Grammaticality as Provability

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Groningen Logic Seminar
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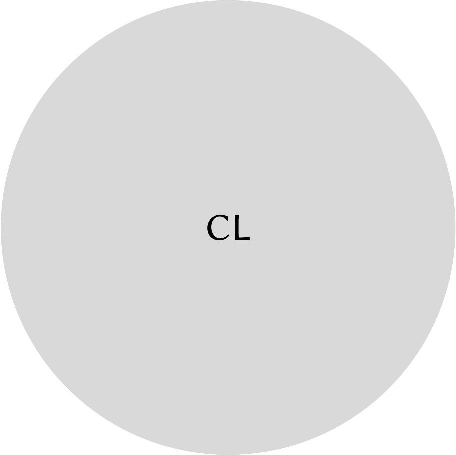
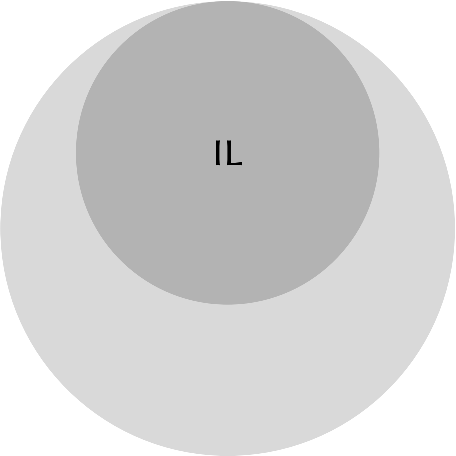
The Big Picture



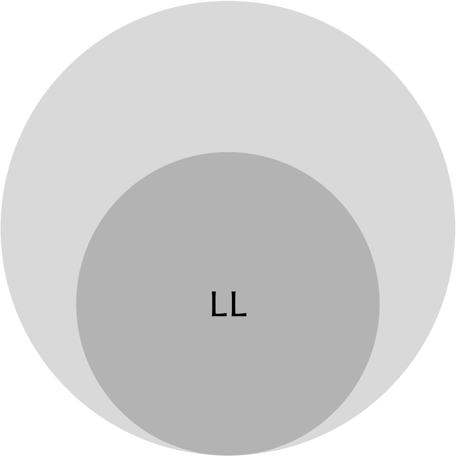
- CL (folklore...)

The Big Picture



- CL (folklore...)
- IL (Heyting, 1930)
  no excluded middle, no involutive negation

The Big Picture



- CL (folklore...)
- IL (Heyting, 1930)
  no excluded middle, no involutive negation
- LL (Girard, 1987)
  truth ≡ resource; no weakening, no contraction

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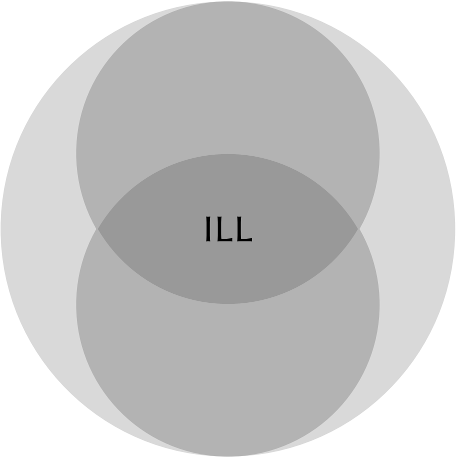
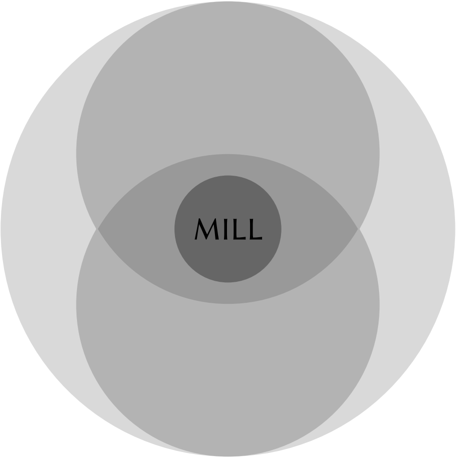


- CL (folklore...)
- IL (Heyting, 1930)
  no excluded middle, no involutive negation
- LL (Girard, 1987)
  truth $\equiv$ resource; no weakening, no contraction
- ILL
  $=$ LL $\cap$ IL

The Big Picture



- CL (folklore...)
- IL (Heyting, 1930)
  no excluded middle, no involutive negation
- LL (Girard, 1987)
  truth ≡ resource; no weakening, no contraction
- ILL
  = LL ∩ IL
- MILL
  = ILL without additives

MILL

typing rules

Types

$A, B, C := p \mid A \to B \mid A \otimes B \qquad (p \in \mathsf{Prim})$

Structures

$\Gamma, \Delta := () \mid \Gamma \cdot A$

$$\frac{}{A \vdash A}\ Ax$$

$$\frac{\Gamma \vdash A \to B \quad \Delta \vdash A}{\Gamma \cdot \Delta \vdash B}\ \to_E \qquad \frac{\Gamma \cdot A \vdash B}{\Gamma \vdash A \to B}\ \to_I$$

$$\frac{\Gamma \vdash A \otimes B \quad \Delta \cdot A \cdot B \vdash C}{\Gamma \cdot \Delta \vdash C}\ \otimes_E \qquad \frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma \cdot \Delta \vdash A \otimes B}\ \otimes_I$$

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MILL

term calculus

Types

$A, B, C := p \mid A \to B \mid A \otimes B \qquad (p \in \mathrm{Prim})$

Structures

$\Gamma, \Delta := () \mid \Gamma \cdot A$

$$\dfrac{}{x : A \vdash x : A}\ Ax$$

$$\dfrac{\Gamma \vdash s : A \to B \quad \Delta \vdash t : A}{\Gamma \cdot \Delta \vdash s\,t : B}\ \to_E \qquad \dfrac{\Gamma \cdot x : A \vdash s : B}{\Gamma \vdash \lambda x.s : A \to B}\ \to_I$$

$$\dfrac{\Gamma \vdash s : A \otimes B \quad \Delta \cdot x : A \cdot y : B \vdash t : C}{\Gamma \cdot \Delta \vdash \text{case } s \text{ of } (x, y) \text{ in } t : C}\ \otimes_E \qquad \dfrac{\Gamma \vdash x : A \quad \Delta \vdash y : B}{\Gamma \cdot \Delta \vdash (x, y) : A \otimes B}\ \otimes_I$$

$$\dfrac{\Gamma \cdot x : A \cdot y : B \cdot \Delta \vdash s : C}{\Gamma \cdot y : B \cdot x : A \cdot \Delta \vdash s : C}\ Ex$$

Invertible Inferences

$$\Gamma_{[A_1 \cdot \ \dots \ \cdot A_n]} \vdash B$$

Invertible Inferences

$$\frac{\Gamma_{[A_1 \cdot \, \ldots \, \cdot A_n]} \vdash B}{() \vdash A_1 \to \ldots \to A_n \to B}$$

Invertible Inferences

$$\dfrac{\dfrac{\Gamma_{[A_1 \cdot \ \dots \ \cdot A_n]} \vdash B}{() \vdash A_1 \to \dots \to A_n \to B}}{() \vdash (A_1 \otimes \cdots \otimes A_n) \to B}$$

Invertible Inferences

$$\cfrac{\cfrac{\cfrac{\Gamma_{[A_1 \cdot\ \dots\ \cdot A_n]} \vdash B}{() \vdash A_1 \to \dots \to A_n \to B}}{() \vdash (A_1 \otimes \cdots \otimes A_n) \to B}}{A_1 \otimes \cdots \otimes A_n \vdash B}$$

Invertible Inferences

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$$\dfrac{\Gamma \vdash B}{\Gamma' \ \vdash B} \ \Gamma' \in \mathrm{S}_n(\Gamma)$$

Invertible Inferences

$$\dfrac{\dfrac{\dfrac{\Gamma_{[A_1\cdot\ \dots\ \cdot A_n]} \vdash B}{() \vdash A_1 \to \dots \to A_n \to B}}{() \vdash (A_1 \otimes \cdots \otimes A_n) \to B}}{A_1 \otimes \cdots \otimes A_n \vdash B}$$

$$\dfrac{\Gamma \vdash B}{\Gamma' \ \vdash B}\ \Gamma' \in S_n(\Gamma)$$

- premises are *multisets*
- $\otimes$ and $\cdot$ are *variadic* and *order-insensitive*

$\vdots$

Invertible Inferences

$$\begin{array}{c} \Gamma_{[A_1\cdot\ \dots\ \cdot A_n]} \vdash B \\ \hline\hline () \vdash A_1 \to \dots \to A_n \to B \\ \hline\hline () \vdash (A_1 \otimes \cdots \otimes A_n) \to B \\ \hline\hline A_1 \otimes \cdots \otimes A_n \vdash B \end{array}$$

$$\cancel{\begin{array}{c} \Gamma \vdash B \\ \hline\hline \Gamma' \vdash B \end{array} \ \Gamma' \in S_n(\Gamma)}$$

- premises are ~~*multisets*~~ *sequences* (n! as many)
- $\otimes$ and $\cdot$ are *variadic* ~~and *order-insensitive*~~

  ⋮

Going Sub$^{(2)}$structural

A world without exchange

Without Ex, $\rightarrow$ branches into two position-refined variants: $/$ and $\backslash$.

Read: B/A – “B over A”
$A\backslash B$ – “A under B”

$$\frac{\Gamma \vdash s : A \quad \Delta \vdash t : A\backslash B}{\Gamma \cdot \Delta \vdash s \triangleright t : B}\ \backslash_E \qquad \frac{x : A, \Gamma \vdash B}{\Gamma \vdash \lambda^l x.s : A\backslash B}\ \backslash_I$$

$$\frac{\Gamma \vdash s : B/A \quad \Delta \vdash t : A}{\Gamma \cdot \Delta \vdash s \triangleleft t : B}\ /_E \qquad \frac{\Gamma \cdot x : A \vdash s : B}{\Gamma \vdash \lambda^r x.s : B/A}\ /_I$$

$$\frac{\Gamma \vdash s : A \otimes B \quad \Delta \cdot x : A, y : B \cdot \Theta \vdash t : C}{\Gamma \cdot \Delta \cdot \Theta \vdash \text{case } s \text{ of } (x, y) \text{ in } t : C}\ \otimes_E$$

Going Sub$^{(2)}$structural

A world without exchange

Now:

$$A_1 \backslash (A_2 \backslash B) \not\equiv A_2 \backslash (A_1 \backslash B) \not\equiv (B / A_2) / A_1 \not\equiv (B / A_1) / A_2 \not\equiv (A_1 \backslash B) / A_2 \not\equiv (A_2 \backslash B) / A_1$$

all these just from $A_1 \to A_2 \to B$ (!)

Going Sub$^{(2)}$structural

A world without exchange

Now:

$$A_1 \backslash (A_2 \backslash B) \not\equiv A_2 \backslash (A_1 \backslash B) \not\equiv (B/A_2)/A_1 \not\equiv (B/A_1)/A_2 \not\equiv (A_1 \backslash B)/A_2 \not\equiv (A_2 \backslash B)/A_1$$

all these just from $A_1 \to A_2 \to B$ (!)

Yet still:

$$(A_1 \backslash B)/A_2 \equiv A_1 \backslash (B/A_2)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \vdash A_1 \backslash (B/A_2)}{A_1 \cdot \Gamma \vdash B/A_2}}{A_1 \cdot \Gamma \cdot A_2 \vdash B}}{\Gamma \cdot A_2 \vdash A_1 \backslash B}}{\Gamma \vdash (A_1 \backslash B)/A_2}$$

The (invisible) culprit?

Going Sub$^{(2)}$structural

A world without exchange or associativity

- premises become *trees* (C_{n-1} as many)
- $\otimes$ and , become *binary*

$\vdots$

$$\frac{\Gamma \vdash s : A \quad \Delta \vdash t : A \backslash B}{(\Gamma \cdot \Delta) \vdash s \triangleright t : B} \backslash_E \qquad \frac{(x : A \cdot \Gamma) \vdash B}{\Gamma \vdash \lambda^l x.s : A \backslash B} \backslash_I$$

$$\frac{\Gamma \vdash s : B / A \quad \Delta \vdash t : A}{(\Gamma \cdot \Delta) \vdash s \triangleleft t : B} /_E \qquad \frac{(\Gamma \cdot x : A) \vdash s : B}{\Gamma \vdash \lambda^r x.s : B / A} /_I$$

$$\frac{\Gamma \vdash s : A \otimes B \quad \Delta[\![(x : A \cdot y : B)]\!] \vdash t : C}{\Delta[\![\Gamma]\!] \vdash \text{case } s \text{ of } (x \cdot y) \text{ in } t : C} \otimes_E$$

The Small Picture

An Alternative Timeline

L

- L (Lambek, 1958)

The Small Picture

An Alternative Timeline

$$\mathsf{L} \xrightarrow{\text{-asso}} \mathsf{NL}$$

- L (Lambek, 1958)
- NL (Lambek, 1961)

The Small Picture

An Alternative Timeline

$$\begin{tikzcd} L \arrow[rr, "\text{-asso}"] \arrow[d, "\text{+comm}"'] & & NL \\ LP & & \end{tikzcd}$$

- L (Lambek, 1958)
- NL (Lambek, 1961)
- LP (van Benthem, 1983)
  = MILL (!)

The Small Picture

An Alternative Timeline

$$\begin{tikzcd}
L \arrow[r, "\text{-asso}"] \arrow[d, "\text{+comm}"'] & NL \arrow[d, "\text{+comm}"] \\
LP & NLP \arrow[l, "\text{+asso}"]
\end{tikzcd}$$

- L (Lambek, 1958)
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The Small Picture

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(N)L(P): Grammar Logics

The Small Picture

An Alternative Timeline

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\mathsf{LP} & \mathsf{NLP} \arrow[l, "\text{+asso}"]
\end{tikzcd}
$$

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(N)L(P): Grammar Logics

"Every mathematical discovery is made twice: once by a logician and once by a computer scientist "
– P. Wadler

The Small Picture

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L \arrow[r, "\text{-asso}"] \arrow[d, "\text{+comm}"'] & NL \arrow[d, "\text{+comm}"] \\
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(N)L(P): Grammar Logics

"Every mathematical discovery is made twice: once by a ~~logician~~ theoretical linguist and once by a ~~computer scientist~~ logician"
– P. Wadler (retrofitted)

(N)L(P)

Executive Summary

Logic	Γ	Asso	Comm
LP	multiset	✓	✓
L	string	✓	✗
NL	tree	✗	✗
NLP	mobile	✗	✓

Type-Logical Grammar 101

The idea

Language	Logic	Computation
grammar	substructural logic	λ-calculus
syntactic category	formula	type
word	hypothesis	variable
phrasal composition	inference rule	computation step
grammaticality	provability	type inhabitation
	$\vdots$	
sentence	proof	program
parsing	deduction	computation

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	⋮	
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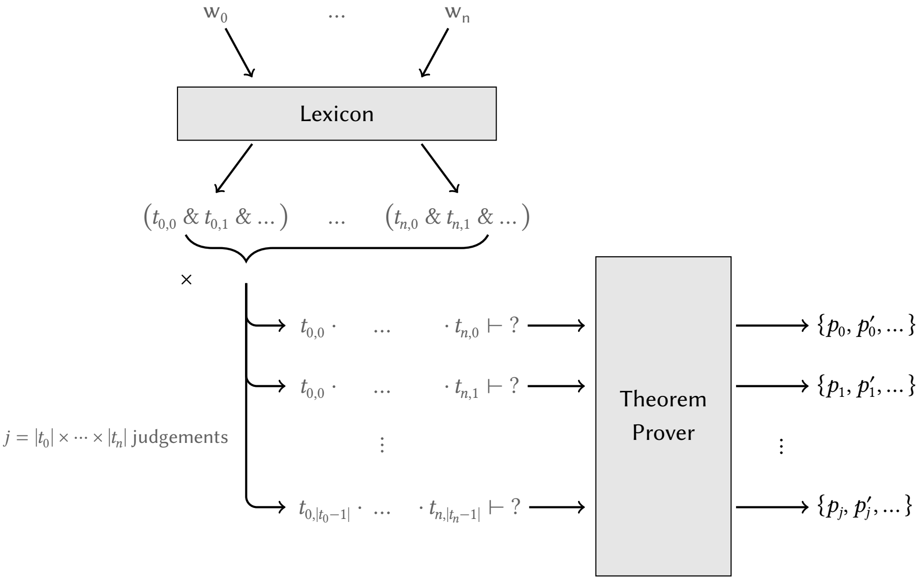
The lexicon – a mapping associating words and types

$Lex : \text{Words} \rightarrow \mathscr{P}(\mathscr{U})$

Pipeline



w_0 … w_n
Lexicon
$(t_{0,0}$ & $t_{0,1}$ & … $)$ … $(t_{n,0}$ & $t_{n,1}$ & … $)$
$\times$
$t_{0,0} \cdot \ \ldots \ \cdot t_{n,0} \vdash ?$
$t_{0,0} \cdot \ \ldots \ \cdot t_{n,1} \vdash ?$
$t_{0,|t_0-1|} \cdot \ \ldots \ \cdot t_{n,|t_n-1|} \vdash ?$
$j = |t_0| \times \cdots \times |t_n|$ judgements
Theorem Prover
$\{p_0, p_0', \ldots\}$
$\{p_1, p_1', \ldots\}$
$\{p_j, p_j', \ldots\}$

Compositionality

A Toy Example

$$\begin{array}{ccccc} \text{syntactic calculus} & \xrightarrow{h_1} & \text{semantic calculus} & \xrightarrow{h_2} & \text{lexical meaning} \\ \text{(N)L}^{(+)} & & \text{MILL}^{(+)}_{\rightarrow,\otimes} & & \text{FOL / IL / ...} \end{array}$$

surface form — *meaning* *derivational* — *meaning* *lexical*

$h_i := \langle \eta_i, \theta_i \rangle$, where:

- η an action on types
- θ an action on proofs (terms)

Compositionality

A Toy Example

$$\text{Source } \Sigma\text{: (N)L} \xrightarrow{\langle \eta, \theta \rangle} \text{Target T: MILL}$$

Base syntactic types Prim_Σ **:** n, np, s
Base semantic types Prim_T **:** e, t

$$\begin{aligned} \eta_0 &: \mathrm{Prim}_\Sigma \to \mathcal{U}_T \\ \eta_0\ np &= e \\ \eta_0\ s &= t \\ \eta_0\ n &= e \to t \end{aligned} \quad \rightsquigarrow \quad \begin{aligned} \eta &: \mathcal{U}_\Sigma \to \mathcal{U}_T \\ \eta\ p &= \eta_0\ p \\ \eta\ (A \backslash B) &= \eta\ (B / A) = (\eta\ A) \to (\eta\ B) \end{aligned}$$

$$\begin{aligned} \theta_0 &: \mathrm{Cons}_\Sigma \to \Lambda_T \\ &\ldots \end{aligned} \quad \rightsquigarrow \quad \begin{aligned} \theta &: \Lambda_\Sigma \to \Lambda_T \\ \theta\ (s \triangleright t) &= \theta\ (t \triangleleft s) = (\theta\ s)\ (\theta\ t) \end{aligned}$$

Warmup

Iterative Composition

$$\text{the} :: np/n$$
$$\text{culling} :: n$$
$$\text{necessary} :: n/n$$

$$\dfrac{\dfrac{\text{the}}{np/n}\,Lex \qquad \dfrac{\dfrac{\text{necessary}}{n/n}\,Lex \quad \dfrac{\text{culling}}{n}\,Lex}{(\text{necessary} \cdot \text{culling}) \vdash n}\,/_E}{(\text{the} \cdot (\text{necessary} \cdot \text{culling})) \vdash np}\,/_E$$

Warmup

Bidirectional F/A Structures

$$\begin{aligned} \text{the} &:: np/n \\ \text{culling} &:: n \\ \text{necessary} &:: n/n \\ \text{was} &:: (np\backslash s)/(n/n) \end{aligned}$$

$$\dfrac{\dfrac{\dfrac{\text{the}}{np/n}\,Lex \quad \dfrac{\text{culling}}{n}\,Lex}{(\text{the}\cdot\text{culling}) \vdash np}\,/_E \quad \dfrac{\dfrac{\text{was}}{(np\backslash s)/(n/n)}\,Lex \quad \dfrac{\text{necessary}}{n/n}\,Lex}{(\text{was}\cdot\text{necessary}) \vdash np\backslash s}\,/_E}{((\text{the}\cdot\text{culling})\cdot(\text{was}\cdot\text{necessary})) \vdash s}\,\backslash_E$$

Warmup

Multirole Functions

$$\begin{aligned}
\text{the} &:: np/n \\
\text{culling} &:: n \\
\text{necessary} &:: n/n \\
\text{was} &:: (np\backslash s)/(n/n) \\
\text{hunters'} &:: n/n
\end{aligned}$$

$$\dfrac{\dfrac{\dfrac{\text{the}}{np/n}\,Lex \quad \dfrac{\dfrac{\text{hunters'}}{n/n}\,Lex \quad \dfrac{\text{culling}}{n}\,Lex}{(\text{hunters'} \cdot \text{culling}) \vdash n}\,/_E}{(\text{the} \cdot (\text{hunters'} \cdot \text{culling})) \vdash np}\,/_E \quad \dfrac{\dfrac{\text{was}}{(np\backslash s)/(n/n)}\,Lex \quad \dfrac{\text{necessary}}{n/n}\,Lex}{(\text{was} \cdot \text{necessary}) \vdash np\backslash s}\,/_E}{((\text{the} \cdot (\text{hunters'} \cdot \text{culling})) \cdot (\text{was} \cdot \text{necessary})) \vdash s}\,\backslash_E$$

Warmup

Constituency Interface

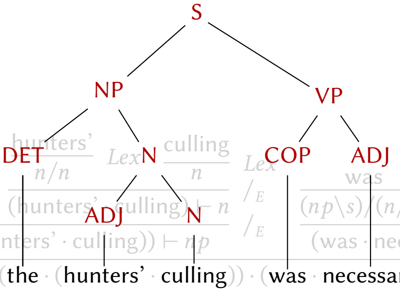
the :: np/n

culling :: n

necessary :: n/n

was :: $(np\backslash s)/(n/n)$

hunters' :: n/n



$$\dfrac{\dfrac{\dfrac{\text{the}}{np/n}\,Lex \quad \dfrac{\dfrac{\text{hunters'}}{n/n}\,Lex \quad \dfrac{\text{culling}}{n}\,Lex}{(\text{hunters'} \cdot \text{culling}) \vdash n}\,/_E}{(\text{the} \cdot (\text{hunters'} \cdot \text{culling})) \vdash np}\,/_E \quad \dfrac{\dfrac{\text{was}}{(np\backslash s)/(n/n)}\,Lex \quad \dfrac{\text{necessary}}{n/n}\,Lex}{(\text{was} \cdot \text{necessary}) \vdash np\backslash s}\,/_E}{((\text{the} \cdot (\text{hunters'} \cdot \text{culling})) \cdot (\text{was} \cdot \text{necessary})) \vdash s}\,\backslash_E$$

Warmup

Lexical Ambiguity

$$
\begin{aligned}
\text{the} &:: np/n \\
\text{culling} &:: n \mathbin{\&} np/np \\
\text{necessary} &:: n/n \\
\text{was} &:: (np\backslash s)/(n/n) \\
\text{hunters'} &:: n/n \mathbin{\&} (np/n)\backslash(np/(np/np))
\end{aligned}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\text{the}}{np/n}\,Lex \quad \dfrac{\text{hunters'}}{(np/n)\backslash(np/(np/np))}\,Lex}{(\text{the}\cdot\text{hunters'}) \vdash np/(np/np)}\,\backslash_E \quad \dfrac{\text{culling}}{np/np}\,Lex}{((\text{the}\cdot\text{hunters'})\cdot\text{culling}) \vdash np}\,/_E \quad \dfrac{\dfrac{\text{was}}{(np\backslash s)/(n/n)}\,Lex \quad \dfrac{\text{necessary}}{n/n}\,Lex}{(\text{was}\cdot\text{necessary}) \vdash np\backslash s}\,/_E}{(((\text{the}\cdot\text{hunters'})\cdot\text{culling})\cdot(\text{was}\cdot\text{necessary})) \vdash s}\,\backslash_E
$$

Warmup

Lexical Ambiguity

$$
\begin{aligned}
\text{the} &:: np/n \\
\text{culling} &:: n \,\&\, np/np \\
\text{necessary} &:: n/n \\
\text{was} &:: (np\backslash s)/(n/n) \\
\text{hunters'} &:: n/n \,\&\, (np/n)\backslash(np/(np/np))
\end{aligned}
$$

$$\theta(((\text{the} \triangleright \text{hunters'}) \triangleleft \text{culling}) \triangleright (\text{was} \triangleleft \text{necessary})) \stackrel{?}{\equiv} \theta((\text{culling} \triangleleft (\text{the} \triangleleft \text{hunters'})) \triangleright (\text{was} \triangleleft \text{necessary}))$$

Warmup

Lexical Ambiguity

$$
\begin{aligned}
\text{the} &:: np/n \\
\text{culling} &:: n \mathbin{\&} np/np \\
\text{necessary} &:: n/n \\
\text{was} &:: (np\backslash s)/(n/n) \\
\text{hunters'} &:: n/n \mathbin{\&} (np/n)\backslash(np/(np/np))
\end{aligned}
$$

$\theta(((\text{the} \triangleright \text{hunters'}) \triangleleft \text{culling}) \triangleright (\text{was} \triangleleft \text{necessary}))$

-- plug in $\theta_0(\text{hunters'}) := \lambda f^{(e\to t)\to e} g^{e\to e}.g\,(f\,\textsc{hunters})$

$\overset{\beta}{\rightsquigarrow}\ \textsc{was}^{((e\to t)\to(e\to t))\to e\to t}\ \textsc{necessary}^{(e\to t)\to e\to t}\ \left(\textsc{culling}^{e\to e}\left(\textsc{the}^{(e\to t)\to e}\ \textsc{hunters}^{e\to t}\right)\right)$

Warmup

Lexical Ambiguity

$$
\begin{aligned}
\text{the} &:: np/n \\
\text{culling} &:: n \mathbin{\&} np/np \\
\text{necessary} &:: n/n \\
\text{was} &:: (np\backslash s)/(n/n) \\
\text{hunters'} &:: n/n \mathbin{\&} (np/n)\backslash(np/(np/np))
\end{aligned}
$$

$$
\begin{aligned}
&\theta(((\text{the} \triangleright \text{hunters'}) \triangleleft \text{culling}) \triangleright (\text{was} \triangleleft \text{necessary})) \\
&\text{-- plug in } \theta_0(\text{hunters'}) \;:=\; \lambda f^{(e\to t)\to e} g^{e\to e}.g\,(f\,\textsc{hunters}) \\
&\quad \overset{\beta}{\rightsquigarrow} \textsc{was}^{((e\to t)\to(e\to t))\to e\to t}\ \textsc{necessary}^{(e\to t)\to e\to t}\ \left(\textsc{culling}^{e\to e}\left(\textsc{the}^{(e\to t)\to e}\ \textsc{hunters}^{e\to t}\right)\right) \\
&\equiv \theta((\text{culling} \triangleleft (\text{the} \triangleleft \text{hunters})) \triangleright (\text{was} \triangleleft \text{necessary}))
\end{aligned}
$$

Troubling Developments

The need for associativity

$$
\begin{prooftree}
\AxiomC{the}
\RightLabel{Lex}
\UnaryInfC{np/n}
\AxiomC{violence}
\RightLabel{Lex}
\UnaryInfC{n}
\AxiomC{that}
\RightLabel{Lex}
\UnaryInfC{$(n\backslash n)/(s/np)$}
\AxiomC{the}
\RightLabel{Lex}
\UnaryInfC{np/n}
\AxiomC{world}
\RightLabel{Lex}
\UnaryInfC{n}
\RightLabel{$/_E$}
\BinaryInfC{$(\text{the} \cdot \text{world}) \vdash np$}
\AxiomC{ignores}
\RightLabel{Lex}
\UnaryInfC{$(np\backslash s)/np$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$np \vdash np$}
\RightLabel{$/_E$}
\BinaryInfC{$(\text{ignores} \cdot np) \vdash np\backslash s$}
\RightLabel{$\backslash_E$}
\BinaryInfC{$((\text{the} \cdot \text{world}) \cdot (\text{ignores} \cdot np)) \vdash s$}
\RightLabel{A}
\UnaryInfC{$(((\text{the} \cdot \text{world}) \cdot \text{ignores}) \cdot np) \vdash s$}
\RightLabel{$/_I$}
\UnaryInfC{$((\text{the} \cdot \text{world}) \cdot \text{ignores}) \vdash s/np$}
\RightLabel{$/_E$}
\BinaryInfC{$(\text{that} \cdot ((\text{the} \cdot \text{world}) \cdot \text{ignores})) \vdash n\backslash n$}
\RightLabel{$\backslash_E$}
\BinaryInfC{$(\text{violence} \cdot (\text{that} \cdot ((\text{the} \cdot \text{world}) \cdot \text{ignores})))$}
\RightLabel{$/_E$}
\BinaryInfC{$(\text{the} \cdot (\text{violence} \cdot (\text{that} \cdot ((\text{the} \cdot \text{world}) \cdot \text{ignores})))) \vdash np$}
\end{prooftree}
$$

Troubling Developments #2

The need for Control

But global associativity:

- too little
  the violence that the world ignores _ happily happily :: $(np\backslash s)\backslash np\backslash s$
- too much
  **the violence that (the state enables genocide) and the world ignores _* and :: $(s\backslash s)/s$

Troubling Developments #2

The need for Control

But global associativity:

- too little
  the violence that the world ignores _ happily happily :: $(np\backslash s)\backslash np\backslash s$
- too much
  **the violence that (the state enables genocide) and the world ignores _* and :: $(s\backslash s)/s$

The solution(s):

- selectively *block* structural manipulation within a *lax* logic, or
- selectively *allow* structural manipulation within a *strict* logic

Troubling Developments #2

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  **the violence that (the state enables genocide) and the world ignores _* and :: $(s\backslash s)/s$

The solution(s):

- selectively *block* structural manipulation within a *lax* logic, or
- selectively *allow* structural manipulation within a *strict* logic

How to distinguish domains where structural rules are (in)admissible?

Modalities to the Rescue

Modal Inference

Rules & Term Imprints

Types

$A, B, C ::= \ldots \mid \Diamond A \mid \Box A$ -- $\Diamond$, $\Box$: residuated pair

Structures

$\Gamma, \Delta ::= \ldots \mid \langle\Gamma\rangle$

$$\frac{\Gamma \vdash A}{\langle\Gamma\rangle \vdash \Diamond A}\ \Diamond_I \qquad \frac{\Gamma \vdash \Box A}{\langle\Gamma\rangle \vdash A}\ \Box_E$$

$$\frac{\langle\Gamma\rangle \vdash A}{\Gamma \vdash \Box A}\ \Box_I \qquad \frac{\Gamma \vdash \Diamond A \quad \Delta[\![\langle A\rangle]\!] \vdash B}{\Delta[\![\Gamma]\!] \vdash B}\ \Diamond_E$$

Modal Inference

Rules & Term Imprints

Types

$A, B, C \ := \ \dots \ | \ \Diamond A \ | \ \Box A$ -- $\Diamond$, $\Box$: residuated pair

Structures

$\Gamma, \Delta \ := \ \dots \ | \ \langle \Gamma \rangle$

$$\frac{\Gamma \vdash A}{\langle \Gamma \rangle \vdash \Diamond A} \Diamond_I \qquad \frac{\Gamma \vdash \Box A}{\langle \Gamma \rangle \vdash A} \Box_E$$

$$\frac{\langle \Gamma \rangle \vdash A}{\Gamma \vdash \Box A} \Box_I \qquad \frac{\Gamma \vdash \Diamond A \quad \Delta[\![\langle A \rangle]\!] \vdash B}{\Delta[\![\Gamma]\!] \vdash B} \Diamond_E$$

Modal Inference

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$$\frac{\Gamma \vdash A}{\langle \Gamma \rangle \vdash \Diamond A} \ \Diamond_I \qquad\qquad \frac{\Gamma \vdash \Box A}{\langle \Gamma \rangle \vdash A} \ \Box_E$$

$$\frac{\langle \Gamma \rangle \vdash A}{\Gamma \vdash \Box A} \ \Box_I \qquad\qquad \frac{\Gamma \vdash \Diamond A \quad \Delta[\![\langle A \rangle]\!] \vdash B}{\Delta[\![\Gamma]\!] \vdash B} \ \Diamond_E$$

Modal Inference

Rules & Term Imprints

Types

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Structures

$\Gamma, \Delta \ := \ \dots \ | \ \langle \Gamma \rangle$

$$\frac{\Gamma \vdash A}{\langle \Gamma \rangle \vdash \Diamond A} \ \Diamond_I \qquad \frac{\Gamma \vdash \Box A}{\langle \Gamma \rangle \vdash A} \ \Box_E$$

$$\frac{\langle \Gamma \rangle \vdash A}{\Gamma \vdash \Box A} \ \Box_I \qquad \frac{\Gamma \vdash \Diamond A \quad \Delta[\![\langle A \rangle]\!] \vdash B}{\Delta[\![\Gamma]\!] \vdash B} \ \Diamond_E$$

Modal Inference

Rules & Term Imprints

Types

$A, B, C ::= \ldots \mid \Diamond A \mid \Box A$ -- $\Diamond$, $\Box$: residuated pair

Structures

$\Gamma, \Delta ::= \ldots \mid \langle \Gamma \rangle$

$$\frac{\Gamma \vdash A}{\langle \Gamma \rangle \vdash \Diamond A} \Diamond_I \qquad \frac{\Gamma \vdash \Box A}{\langle \Gamma \rangle \vdash A} \Box_E$$

$$\frac{\langle \Gamma \rangle \vdash A}{\Gamma \vdash \Box A} \Box_I \qquad \frac{\Gamma \vdash \Diamond A \quad \Delta[\![\langle A \rangle]\!] \vdash B}{\Delta[\![\Gamma]\!] \vdash B} \Diamond_E$$

Modal Inference

Rules & Term Imprints

Types

$A, B, C \;:=\; \ldots \mid \Diamond A \mid \Box A$ -- $\Diamond$, $\Box$: residuated pair

Structures

$\Gamma, \Delta \;:=\; \ldots \mid \langle \Gamma \rangle$

$$\frac{\Gamma \vdash A}{\langle \Gamma \rangle \vdash \Diamond A}\ \Diamond_I \qquad \frac{\Gamma \vdash \Box A}{\langle \Gamma \rangle \vdash A}\ \Box_E$$

$$\frac{\langle \Gamma \rangle \vdash A}{\Gamma \vdash \Box A}\ \Box_I \qquad \frac{\Gamma \vdash \Diamond A \quad \Delta[\![\langle A \rangle]\!] \vdash B}{\Delta[\![\Gamma]\!] \vdash B}\ \Diamond_E$$

Modal Inference

Rules & Term Imprints

Types

$A, B, C \;:=\; \dots \mid \Diamond A \mid \Box A$ -- $\Diamond$, $\Box$: residuated pair

Structures

$\Gamma, \Delta \;:=\; \dots \mid \langle \Gamma \rangle$

$$\frac{\Gamma \vdash A}{\langle \Gamma \rangle \vdash \Diamond A}\ \Diamond_I \qquad \frac{\Gamma \vdash \Box A}{\langle \Gamma \rangle \vdash A}\ \Box_E$$

$$\frac{\langle \Gamma \rangle \vdash A}{\Gamma \vdash \Box A}\ \Box_I \qquad \frac{\Gamma \vdash \Diamond A \quad \Delta[\![\langle A \rangle]\!] \vdash B}{\Delta[\![\Gamma]\!] \vdash B}\ \Diamond_E$$

Modal Inference

Rules & Term Imprints

Types

$A, B, C \ := \ \dots \mid \Diamond A \mid \Box A$ -- $\Diamond$, $\Box$: residuated pair

Structures

$\Gamma, \Delta \ := \ \dots \mid \langle \Gamma \rangle$

$$\frac{\Gamma \vdash s : A}{\langle \Gamma \rangle \vdash {\vartriangle} s : \Diamond A} \ \Diamond_I \qquad \frac{\Gamma \vdash s : \Box A}{\langle \Gamma \rangle \vdash {\blacktriangledown} s : A} \ \Box_E$$

$$\frac{\langle \Gamma \rangle \vdash s : A}{\Gamma \vdash {\blacktriangle} s : \Box A} \ \Box_I \qquad \frac{\Gamma \vdash s : \Diamond A \quad \Delta[\![\langle x : A \rangle]\!] \vdash t : B}{\Delta[\![\Gamma]\!] \vdash \mathsf{case}\ {\triangledown} s\ \mathsf{of}\ x\ \mathsf{in}\ t : B} \ \Diamond_E$$

Modal Inference

Derived Properties 1: Tonicity

$$\dfrac{\dfrac{}{x' : \Diamond A \vdash x' : \Diamond A}\,Ax \qquad \dfrac{\dfrac{\vdots\, s}{x : A \vdash s : B}}{\langle x : A \rangle \vdash \vartriangle s : \Diamond B}\,\Diamond_I}{x' : \Diamond A \vdash \text{case } \triangledown x' \text{ of } x \text{ in } \vartriangle s : \Diamond B}\,\Diamond_E$$

$$\dfrac{\dfrac{x : A \vdash s : B}{\langle x' : \Box A \rangle \vdash s[\blacktriangledown x' / x] : B}\,\to_\beta}{x' : \Box A \vdash \blacktriangle s[\blacktriangledown x' / x] : \Box B}\,\Box_I$$

$$(A \implies B) \implies (\Box A \implies \Box B)$$
$$(A \implies B) \implies (\Diamond A \implies \Diamond B)$$

Modal Inference

Derived Properties 2: Residuation

$$
\dfrac{x' : \Diamond A \vdash x' : \Diamond A \qquad \dfrac{\dfrac{\vdots\, s}{x : A \vdash s : \Box B}}{\langle x : A \rangle \vdash \blacktriangledown s : B}\,\Box_E}{x' : \Diamond A \vdash \text{case } \triangledown x' \text{ of } x \text{ in } \blacktriangledown s : B}\,\Diamond_E
\qquad\qquad
\dfrac{\dfrac{\dfrac{\vdots\, s}{x : \Diamond A \vdash s : B}}{\langle x' : A \rangle \vdash s[\vartriangle x'/x] : B}\,\rightarrow_\beta}{x' : A \vdash \blacktriangle s : \Box B}\,\Box_E
$$

$$(A \implies \Box B) \iff (\Diamond A \implies B)$$

Modal Inference

Derived Properties 3: Interior & Closure

$$\dfrac{\dfrac{\Gamma \vdash s : A}{\langle \Gamma \rangle \vdash \vartriangle s : \Diamond A}\ \Diamond_I}{\Gamma \vdash \blacktriangle\vartriangle s : \Box\Diamond A}\ \Box_I$$

$$\dfrac{\Gamma \vdash s : \Diamond\Box A \qquad \dfrac{x : \Box A \vdash x : \Box A}{\langle x : \Box A \rangle \vdash \blacktriangledown x : A}\ \Box_E}{\Gamma \vdash \mathsf{case}\ \triangledown s\ \mathsf{of}\ x\ \mathsf{in}\ \blacktriangledown x : A}\ \Diamond_E$$

$$\Diamond\Box A \implies A \implies \Box\Diamond A$$

Modal Inference

Derived Properties 4 : Triple Laws

$$\dfrac{\dfrac{\dfrac{}{x : \Box\Diamond\Box A \vdash x : \Box\Diamond\Box A}\,Ax}{\langle x : \Box\Diamond\Box A\rangle \vdash \blacktriangledown x : \Diamond\Box A}\,\Box_E \qquad \dfrac{\dfrac{}{x' : \Box A \vdash x' : \Box A}\,Ax}{\langle x' : \Box A\rangle \vdash \blacktriangledown x' : A}\,\Box_E}{\dfrac{\langle x : \Box\Diamond\Box A\rangle \vdash \text{case } \triangledown\blacktriangledown x' \text{ of } x \text{ in } \blacktriangledown x : A}{x : \Box\Diamond\Box A \vdash \blacktriangle(\text{case } \triangledown\blacktriangledown x' \text{ of } x \text{ in } \blacktriangledown x) : \Box A}\,\Box_I}\,\Diamond_E$$

$$\dfrac{\dfrac{\dfrac{}{x : \Box A \vdash x : \Box A}\,Ax}{\langle x : \Box A\rangle \vdash \vartriangle x : \Diamond\Box A}\,\Diamond_I}{x : \Box A \vdash \blacktriangle\vartriangle x : \Box\Diamond\Box A}\,\Box_I$$

$$\dfrac{\dfrac{}{x' : \Box\Diamond\Box A \vdash x' : \Box\Diamond\Box A}\,Ax \qquad \dfrac{\dfrac{}{x : \Box\Diamond A \vdash x : \Box\Diamond A}\,Ax}{\langle x : \Box\Diamond A\rangle \vdash \blacktriangledown x : \Diamond A}\,\Box_E}{x' : \Diamond\Box\Diamond A \vdash \text{case } \triangledown x' \text{ of } x \text{ in } \blacktriangledown x : \Diamond A}\,\Diamond_E$$

$$\dfrac{\dfrac{}{x' : \Diamond A \vdash x' : \Diamond A}\,Ax \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{x : A \vdash x}\,Ax}{\langle x : A\rangle \vdash \vartriangle x : \Diamond A}\,\Diamond_I}{x : A \vdash \blacktriangle\vartriangle x : \Box\Diamond A}\,\Box_I}{\langle x : A\rangle \vdash \vartriangle\blacktriangle\vartriangle x : \Diamond\Box\Diamond A}\,\Diamond_I}{x' : \Diamond A \vdash \text{case } \triangledown x' \text{ of } x \text{ in } \vartriangle\blacktriangle\vartriangle x : \Diamond\Box\Diamond A}\,\Diamond_E$$

$$\Box\Diamond\Box A \iff \Box A$$
$$\Diamond\Box\Diamond A \iff \Diamond A$$

Take 1

Structural Postulates

$$\frac{\Gamma[\![(A_1 \cdot \langle A_2 \rangle) \cdot A_3]\!] \vdash B}{\Gamma[\![(A_1 \cdot A_3) \cdot \langle A_2 \rangle]\!] \vdash B}\ C^r_\diamond \qquad \frac{\Gamma[\![A_1 \cdot (A_2 \cdot \langle A_3 \rangle)]\!] \vdash B}{\Gamma[\![(A_1 \cdot A_2) \cdot \langle A_3 \rangle]\!] \vdash B}\ A^r_\diamond$$

Take 1

Structural Postulates

$$\frac{\Gamma[\![(A_1 \cdot \langle A_2 \rangle) \cdot A_3]\!] \vdash B}{\Gamma[\![(A_1 \cdot A_3) \cdot \langle A_2 \rangle]\!] \vdash B}\; C^r_\diamond \qquad \frac{\Gamma[\![A_1 \cdot (A_2 \cdot \langle A_3 \rangle)]\!] \vdash B}{\Gamma[\![(A_1 \cdot A_2) \cdot \langle A_3 \rangle]\!] \vdash B}\; A^r_\diamond$$

Rule form (syntactic) equivalents of formula-level postulates

$$\begin{aligned} C^r_\diamond &: A_1 \otimes (\diamond A_2 \otimes A_3) \implies (A_1 \otimes A_3) \otimes \diamond A_2 \\ A^r_\diamond &: A_1 \otimes (A_2 \otimes \diamond A_3) \implies (A_1 \otimes A_2) \otimes \diamond A_3 \end{aligned}$$

Structural Postulates

Modalities Licensing Movement

that :: $\cancel{(n\backslash n)/(s/np)}$ $(n\backslash n)/(s/\Diamond\Box np)$

$$
\begin{prooftree}
\AxiomC{that}
\RightLabel{Lex}
\UnaryInfC{$(n\backslash n)/(s/\Diamond\Box np)$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Diamond\Box np \vdash \Diamond\Box np$}
\AxiomC{the}
\RightLabel{Lex}
\UnaryInfC{np/n}
\AxiomC{world}
\RightLabel{Lex}
\UnaryInfC{n}
\RightLabel{$/_E$}
\BinaryInfC{$(\text{the} \cdot \text{world}) \vdash np$}
\AxiomC{ignores}
\RightLabel{Lex}
\UnaryInfC{$(np\backslash s)/np$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Box np \vdash \Box np$}
\RightLabel{$\Box_E$}
\UnaryInfC{$\langle \Box np \rangle \vdash np$}
\RightLabel{$/_E$}
\BinaryInfC{$(\text{ignores} \cdot \langle \Box np \rangle) \vdash np\backslash s$}
\AxiomC{happily}
\RightLabel{Lex}
\UnaryInfC{$(np\backslash s)\backslash np\backslash s$}
\RightLabel{$\backslash_E$}
\BinaryInfC{$((\text{ignores} \cdot \langle \Box np \rangle) \cdot \text{happily}) \vdash np\backslash s$}
\RightLabel{$\backslash_E$}
\BinaryInfC{$((\text{the} \cdot \text{world}) \cdot ((\text{ignores} \cdot \langle \Box np \rangle) \cdot \text{happily}) \vdash s$}
\RightLabel{$C^r_\Diamond$}
\UnaryInfC{$((\text{the} \cdot \text{world}) \cdot ((\text{ignores} \cdot \text{happily}) \cdot \langle \Box np \rangle)) \vdash s$}
\RightLabel{$A^r_\Diamond$}
\UnaryInfC{$(((\text{the} \cdot \text{world}) \cdot (\text{ignores} \cdot \text{happily})) \cdot \langle \Box np \rangle) \vdash s$}
\RightLabel{$\Diamond_E$}
\BinaryInfC{$(((\text{the} \cdot \text{world}) \cdot ((\text{ignores} \cdot \text{happily})) \cdot \Diamond\Box np) \vdash s$}
\RightLabel{$/_I$}
\UnaryInfC{$((\text{the} \cdot \text{world}) \cdot ((\text{ignores} \cdot \text{happily})) \vdash s/\Diamond\Box np$}
\RightLabel{$/_E$}
\BinaryInfC{$(\text{that} \cdot ((\text{the} \cdot \text{world}) \cdot ((\text{ignores} \cdot \text{happily}))) \vdash n\backslash n$}
\end{prooftree}
$$

Take 2

Modalities Blocking Movement

and :: $\cancel{(s\backslash s)/s}$ $(s\backslash\Box s)/s$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{\cdots\vdash s}\qquad \dfrac{\dfrac{\text{and}}{(s\backslash\Box s)/s}\,\mathit{Lex}\qquad \dfrac{\dfrac{\dfrac{\text{the}}{np/n}\,\mathit{Lex}\quad \dfrac{\text{world}}{n}\,\mathit{Lex}}{(\text{the}\cdot\text{world})\vdash np}\,/_E\qquad \dfrac{\dfrac{\text{ignores}}{(np\backslash s)/np}\,\mathit{Lex}\quad \dfrac{\dfrac{}{\Box np\vdash\Box np}\,\mathit{Ax}}{\langle\Box np\rangle\vdash np}\,\Box_E}{(\text{ignores}\cdot\langle\Box np\rangle)\vdash np\backslash s}\,/_E}{((\text{the}\cdot\text{world})\cdot(\text{ignores}\cdot\langle\Box np\rangle))\vdash s}\,\backslash_E}{(\text{and}\cdot((\text{the}\cdot\text{world})\cdot(\text{ignores}\cdot\langle\Box np\rangle)))\vdash s\backslash\Box s}\,/_E}{(\cdots\cdot(\text{and}\cdot((\text{the}\cdot\text{world})\cdot(\text{ignores}\cdot\langle\Box np\rangle))))\vdash\Box s}\,\backslash_E}{\langle(\cdots\cdot(\text{and}\cdot((\text{the}\cdot\text{world})\cdot(\text{ignores}\cdot\langle\Box np\rangle))))\rangle\vdash s}\,\Box_E}{((\langle(\cdots\cdot(\text{and}\cdot((\text{the}\cdot\text{world})\cdot\text{ignores})))\rangle\cdot\langle\Box np\rangle)\vdash s}\,\lightning
$$

Typelogical Grammar 102

The Multimodal View

Syntax

- choose a “language-neutral” logical core

Typelogical Grammar 102

The Multimodal View

Syntax

- choose a “language-neutral” logical core
- if too strict
  - implement language-specific structural postulates by pattern matching on modal structure
  - adjust the lexicon -- *which words elicit movement and / or rebracketing?*

Typelogical Grammar 102

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- choose a “language-neutral” logical core
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  - adjust the lexicon -- *which words elicit movement and / or rebracketing?*
- if too lax
  - use modal structure to demarcate structurally strict / externally impervious domains
  - (re)adjust the lexicon -- *which words impose syntactic islands?*

Typelogical Grammar 102

The Multimodal View

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  - use modal structure to demarcate structurally strict / externally impervious domains
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Semantics

Just forget about modalities.

- $\eta\,(\Diamond A) = \eta\,(\Box A) = \eta\; A$
- $\theta\,(\vartriangle s) = \theta\,(\blacktriangle s) = \theta\,(\blacktriangledown s) = \theta\; s$
- $\theta\,(\text{case} \triangledown s \text{ of } x \text{ in } t) = (\theta\, t)[\theta\, s / \theta\, x]$

Typelogical Grammar 102

The Multimodal View

Syntax

- choose a "language-neutral" logical core
- if too strict $\Longrightarrow$ modal structure is transient
  - implement language-specific structural postulates by pattern matching on modal structure
  - adjust the lexicon -- *which words elicit movement and / or rebracketing?*
- if too lax $\Longrightarrow$ modal structure is persistent
  - use modal structure to demarcate structurally strict / externally impervious domains
  - (re)adjust the lexicon -- *which words impose syntactic islands?*

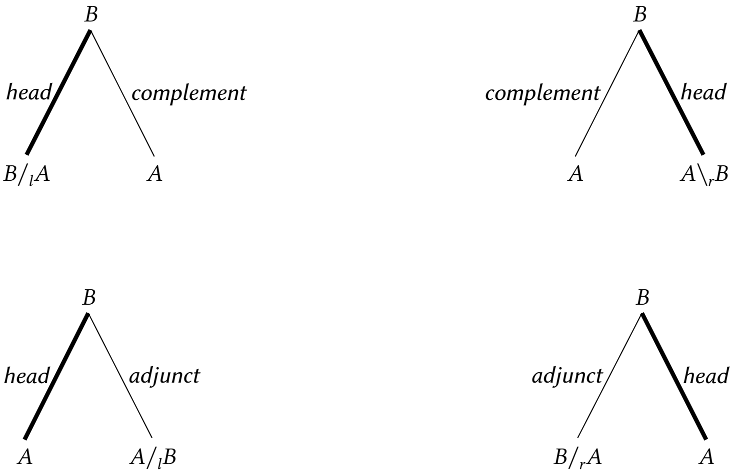
Semantics

Just forget about modalities.

- $\eta\,(\Diamond A) = \eta\,(\Box A) = \eta\; A$
- $\theta\,(\vartriangle s) = \theta\,(\blacktriangle s) = \theta\,(\blacktriangledown s) = \theta\; s$
- $\theta\,(\text{case } \triangledown s \text{ of } x \text{ in } t) = (\theta\, t)[\theta\, s / \theta\, x]$

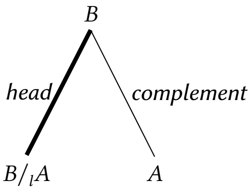
Persistent Modal Structure

An Alternative Usecase

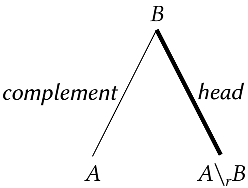


Persistent Modal Structure

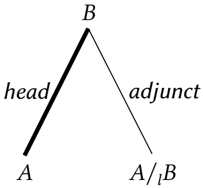
An Alternative Usecase



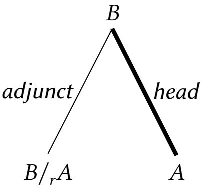
$$B/_l A \equiv \Box(B/A)$$



$$A\backslash_r B \equiv \Box(A\backslash B)$$



$$A/_l B \equiv \Diamond A/B$$



$$B/_r A \equiv B/\Diamond A$$

Persistent Modal Structure

A Modern Refinement

- Let $\mathrm{D} := \mathrm{C} \cup \mathrm{A}$, where:
  - $\mathrm{C} := \{\mathrm{su}, \mathrm{obj1}, \mathrm{obj2}, \mathrm{pc}, \ldots\}$ -- *mandatory complements*
  - $\mathrm{A} := \{\mathrm{det}, \mathrm{mod}, \ldots\}$ -- *optional adjuncts*
- Instantiate a residuated pair $(\blacklozenge^{\mathrm{d}}, \blacksquare^{\mathrm{d}})$ for each $\mathrm{d} \in \mathrm{D}$.
- Type grammatical functors as:
  - $\blacklozenge^{\mathrm{d}} A \to B$ -- *head assigning dependency role* d *to its complement* A
  - $\blacksquare^{\mathrm{d}}(A \to B)$ -- *adjunct projecting its own role* d

Dependency Modalities

Labeled FA Structures

$$\begin{aligned} \text{the} &:: \blacksquare^{\text{det}}(np/n) \\ \text{culling} &:: n \\ \text{necessary} &:: \blacksquare^{\text{mod}}(n/n) \end{aligned}$$

$$\dfrac{\dfrac{\dfrac{\text{the}}{\blacksquare^{\text{det}}(np/n)}\,Lex}{\langle\text{the}\rangle^{\text{det}} \vdash np/n}\,\blacksquare_E \qquad \dfrac{\dfrac{\dfrac{\text{necessary}}{\blacksquare^{\text{mod}} n/n}\,Lex}{\langle\text{necessary}\rangle^{\text{mod}} \vdash n/n}\,\blacksquare_E \quad \dfrac{\text{culling}}{n}\,Lex}{(\langle\text{necessary}\rangle^{\text{mod}} \cdot \text{culling}) \vdash n}\,/_E}{(\langle\text{the}\rangle^{\text{det}} \cdot (\langle\text{necessary}\rangle^{\text{mod}} \cdot \text{culling})) \vdash np}\,/_E$$

Dependency Modalities

Labeled FA Structures

$$
\begin{aligned}
\text{the} &:: \blacksquare^{\text{det}}(np/n)\\
\text{culling} &:: n\\
\text{necessary} &:: \blacksquare^{\text{mod}}(n/n)
\end{aligned}
$$

$$
\dfrac{
\dfrac{\dfrac{\text{the}}{\blacksquare^{\text{det}}(np/n)}\,Lex}{\langle\text{the}\rangle^{\text{det}} \vdash np/n}\,\blacksquare_E
\qquad
\dfrac{
\dfrac{\dfrac{\text{necessary}}{\blacksquare^{\text{mod}} n/n}\,Lex}{\langle\text{necessary}\rangle^{\text{mod}} \vdash n/n}\,\blacksquare_E
\quad
\dfrac{\text{culling}}{n}\,Lex
}{(\langle\text{necessary}\rangle^{\text{mod}} \cdot \text{culling}) \vdash n}\,/_E
}{(\langle\text{the}\rangle^{\text{det}} \cdot (\langle\text{necessary}\rangle^{\text{mod}} \cdot \text{culling})) \vdash np}\,/_E
$$

mod

det

Dependency Modalities

Labeled FA Structures

$$
\begin{aligned}
\text{the} &:: \blacksquare^{\text{det}}(np/n)\\
\text{culling} &:: n\\
\text{necessary} &:: \blacksquare^{\text{mod}}(n/n)\\
\text{was} &:: (\blacklozenge^{\text{su}}np\backslash s)/(\blacklozenge^{\text{pc}}\blacksquare^{\text{mod}}(n/n))
\end{aligned}
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{the}}{\blacksquare^{\text{det}}(np/n)}\,\mathit{Lex}}{\langle\text{the}\rangle^{\text{det}} \vdash np/n}\,\blacksquare_E \quad \dfrac{\text{culling}}{n}\,\mathit{Lex}}{(\langle\text{the}\rangle^{\text{det}} \cdot \text{culling}) \vdash np}\,/_E}{\langle(\langle\text{the}\rangle^{\text{det}} \cdot \text{culling})\rangle^{\text{su}} \vdash \blacklozenge^{\text{su}}np}\,\blacklozenge_I \quad \dfrac{\dfrac{\text{was}}{(\blacklozenge^{\text{su}}np\backslash s)/(\blacklozenge^{\text{pc}}\blacksquare^{\text{mod}}(n/n))}\,\mathit{Lex} \quad \dfrac{\dfrac{\text{necessary}}{\blacksquare^{\text{mod}}(n/n)}\,\mathit{Lex}}{\langle\text{necessary}\rangle^{\text{pc}} \vdash \blacklozenge^{\text{pc}}\blacksquare^{\text{mod}}(n/n)}\,\blacklozenge_I}{(\text{was} \cdot \langle\text{necessary}\rangle^{\text{pc}}) \vdash \blacklozenge^{\text{su}}np\backslash s}\,/_E}{(\langle(\langle\text{the}\rangle^{\text{det}} \cdot \text{culling})\rangle^{\text{su}} \cdot (\text{was} \cdot \langle\text{necessary}\rangle^{\text{pc}})) \vdash s}\,\backslash_E
$$

Dependency Modalities

Labeled FA Structures

$$
\begin{aligned}
\text{the} &:: \blacksquare^{\text{det}}(np/n)\\
\text{culling} &:: n\\
\text{necessary} &:: \blacksquare^{\text{mod}}(n/n)\\
\text{was} &:: (\blacklozenge^{\text{su}}np\backslash s)/(\blacklozenge^{\text{pc}}\blacksquare^{\text{mod}}(n/n))
\end{aligned}
$$

$$
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{\text{the}}{\blacksquare^{\text{det}}(np/n)}\,Lex
}{\langle\text{the}\rangle^{\text{det}} \vdash np/n}\,\blacksquare_E
\quad
\dfrac{\text{culling}}{n}\,Lex
}{(\langle\text{the}\rangle^{\text{det}}\cdot\text{culling}) \vdash np}\,/_E
}{\langle(\langle\text{the}\rangle^{\text{det}}\cdot\text{culling})\rangle^{\text{su}} \vdash \blacklozenge^{\text{su}}np}\,\blacklozenge_I
\quad
\dfrac{
\dfrac{\text{was}}{(\blacklozenge^{\text{su}}np\backslash s)/(\blacklozenge^{\text{pc}}\blacksquare^{\text{mod}}(n/n))}\,Lex
\quad
\dfrac{
\dfrac{\text{necessary}}{\blacksquare^{\text{mod}}(n/n)}\,Lex
}{\langle\text{necessary}\rangle^{\text{pc}} \vdash \blacklozenge^{\text{pc}}\blacksquare^{\text{mod}}(n/n)}\,\blacklozenge_I
}{(\text{was}\cdot\langle\text{necessary}\rangle^{\text{pc}}) \vdash \blacklozenge^{\text{su}}np\backslash s}\,/_E
}{(\langle(\langle\text{the}\rangle^{\text{det}}\cdot\text{culling})\rangle^{\text{su}}\cdot(\text{was}\cdot\langle\text{necessary}\rangle^{\text{pc}})) \vdash s}\,\backslash_E
$$

det su pc

Dependency Modalities

Variations on a Theme

- Dependency domains $\supseteq$ constituency structures?
  -- a bottleneck of (and argument for) free associativity

$$(\langle(\langle \mathrm{the}\rangle^{\mathrm{det}} \cdot \mathrm{culling})\rangle^{\mathrm{su}} \cdot (\mathrm{was} \cdot \langle \mathrm{necessary}\rangle^{\mathrm{pc}})) \rightsquigarrow \langle\langle \mathrm{the}\rangle^{\mathrm{det}} \cdot \mathrm{culling}\rangle^{\mathrm{su}} \cdot \mathrm{was} \cdot \langle \mathrm{necessary}\rangle^{\mathrm{pc}}$$

Dependency Modalities

Variations on a Theme

- Dependency domains ⊇ constituency structures?
  -- a bottleneck of (and argument for) free associativity
- Dependency cued by morphology?
  -- lexically pre-assigned complements

luin (read) :: $(\blacklozenge^{\mathsf{su}} np \backslash s)/\blacklozenge^{\mathsf{obj1}} np$
minä (I-nom) :: $\blacksquare^{\mathsf{su}}\blacklozenge^{\mathsf{su}} np$
kirjan (book-acc) :: $\blacksquare^{\mathsf{obj1}}\blacklozenge^{\mathsf{obj1}} np$

$$
\dfrac{
  \dfrac{\dfrac{\text{minä}}{\blacksquare^{\mathsf{su}}\blacklozenge^{\mathsf{su}} np}\ \mathit{Lex}}{\langle \text{minä}\rangle^{\mathsf{su}} \vdash \blacklozenge^{\mathsf{su}} np}\ \blacksquare_E
  \qquad
  \dfrac{
    \dfrac{\text{luin}}{(\blacklozenge^{\mathsf{su}} np \backslash s)/\blacklozenge^{\mathsf{obj1}} np}\ \mathit{Lex}
    \qquad
    \dfrac{\dfrac{\text{kirjan}}{\blacksquare^{\mathsf{obj1}}\blacklozenge^{\mathsf{obj1}} np}\ \mathit{Lex}}{\langle \text{kirjan}\rangle^{\mathsf{obj1}} \vdash \blacklozenge^{\mathsf{obj1}} np}\ \blacksquare_E
  }{\text{luin} \cdot \langle \text{kirjan}\rangle^{\mathsf{obj1}} \vdash \blacklozenge^{\mathsf{su}} np \backslash s}\ /_E
}{\langle \text{minä}\rangle^{\mathsf{su}} \cdot \text{luin} \cdot \langle \text{kirjan}\rangle^{\mathsf{obj1}} \vdash s}\ \backslash_E
$$

Dependency Modalities

Variations on a Theme

- Dependency domains $\supseteq$ constituency structures?
  -- a bottleneck of (and argument for) free associativity
- Dependency cued by morphology?
  -- lexically pre-assigned complements
- Movement-like phenomena limited to certain dependencies?
  -- dependency-dependent structural rules

$$\dfrac{\dfrac{\Gamma[\![(\langle\Delta\rangle^{\mathsf{su}} \cdot (\Theta \cdot \langle\Xi\rangle^{\mathsf{obj1}}))]\!] \vdash A}{\Gamma[\![(\langle\Delta\rangle^{\mathsf{su}} \cdot (\langle\Xi\rangle^{\mathsf{obj1}} \cdot \Theta))]\!] \vdash A}\ \mathit{top}}{\vdots}$$

or perhaps: Ex holds within the sentential domain (yet no word salad...)

Dependency Modalities

Variations on a Theme

- Dependency domains $\supseteq$ constituency structures?
  -- a bottleneck of (and argument for) free associativity
- Dependency cued by morphology?
  -- lexically pre-assigned complements
- Movement-like phenomena limited to certain dependencies?
  -- dependency-dependent structural rules
- Dependency resolves ambiguity?
  -- derivational ambiguity $\rightsquigarrow$ lexical ambiguity

Dutch embedded clauses are verb-final; e.g. haten :: $\blacklozenge^{\mathsf{obj1}} np \backslash (\blacklozenge^{\mathsf{su}} np \backslash s)$

Dependency Modalities

Variations on a Theme

- Dependency domains $\supseteq$ constituency structures?
  -- a bottleneck of (and argument for) free associativity
- Dependency cued by morphology?
  -- lexically pre-assigned complements
- Movement-like phenomena limited to certain dependencies?
  -- dependency-dependent structural rules
- Dependency resolves ambiguity?
  -- derivational ambiguity $\rightsquigarrow$ lexical ambiguity

Dutch embedded clauses are verb-final; e.g. haten :: $\blacklozenge^{\mathsf{obj1}} np \backslash (\blacklozenge^{\mathsf{su}} np \backslash s)$

gender-matched relative clauses are derivationally ambiguous:

"*mannen die vrouwen haten*" $\rightsquigarrow$ "men that hate women" | "men thate women hate"

consider instead; die :: $\blacksquare^{\mathsf{mod}}(np\backslash np)/\blacklozenge^{\mathsf{body}}(\blacklozenge^{\mathsf{su}} np\backslash ssub)$ & $\blacksquare^{\mathsf{mod}}(np\backslash np)/\blacklozenge^{\mathsf{body}}(\lozenge\square\blacklozenge^{\mathsf{obj1}} np\backslash ssub)$

Dependency Modalities

Hypothetical Reasoning 1

die :: $\blacksquare^{\mathsf{mod}}(np\backslash np)/\blacklozenge^{\mathsf{body}}(\blacklozenge^{\mathsf{su}}np\backslash ssub)$

$$
\dfrac{
  \dfrac{
    \dfrac{}{\blacklozenge^{\mathsf{su}}np \vdash \blacklozenge^{\mathsf{su}}np}\,Ax
    \qquad
    \dfrac{
      \dfrac{
        \dfrac{}{np \vdash np}\,Ax
      }{\langle np\rangle^{\mathsf{su}} \vdash \blacklozenge^{\mathsf{su}}np}\,\blacklozenge_I
      \qquad
      \dfrac{
        \dfrac{
          \dfrac{\text{vrouwen}}{np}\,Lex
        }{\langle \text{vrouwen}\rangle^{\mathsf{obj1}} \vdash \blacklozenge^{\mathsf{obj1}}np}\,\blacklozenge_I
        \qquad
        \dfrac{\text{haten}}{\blacklozenge^{\mathsf{obj1}}np\backslash(\blacklozenge^{\mathsf{su}}np\backslash ssub)}\,Lex
      }{(\langle \text{vrouwen}\rangle^{\mathsf{obj1}} \cdot \text{haten}) \vdash \blacklozenge^{\mathsf{su}}np\backslash ssub}\,\backslash_E
    }{(\langle np\rangle^{\mathsf{su}} \cdot (\langle \text{vrouwen}\rangle^{\mathsf{obj1}} \cdot \text{haten}) \vdash ssub}\,\backslash_E
  }{
    \dfrac{(\blacklozenge^{\mathsf{su}}np \cdot (\langle \text{vrouwen}\rangle^{\mathsf{obj1}} \cdot \text{haten})) \vdash ssub}{(\langle \text{vrouwen}\rangle^{\mathsf{obj1}} \cdot \text{haten}) \vdash \blacklozenge^{\mathsf{su}}np\backslash ssub}\,\backslash_I
  }\,\blacklozenge_E
}{\langle(\langle \text{vrouwen}\rangle^{\mathsf{obj1}} \cdot \text{haten})\rangle^{\mathsf{body}} \vdash \blacklozenge^{\mathsf{body}}(\blacklozenge^{\mathsf{su}}np\backslash ssub)}\,\blacklozenge_I
$$

$$\vdots$$

Dependency Modalities

Hypothetical Reasoning 2: Horizontal Movement

die :: $\blacksquare^{\mathsf{mod}}(np\backslash np)/\blacklozenge^{\mathsf{body}}(\Diamond\Box\blacklozenge^{\mathsf{obj1}}np\backslash ssub)$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Diamond\Box\blacklozenge^{\mathsf{obj1}}np \vdash \Diamond\Box\blacklozenge^{\mathsf{obj1}}np$}
\AxiomC{$\mathsf{vrouwen}$}
\RightLabel{Lex}
\UnaryInfC{np}
\RightLabel{$\blacklozenge_I$}
\UnaryInfC{$\langle\mathsf{vrouwen}\rangle^{\mathsf{su}} \vdash \blacklozenge^{\mathsf{su}}np$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Box\blacklozenge^{\mathsf{obj1}}np \vdash \Box\blacklozenge^{\mathsf{obj1}}np$}
\RightLabel{$\Box_E$}
\UnaryInfC{$\langle\Box\blacklozenge^{\mathsf{obj1}}np\rangle \vdash \blacklozenge^{\mathsf{obj1}}np$}
\AxiomC{haten}
\RightLabel{Lex}
\UnaryInfC{$\blacklozenge^{\mathsf{obj1}}np\backslash(\blacklozenge^{\mathsf{su}}np\backslash ssub)$}
\RightLabel{$\backslash_E$}
\BinaryInfC{$(\langle\blacklozenge^{\mathsf{obj1}}np\rangle \cdot \mathsf{haten}) \vdash \blacklozenge^{\mathsf{su}}np\backslash ssub$}
\RightLabel{$\backslash_E$}
\BinaryInfC{$(\langle\mathsf{vrouwen}\rangle^{\mathsf{su}} \cdot (\langle\Box\blacklozenge^{\mathsf{obj1}}np\rangle \cdot \mathsf{haten})) \vdash ssub$}
\RightLabel{$C^{l}_{\Diamond}$}
\UnaryInfC{$(\langle\Box\blacklozenge^{\mathsf{obj1}}np\rangle \cdot (\langle\mathsf{vrouwen}\rangle^{\mathsf{su}} \cdot \mathsf{haten})) \vdash ssub$}
\RightLabel{$\blacklozenge_E$}
\BinaryInfC{$(\Diamond\Box\blacklozenge^{\mathsf{obj1}}np \cdot (\langle\mathsf{vrouwen}\rangle^{\mathsf{su}} \cdot \mathsf{haten})) \vdash ssub$}
\RightLabel{$\backslash_I$}
\UnaryInfC{$(\langle\mathsf{vrouwen}\rangle^{\mathsf{su}} \cdot \mathsf{haten}) \vdash \Diamond\Box\blacklozenge^{\mathsf{obj1}}np\backslash ssub$}
\RightLabel{$\blacklozenge_I$}
\UnaryInfC{$\langle(\langle\mathsf{vrouwen}\rangle^{\mathsf{su}} \cdot \mathsf{haten})\rangle^{\mathsf{body}} \vdash \blacklozenge^{\mathsf{body}}(\Diamond\Box\blacklozenge^{\mathsf{obj1}}np\backslash ssub)$}
\UnaryInfC{$\vdots$}
\end{prooftree}
$$

Dependency Modalities

Hypothetical Reasoning 3: Higher-Order

waarom :: $\blacklozenge^{\mathrm{body}}(\blacklozenge^{\mathrm{mod}}\blacksquare^{\mathrm{mod}}(sv1/sv1)\backslash sv1)/whq$

$$
\dfrac{
  \dfrac{}{\blacklozenge^{\mathrm{mod}}\blacksquare^{\mathrm{mod}}(sv1/sv1)}\,Ax
  \qquad
  \dfrac{
    \dfrac{
      \dfrac{}{\blacksquare^{\mathrm{mod}}(sv1/sv1)}\,Ax
    }{\langle\blacksquare^{\mathrm{mod}}(sv1/sv1)\rangle^{\mathrm{mod}} \vdash sv1/sv1}\,\blacksquare^{\mathrm{mod}}{}_E
    \qquad
    \dfrac{\vdots}{\mathrm{eet}\cdot\langle \mathrm{je}\rangle^{\mathrm{su}}\cdot\langle\langle \mathrm{mijn}\rangle^{\mathrm{det}}\cdot \mathrm{ijs}\rangle^{\mathrm{obj1}} \vdash sv1}
  }{\langle\blacksquare^{\mathrm{mod}}(sv1\backslash sv1)\rangle^{\mathrm{mod}}\cdot\mathrm{eet}\cdot\langle \mathrm{je}\rangle^{\mathrm{su}}\cdot\langle\langle \mathrm{mijn}\rangle^{\mathrm{det}}\cdot \mathrm{ijs}\rangle^{\mathrm{obj1}} \vdash sv1}\,/_E
}{
  \dfrac{
    \dfrac{\blacklozenge^{\mathrm{mod}}\blacksquare^{\mathrm{mod}}(sv1/sv1)\cdot\mathrm{eet}\cdot\langle \mathrm{je}\rangle^{\mathrm{su}}\cdot\langle\langle \mathrm{mijn}\rangle^{\mathrm{det}}\cdot \mathrm{ijs}\rangle^{\mathrm{obj1}} \vdash sv1}{\mathrm{eet}\cdot\langle \mathrm{je}\rangle^{\mathrm{su}}\cdot\langle\langle \mathrm{mijn}\rangle^{\mathrm{det}}\cdot \mathrm{ijs}\rangle^{\mathrm{obj1}} \vdash \blacklozenge^{\mathrm{mod}}\blacksquare^{\mathrm{mod}}(sv1/sv1)\backslash sv1}\,\backslash_I
  }{
    \dfrac{\langle\mathrm{eet}\cdot\langle \mathrm{je}\rangle^{\mathrm{su}}\cdot\langle\langle \mathrm{mijn}\rangle^{\mathrm{det}}\cdot \mathrm{ijs}\rangle^{\mathrm{obj1}}\rangle^{\mathrm{body}} \vdash \blacklozenge^{\mathrm{body}}(\blacklozenge^{\mathrm{mod}}\blacksquare^{\mathrm{mod}}(sv1/sv1)\backslash sv1)}{\vdots}
  }\,\blacklozenge_I
}\,\blacklozenge_E
$$

Dependency Modalities

Hypothetical Reasoning 4: Vertical Movement

$$
\begin{prooftree}
\AxiomC{$\vdots$}
\noLine
\UnaryInfC{$(\mathrm{op} \cdot \mathrm{wie}) \vdash whq \backslash \blacklozenge^{\mathrm{body}}(sv1 \backslash {\color{red}???} \blacklozenge^{\mathrm{pc}} prp)$}
\AxiomC{heb}
\RightLabel{Lex}
\UnaryInfC{$(sv1/\blacklozenge^{\mathrm{vc}} ppart)/\blacklozenge^{\mathrm{su}} np)$}
\AxiomC{je}
\RightLabel{Lex}
\UnaryInfC{np}
\RightLabel{$\blacklozenge_I$}
\UnaryInfC{$\langle \mathrm{je} \rangle^{\mathrm{su}} \vdash \blacklozenge^{\mathrm{su}} np$}
\RightLabel{$/_E$}
\BinaryInfC{$(\mathrm{heb} \cdot \langle \mathrm{je} \rangle^{\mathrm{su}}) \vdash sv1/\blacklozenge^{\mathrm{vc}} ppart$}
\AxiomC{$\mathrm{gestemd}$}
\RightLabel{Lex}
\UnaryInfC{$ppart/\blacklozenge^{\mathrm{pc}} prp$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\blacklozenge^{\mathrm{pc}} prp \vdash \blacklozenge^{\mathrm{pc}} prp$}
\RightLabel{$/_E$}
\BinaryInfC{$(\mathrm{gestemd} \cdot \blacklozenge^{\mathrm{pc}} prp) \vdash ppart$}
\RightLabel{$/_E$}
\BinaryInfC{$((\mathrm{heb} \cdot \langle \mathrm{je} \rangle^{\mathrm{su}}) \cdot (\mathrm{gestemd} \cdot \blacklozenge^{\mathrm{pc}} prp)) \vdash sv1$}
\RightLabel{$\blacklozenge_I$}
\UnaryInfC{$\langle ((\mathrm{heb} \cdot \langle \mathrm{je} \rangle^{\mathrm{su}}) \cdot (\mathrm{gestemd} \cdot \blacklozenge^{\mathrm{pc}} prp)) \rangle^{\#1} \vdash \blacklozenge^{\mathrm{body}} sv1$}
\noLine
\UnaryInfC{$\vdots \; {\color{red}?}$}
\noLine
\UnaryInfC{$\langle ((\mathrm{heb} \cdot \langle \mathrm{je} \rangle^{\mathrm{su}}) \cdot \mathrm{gestemd}) \rangle^{\mathrm{body}} \vdash \blacklozenge^{\mathrm{body}} sv1/{\color{red}???}\blacklozenge^{\mathrm{pc}} prp$}
\RightLabel{$/_E$}
\BinaryInfC{}
\end{prooftree}
$$

Dependency Modalities

Hypothetical Reasoning 4: Vertical Movement

$$
\dfrac{
\dfrac{\vdots}{(\mathrm{op}\cdot\mathrm{wie}) \vdash whq\backslash\lozenge^{\mathrm{body}}(sv1\backslash ???\lozenge^{\mathrm{pc}}prp)}
\qquad
\dfrac{
\dfrac{
\dfrac{
\dfrac{
\dfrac{\mathrm{heb}}{(sv1/\lozenge^{\mathrm{vc}}ppart)/\lozenge^{\mathrm{su}}np}\,Lex
\quad
\dfrac{\dfrac{\mathrm{je}}{np}\,Lex}{\langle\mathrm{je}\rangle^{\mathrm{su}} \vdash \lozenge^{\mathrm{su}}np}\,\lozenge_I
}{(\mathrm{heb}\cdot\langle\mathrm{je}\rangle^{\mathrm{su}}) \vdash sv1/\lozenge^{\mathrm{vc}}ppart}\,/_E
\quad
\dfrac{
\dfrac{\mathrm{gestemd}}{ppart/\lozenge^{\mathrm{pc}}prp}\,Lex
\quad
\dfrac{}{\lozenge^{\mathrm{pc}}prp \vdash \lozenge^{\mathrm{pc}}prp}\,Ax
}{(\mathrm{gestemd}\cdot\lozenge^{\mathrm{pc}}prp) \vdash ppart}\,/_E
}{((\mathrm{heb}\cdot\langle\mathrm{je}\rangle^{\mathrm{su}})\cdot(\mathrm{gestemd}\cdot\lozenge^{\mathrm{pc}}prp)) \vdash sv1}\,/_E
}{\langle((\mathrm{heb}\cdot\langle\mathrm{je}\rangle^{\mathrm{su}})\cdot(\mathrm{gestemd}\cdot\lozenge^{\mathrm{pc}}prp))\rangle^{\mathrm{body}} \vdash \lozenge^{\mathrm{body}}sv1}\,\lozenge_I
}{\vdots\;?}
}{\langle((\mathrm{heb}\cdot\langle\mathrm{je}\rangle^{\mathrm{su}})\cdot\mathrm{gestemd})\rangle^{\mathrm{body}} \vdash \lozenge^{\mathrm{body}}sv1/???\lozenge^{\mathrm{pc}}prp}
}{}\,/_E
$$

Dependency Modalities

Hypothetical Reasoning 4: Vertical Movement

Structural blockades backfiring...

Perhaps:

$$\frac{\Gamma[\![\langle \Delta \cdot \langle A \rangle^{\nu} \rangle^{\mu}]\!] \vdash B}{\Gamma[\![\langle \Delta \rangle^{\mu} \cdot \langle A \rangle^{\nu}]\!] \vdash B} \; X$$

Dependency Modalities

Semantics (?)

Open question {- - *read: no idea* - -} possible options:

- dependency marks as “thematic roles”?

Dependency Modalities

Semantics (?)

Open question {- - *read: no idea* - -} possible options:

- dependency marks as “thematic roles”?
- S4 modalities $\rightsquigarrow$ monads

Dependency Modalities

Semantics (?)

Open question {- - *read: no idea* - -} possible options:

- dependency marks as "thematic roles"?
- S4 modalities $\rightsquigarrow$ monads

  these modalities $\rightsquigarrow$ ...?

Dependency Modalities

Semantics (?)

Open question {- - *read: no idea* - -} possible options:

- dependency marks as "thematic roles"?
- S4 modalities $\rightsquigarrow$ monads

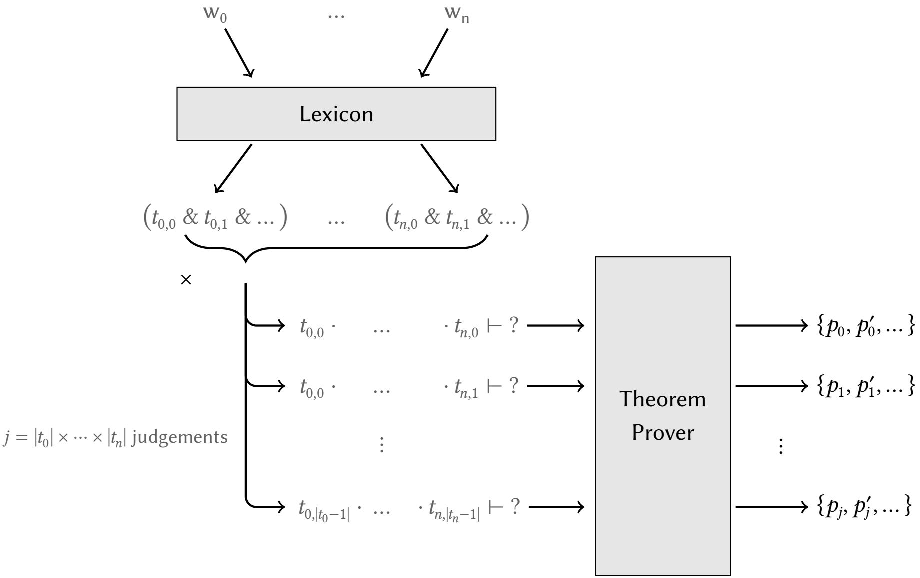
  these modalities $\rightsquigarrow$ ...?
- passageway to DTT proof-theoretic semantics?

TLDR

Today's agenda (a posteriori)

- MILL (from above & below)
- (N)L(P) and linguistic applications
- Traversing between (N)L(P) with modalities
- Modalities for dependency demarcation

The Real World



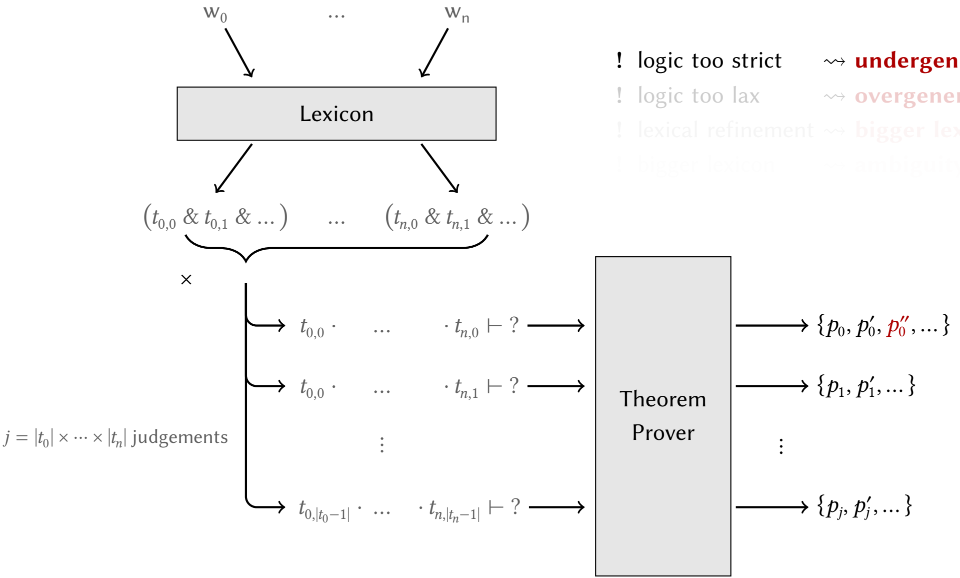
The Real World

! logic too strict $\rightsquigarrow$ **undergeneration**

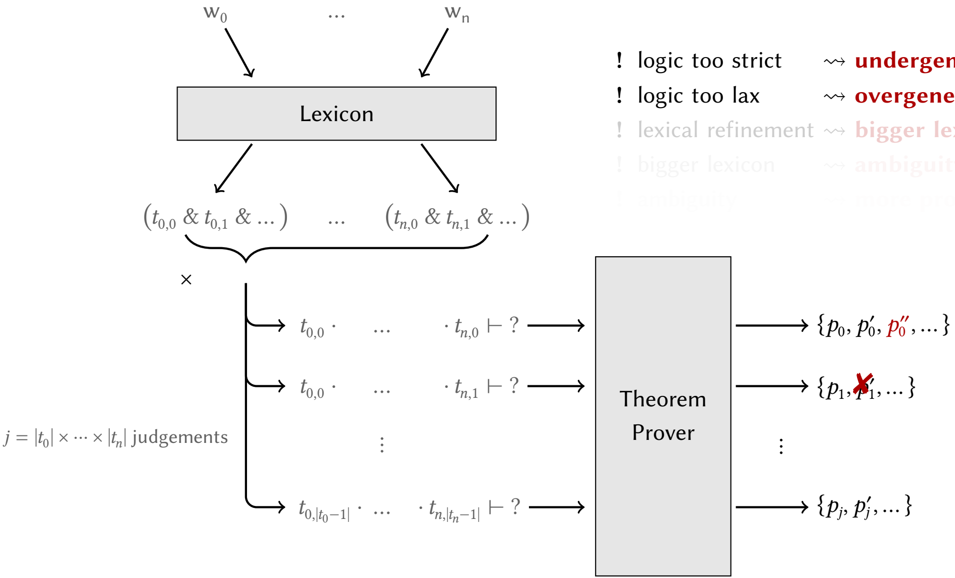
! logic too lax $\rightsquigarrow$ **overgeneration**

! lexical refinement $\rightsquigarrow$ **bigger lexicon**

! bigger lexicon $\rightsquigarrow$ **ambiguity & sparsity**

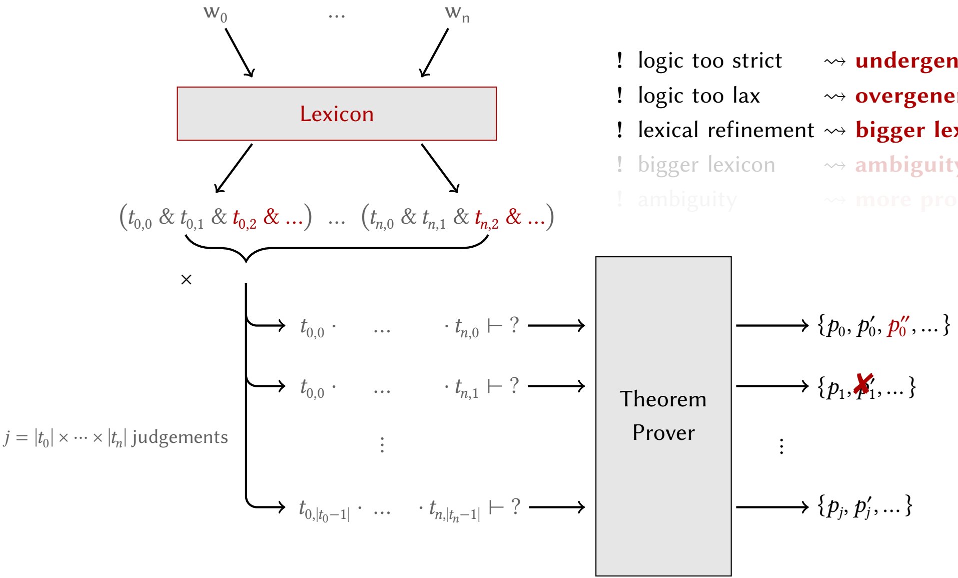


The Real World



!	logic too strict	$\rightsquigarrow$	**undergeneration**
!	logic too lax	$\rightsquigarrow$	**overgeneration**
!	lexical refinement	$\rightsquigarrow$	**bigger lexicon**
!	bigger lexicon	$\rightsquigarrow$	**ambiguity & sparsity**
!	ambiguity	$\rightsquigarrow$	**more proof candidates**

The Real World



- ! logic too strict $\rightsquigarrow$ **undergeneration**
- ! logic too lax $\rightsquigarrow$ **overgeneration**
- ! lexical refinement $\rightsquigarrow$ **bigger lexicon**
- ! bigger lexicon $\rightsquigarrow$ **ambiguity & sparsity**
- ! ambiguity $\rightsquigarrow$ **more proof candidates**

The Real World

! logic too strict $\rightsquigarrow$ **undergeneration**

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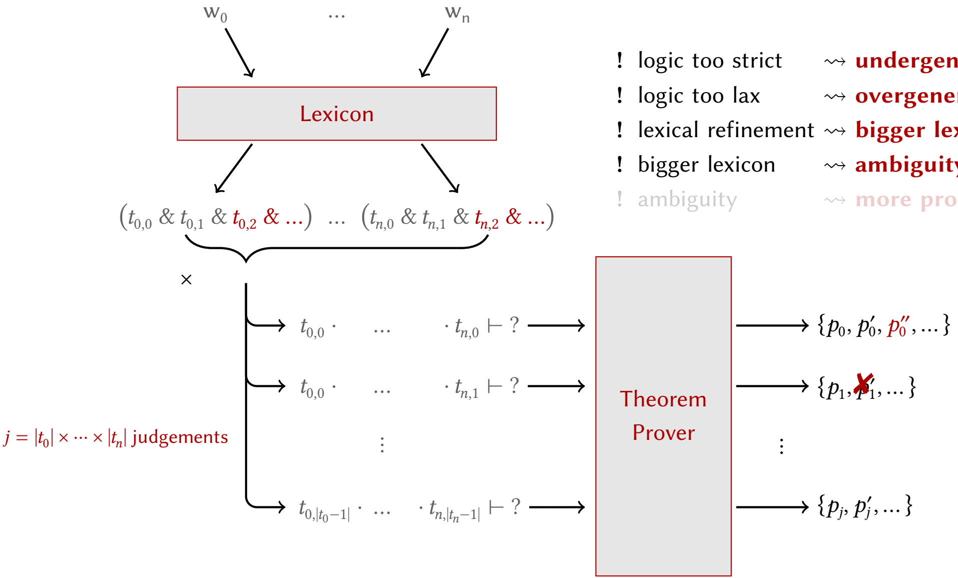
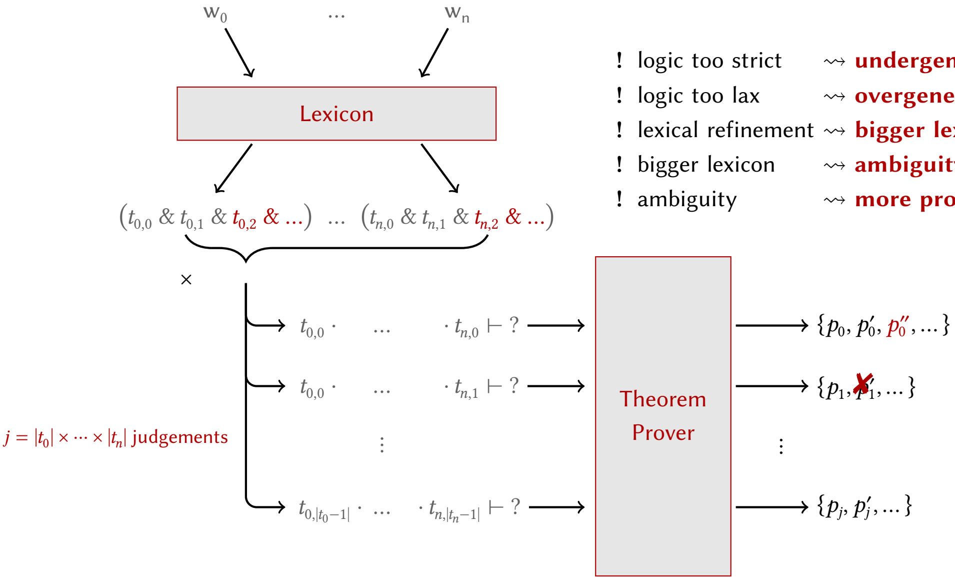
! lexical refinement $\rightsquigarrow$ **bigger lexicon**

! bigger lexicon $\rightsquigarrow$ **ambiguity & sparsity**

! ambiguity $\rightsquigarrow$ **more proof candidates**



The Real World



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! bigger lexicon $\rightsquigarrow$ **ambiguity & sparsity**

! ambiguity $\rightsquigarrow$ **more proof candidates** fin.