
Logic-Based Parsing with Neural Networks

Compositional Models of Vector-based Semantics

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Types

ILL_{\multimap} plus \diamond, \square modalities for *dependency domain demarkation*.

Types inductively defined by:

$$\mathbb{T} := A \mid T \multimap T \mid \diamond^d T \mid \square^d T \quad A \in \mathbb{A}, T \in \mathbb{T}$$

\multimap – linear function builder

\diamond – reserved for "necessary arguments", i.e. complements

\square – reserved for "optional functors", i.e. adjuncts

Rules & Terms

☺ function/argument structures

$$\frac{}{\mathbf{c} : T \vdash \mathbf{c} : T} \textit{Lex} \qquad \frac{\Gamma \vdash \mathbf{s} : T_1 \multimap T_2 \quad \Delta \vdash \mathbf{t} : T_1}{\Gamma, \Delta \vdash \mathbf{s} \mathbf{t} : T_2} \multimap E$$

Rules & Terms

☺ function/argument structures

$$\frac{}{c : T \vdash c : T} Lex \qquad \frac{\Gamma \vdash s : T_1 \multimap T_2 \quad \Delta \vdash t : T_1}{\Gamma, \Delta \vdash s \ t : T_2} \multimap E$$

☺ simple dependency demarkation

$$\frac{\Gamma \vdash t : T}{\langle \Gamma \rangle^d \vdash_{\Delta^d} t : \Diamond^d T} \Diamond^d I \qquad \frac{\Gamma \vdash s : \Box^d T}{\langle \Gamma \rangle^d \vdash \blacktriangledown^d s : T} \Box^d E$$

Rules & Terms

☺ function/argument structures

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☺ hypothetical reasoning

$$\frac{}{x : T \vdash x : T} Ax \qquad \frac{\Gamma, x : T_1 \vdash s : T_2}{\Gamma \vdash \lambda x. s : T_1 \multimap T_2} \multimap I$$

Rules & Terms

☺ function/argument structures

$$\frac{}{c : T \vdash c : T} Lex \qquad \frac{\Gamma \vdash s : T_1 \multimap T_2 \quad \Delta \vdash t : T_1}{\Gamma, \Delta \vdash s \ t : T_2} \multimap E$$

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👁 cosmic horror from the great beyond

$$\frac{\langle \Gamma \rangle^d \vdash s : T}{\Gamma \vdash \blacktriangle^d s : \Box^d T} \Box^d I \qquad \frac{\Gamma[\langle x : T_1 \rangle^d] \vdash t : T_2 \quad \Delta \vdash s : \Diamond^d T_1}{\Gamma[\Delta] \vdash t[x \mapsto \nabla^d s] : T_2} \Diamond^d E$$

Rules & Terms

Bonus:

ad-hoc extraction

$$\frac{\langle \Gamma, \langle \mathbf{x} : T_1 \rangle^{\mathbf{x}}, \Delta \rangle^d \vdash \mathbf{s} : T_2}{\langle \Gamma, \Delta \rangle^d, \langle \mathbf{x} : T_1 \rangle^{\mathbf{x}} \vdash \mathbf{s} : T_2} \mathbf{x}$$

Example

alt-tab!

Formula Decomposition: Types \equiv Trees

The type assignment of “*where*”:

$$\diamond^{relcl}(\diamond^x \square^x \diamond^{mod} \square^{mod} (ppart \multimap ppart) \multimap ssub) \multimap \square^{mod} (np \multimap np)$$

Formula Decomposition: Types \equiv Trees

The type assignment of “*where*”:

$$\diamond^{relcl}(!x!^{mod}(ppart \multimap ppart) \multimap ssub) \multimap \square^{mod}(np \multimap np)$$

$$!(_) := \diamond\square(_)$$

Formula Decomposition: Types \equiv Trees

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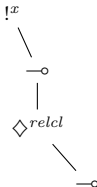
$$\diamond^{relcl}(!x!^{mod}(ppart \multimap ppart) \multimap_{ssub}) \multimap \square^{mod}(np \multimap np)$$



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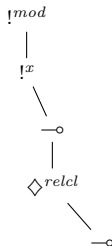
$$\diamond^{relcl}(\textcolor{red}{!}x!^{mod}(ppart \multimap ppart) \multimap ssub) \multimap \square^{mod}(np \multimap np)$$



Formula Decomposition: Types \equiv Trees

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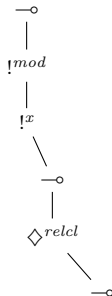
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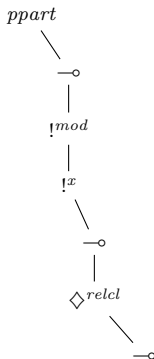
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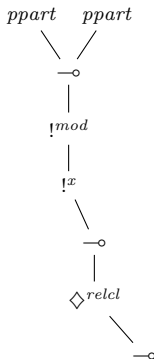
$$\diamond^{relcl}(!x!^{mod}(\textcolor{red}{ppart} \multimap ppart) \multimap ssub) \multimap \square^{mod}(np \multimap np)$$



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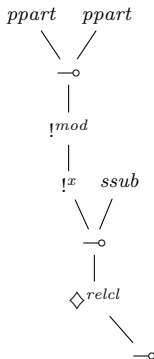
$$\diamond^{relcl}(!x!^{mod}(ppart \multimap \textcolor{red}{ppart}) \multimap ssub) \multimap \square^{mod}(np \multimap np)$$



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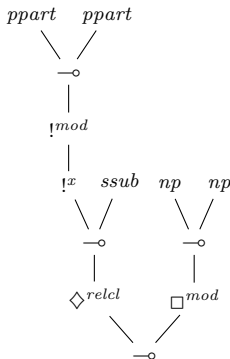
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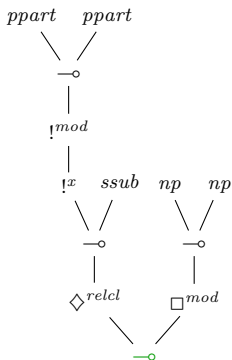


Formula Decomposition: Polarity Induction

subtree polarity (*preserved* to the right, *inverted* to the left)

+ we **have**

- we **miss**

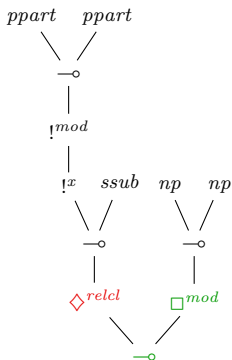


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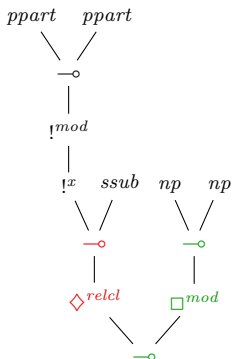


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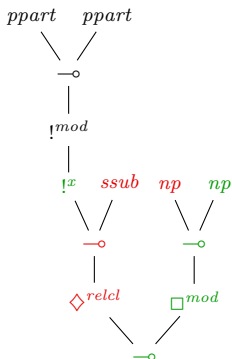


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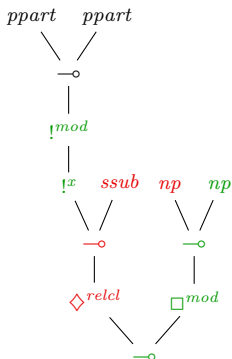


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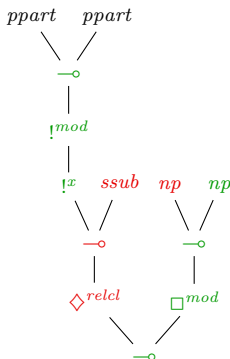


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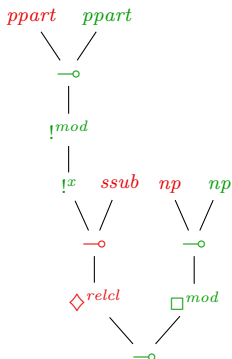


Formula Decomposition: Polarity Induction

subtree polarity (*preserved* to the right, *inverted* to the left)

+ we **have**

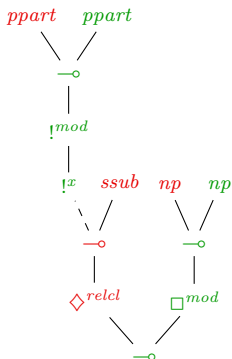
- we **miss**



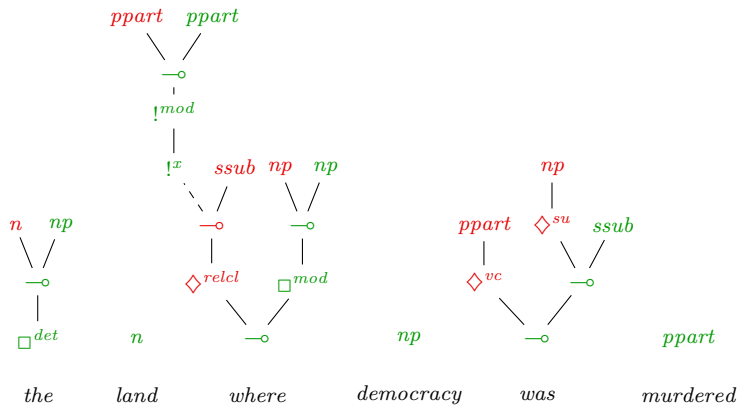
Formula Decomposition: Polarity Induction

subtree polarity (*preserved* to the right, *inverted* to the left)

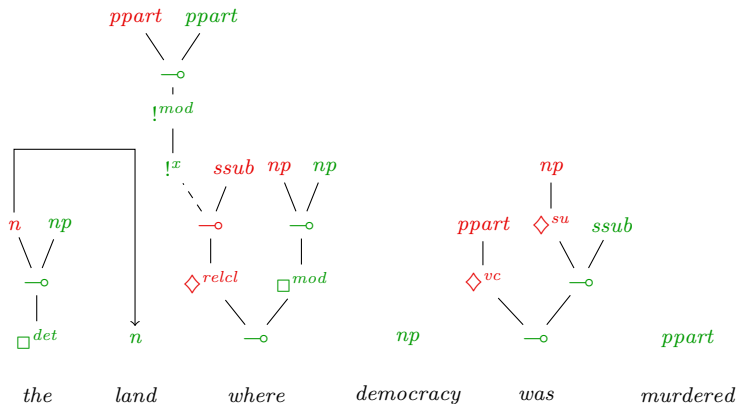
- + we have
- we miss



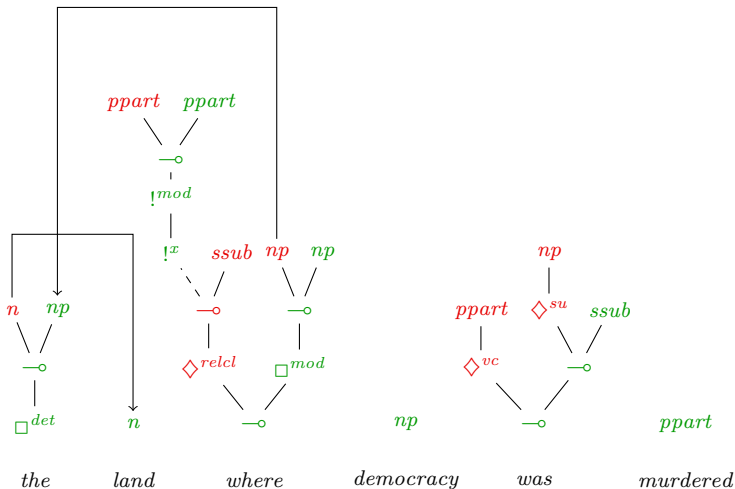
Proof Frames



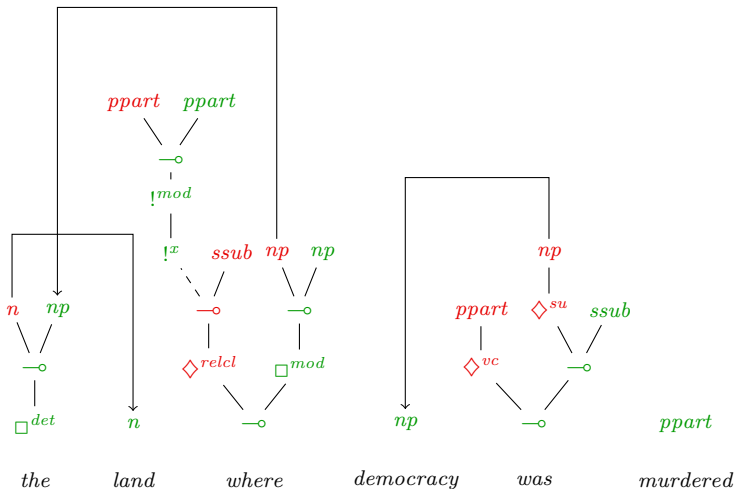
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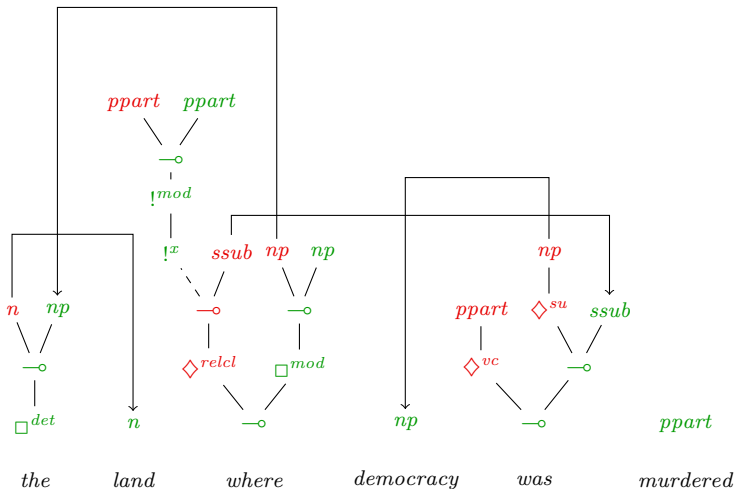
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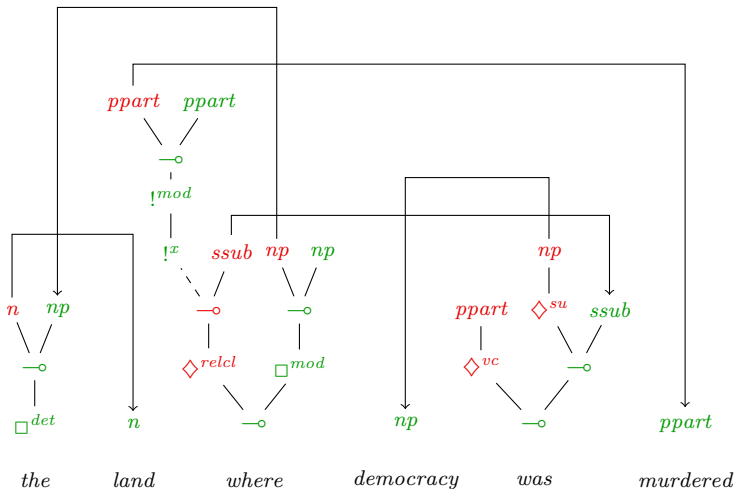
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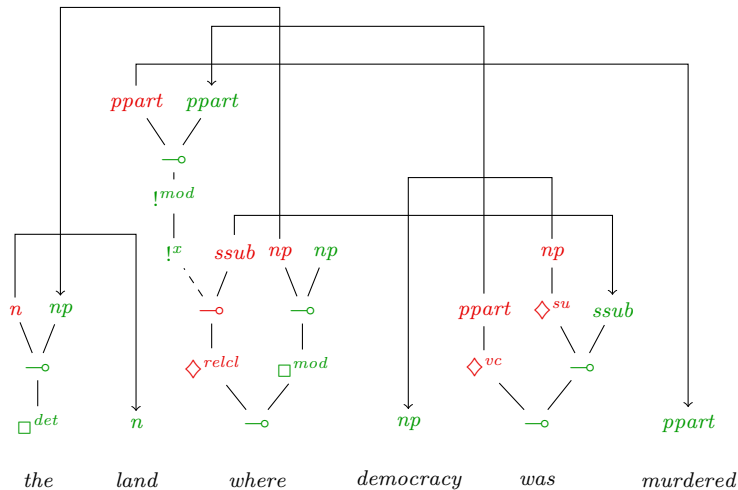


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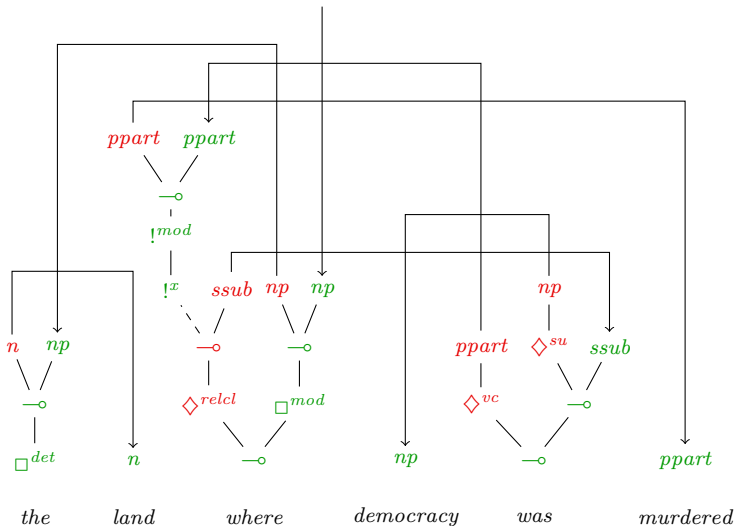


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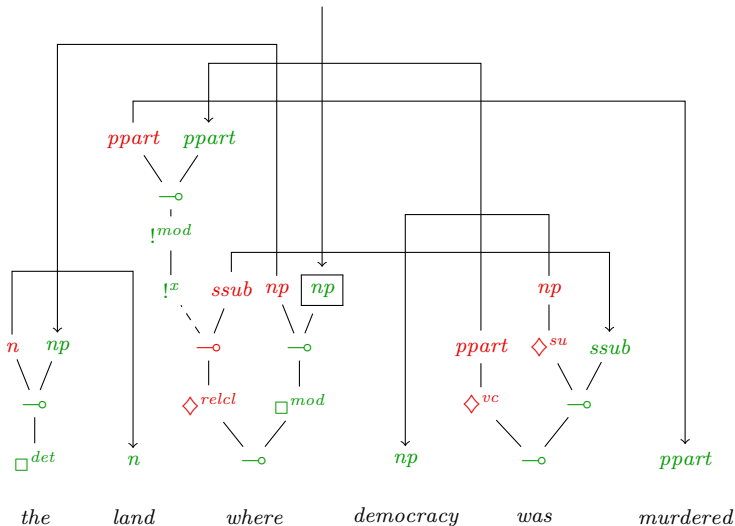




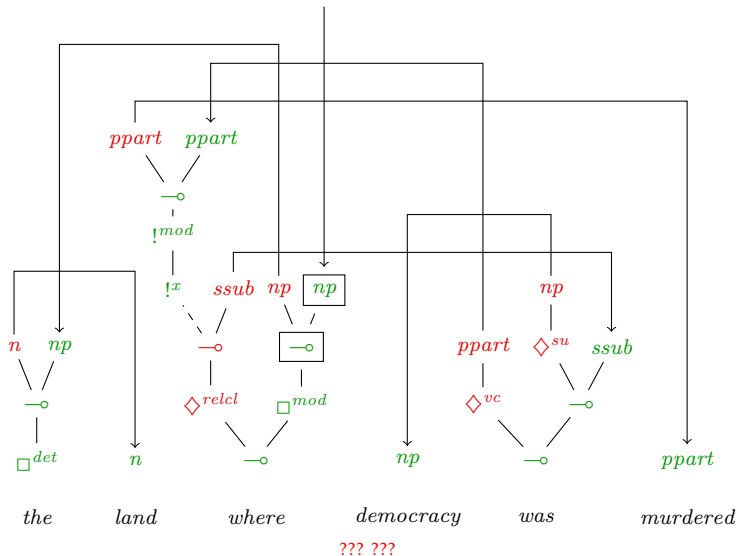
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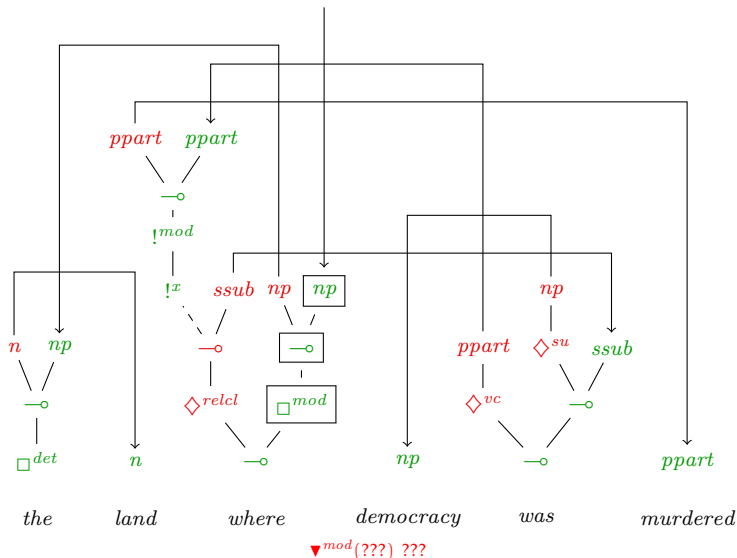
Proof Frames Structures



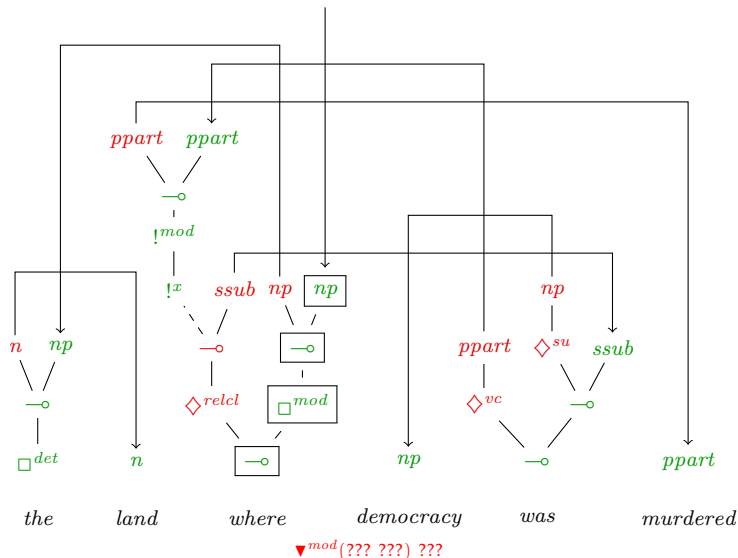
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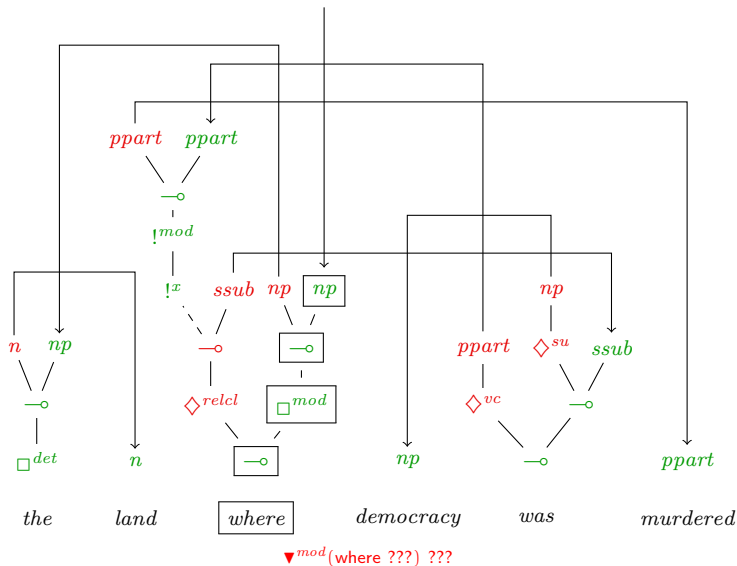
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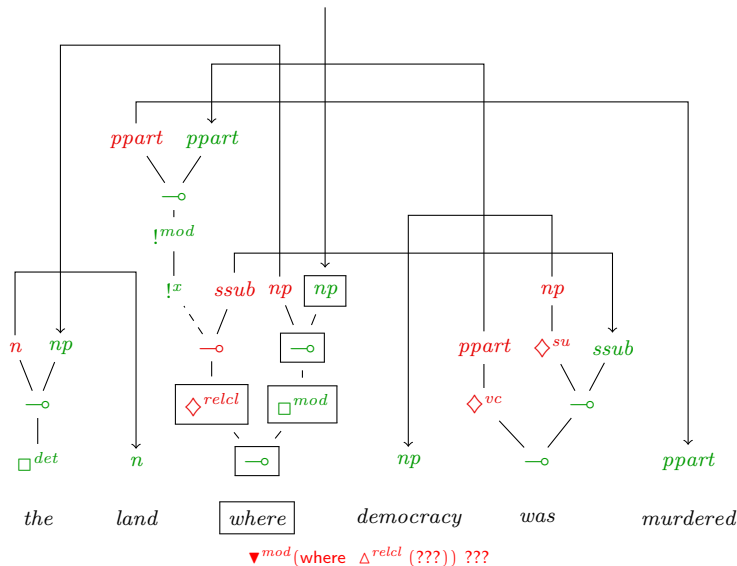
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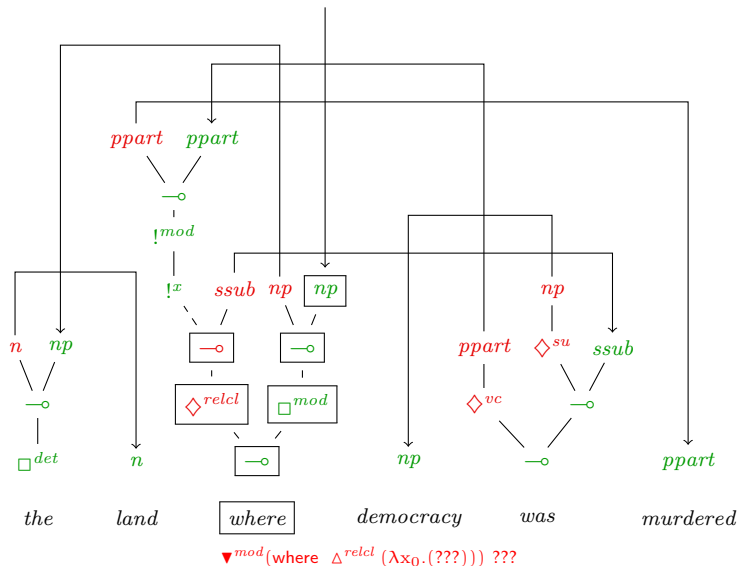
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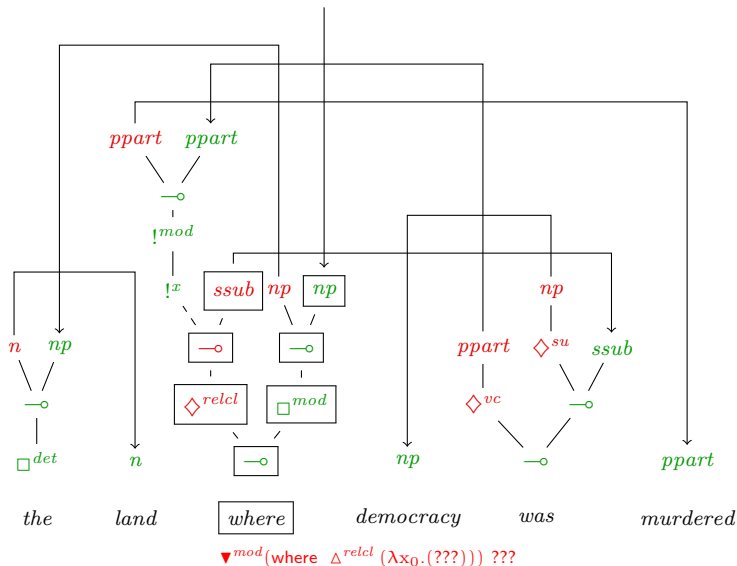
Proof Frames Structures



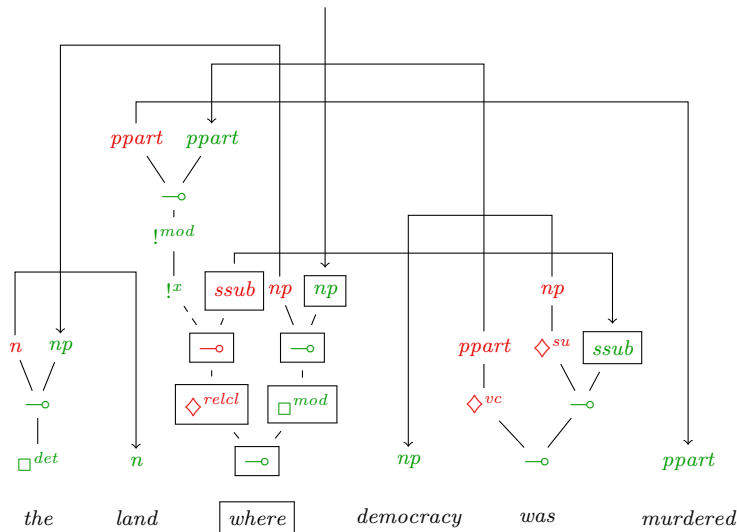
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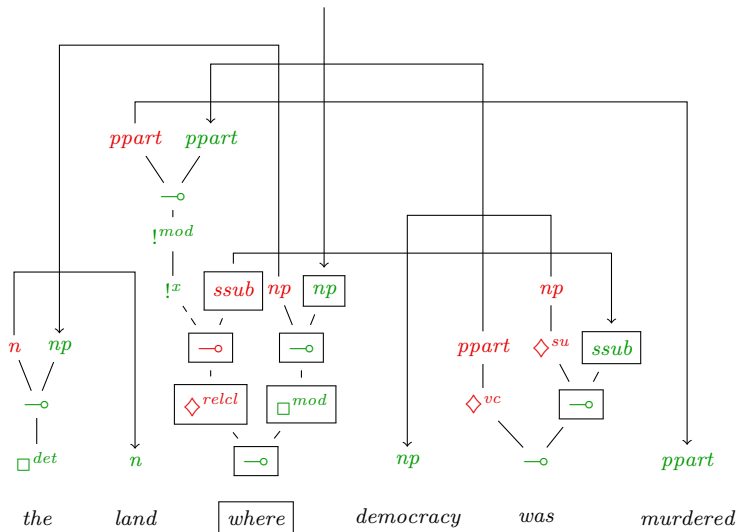


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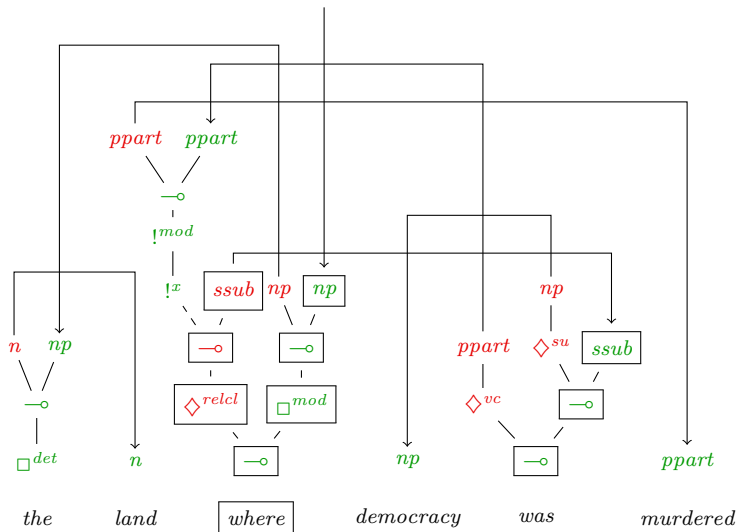
▼^{mod}(where Δ^{relcl}(λx₀.(?))) ??

Proof Frames Structures



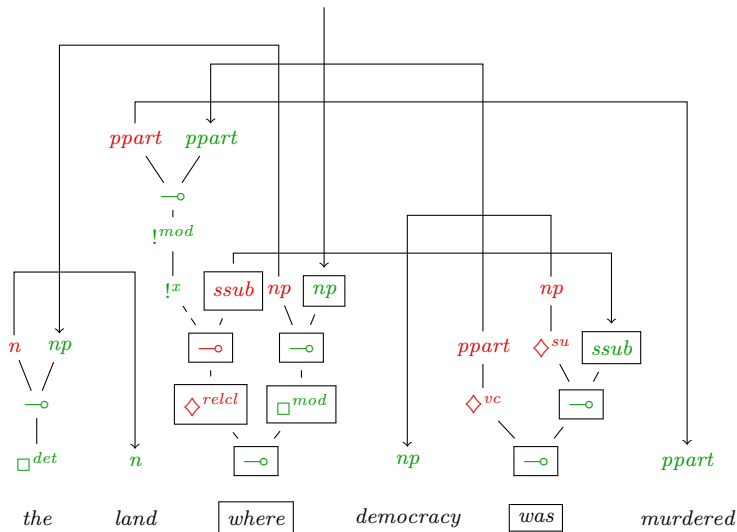
▼^{mod}(where $\Delta^{relcl}(\lambda x_0. (??? \ ???))$) ???

Proof Frames Structures



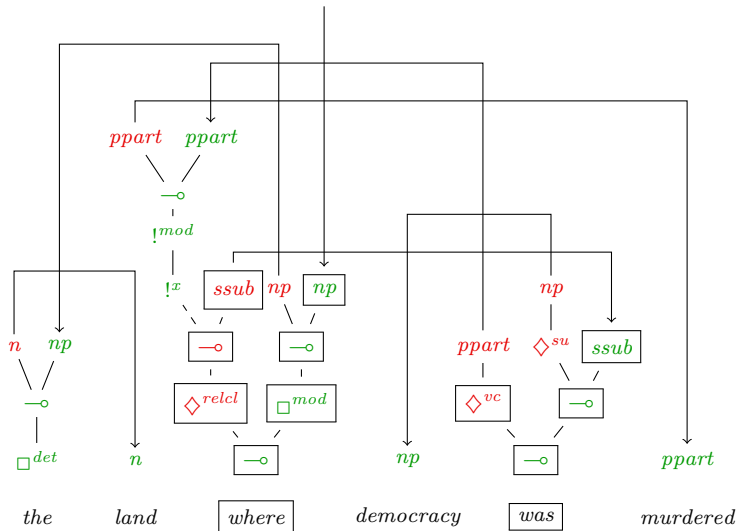
▼^{mod}(where Δ^{relcl}(λx₀.(??? ??? ???))) ???

Proof Frames Structures



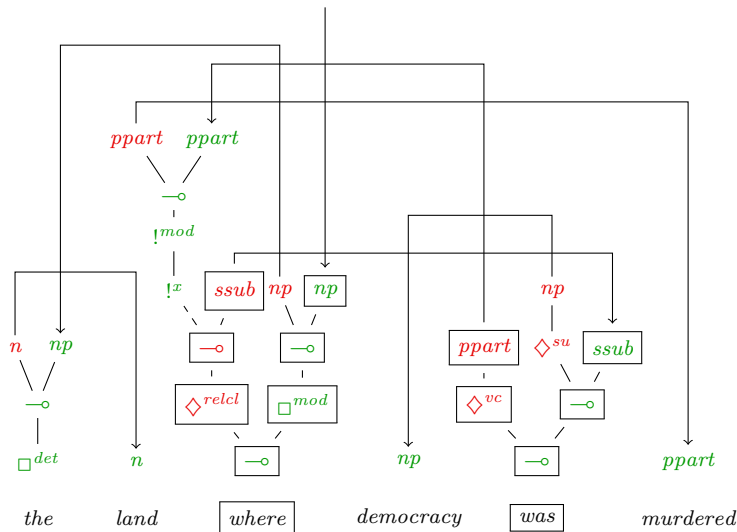
▼^{mod}(where Δ^{relcl}(λx₀.(was ??? ???))) ???

Proof Frames Structures



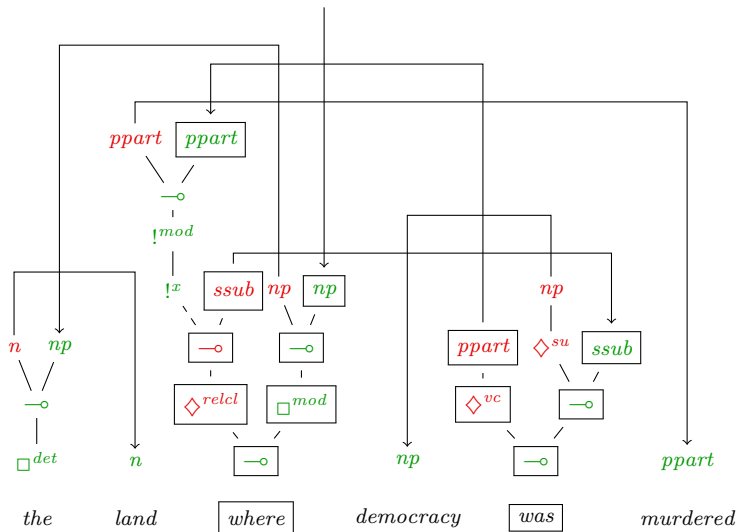
▼^{mod}(where Δ^{relcl} (λx₀.(was Δ^{vc} (???) ???))) ???

Proof Frames Structures



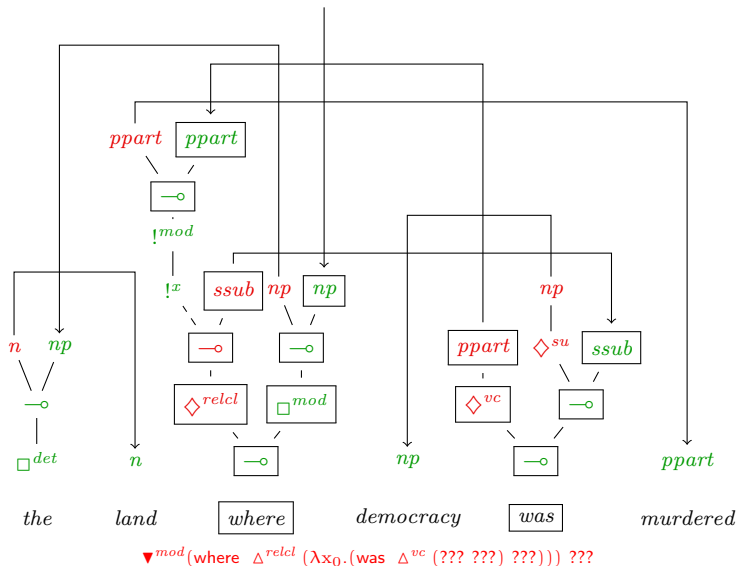
▼^{mod}(where $\Delta^{relcl}(\lambda x_0.(\text{was } \Delta^{vc}(\text{???}) \text{???}))$) ???

Proof Frames Structures

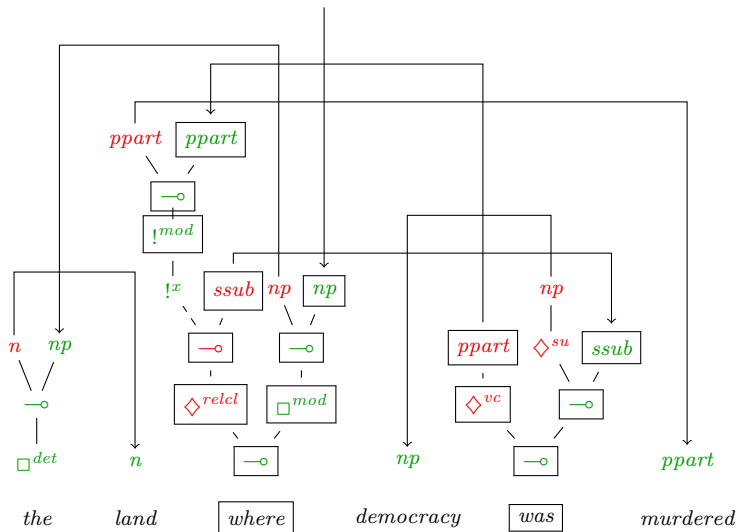


▼^{mod}(where $\Delta^{relcl}(\lambda x_0.(\text{was } \Delta^{vc}(\text{???}) \text{???})) \text{???}$)

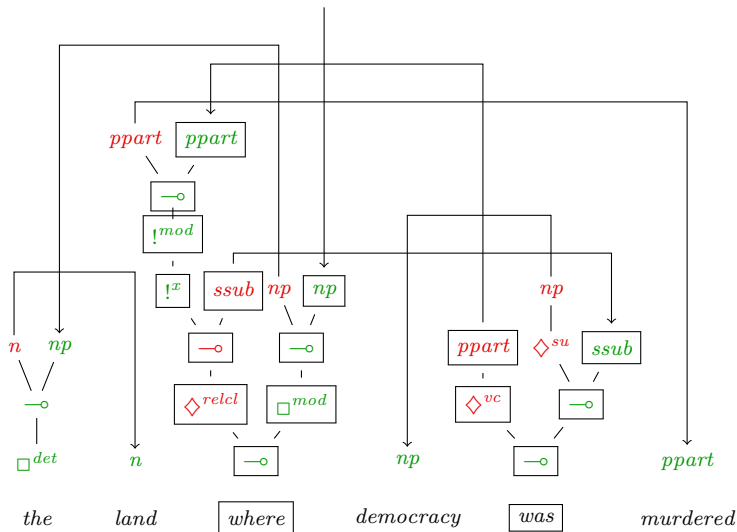
Proof Frames Structures



Proof Frames Structures

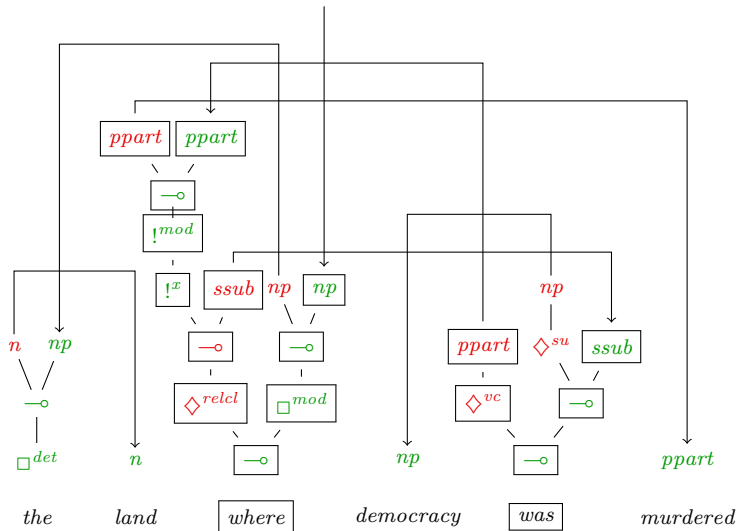

$$\nabla^{mod}(\text{where } \Delta^{relcl}(\lambda x_0. (\text{was } \Delta^{vc}(\nabla^{mod} \nabla^{mod}(???) ???) ???)) ???$$

Proof Frames Structures



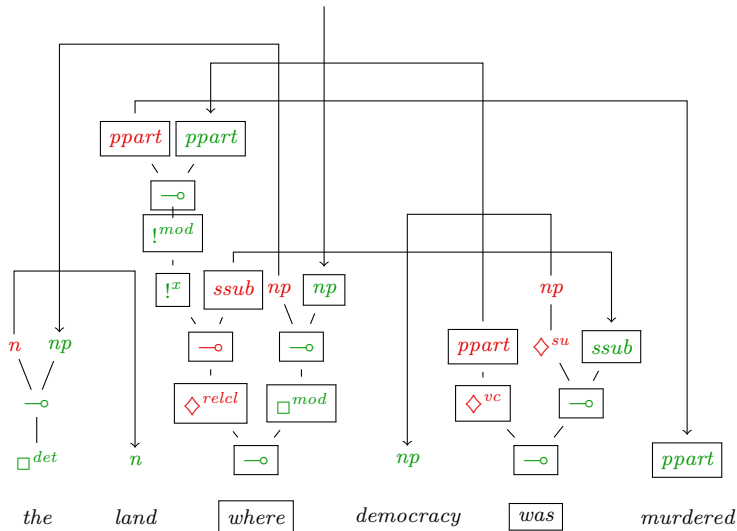
$\nabla^{mod}(\text{where } \Delta^{relcl}(\lambda x_0.(\text{was } \Delta^{vc}(\nabla^{mod}\nabla^{mod}\nabla^x\nabla^x???))) ???)$

Proof Frames Structures



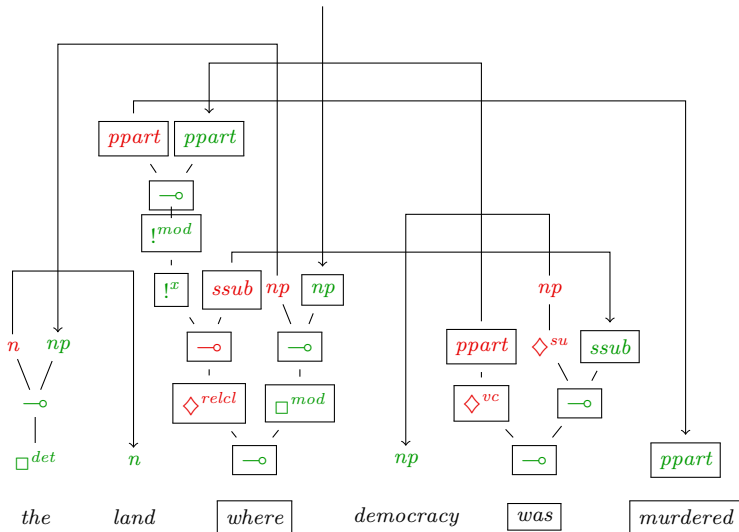
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Proof Frames Structures



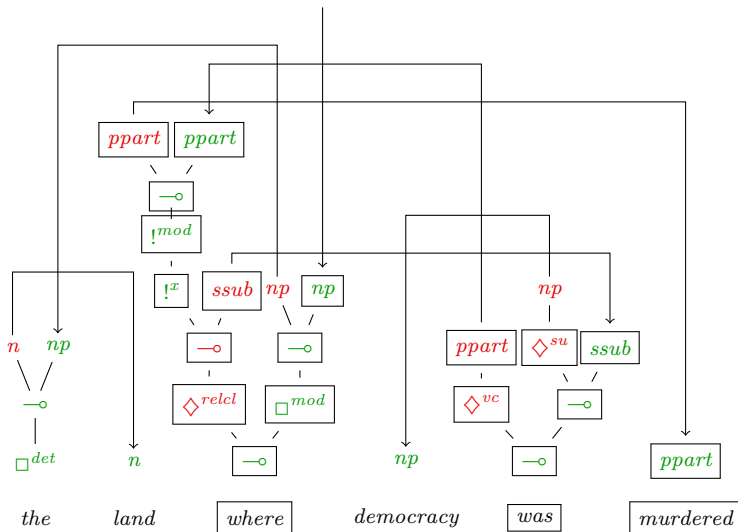
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Proof Frames Structures



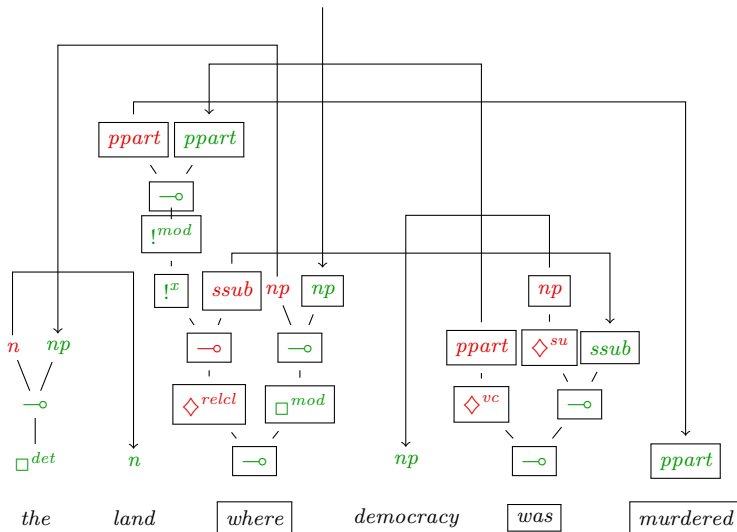
$\nabla^{mod}(\text{where } \Delta^{relcl}(\lambda x_0.(\text{was } \Delta^{vc}(\nabla^{mod}\nabla^{mod}\nabla^x\nabla^x x_0 \text{ murdered}) ???)) ???$

Proof Frames Structures



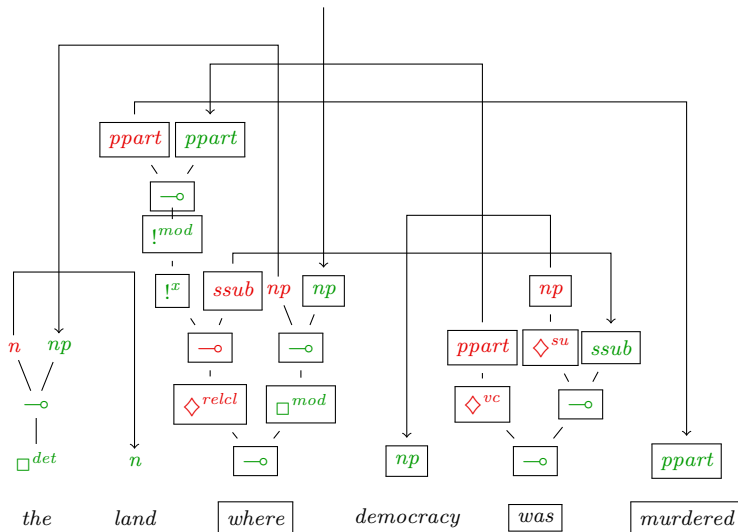
$$\nabla^{mod}(\text{where } \Delta^{relcl}(\lambda x_0. (\text{was } \Delta^{vc}(\nabla^{mod} \nabla^{mod} \nabla^x \nabla^x x_0 \text{ murdered}) \Delta^{su} ???))) ???$$

Proof Frames Structures



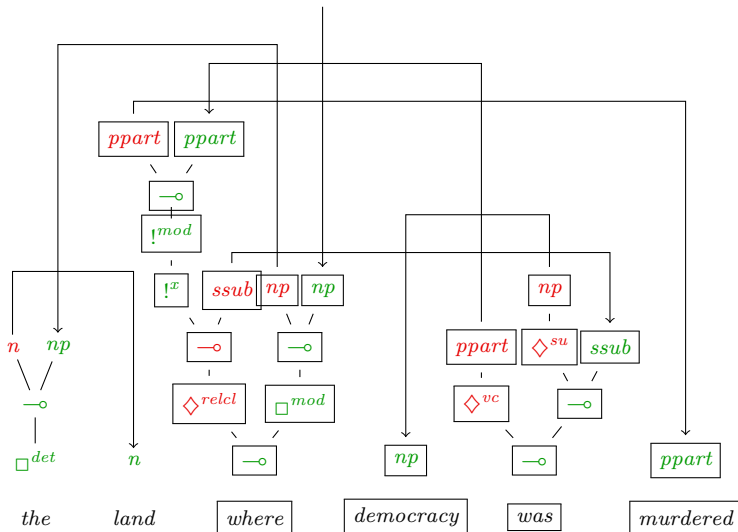
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Proof Frames Structures



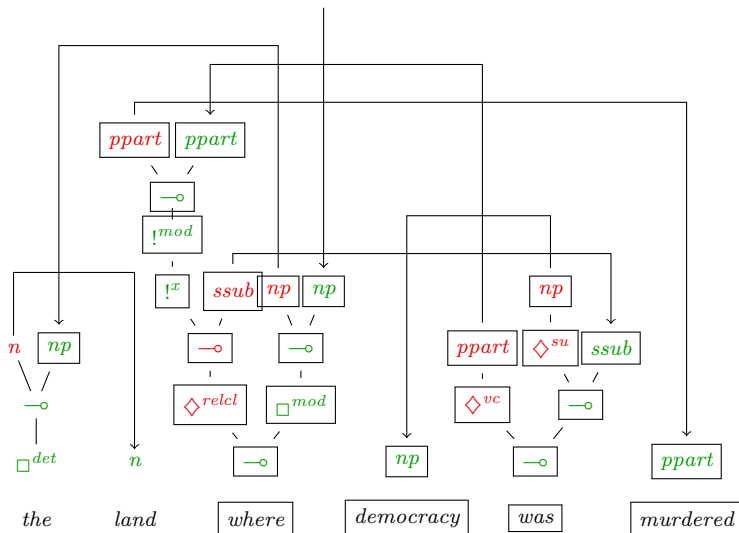
$\nabla^{mod}(\text{where } \Delta^{relcl}(\lambda x_0.(\text{was } \Delta^{vc}(\nabla^{mod}\nabla^{mod}\nabla^x\nabla^x x_0 \text{ murdered}) \Delta^{su} \text{ democracy}))) ???$

Proof Frames Structures



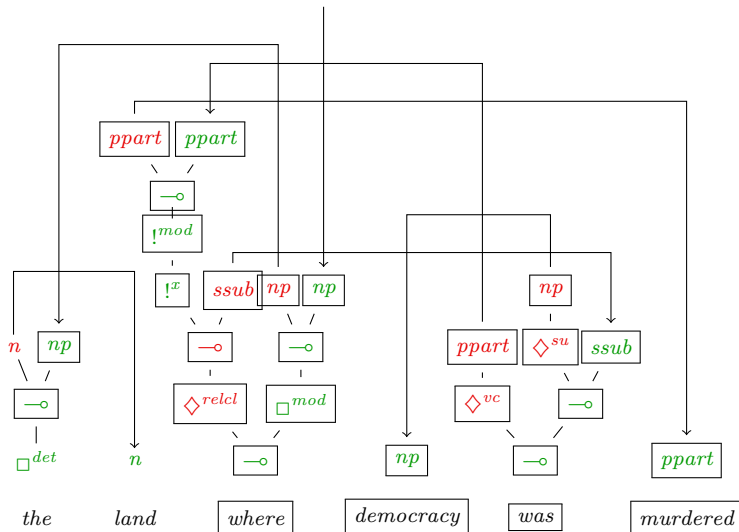
$\nabla^{mod}(\text{where } \Delta^{relcl} (\lambda x_0. (\text{was } \Delta^{vc} (\nabla^{mod} \nabla^{mod} \nabla^x \nabla^x x_0 \text{ murdered}) \Delta^{su} \text{ democracy}))) ???$

Proof Frames Structures



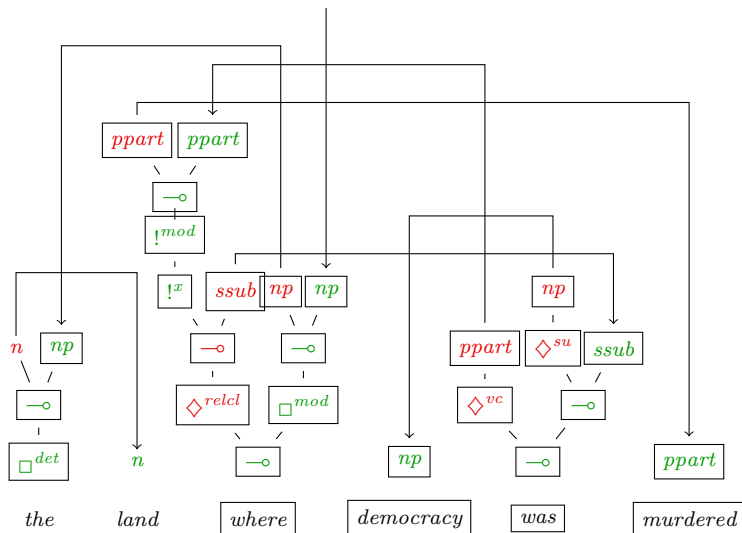
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Proof Frames Structures



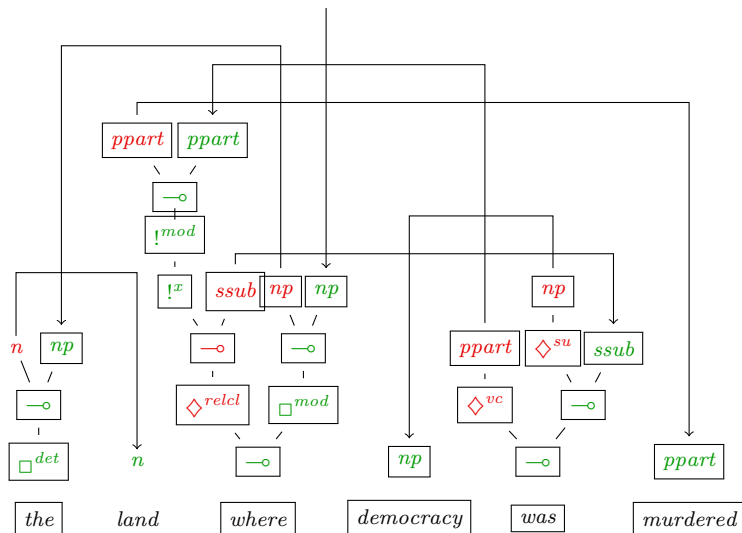
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Proof Frames Structures



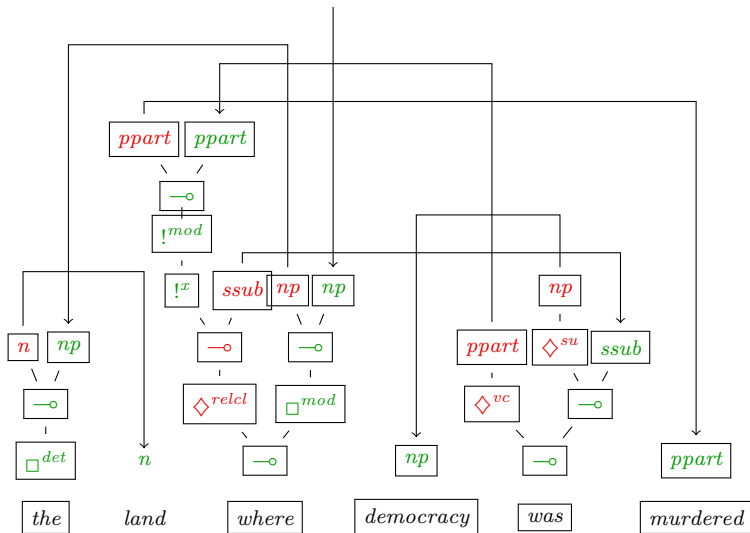
$\nabla^{mod}(\text{where } \Delta^{relcl}(\lambda x_0.(\text{was } \Delta^{vc}(\nabla^{mod}\nabla^{mod}\nabla^x\nabla^x x_0 \text{ murdered}) \Delta^{su} \text{ democracy}))) (\nabla^{det} ??? ???)$

Proof Frames Structures



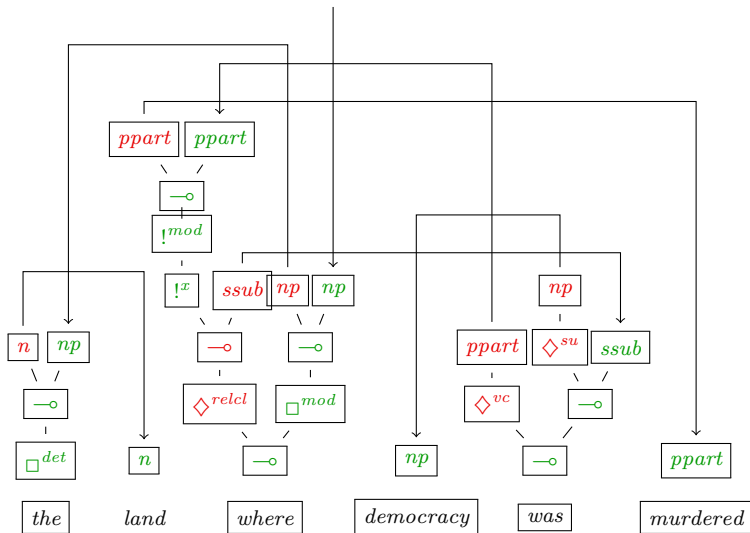
$\nabla^{mod}(\text{where } \Delta^{relcl}(\lambda x_0.(\text{was } \Delta^{vc}(\nabla^{mod}\nabla^{mod}\nabla^x\nabla^x x_0 \text{ murdered}) \Delta^{su} \text{ democracy}))) (\nabla^{det} \text{the} ???)$

Proof Frames Structures



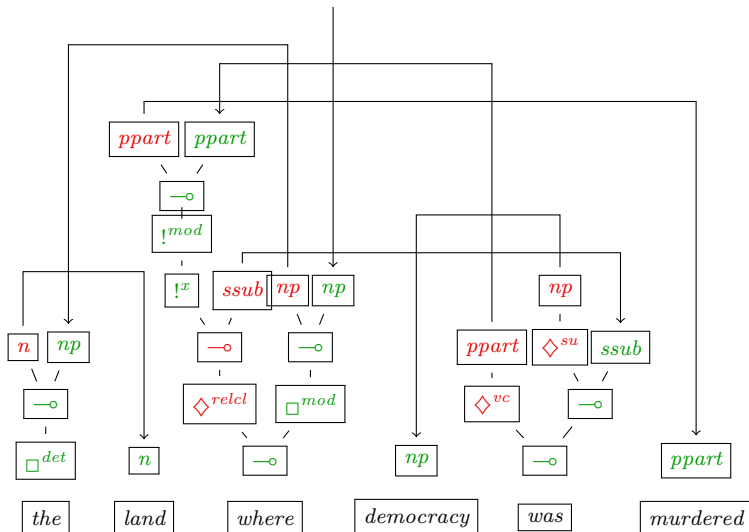
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Proof Frames Structures



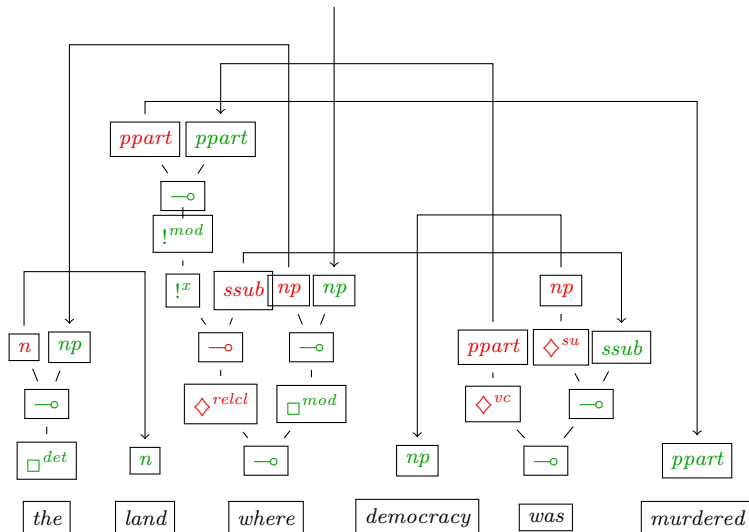
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Proof Frames Structures



$\nabla^{mod}(\text{where } \Delta^{relcl}(\lambda x_0. (\text{was } \Delta^{vc}(\nabla^{mod} \nabla^{mod} \nabla^x \nabla^x x_0 \text{ murdered}) \Delta^{su} \text{ democracy}))) (\nabla^{det} \text{the land})$

Proof Frames Structures Nets



$\nabla^{mod}(\text{where } \Delta^{relcl}(\lambda x_0. (\text{was } \Delta^{vc}(\nabla^{mod} \nabla^{mod} \nabla^x \nabla^x x_0 \text{ murdered}) \Delta^{su} \text{ democracy}))) (\nabla^{det} \text{the land})$

written and directed by
KOKOS

“Lets neural shit up”
– Alonzo Church, 1973

Main ingredients

- ▶ Type assignment (supertagging)
parallel tree decoding with dynamic graph convolutions
- ▶ Axiom linking (neural bijections)
optimal transport with Sinkhorn iterations
- ▶ Formal verification
proof net traversal

Supertagging 101

goal

maximize $p(T_1, \dots, T_n)$ conditional on some input (w_1, \dots, w_n)

the catch

types are sparse \implies fixed vocabulary classification = no good

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Supertagging as parallel graph completion

???

???

???

???

???

???

the

land

where

democracy

was

murdered

Supertagging as parallel graph completion

???

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???

???

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the

land

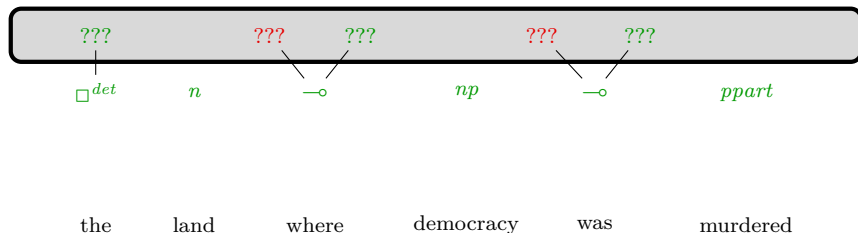
where

democracy

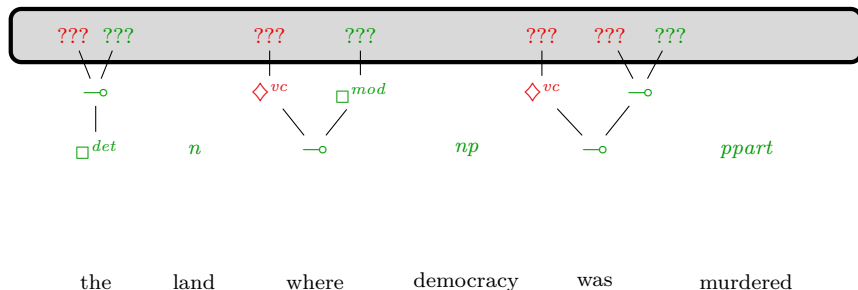
was

murdered

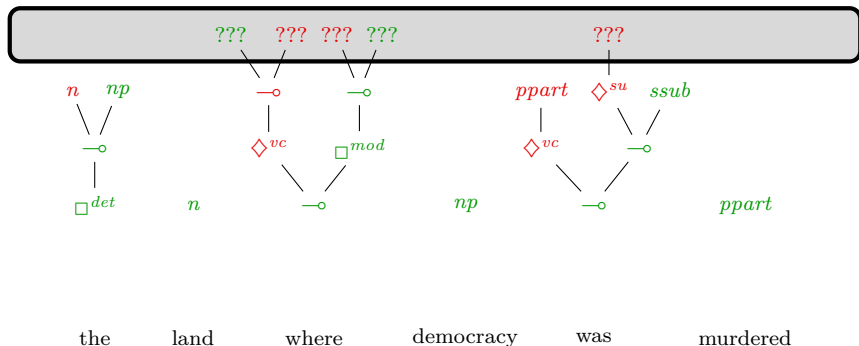
Supertagging as parallel graph completion



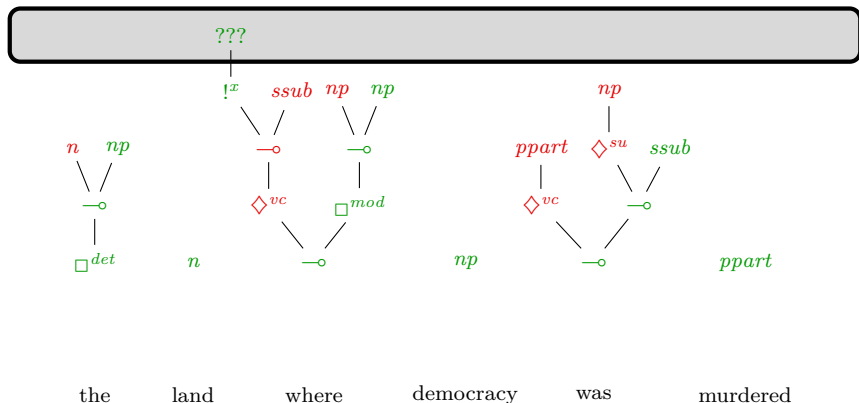
Supertagging as parallel graph completion



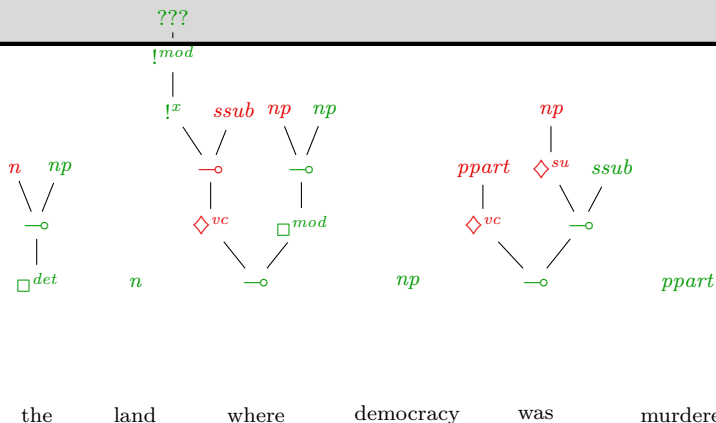
Supertagging as parallel graph completion



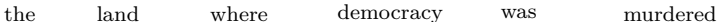
Supertagging as parallel graph completion



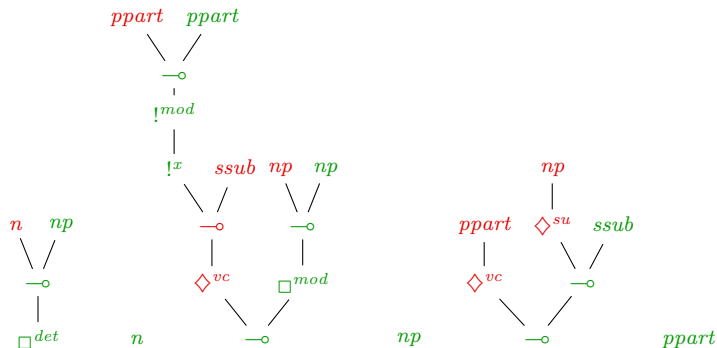
Supertagging as parallel graph completion



???



Supertagging as parallel graph completion



the

land

where

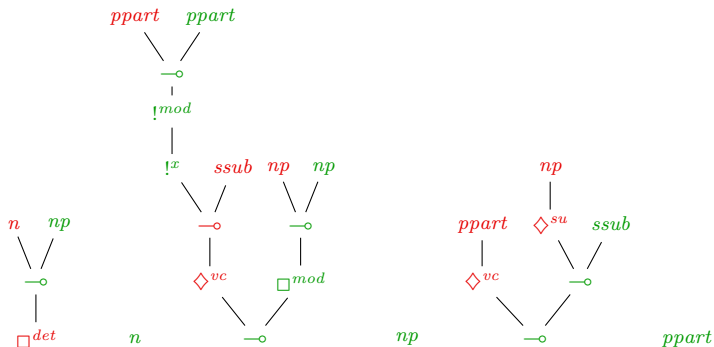
democracy

was

murdered

Axiom Linking

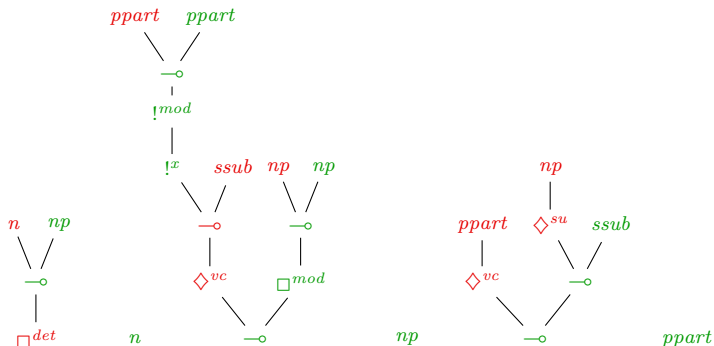
missing edges ☹



Axiom Linking

missing edges ☹

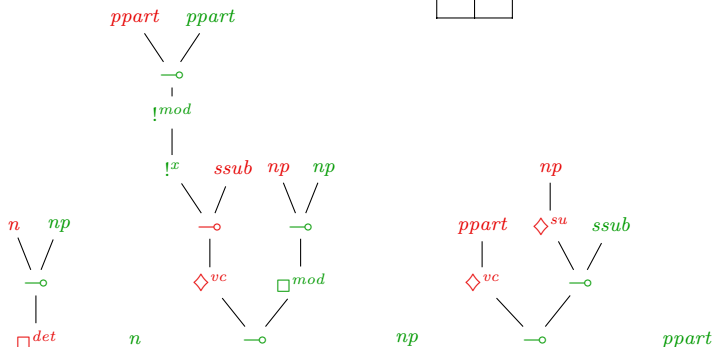
- ! only consider edges between atoms of the *same* sign and *different* polarity
- ! each atom can only be used *once*



Axiom Linking

missing edges ☹

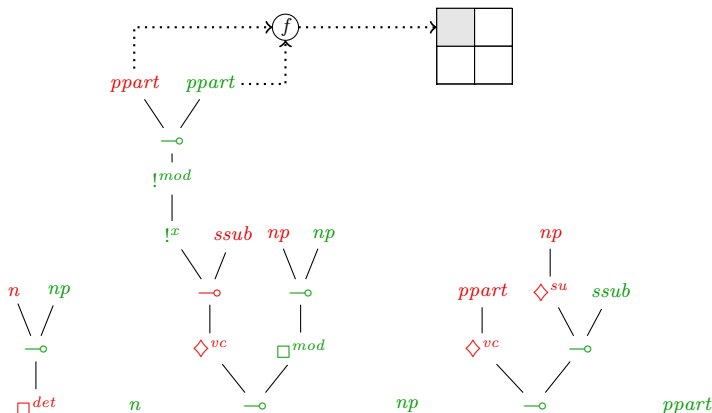
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Axiom Linking

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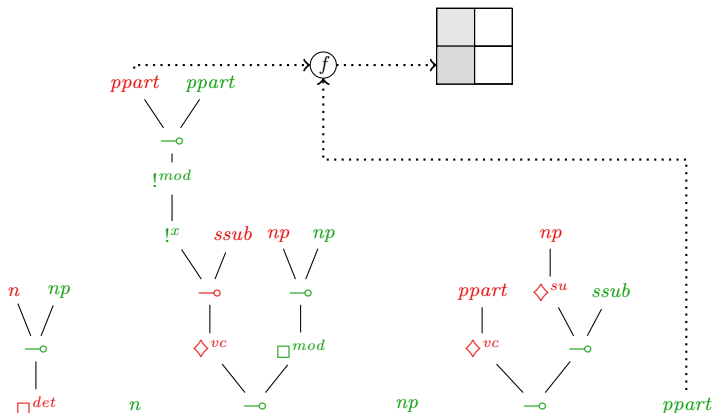
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Axiom Linking

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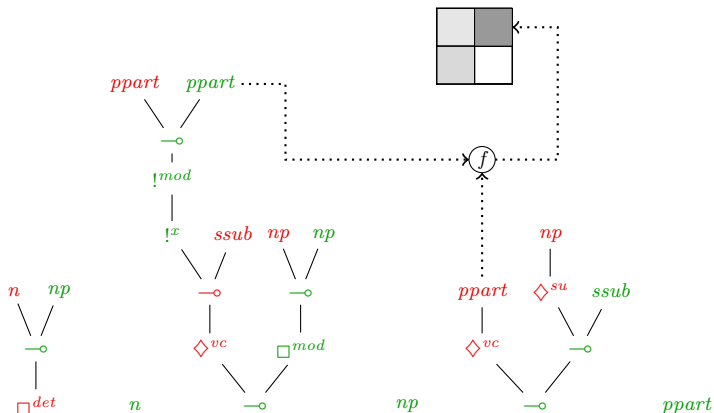
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Axiom Linking

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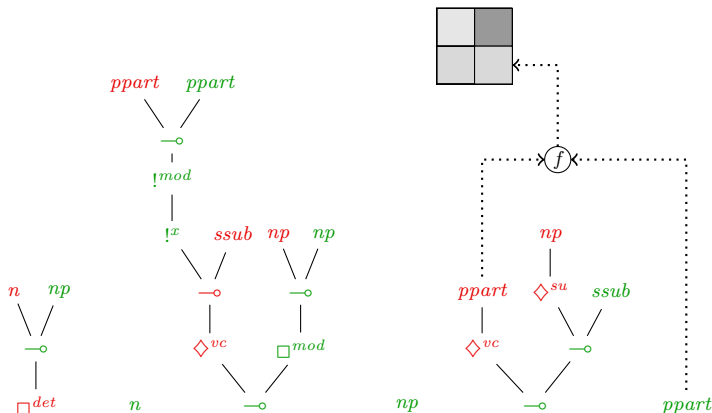
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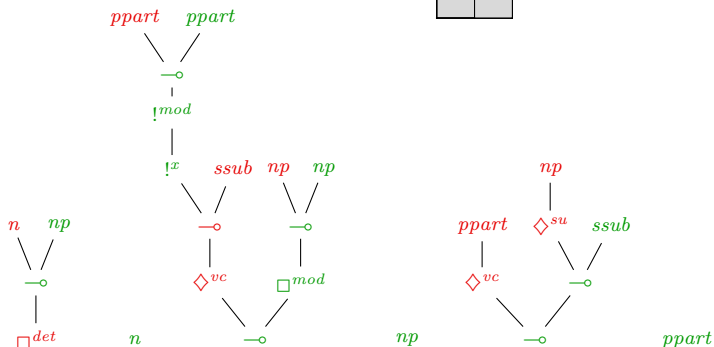
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Axiom Linking

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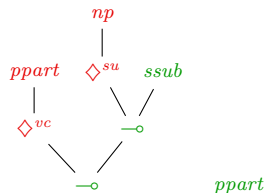
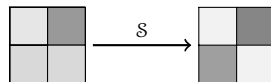
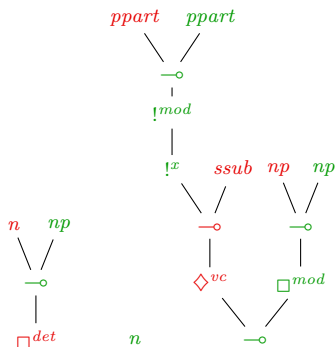
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Axiom Linking

missing edges ☹

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Axiom Linking

no missing edges! ☺

