Syntax-Semantics Interface

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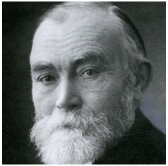
Two Key Ideas



Gottlob Frege

Principle of Compositionality
The meaning of a complex expression is derived by its constituent components and their interactions.

Universal Grammar
Homomorphism between a syntactic and a semantic algebra associates syntactic operations to semantic operations.

Richard Montague

In our case

syntactic calculus	$\xrightarrow{h_1}$	semantic calculus	$\xrightarrow{h_2}$	lexical meaning
NL / L / ..		$\mathrm{MILL}_{\multimap,\otimes}$ /..		$\mathrm{IL}_{\rightarrow,\times}$
surface form		*meaning* (computational)		*meaning* (lexical)

For each h_i we will need:

- A map to associate source types to target types
- A map to associate source terms to target terms

remember: $\mathfrak{L}$, η, θ of ACGs

Linear Syntax, Intuitionistic Semantics?

Minimal term syntax for NL

$$\frac{\Gamma \vdash \mathtt{f} : B/A \quad \Delta \vdash \mathtt{x} : A}{(\Gamma, \Delta) \vdash \mathtt{f} \triangleleft \mathtt{x} : B} \; /E \qquad \frac{\Gamma \vdash \mathtt{x} : B \quad \Delta \vdash \mathtt{f} : A \backslash B}{(\Gamma, \Delta) \vdash \mathtt{x} \triangleright \mathtt{f} : B} \; \backslash E$$

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- Syntactic Term: `(the` $\lhd$ `worker)` $\rhd$ `(liberates` $\rhd$ `herself)`

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- Syntactic Term: `(the` $\lhd$ `worker)` $\rhd$ `(liberates` $\rhd$ `herself)`
- Computational Term: `(herself liberates) (the worker)`

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- Syntactic Term: `(the` $\triangleleft$ `worker)` $\triangleright$ `(liberates` $\triangleright$ `herself)`
- Computational Term: `(herself liberates) (the worker)`
- Lexical Term: `(liberates (the worker)) (the worker)`
  assigning `herself` $:= \lambda \mathtt{fx}.((\mathtt{f}\ \mathtt{x})\ \mathtt{x})$ non-linear!

Interpretation Domains

What is the meaning of life?

Interpretation Domains

What is the meaning of life?

`life`

Interpretation Domains

What is the meaning of life?

`life`

Available Options:

- Truth-Conditional
  Sentences as truth-values, properties as predicates, entities as set elements . . .
- Type-Theoretic
  Sentences as judgements, properties as dependent types, entities as simple types . . .
- Distributional/Statistical
  Words and phrases as tensors populated by co-occurrence counts
- Neural
  Words and phrases as tensors populated by optimization of objective function

Truth-Conditional Semantics: take 1

Truth-conditional semantics

consist of two *semantic primitives*: e (for entities) and t (for truth-values) with $\multimap$ as the type-forming operator, shorthand xy for $x \multimap y$[1]

[1] left-associative

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Simple Translation

Let $\lceil . \rceil$ a mapping, s.t. $\lceil np \rceil = e$, $\lceil s \rceil = t$, $\lceil B/A \rceil = \lceil A \backslash B \rceil = \lceil A \rceil \multimap \lceil B \rceil$

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☺ Fine for simple clauses:

$$\text{Joseph hates Leon} \rightsquigarrow ((\mathtt{hates}^{eet}\ \mathtt{Leon}^{e})^{et}\ \mathtt{Joseph}^{e})^{t}$$

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☺ Fine for simple clauses:

$$\text{Joseph hates Leon} \rightsquigarrow ((\texttt{hates}^{eet}\ \texttt{Leon}^{e})^{et}\ \texttt{Joseph}^{e})^{t}$$

☹ ..less so for quantifiers:

$$\text{Joseph hates everybody} \overset{?}{\rightsquigarrow} (\forall\ \lambda x.\texttt{hates}\ x\ \texttt{Joseph})^{t}$$

Impossible to derive with simple translation!

[1] left-associative

Truth-Conditional Semantics: Three Translations

We examine three translations:

- Flexible Interpretation (Hendriks 1993)
- Incremental CPS Interpretation (Barker 2004)
- Plotkin's CPS Interpretation (Lebedeva 2012)

(1) Flexible Interpretation

The type map η is lifted from a function to a *relation*: Each syntactic source type is associated with a *set* of semantic target types.

$$\eta(A) = \{B \mid B \in \mathrm{shift}(\lceil A \rceil)\}$$

where shift the reflexive, transitive closure of $\lceil . \rceil$ under the laws:

- **Value Raising**

  From $f : \vec{A} \multimap B$ derive:
  $$\lambda \vec{x} w.(w\ (f\ \vec{x})) : \vec{A} \multimap (B \multimap D) \multimap D$$

- **Argument Raising**

  From $f : \vec{A} \multimap B \multimap \vec{C} \multimap D$ derive:
  $$\lambda \vec{x} w \vec{y}.(w\ \lambda z.(f\ \vec{x}\ z\ \vec{y})) : \vec{A} \multimap ((B \multimap D) \multimap D) \multimap \vec{C} \multimap D$$

- **Argument Lowering**

  From $f : \vec{A} \multimap ((B \multimap D) \multimap D) \multimap \vec{C} \multimap E$ derive:
  $$\lambda \vec{x} w \vec{y}.(f\ \vec{x}\ (\lambda z.(z\ w))\vec{y}) : \vec{A} \multimap B \multimap \vec{C} \multimap D$$

(1) Flexible Interpretation: Example

Lexical Translation

Source	Target
Joseph :: np	$\texttt{Joseph}$ :: e
hates :: $(np\backslash s)/np$	$\texttt{hates}$ :: eet
everybody :: np	$\forall$ :: $(et)t$

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everybody: np
$\lceil np \rceil = e$

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everybody: np

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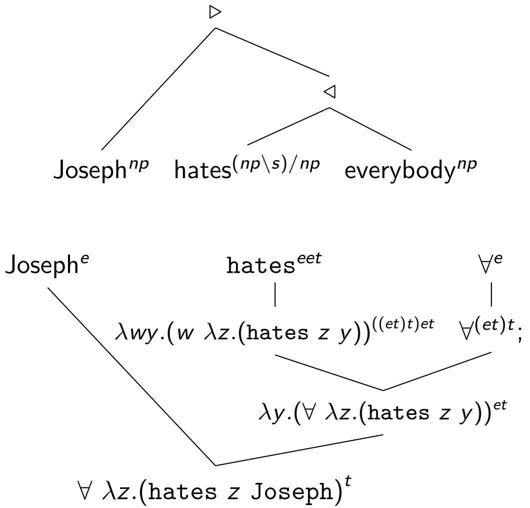
everybody: np
$\lceil np \rceil = e \xrightarrow{vr} \forall : (et)t$
hates: $(np\backslash s)/np$
$\lceil (np\backslash s)/np \rceil = eet \xrightarrow{ar} \lambda wy.(w\ \lambda z.(\texttt{hates}\ z\ y)) : ((et)t)et$

(1) Flexible Interpretation: Example



(2) Incremental CPS

Two translations left-to-right $(.)^{\rightsquigarrow}$ and right-to-left incremental $(.)^{\leftrightsquigarrow}$
For A atomic, $A^{\rightsquigarrow} = A^{\leftrightsquigarrow} = (\lceil A \rceil \multimap \bot) \multimap \bot$[2]

For complex types, we have:

- Left-to-right $(.)^{\rightsquigarrow}$

$$(N \triangleright M)^{\rightsquigarrow} = \lambda k.(N^{\rightsquigarrow}\ \lambda n.(M^{\rightsquigarrow}\ \lambda m.(k\ (m\ n))))$$
$$(M \triangleleft N)^{\rightsquigarrow} = \lambda k.(M^{\rightsquigarrow}\ \lambda m.(N^{\rightsquigarrow}\ \lambda n.(k\ (m\ n))))$$

- Right-to-left $(.)^{\leftrightsquigarrow}$

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[2] $\bot$ the *response* type, here t

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Lexical Translation

Source	Target
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Left-to-right translation

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Left-to-right translation

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$$(\mathrm{h} \triangleleft \mathrm{e})^{\leadsto} = \lambda k_1.(\mathrm{hates}^{\leadsto}\ \lambda n_1.(\mathrm{everybody}^{\leadsto}\ \lambda m_1.(k_1\ (m_1\ n_1))))$$

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$$\begin{aligned}(\mathrm{h} \triangleleft \mathrm{e})^{\rightsquigarrow} &= \lambda k_1.(\mathrm{hates}^{\rightsquigarrow}\ \lambda n_1.(\mathrm{everybody}^{\rightsquigarrow}\ \lambda m_1.(k_1\ (m_1\ n_1)))) \\ &= \lambda k_1.(\lambda k_0.(k_0\ \texttt{hates})\ \lambda n_1.(\forall\ \lambda m_1.(k_1\ (m_1\ n_1))))\end{aligned}$$

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Source	Lexical Translation	Target
Joseph :: np		$\lambda k.(k\ \texttt{Joseph})$ :: $(et)t$
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&\overset{\beta}{\leadsto} \lambda k_1.(\forall\ \lambda m_1.(k_1\ (\texttt{hates}\ m_1)))
\end{aligned}$$

(2) Incremental CPS: Example

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$$(\mathrm{J} \triangleright (\mathrm{h} \triangleleft \mathrm{e}))^{\rightsquigarrow} = \lambda k_2.(\mathrm{Joseph}^{\rightsquigarrow}\ \lambda n_2.((\mathrm{hates} \triangleleft \mathrm{everybody})^{\rightsquigarrow}\ \lambda m_2.(k_2\ (m_2\ n_2)))))$$

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$$\begin{aligned}
(\mathsf{J} \triangleright (\mathsf{h} \triangleleft \mathsf{e}))^{\leadsto} &= \lambda k_2.(\mathsf{Joseph}^{\leadsto}\ \lambda n_2.((\mathsf{hates} \triangleleft \mathsf{everybody})^{\leadsto}\ \lambda m_2.(k_2\ (m_2\ n_2)))))\\
&= \lambda k_2.(\lambda k_3.(k_3\ \texttt{Joseph})\ \lambda n_2.((\mathsf{hates} \triangleleft \mathsf{everybody})^{\leadsto}\ \lambda m_2.(k_2\ (m_2\ n_2)))\\
&\overset{\beta}{\leadsto} \lambda k_2.(\lambda n_2.((\mathsf{hates} \triangleleft \mathsf{everybody})^{\leadsto}\ \lambda m_2.(k_2\ (m_2\ n_2))\ \texttt{Joseph})\\
&\overset{\beta}{\leadsto} \lambda k_2.((\mathsf{hates} \triangleleft \mathsf{everybody})^{\leadsto}\ \lambda m_2.(k_2\ (m_2\ \texttt{Joseph}))\\
&= \lambda k_2.(\lambda k_1.(\forall\ \lambda m_1.(k_1\ (\texttt{hates}\ m_1)))\ \lambda m_2.(k_2\ (m_2\ \texttt{Joseph}))\\
&\overset{\beta}{\leadsto} \lambda k_2.(\forall\ \lambda m_1.(\lambda m_2.(k_2\ (m_2\ \texttt{Joseph}))\ (\texttt{hates}\ m_1)))\\
&\overset{\beta}{\leadsto} \lambda k_2.(\forall\ \lambda m_1.(k_2\ (\texttt{hates}\ m_1\ \texttt{Joseph}))) \xrightarrow{\text{eval}} \forall \lambda m_1.(\texttt{hates}\ m_1\ \texttt{Joseph})
\end{aligned}$$

(3) Plotkin's CPS Interpretation

New *value* translation $\lceil . \rceil$:

$$\lceil np \rceil = e,\ \lceil s \rceil = t,\ \lceil B/A \rceil = \lceil A \backslash B \rceil = \lceil A \rceil \multimap (\lceil B \rceil \multimap \bot) \multimap \bot$$

And *computation* translation $\overline{A} = (\lceil A \rceil \multimap \bot) \multimap \bot$ (here: $\bot = t$).

Lexical Translation

Source	$\lceil . \rceil$	Target
Joseph :: np	e	$\lambda k.(k\ \texttt{Joseph}) :: (et)t$
hates :: $(np \backslash s)/np$	?	?
everybody :: np	e	$\forall :: (et)t$

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$$\lceil np \backslash s \rceil = \lceil np \rceil \multimap (\lceil s \rceil \multimap \bot) \multimap \bot = e \multimap (t \multimap t) \multimap t = e\,(tt)\,t$$

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Lexical Translation

Source	$\lceil . \rceil$	Target
Joseph :: np	e	$\lambda k.(k\ \texttt{Joseph}) :: (et)t$
hates :: $(np \backslash s)/np$	$e\,((e\,(tt)\,t)\,t)\,t$	?
everybody :: np	e	$\forall :: (et)t$

$$\lceil np \backslash s \rceil = \lceil np \rceil \multimap (\lceil s \rceil \multimap \bot) \multimap \bot = e \multimap (t \multimap t) \multimap t = e\,(tt)\,t$$

$$\lceil (np \backslash s)/np \rceil = \lceil np \rceil \multimap (\lceil np \backslash s \rceil \multimap \bot) \multimap \bot = e\,((e\,(tt)\,t)\,t)\,t$$

(3) Plotkin's CPS Interpretation

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Lexical Translation

Source	$\lceil . \rceil$	Target
Joseph :: np	e	$\lambda k.(k\ \texttt{Joseph}) :: (et)t$
hates :: $(np \backslash s)/np$	$e\,((e\,(tt)\,t)\,t)\,t$	$? :: ((e\,((e\,(tt)\,t)\,t)\,t)\,t)\,t$
everybody :: np	e	$\forall :: (et)t$

$$\lceil np \backslash s \rceil = \lceil np \rceil \multimap (\lceil s \rceil \multimap \bot) \multimap \bot = e \multimap (t \multimap t) \multimap t = e\,(tt)\,t$$

$$\lceil (np \backslash s)/np \rceil = \lceil np \rceil \multimap (\lceil np \backslash s \rceil \multimap \bot) \multimap \bot = e\,((e\,(tt)\,t)\,t)\,t$$

$$\overline{(np \backslash s)/np} = ((e\,((e\,(tt)\,t)\,t)\,t)\,t)\,t$$

(3) Plotkin's CPS Interpretation

New *value* translation $\lceil . \rceil$:

$$\lceil np \rceil = e,\ \lceil s \rceil = t,\ \lceil B/A \rceil = \lceil A \backslash B \rceil = \lceil A \rceil \multimap (\lceil B \rceil \multimap \bot) \multimap \bot$$

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Lexical Translation

Source	$\lceil . \rceil$	Target
Joseph :: np	e	$\lambda k.(k\ \texttt{Joseph}) :: (et)t$
hates :: $(np \backslash s)/np$	$e\,((e\,(tt)\,t)\,t)\,t$	? :: $((e\,((e\,(tt)\,t)\,t)\,t)\,t)\,t$
everybody :: np	e	$\forall :: (et)t$

Term Assignment

$$\underbrace{\left(\left(\overbrace{e}^{x}\ \underbrace{\left(\left(\overbrace{e}^{y}\ \overbrace{(tt)}^{\tau}\ t\right) t\right)}_{h}\ t\right) t\right)}_{f}\ t$$

(3) Plotkin's CPS Interpretation

New *value* translation $\lceil . \rceil$:

$$\lceil np \rceil = e,\ \lceil s \rceil = t,\ \lceil B/A \rceil = \lceil A \backslash B \rceil = \lceil A \rceil \multimap (\lceil B \rceil \multimap \bot) \multimap \bot$$

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Lexical Translation

Source	$\lceil . \rceil$	Target
Joseph :: np	e	$\lambda k.(k\ \mathtt{Joseph}) :: (et)t$
hates :: $(np\backslash s)/np$	$e\,((e\,(tt)\,t)\,t)\,t$	$? :: ((e\,((e\,(tt)\,t)\,t)\,t)\,t)\,t$
everybody :: np	e	$\forall :: (et)t$

Term Assignment

$$\underbrace{\left(\left(\overbrace{e}^{x}\,\underbrace{\left(\left(\overbrace{e}^{y}\,\overbrace{(tt)}^{\tau}\,t\right)t\right)}_{h}\,t\right)t\right)}_{f}\,t \qquad \lambda f.(f\ \lambda xh.(h\ \lambda y\tau.(\tau\ (\mathtt{hates}\ x\ y))))$$

(3) Plotkin's CPS Interpretation

Lexical Translation

Source	Target
Joseph :: np	$\lambda k.(k\ \texttt{Joseph}) :: (et)t$
hates :: $(np\backslash s)/np$	$\lambda f.(f\ \lambda xh.(h\ \lambda y\tau.(\tau\ (\texttt{hates}\ x\ y)))) :: ((e((e(tt)t)t)t)t)t$
everybody :: np	$\forall :: (et)t$

(3) Plotkin's CPS Interpretation

Lexical Translation

Source	Target
Joseph :: np	$\lambda k.(k\ \texttt{Joseph}) :: (et)t$
hates :: $(np\backslash s)/np$	$\lambda f.(f\ \lambda xh.(h\ \lambda y\tau.(\tau\ (\texttt{hates}\ x\ y)))) :: ((e((e(tt)t)t)t)t)t$
everybody :: np	$\forall :: (et)t$

Term Translation

$$\overline{M \triangleleft N} = \overline{N \triangleright M} = \lambda k.(\overline{M}\ \lambda m.(\overline{N}\ \lambda n.(m\ n\ k)))$$

$$\begin{aligned}
\overline{\mathrm{h} \triangleleft \mathrm{e}} &= \lambda k.\,(\lambda f.\,(f\ \lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y))))\ \lambda m.\,(\forall\ \lambda n.\,(m\ n\ k)))\\
&= \lambda k.\,(\lambda m.\,(\forall\ \lambda n.\,(m\ n\ k))\ \lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y))))\\
&= \lambda k.\,(\forall\ \lambda n.\,(\lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y)))\ n\ k))\\
&= \lambda k.\,(\forall\ \lambda n.\,(k\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ n\ y))))
\end{aligned}$$

(3) Plotkin's CPS Interpretation

Lexical Translation

Source	Target
Joseph :: np	$\lambda k.(k\ \texttt{Joseph}) :: (et)t$
hates :: $(np\backslash s)/np$	$\lambda f.(f\ \lambda xh.(h\ \lambda y\tau.(\tau\ (\texttt{hates}\ x\ y)))) :: ((e((e(tt)t)t)t)t)t$
everybody :: np	$\forall :: (et)t$

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$$\begin{aligned}
\overline{\mathrm{h} \triangleleft \mathrm{e}} &= \lambda k.\,(\lambda f.\,(f\ \lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y))))\ \lambda m.\,(\forall\ \lambda n.\,(m\ n\ k)))\\
&= \lambda k.\,(\lambda m.\,(\forall\ \lambda n.\,(m\ n\ k))\ \lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y))))\\
&= \lambda k.\,(\forall\ \lambda n.\,(\lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y)))\ n\ k))\\
&= \lambda k.\,(\forall\ \lambda n.\,(k\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ n\ y))))
\end{aligned}$$

(3) Plotkin's CPS Interpretation

Lexical Translation

Source	Target
Joseph :: np	$\lambda k.(k\ \texttt{Joseph}) :: (et)t$
hates :: $(np\backslash s)/np$	$\lambda f.(f\ \lambda xh.(h\ \lambda y\tau.(\tau\ (\texttt{hates}\ x\ y)))) :: ((e((e(tt)t)t)t)t)t$
everybody :: np	$\forall :: (et)t$

Term Translation

$$\overline{M \triangleleft N} = \overline{N \triangleright M} = \lambda k.(\overline{M}\ \lambda m.(\overline{N}\ \lambda n.(m\ n\ k)))$$

$$\begin{aligned}
\overline{\mathrm{h} \triangleleft \mathrm{e}} &= \lambda k.\,(\lambda f.\,(f\ \lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y))))\ \lambda m.\,(\forall\ \lambda n.\,(m\ n\ k))) \\
&= \lambda k.\,(\lambda m.\,(\forall\ \lambda n.\,(m\ n\ k))\ \lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y)))) \\
&= \lambda k.\,(\forall\ \lambda n.\,(\lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y)))\ n\ k)) \\
&= \lambda k.\,(\forall\ \lambda n.\,(k\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ n\ y))))
\end{aligned}$$

(3) Plotkin's CPS Interpretation

Lexical Translation

Source	Target
Joseph :: np	$\lambda k.(k\ \texttt{Joseph}) :: (et)t$
hates :: $(np\backslash s)/np$	$\lambda f.(f\ \lambda xh.(h\ \lambda y\tau.(\tau\ (\texttt{hates}\ x\ y)))) :: ((e((e(tt)t)t)t)t)t$
everybody :: np	$\forall :: (et)t$

Term Translation

$$\overline{M \lhd N} = \overline{N \rhd M} = \lambda k.(\overline{M}\ \lambda m.(\overline{N}\ \lambda n.(m\ n\ k)))$$

$$\begin{aligned}\overline{\mathrm{h} \lhd \mathrm{e}} &= \lambda k.\,(\lambda f.\,(f\ \lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y))))\ \lambda m.\,(\forall\ \lambda n.\,(m\ n\ k)))\\ &= \lambda k.\,(\lambda m.\,(\forall\ \lambda n.\,(m\ n\ k))\ \lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y))))\\ &= \lambda k.\,(\forall\ \lambda n.\,(\lambda xh.\,(h\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ x\ y)))\ n\ k))\\ &= \lambda k.\,(\forall\ \lambda n.\,(k\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ n\ y))))\end{aligned}$$

(3) Plotkin's CPS Interpretation

Lexical Translation

Source	Target
Joseph :: np	$\lambda k.(k\ \texttt{Joseph}) :: (et)t$
hates :: $(np\backslash s)/np$	$\lambda f.(f\ \lambda xh.(h\ \lambda y\tau.(\tau\ (\texttt{hates}\ x\ y)))) :: ((e((e(tt)t)t)t)t)t$
everybody :: np	$\forall :: (et)t$

Term Translation

$$\overline{M \triangleleft N} = \overline{N \triangleright M} = \lambda k.(\overline{M}\ \lambda m.(\overline{N}\ \lambda n.(m\ n\ k)))$$

$$\begin{aligned}
\overline{\mathsf{J} \triangleright (\mathsf{h} \triangleleft \mathsf{e})} &= \lambda k_2.\,(\overline{\mathsf{h} \triangleleft \mathsf{e}}\ \lambda m_2.\,(\lambda k_3.\,(k_3\ \texttt{Joseph})\ \lambda n_2.\,(m_2\ n_2\ k_2)))\\
&\overset{\beta}{\leadsto} \lambda k_2.\,(\overline{\mathsf{h} \triangleleft \mathsf{e}}\ \lambda m_2.\,(\lambda n_2.\,(m_2\ n_2\ k_2)\ \texttt{Joseph}))\\
&\overset{\beta}{\leadsto} \lambda k_2.\,(\overline{\mathsf{h} \triangleleft \mathsf{e}}\ \lambda m_2.\,(m_2\ \texttt{Joseph}\ k_2))\\
&= \lambda k_2.\,(\lambda k.\,(\forall\ \lambda n.\,(k\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ n\ y))))\ \lambda m_2.\,(m_2\ \texttt{Joseph}\ k_2))\\
&= \lambda k_2.\,(\forall\ \lambda n.\,(\lambda m_2.\,(m_2\ \texttt{Joseph}\ k_2)\ \lambda y\tau.\,(\tau\ \ (\texttt{hates}\ n\ y))))\\
&= \lambda k_2.\,(\forall\ \lambda n.\,(\lambda y\tau.\,(\tau\ \ (\texttt{hates}\ n\ y))\ \texttt{Joseph}\ k_2))\\
&= \lambda k_2.\,(\forall\ \lambda n.\,(k_2\ \ (\texttt{hates}\ n\ \texttt{Joseph}))) \xrightarrow{eval} \forall\ \lambda n.\,(\texttt{hates}\ n\ \texttt{Joseph})
\end{aligned}$$