Data-driven Logic

or: type theory for the working engineer

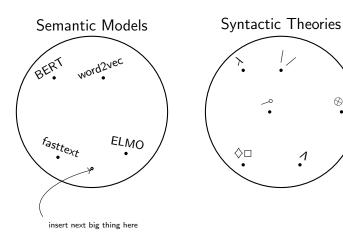
Kokos

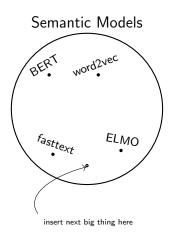
Logic & Language $\frac{2020}{2020}$

16/12/21

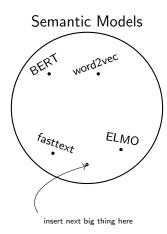


¹totally not the same slides as last year

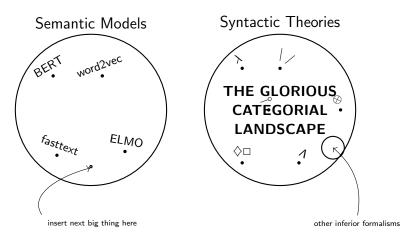




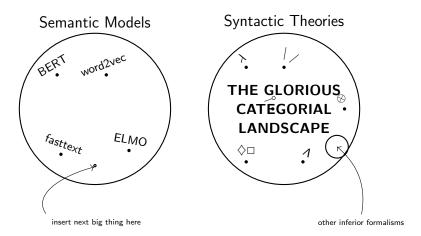








What's missing?



What's missing?

No (working) unifying theory or application.

Neural Type-Driven Representations

The agenda:

- λ Choosing the logic
- λ Making a dataset: proofs and lexical type assignments
- λ Learning the type assignment process
- λ Navigating the proof space
- λ Syntax-aware & type-correct text representations

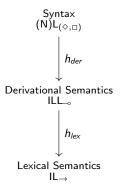
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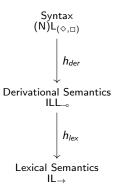
Syntax-Semantics Interface

Type-logical perspective

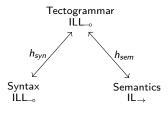


Syntax-Semantics Interface

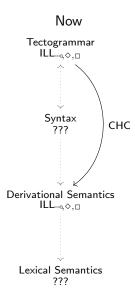
Type-logical perspective



ACG perspective



A twist



Grammar

 ILL_{\multimap} plus \lozenge , \square modalities for *dependency domain demarkation*.

Types inductively defined by:

$$\mathcal{T} := A \mid \mathcal{T} \multimap \mathcal{T}' \mid \diamondsuit^d \mathcal{T} \mid \square^d \mathcal{T} \qquad A \in \mathcal{A}, \mathcal{T} \in \mathcal{T}$$

Rules

$$\frac{\Gamma \vdash \mathbf{s} : A \multimap B \quad \Delta \vdash \mathbf{t} : A}{\Gamma, \Delta \vdash \mathbf{s} \mathbf{t} : B} \multimap E \qquad \frac{\Gamma, \mathbf{x} : A \vdash \mathbf{s} : B}{\Gamma \vdash \lambda \mathbf{x} . \mathbf{s} : A \multimap B} \multimap I$$

$$\frac{\Gamma \vdash \mathbf{t} : A}{\langle \Gamma \rangle^d \vdash \Delta^d \mathbf{t} : \diamondsuit^d A} \diamondsuit^d I \qquad \frac{Y \vdash \mathbf{s} : \diamondsuit^d A \quad X[\langle \mathbf{x} : A \rangle^d] \vdash \mathbf{t} : B}{X[Y] \vdash \mathbf{t} [\nabla^d \mathbf{s} / \mathbf{x}] : B} \diamondsuit^d E$$

$$\frac{\langle X \rangle^d \vdash \mathbf{s} : A}{X \vdash \blacktriangle^d \mathbf{s} : \Box^d A} \Box^d I \qquad \frac{X \vdash \mathbf{s} : \Box^d A}{\langle X \rangle^d \vdash \blacktriangledown^d \mathbf{s} : A} \Box^d E$$

Lexicon $\ensuremath{\mathcal{L}}$ assigning words types from:

Lexicon $\mathcal L$ assigning words types from: A

animals, ducks, I : np

 $\frac{\text{ducks}}{\text{ducks}:\textit{np}}~\mathcal{L}$

Lexicon $\mathcal L$ assigning words types from: $A \mid \diamondsuit^d T \multimap T'$

animals, ducks, I : np fly, swim : $\diamondsuit^{su}np - \circ s$ like : $\diamondsuit^{obj}np - \circ \diamondsuit^{obj}np - \circ s$

$$\frac{\frac{\mathsf{fly}}{\diamondsuit^{su} np \multimap s} \ \mathcal{L} \ \frac{\frac{\mathsf{ducks}}{np} \ \mathcal{L}}{\langle \mathsf{ducks} \rangle^{su} \ \vdash \diamondsuit^{su} np} \ \diamondsuit^{su} I}{\langle \mathsf{ducks} \rangle^{su} \ \mathsf{fly} \vdash s} \ \diamondsuit^{su} I$$

fly \triangle^{su} ducks

Lexicon $\mathcal L$ assigning words types from: $A \mid \diamondsuit^d T \multimap T' \mid \Box^d (T \multimap T')$

```
animals, ducks, I : np fly, swim : \diamondsuit^{su} np - \circ s like : \diamondsuit^{obj} np - \circ \diamondsuit^{obj} np - \circ s gracefully : \Box^{mod} (s - \circ s)
```

$$\frac{\frac{\operatorname{gracefully}}{\Box^{mod}\left(s \multimap s\right)} \ \mathcal{L}}{\frac{\langle \operatorname{gracefully} \rangle^{mod} \vdash s \multimap s}{\langle \operatorname{ducks} \rangle^{su} \ \operatorname{fly} \vdash s}}{\langle \operatorname{ducks} \rangle^{su} \ \operatorname{fly} \ \langle \operatorname{gracefully} \rangle^{mod} \vdash s}} \multimap \mathcal{E}$$

$$\mathsf{P}^{mod} \mathsf{gracefully} \ (\mathsf{fly} \ \triangle^{su} \ \operatorname{ducks})$$

Lexicon \mathcal{L} assigning words types from: $A \mid \diamondsuit^d T \multimap T' \mid \Box^d (T \multimap T')$ ks, I : np fly, swim : $\diamondsuit^{su}np \multimap s$ like : $\diamondsuit^{obj}np \multimap \diamondsuit^{obj}np \multimap s$ gracefully : $\Box^{mod}(s \multimap s)$ that : $\diamondsuit^{body}(\diamondsuit^{su}np \multimap s) \multimap \Box^{mod}(np \multimap np)$ animals, ducks, I : np $\frac{\text{that}}{\diamondsuit^{body}\left(\diamondsuit^{obj}np \multimap s\right) \multimap \Box^{mod}\left(np \multimap np\right)} \ \mathcal{L} \quad \frac{\langle I \rangle^{su} \ \text{like} \vdash \lambda x.(\texttt{like} \ x \ \triangle^{su}I) : \diamondsuit^{obj}np \multimap s}{\langle \langle I \rangle^{su} \ \text{like} \rangle^{body} \vdash \diamondsuit^{body}\left(\diamondsuit^{obj}np \multimap s\right)} \ \diamondsuit^{body}I$ $\frac{\mathsf{that}\ \langle\langle \mathsf{I}\rangle^{su}\ \mathsf{like}\rangle^{body} \vdash \Box^{mod}(np \multimap np)}{\langle\mathsf{that}\ \langle\langle \mathsf{I}\rangle^{su}\ \mathsf{like}\rangle^{body}\rangle^{mod}\vdash np \multimap np} \ \Box^{mod}E$ $\frac{\text{animals}}{np} \mathcal{L}$

$$lacktriangledown^{mod}\left({
m that} \ \triangle^{body}\left(\lambda x. ({
m like} \ {
m x} \ \triangle^{su}I) \right)
ight)$$
 animals

animals $\langle \text{that } \langle \langle \text{I} \rangle^{su} \text{ like} \rangle^{body} \rangle^{mod} \vdash np$

Why ILL_{⊸,⇔,□}?

Why ILL..?

- ► Easier to extract from corpora
- Massive reduction in lexical ambiguity (more later)
- Abstract away from trivial word-order permutations
- Surface syntax matters little to semantics

Why ILL_{⊸,♦,□}?

Why ILL..?

- ► Easier to extract from corpora
- Massive reduction in lexical ambiguity (more later)
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Why \diamondsuit , \square ?

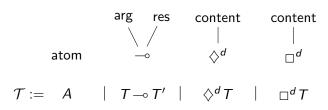
- ► More interpretation options
- Subsume dependency parsing
- More informative for semantics
- Modalities can regulate non-logical parsing

Proof Nets

A graphical, diagrammatical representation of (intuitionitic) linear logic proofs.

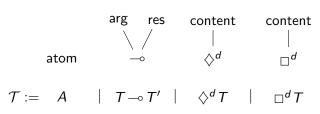
Formula decomposition

Type formation rules \equiv tree constructors



Formula decomposition

Type formation rules \equiv tree constructors

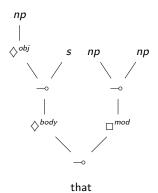


or, overloading \Diamond and \Box as binary operators for brevity



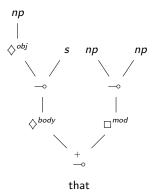
Tree Polarization

Let + stand for resources we have , - for ones we seek:



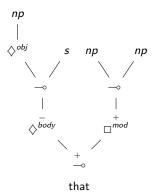
Tree Polarization

Let + stand for resources we *have*, - for ones we seek:



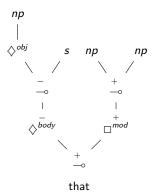
Tree Polarization

Let + stand for resources we *have*, - for ones we seek:



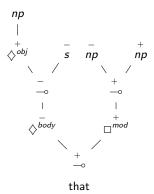
Tree Polarization

Let + stand for resources we *have*, - for ones we seek:



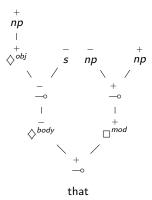
Tree Polarization

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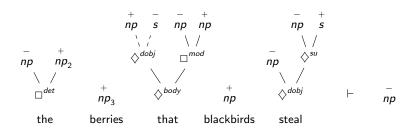
Tree Polarization

Let + stand for resources we *have*, - for ones we seek:



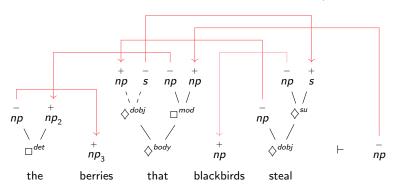
Proof Frame

A sequence of polarized unfolded formulas



Proof Structure

A proof net together with axiom links: bijection between pos/neg atoms



Traversing a Proof Net

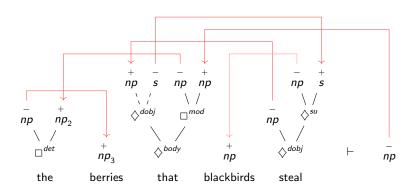
positive (down) mode: start from positive leaf, follow positive nodes to subtree root

- □ wrap future with ▼
- each is an application on reaching root, write word or variable name and perform negative traversal on negative child of each —

negative (up) mode: start from negative root, follow negatives to subtree leaf

- □ wrap future with ▲
- ♦ wrap future with △
- each is an abstraction: assign a fresh variable to positive subtree on reaching a leaf, cross the axiom link and switch to positive mode

start from conclusion in negative mode



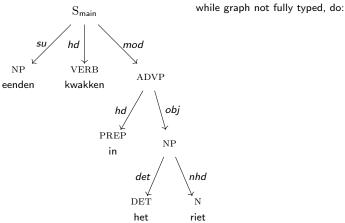
From parse graphs to $ILL_{\neg,\Diamond,\Box}$ types

algorithm: graph flooding on dags init with maps

- from pos & phrasal categories to \mathcal{A} e.g. NP \rightarrow np, INF \rightarrow inf,...
- from grammatical roles to \diamondsuit (complements) and \square (adjuncts) e.g. $su \to \diamondsuit^{su}$, $obj \to \diamondsuit^{obj}$, ..., $mod \to \square^{mod}$, $det \to \square^{det}$

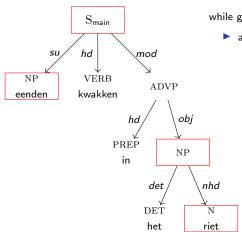
and a strict total order over \diamondsuit , e.g. $\diamondsuit^{su} > \diamondsuit^{obj}$

Simple case: trees



"eenden kwakken in het riet"

Simple case: trees

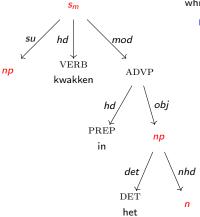


"eenden kwakken in het riet"

while graph not fully typed, do:

assign stand-alone nodes no incoming adjunct or head edge

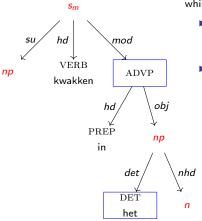
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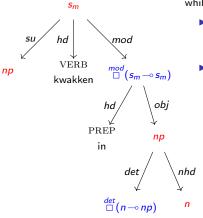
assign stand-alone nodes no incoming adjunct or head edge type via the A-map



"eenden kwakken in het riet"

while graph not fully typed, do:

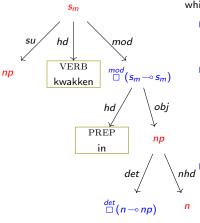
- assign stand-alone nodes no incoming adjunct or head edge type via the A-map
- assign adjuncts
 incoming adjunct edge
 parent is typed



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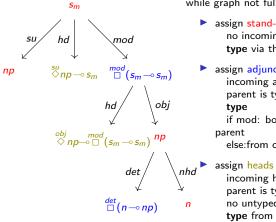
- assign stand-alone nodes no incoming adjunct or head edge type via the A-map
- assign adjuncts
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 parent is typed
 type
 if mod: boxed endofunctor of
 parent
 else:from comp sibs to parent



"eenden kwakken in het riet"

while graph not fully typed, do:

- assign stand-alone nodes no incoming adjunct or head edge type via the A-map
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 type
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 - assign heads incoming head edge parent is typed no untyped complement sibs



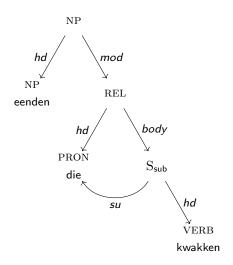
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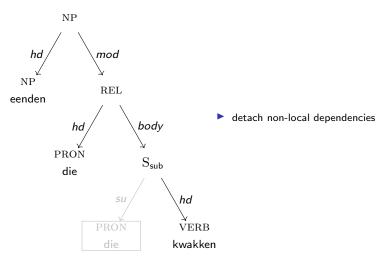
while graph not fully typed, do:

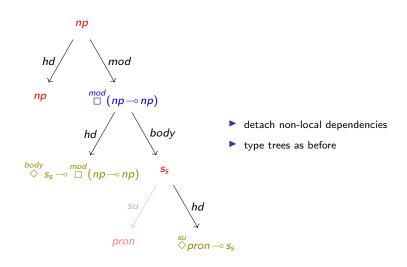
- assign stand-alone nodes no incoming adjunct or head edge type via the A-map
- assign adjuncts incoming adjunct edge parent is typed type if mod: boxed endofunctor of parent else:from comp sibs to parent incoming head edge

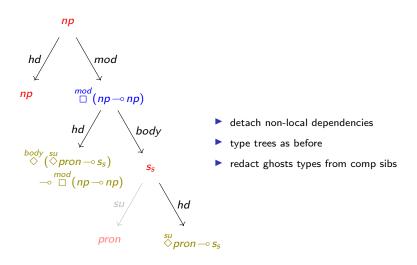
no untyped complement sibs type from comp sibs to parent

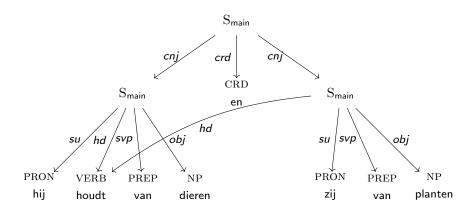
parent is typed



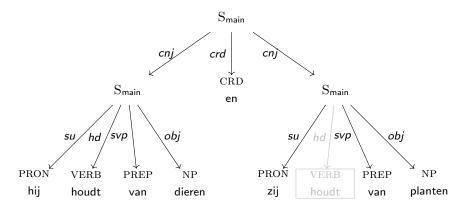






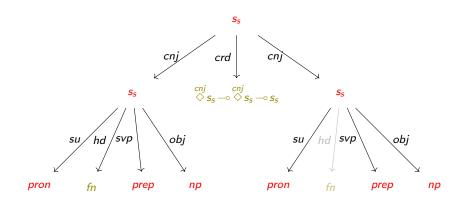


"hij houdt van dieren en zij van planten"



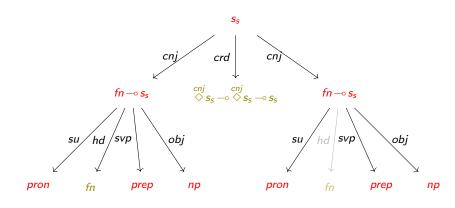
"hij houdt van dieren en zij van planten"

detach and type trees as usual



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detach and type trees as usual



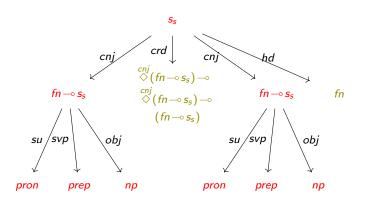
"hij houdt van dieren en zij van planten"

- detach and type trees as usual
- redact missing types from both conjuncts

$$\textit{fn} := \overset{\textit{svp}}{\lozenge} \textit{prep} \mathop{\multimap} \overset{\textit{obj}}{\lozenge} \textit{np} \mathop{\multimap} \overset{\textit{su}}{\lozenge} \textit{pron} \mathop{\multimap} \textit{s}_{\textit{s}}$$

ACG Flashback

- each conjunct represents a tuple of types $c = (t_1, t_2, \dots t_n) \equiv t_1 \otimes t_2 \otimes \dots \otimes t_n$
- ▶ encoded as the higher-order function $(c \multimap r) \multimap r$ and curried into $(t_1 \multimap t_2 \multimap \ldots \multimap t_n \multimap r) \multimap r$



"hij houdt van dieren en zij van planten"

- detach and type trees as usual
- redact missing types from both conjuncts
- update coord type & attach copies at top level

$$\textit{fn} := \overset{\textit{svp}}{\lozenge} \textit{prep} - \overset{\textit{obj}}{\lozenge} \textit{np} - \overset{\textit{su}}{\lozenge} \textit{pron} - \hspace{-.5em} \cdot \hspace{-.5em} s_{s}$$



A glimpse at a higher universe

Second-order IL (system F or polymorphic λ -calculus)

$$\frac{\Gamma \vdash \mathtt{M} : \forall \alpha. \sigma}{\Gamma \vdash \mathtt{M} : \sigma[\tau/\alpha]} \qquad \frac{\Gamma \vdash \mathtt{M} : \sigma}{\Gamma \vdash \mathtt{\Lambda} \alpha. \mathtt{M} : \forall \alpha. \sigma}$$

A glimpse at a higher universe

Second-order IL (system F or polymorphic λ -calculus)

$$\frac{\Gamma \vdash \mathtt{M} : \forall \alpha.\sigma}{\Gamma \vdash \mathtt{M}\tau : \sigma[\tau/\alpha]} \qquad \frac{\Gamma \vdash \mathtt{M} : \sigma}{\Gamma \vdash \Lambda\alpha.\mathtt{M} : \forall \alpha.\sigma}$$

In that universe, modifiers and coordinators are polymorphic types:

$$\mathsf{mod} := \mathsf{\Lambda}\alpha.\mathtt{w} : \forall \alpha. \square^{\mathit{mod}} \left(\alpha \! \multimap \! \alpha\right)$$

and

$$\operatorname{crd} := \operatorname{\Lambda}\!\alpha.\operatorname{w} : \forall \alpha. \diamondsuit^{\operatorname{cnj}}\alpha \mathop{\multimap} \diamondsuit^{\operatorname{cnj}}\alpha \mathop{\multimap} \alpha$$

Coordinators as derived types

Elliptical coordinators can also be seen as a transformation of basic types. If $c = (t_1 \otimes t_2 \otimes \ldots \otimes t_N)$ the conjoined tuples,

$$crd = c - \circ c - \circ c$$

$$\xrightarrow{vr} c - \circ c - \circ (c - \circ s) - \circ s$$

$$\xrightarrow{ar^0} ((c - \circ s) - \circ s) - \circ c - \circ (c - \circ s) - \circ s$$

$$\xrightarrow{ar^1} ((c - \circ s) - \circ s) - \circ ((c - \circ s) - \circ s) - \circ (c - \circ s) - \circ s$$

$$\equiv ((t_1 - \circ t_2 - \dots - \circ t_n - \circ s) - \circ s) - \circ$$

$$((t_1 - \circ t_2 - \dots - \circ t_n - \circ s) - \circ s) - \circ$$

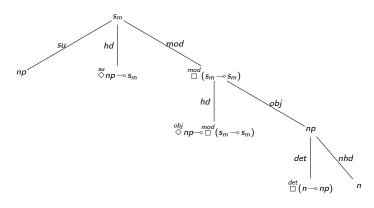
$$(t_1 - \circ t_2 - \dots - \circ t_n - \circ s) - \circ s$$

$$\diamondsuit^{cnj} S_{main} - \diamondsuit^{cnj} S_{main} - \diamondsuit S_{main}$$

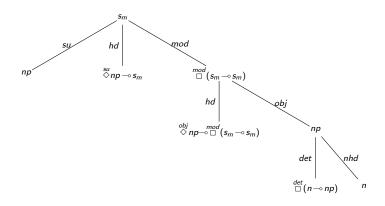
$$\xrightarrow{vr^*} \diamondsuit^{cnj} S_{main} - \diamondsuit^{cnj} S_{main} - \circ fn - \circ S_{main}$$

$$\xrightarrow{ar^1} \diamondsuit^{cnj} (fn - \circ S_{main}) - \diamondsuit^{cnj} S_{main} - \circ fn - \circ S_{main}$$

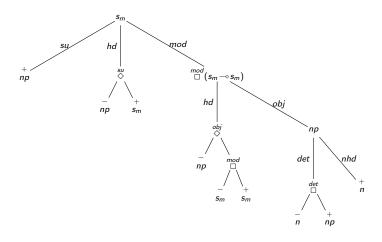
$$\xrightarrow{ar^2} \diamondsuit^{cnj} (fn - \circ S_{main}) - \circ \diamondsuit^{cnj} (fn - \circ S_{main}) - \circ fn - \circ S_{main}$$



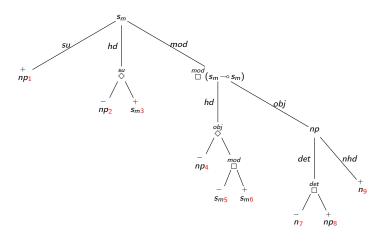
given a typed graph:



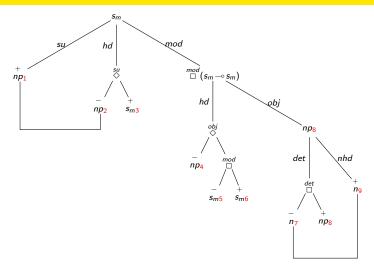
(1) convert types to binary trees and assign polarities



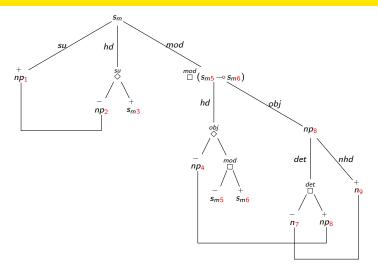
(2) assign identifying indices



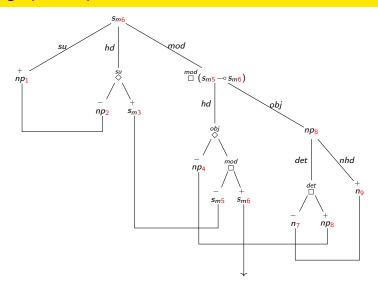
$$\{2\mapsto ?, 4\mapsto ?, 5\mapsto ?, 7\mapsto ?\}$$



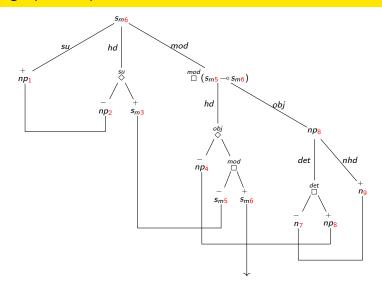
$$\{2\mapsto 1, 4\mapsto ?, 5\mapsto ?, 7\mapsto 9\}$$



$$\{2\mapsto 1, 4\mapsto 8, 5\mapsto ?, 7\mapsto 9\}$$



$$\big\{2\mapsto 1, 4\mapsto 8, 5\mapsto 3, 7\mapsto 9\big\}_{\tiny \scriptsize \textbf{COS}} + \tiny \scriptsize \textbf{COS} + \tiny \scriptsize \textbf{COS}}$$



the resulting structure is a proof net

..continuing

The agenda:

- λ Choosing the logic
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Lexical type ambiguity

Type assignments are often *ambiguous* and context-dependent:

very realistic lexicon

Lexical type ambiguity

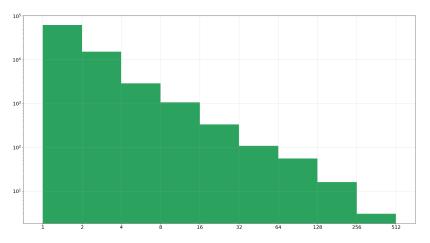
generally:

$$w :: t_1 \& t_2 \& t_3 \& \dots \& t_n$$

more refined grammar \implies

- harder lexical disambiguation (more n per word)
- © easier parsing (if one starts from correct type)

Lexical type ambiguity



types (log2-transformed) per word

Supertagging

General idea

Given a sentence w_1, w_2, \ldots, w_n find the type sequence $t_1, t_2, \ldots t_n$ that maximimizes

$$p(t_1,t_2,\ldots t_n|w_1,w_2,\ldots w_n,\theta)$$

where $\boldsymbol{\theta}$ the trainable parameter space

Paleolithic ages

Indipendence Assumption

$$max(p(t_1, t_2, \dots t_n | w_1, w_2, \dots w_n, \theta)) \approx \prod_{i=1}^n max(p(t_i, w_i, \theta))$$

Implemented as naive bayes/maximum entropy models, later using feed-forward networks on distributional vectors

- © Beats hand-writing a lexicon
- © Small (if any) context/receptive field
- © No domain generalization
- © Severe sample sparsity when using windows

Discriminative approach

Markov assumption

$$p(t_1, t_2, \dots t_n | w_1, w_2, \dots w_n, \theta)$$

$$\approx \prod_{i=1}^n p(t_i | w_1, w_2, \dots w_n, \theta)$$

Implemented as contextualized token classification using RNNs

- global context (lossy)
- domain generalization
- © single answer
- © sample sparsity
- ② no codomain generalization (closed world assumption)

Generative approach

Markov assumption

$$p(t_1, t_2, \dots t_n | w_1, w_2, \dots w_n, \theta)$$

$$\approx \prod_{i=1}^n p(t_i | t_1, t_2, \dots, t_{i-1}, w_1, w_2, \dots w_n, \theta)$$

(could be) implemented as a transducer/seq2seq model

- global context (lossless)
- many answers
- © sample sparsity
- © no codomain generalization

Generative approach: one more step

The type syntax:

$$cod(\mathcal{L}) := A \mid \stackrel{d}{\diamondsuit} T \multimap T' \mid \stackrel{d}{\Box} (T \multimap T')$$

corresponds to a simple cfg (in prefix notation) with meta-rules:

$$S \to A$$
 $\forall A \in \mathcal{A}$ $S \to \diamondsuit^d S S$ $\forall d \in \text{comps}$ $S \to \Box^d S S$ $\forall d \in \text{adjns}$

i.e. each type t_i can be written as a sequence of primitive symbols $\vec{\sigma}$

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i.e. each type t_i can be written as a sequence of primitive symbols $\vec{\sigma}$

$$p(t_1, t_2, \dots t_n | w_1, w_2, \dots w_n, \theta)$$

$$\approx \prod_{i=1}^n p(\sigma_i | \sigma_1, \sigma_2, \dots, \sigma_{i-1}, w_1, w_2, \dots w_n, \theta)$$

- each type contains many subtypes (less sparsity)
- any valid types can be inductively constructed (open codomain)



Supertagging today

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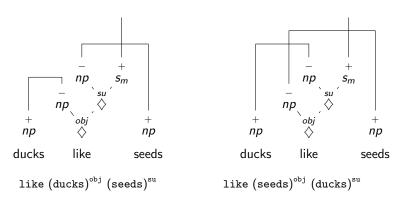
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- ? left-right decoding does not make use of easy types first
- ? logical constraints are not easy to integrate with neural decoding
- ! new possibilities for parsing and meaning representation

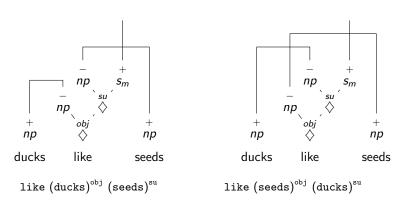
Proof Ambiguity

The type system (being non-directional) permits too many parses:

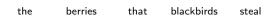


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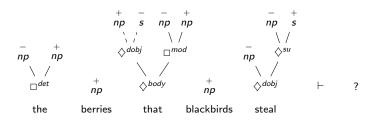
How can we select the correct one?



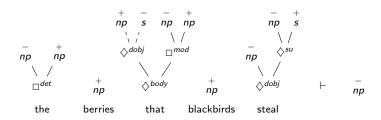
1. pass the sentence through the supertagger

$$\Box^{det}, np, np, \#, np, \#, \diamondsuit^{body}, \diamondsuit^{dobj}, np, s, \Box^{mod}, np, np, \#, \diamondsuit^{dobj}, np, \diamondsuit^{su}, np, s$$
 the berries that blackbirds steal

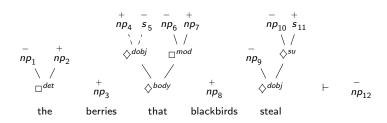
2. parse types to trees and assign polarity information



3. find conclusion as the singleton $\mathcal{A}^+ - \mathcal{A}^-$

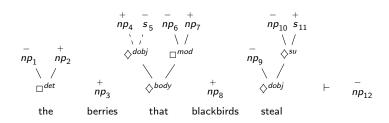


4. index pos/neg occurrences and arrange in a table



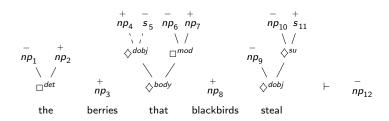
5. fill table with pair-wise agreement scores





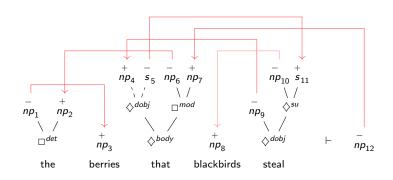
6. discretize result with Sinkhorn





7. train against ground-truth axiom links





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The future

- λ Choosing the logic
- λ Making a dataset: proofs and lexical type assignments
- λ Learning the type assignment process
- λ Navigating the proof space
- λ Syntax-aware & type-correct text representations

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- λ Choosing the logic
- λ Making a dataset: proofs and lexical type assignments
- λ Learning the type assignment process
- λ Navigating the proof space
- λ Syntax-aware & type-correct text representations
 - meaning representation from proofs: graph traversal/encoding
 - types as recipes/functions on word embeddings, pure interpretations of the system
 - ...
 - [your project/internship/thesis idea here]

diy: github.com/konstantinosKokos/neural-proof-nets