbol{\pi}_t^i[3]L^\top\boldsymbol{w}_t^{r,i}}_{\text{Backward shift}}$$

# Conclusion

Recap

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We can simulate computation and expand upon it by using differentiable, continuous operations.

### Takeaway

We can automatically infer simple functions over complex data structures in the form of probability distributions, just by using examples!

# Appendix: LSTM Equations

$$\boldsymbol{i}_t^l = \sigma(W_i^l[\boldsymbol{\mathcal{X}}_t; \boldsymbol{h}_{t-1}^l; \boldsymbol{h}_t^{l-1}] + \boldsymbol{b}_i^l) \qquad \text{(input gate)}$$

$$\boldsymbol{f}_t^l = \sigma(W_f^l[\boldsymbol{\mathcal{X}}_t; \boldsymbol{h}_{t-1}^l; \boldsymbol{h}_t^{l-1}] + \boldsymbol{b}_f^l) \qquad \text{(forget gate)}$$

$$\boldsymbol{s}_t^l = \boldsymbol{f}_t^l \boldsymbol{s}_{t-1}^l + \boldsymbol{i}_t^l tanh(W_s^l[\boldsymbol{\mathcal{X}}_t; \boldsymbol{h}_{t_1}^l; \boldsymbol{h}_t^{l-1}] + \boldsymbol{b}_s^l) \qquad \text{(state)}$$

$$\boldsymbol{o}_t^l = \sigma(W_o^l[\boldsymbol{\mathcal{X}}_t; \boldsymbol{h}_{t-1}^l; \boldsymbol{h}_t^{l-1}] + \boldsymbol{b}_o^l) \qquad \text{(output gate)}$$

$$\boldsymbol{h}_t^l = \boldsymbol{o}_t^l tanh(\boldsymbol{s}_t^l) \qquad \text{(hidden)}$$

$$\boldsymbol{\upsilon}_t = W_y[\boldsymbol{h}_t^1; \ldots; \boldsymbol{h}_t^L] \qquad \text{(output vector)}$$

$$\boldsymbol{\xi}_t = W_\xi[\boldsymbol{h}_t^1; \ldots; \boldsymbol{h}_t^L] \qquad \text{(interface vector)}$$

# Appendix: Further Reading

## Neural Architectures

- Learning to Forget (Gers, Schmidhuber, Cummins)
- Neural Turing Machines (Graves, Wayne, Danihelka)
- Entity Networks (Henaff, Weston, Szlam, Bordes, LeCun)
- End-to-End Memory Networks (Sukhbaatar, Szlam, Weston, Fergus)
- Jointly Learning to Align and Translate (Bahdanau, Cho, Bengio)

## Probabilistic Programming

- Principles of Probabilistic Programming Languages (Goodman)
- Backprop as a Functor (Fong, Spivak, Tuyras)
- Formal Methods for Probabilistic Programming (Selsam, Liang, Dill)