SUPPLEMENTARY MATERIAL

"Crypto VS Wall Street: Decoding the Effect of Bitcoin halving"

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Abstract

This Supplement documents, in a fully reproducible manner, the data construction, alignment, and estimation procedures for the *Finance Research Letters* article titled: "Crypto VS Wall Street: Decoding the Effect of Bitcoin halving'. We (i) clarify variable definitions and state clearly that all econometric work uses per-period log returns and stationary variables; (ii) reconcile mixed trading calendars by sampling on a daily calendar and averaging prices across non-trading spans, preserving close-to-close moves, avoiding reopen discontinuities, and retaining weekend information for crypto; (iii) explain why dropping days or zero-return fills would distort volatility, autocorrelation, and frequency-domain inference; (iv) provide the frequency-to-time mapping used in the Breitung-Candelon tests to interpret short-, and long-horizon causality and event occurrences. The replication package includes the dataset (**.dta**) and **STATA** code that reproduces Table 1 and all estimation results. In this context, time-stamped outputs are provided. The results of the article are unchanged. Our aim is that an independent researcher can follow each step and replicate the analysis end-to-end, and accordingly we adopt a clear expository style that minimizes technicalities. Of course, as in all sciences, alternative, defensible choices may yield different results. The current Supplement ($\approx$ 3,100 words) is longer than the article ($\approx$ 2,000 words).

Introduction

The authors, as a team of Applied Mathematicians, Physicists, and Engineers, with Master's Degrees in *Physics*, *Mathematics*, *Economics*, and *Finance*, and three (3) of us with PhDs in *Quantitative Economics*, we place the highest value on transparency and reproducibility.

Actually, we view transparency and reproducibility as essential across the sciences. Since this published article appeared in a "Letters" format with very strict length limits, several implementation details could not be reported in print.

To ensure full reproducibility, we are supplying supplementary material that:

(i) clarifies the variables used;

(ii) explains the handling of non-trading days; and

(iii) provides code that reproduces Table 1 and all estimations.

Finally, to further enhance transparency, we:

(iv) re-run the complete workflow; and

(v) provide time-stamped, real-time screenshots of outputs (tables, tests, and frequency-domain plots) alongside the replication code.

Our aim is to enable an independent researcher to follow every step and replicate the results end-to-end. Accordingly, we adopt a clear expository style that minimizes technicalities, so readers from diverse backgrounds can engage with the material and ideas in this Supplement.

Finally, as in all sciences, our findings are conditional on: (a) the maintained assumptions, (b) data-construction choices, and (c) model specification. Of course, alternative, defensible choices may yield different results.

I. Variables & Definitions

In Table 1, the upper part of the table reports the prices of BTC, ETH, SP500, and gold, followed by the market capitalizations of BTC and ETH. Then, we present the return series: “ReturnsBonds”, “returnBTC”, “returnETH”, “returnSP500”, and “returnGold”. The variable “return SP500” (lowercase) is the per-period log return, i.e.

$$\Delta \ln P_t = \ln P_t - \ln P_{t-1}$$

and the same convention applies to the other return variables. Note that there is a small typo: “Return SP500” (capital “R”) corresponds to the log level $\ln P_t$, not a return.

Now, for the “returnBonds” variable, these are yields that were found to be stationary for both periods (see Replication Section). Our choice was the use of Bond yields directly in the analysis. Our modeling choice is based on two (2) main factors:

(i) The 10-year Treasury yield is a primitive state variable in asset-pricing and macro-finance. Our hypotheses concern how this rate co-moves with other markets of interest, not the profit and loss (P&L) of a particular duration-targeted portfolio.

(ii) Daily bond “returns” mix price changes (from rate moves, scaled by duration) with coupon accrual. Using the yield itself avoids this mix.

No Implications for the Results.

All econometric work, including (i) unit root tests and (ii) spectral analysis, was conducted on the variables defined above, which were all found to be stationary (see Replication Section). Note that the variable “Return SP500” (capital “R”), which corresponds to the log level $\ln P_t$, does ***not*** enter any estimation at all. Hence, the empirical results and their interpretation are completely ***unchanged***.

II. Calendar or Trading Days?

Table 1 reports 1,587 observations, which equals the number of calendar days in our window (2017-11-09 to 2022-03-14, inclusive). This was a deliberate design choice. Crypto markets trade seven (7) days per week, whereas equities and bonds follow five (5) day schedules.

Sampling on a calendar-day grid allows us to:

(i) retain weekend dynamics in Crypto, rather than discarding $\frac{2}{7} \approx 29\%$ of valid variation, and

(ii) preserve sample length, which improves statistical power and frequency resolution, without removing $\frac{2}{7} \approx 29\%$ of its length.

So:

a) We created a calendar-day index for the full window, for weekends and holidays, with no official close for equities and bonds, and we constructed the price average between the preceding and subsequent trading-day closes (e.g. Friday $\rightarrow$ Monday).

b) Then, the returns were computed as log returns on this uniform grid.

c) For estimation purposes, we used the aligned return series.

Why we chose averaging than i) dropping or ii) zero-return fill

A. Irregular sampling, i.e. dropping days:

(i) removes 29% of valid crypto variation

(ii) breaks the regular grid required for standard tools

(iii) reduces statistical power

(iv) renders the mapping from radians sample to "days" ill-defined.

B. Zero-return fills, i.e. mechanically setting weekend returns to zero, creates:

(i) Variance distortion, since the latent increments are replaced by zeros, concentrating the 3-day move into Monday. This modeling choice depresses average daily variance.

(ii) Autocorrelation and seasonality, since the deterministic pattern (0, 0, large) creates spurious spikes, unrelated to the real underlying process.

(iii) Spectral contamination, because the induced weekly seasonality concentrates power at

$$\omega = \frac{2\pi}{7}$$

and its harmonics (i.e. a pattern that repeats every 7 days shows up at the weekly frequency, and also at its harmonics $2\times$, $3\times$, the weekly rate), yielding artificial peaks in the periodogram, thus biasing frequency-domain inference.

(iv) Artifacts, in case equities and bonds are zero-filled but crypto is not, then cross-spectral measures reflect calendar artifacts rather than economic and financial comovement. In other words, those artificial peaks come from preprocessing, not the economics. So any frequency-domain inference (e.g. causality at weekly horizons) gets biased toward false signals at the weekly frequency and its harmonics.

Therefore, price averaging between the preceding and subsequent trading-day closes (e.g. Friday→Monday):

- preserves the exact cumulative close-to-close move
- avoids reopen discontinuities and maintains a regular daily grid, and
- is consistent with a latent continuous-time price process observed intermittently, thereby yielding more reliable frequency-domain inference.

III. Frequency-domain causality in time

Now, let's analyze the interpretation of the frequency – domain causality in time units.

First, let the sampling interval be $\Delta_t = 1$ and let $\omega \in (0,\ \pi]$ denote angular frequency in radians (rad) per day. The Breitung – Candelon test evaluates, at each ω, the null of "no Granger causality at frequency ω".

As we know, the frequency and period are related as follows:

$$T_{\text{days}} = \frac{2\pi}{\omega}\,\Delta_t = \frac{2\pi}{\omega}.$$

This converts each frequency into the length of one full cycle in days. Then, the number of occurrences O (in days) of the periodical phenomenon (i.e. cycles) is equal to:

$$O = \frac{365}{T_{\text{days}}}\,.$$

Therefore, high frequencies (ω near π, equivalent to T near 2) imply short horizon dynamics. Whereas, low frequency (ω small, and T large) capture long horizon components.

Of course, the usual sanity checks are in force:

$\omega = \pi \Rightarrow T = 2$ days

$\omega = 2\pi/7 \Rightarrow T = 7$ days

and so on.

For example, when the test reports, say, $\omega = 0.125$ rad/day, it implies $T = 2\pi/0.125 \approx 50$ days, and we report about $O = 365/T_{\text{days}} = 365/50 \approx 7.3$ occurrences per year (in days).

What is the case in our paper?

For the case of $\omega = \pi \approx 3.14$ rad/day appearing – among others – in our paper, we have that:

$T = 2\pi/\omega = 2$ days.

So, on a 365-day calendar that is $\frac{365}{T} = \frac{365}{2} = 182{,}5$ occurrences (in days) per year. This is exactly how the number 182 comes from in our analysis, with $\omega \approx 3.14$.

Applying this mapping across BTC, ETH, gold, SP500, and bonds before and after the halving yields a clear time-domain of causal links changed around the event.

IV. Per Period Log Returns & Summation

In our paper, we use per-period log returns $r_t = \ln\left(\frac{X_t}{X_{t-1}}\right)$. In addition to these baseline returns, we computed, for each period, the cumulative log change (i.e. cumulative log return), $\sum_t \ln\left(\frac{X_t}{X_{t-1}}\right)$ (Equation 17, in the manuscript) to assess whether the pre- and post-halving subsamples exhibit materially different aggregate behavior. This diagnostic did ***not*** enter any of the econometric tests or estimations.

For all the analyses, only the daily log returns r_t was used. More precisely, as stated on page 5, of the paper: *"Throughout the paper, "return" refers to the daily log return* $r_t = \ln\left(\frac{X_t}{X_{t-1}}\right)$ *for each series."* Also, on the same page: *"For consistency and to avoid ambiguity, we use "return" when we report or analyze* r_t *[...]".*

So, Table 1 and all estimations are based on per-period r_t, and not cumulative log return.

V. The Workflow

Here is a very brief workflow of our work:

(a) Sampling convention: Daily calendar.

(b) We downloaded and made initial checks.

- We collected daily close prices P_t for all assets over 2017-11-09 to 2022-03-14.
- We retained the official close for each trading day.

(c) We created a common calendar-day grid (union of dates)

- We constructed a complete calendar-day index for the full window.
- Crypto has observations on all days, whereas equities and bonds are missing on weekends and market holidays.

(d) For assets without a trading close on day t, we imputed the price by averaging. For simplicity, the price averaging across non-trading spans (e.g. Friday→Monday) was performed upstream in MS-Excel prior to import.

- For each asset and for each period, we reported relevant summary statistics.

(e) We computed cumulative log return by period for each period i.e. pre- & post-halving in order to check whether a regime analysis would be relevant.

(v) Unit-root diagnostics (returns)

- We ran unit root tests on r_t within each period, using the popular ADF test, for all variables.
- Conclusion: returns have no unit roots, i.e. $r_t \sim I(0)$.

(vi) Estimations: spectral (frequency-domain) causality

- We applied Breitung–Candelon frequency-domain Granger causality on return series r_t for each period.
- We reported results of occurrences per year (in days).

VI. Dataset

We hereby provide the source links, and, below, the full **STATA** code that fully reproduces Table 1 and the estimation results. A **.dta** file with the data accompanies this Supplement to ease replication.

More precisely, the data come from:

- `https://coinmarketcap.com/`

  for Bitcoin (BTC) and Ethereum (ETH) prices, and Market Cap

- `https://finance.yahoo.com/`

  for the SPDR S&P 500 ETF (SPY), and US 10-year bond prices indexed as (TNX). Specifically, for the US 10-year bond we downloaded the series CBOE Interest Rate 10 Year T No (TNX).

- `https://www.investing.com/`

  for gold prices we used the GCQ2 series. Investing.com now lists the available series as GCZ5. Over our sample period the two series are nearly identical.

The dataset spans the period 9 November 2017 till 14 March 2022.

And the **STATA** 15.1 code is provided in the Appendix.

VII. Replication

Based on the data file provided (**dta**), and the **STATA** code provided below, we replicated the results. Screenshots of the estimated results with relevant captions are presented below.

The analysis begins with the per period Descriptive Statistics.

The **STATA** 15.1 code used for producing Figure 1, which corresponds to Table 1 in the published manuscript, is the following:

```
summarize Price_BTC MarketCap_BTC Price_ETH MarketCap_ETH Price_SP500
Return_SP500 Price_GOLD Return_Bonds return_BTC return_ETH return_SP500
return_Gold in 1/547// Pre-Halving Period
summarize Price_BTC MarketCap_BTC Price_ETH MarketCap_ETH Price_SP500
Return_SP500 Price_GOLD Return_Bonds return_BTC return_ETH return_SP500
return_Gold in 548/1587//Post-Halving Period
```

Then, the analysis turns to the to unit root testing of the time series. Figures 2 & 3 present the outputs of **STATA** for ADF testing for the pre-halving period, whereas Figures 4 & 5 refer to the post-halving period. The **STATA** code used is:

```
dfuller return_BTC in 1/547, drift lags(3)
dfuller return_ETH in 1/547, drift lags(3)
dfuller return_SP500 in 1/547, drift lags(3)
dfuller return_Gold in 1/547, drift lags(3)
dfuller Return_Bonds in 1/547, drift lags(3)
dfuller return_BTC in 548/1587, drift lags(3)
dfuller return_ETH in 548/1587, drift lags(3)
dfuller return_SP500 in 548/1587, drift lags(3)
dfuller return_Gold in 548/1587, drift lags(3)
dfuller Return_Bonds in 548/1587, drift lags(3)
```

Next, we present the replication of the manuscript's Correlation Matrices across the two periods. Figures 6 & 7 correspond to the results presented in Table 2 of the published paper. The **STATA** code used is:

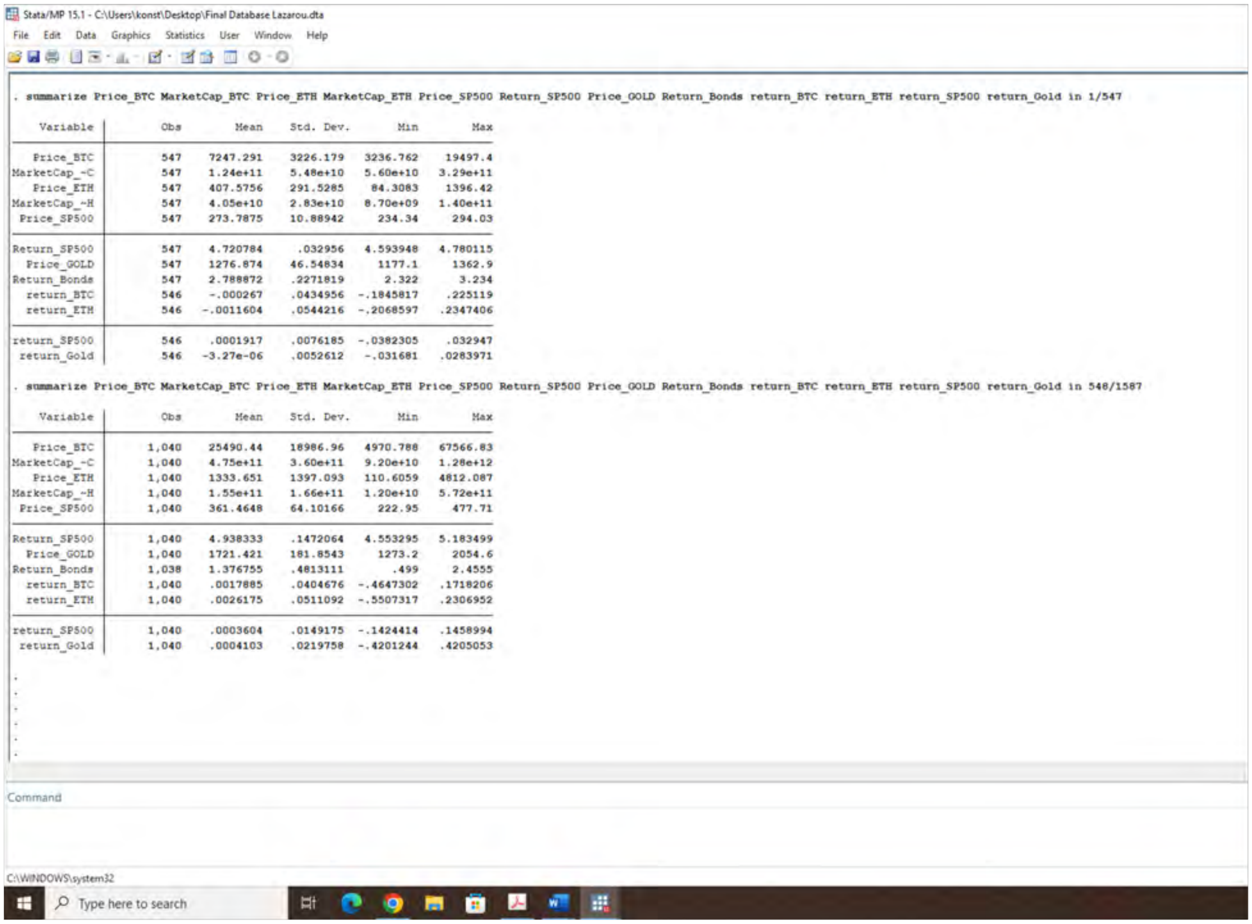
```
pwcorr Return_Bonds return_BTC return_ETH return_SP500 return_Gold in 1/547, sig
pwcorr Return_Bonds return_BTC return_ETH return_SP500 return_Gold in 548/1587, sig
```

Finally, the replication of selected but indicative spectral causalities for the Pre-Halving period are presented in Figures 8- 10. These Figures correspond to the results presented in Figure 1 of the published manuscript for the designated spectral causality tests between asset returns. Spectral causalities for the Post-Halving period are presented in Figures 11-13. These Figures correspond to the figures presented in Figure 2 of the published manuscript for the designated spectral causality tests between asset returns. Please note that in Figures 8-13 we also present the t-statistic p-value and radian list table for all spectral frequencies. These spectral frequencies correspond to the Table 3 of the published manuscript for the designated radian range for the spectral frequencies. The **STATA** code used is:

```
bcgcausality return_BTC return_ETH in 1/547
bcgcausality return_BTC return_SP500 in 1/547
matrix list r(W)
bcgcausality return_SP500 Return_Bonds in 1/547
matrix list r(W)
bcgcausality return_BTC return_ETH in 548/1587
matrix list r(W)
bcgcausality return_SP500 return_Gold in 548/1587
matrix list r(W)
bcgcausality return_SP500 Return_Bonds in 548/1587
matrix list r(W)
```

Descriptive Statistics

Figure 1 Descriptive Statistics



Stata/MP 15.1 - C:\Users\konst\Desktop\Final Database Lazarou.dta

```
. summarize Price_BTC MarketCap_BTC Price_ETH MarketCap_ETH Price_SP500 Return_SP500 Price_GOLD Return_Bonds return_BTC return_ETH return_SP500 return_Gold in 1/547
```

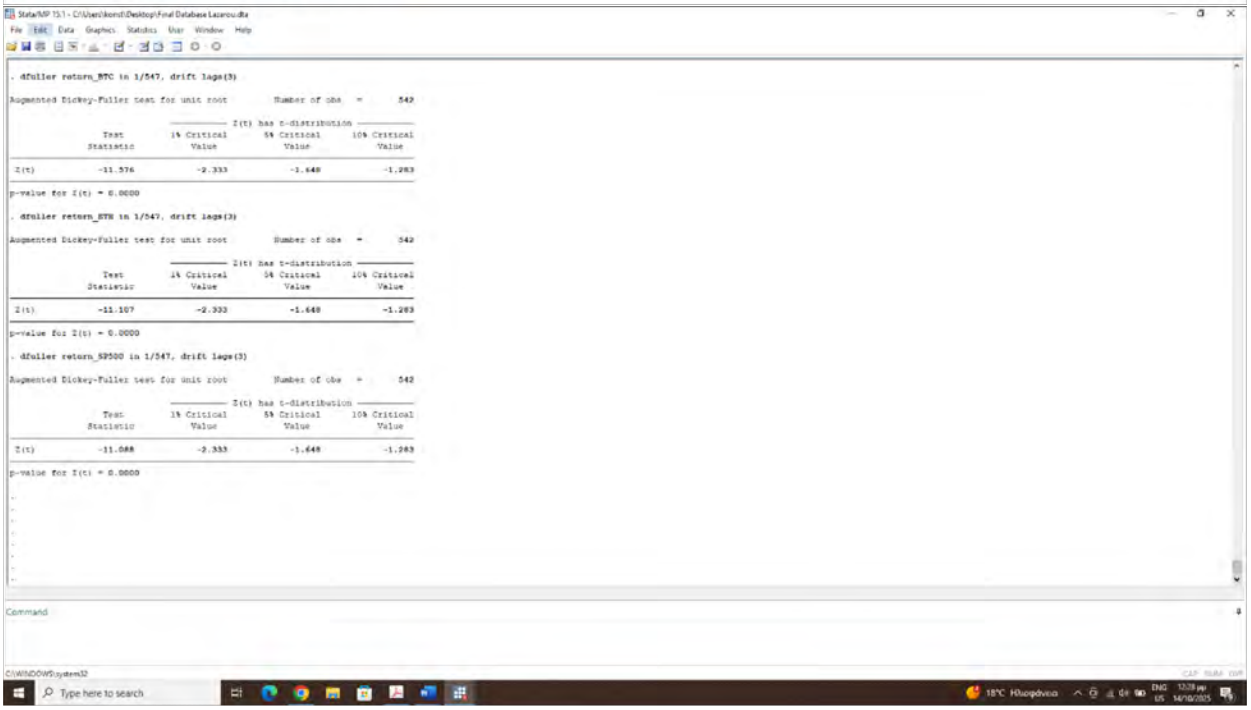
Variable	Obs	Mean	Std. Dev.	Min	Max
Price_BTC	547	7247.291	3226.179	3236.762	19497.4
MarketCap_~C	547	1.24e+11	5.48e+10	5.60e+10	3.29e+11
Price_ETH	547	407.5756	291.5285	84.3083	1396.42
MarketCap_~H	547	4.05e+10	2.83e+10	8.70e+09	1.40e+11
Price_SP500	547	273.7875	10.88942	234.34	294.03
Return_SP500	547	4.720784	.032956	4.593948	4.780115
Price_GOLD	547	1276.874	46.54834	1177.1	1362.9
Return_Bonds	547	2.788872	.2271819	2.322	3.234
return_BTC	546	-.000267	.0434956	-.1845817	.225119
return_ETH	546	-.0011604	.0544216	-.2068597	.2347406
return_SP500	546	.0001917	.0076185	-.0382305	.032947
return_Gold	546	-3.27e-06	.0052612	-.031681	.0283971

```
. summarize Price_BTC MarketCap_BTC Price_ETH MarketCap_ETH Price_SP500 Return_SP500 Price_GOLD Return_Bonds return_BTC return_ETH return_SP500 return_Gold in 548/1587
```

Variable	Obs	Mean	Std. Dev.	Min	Max
Price_BTC	1,040	25490.44	18986.96	4970.788	67566.83
MarketCap_~C	1,040	4.75e+11	3.60e+11	9.20e+10	1.28e+12
Price_ETH	1,040	1333.651	1397.093	110.6059	4812.087
MarketCap_~H	1,040	1.55e+11	1.66e+11	1.20e+10	5.72e+11
Price_SP500	1,040	361.4648	64.10166	222.95	477.71
Return_SP500	1,040	4.938333	.1472064	4.553295	5.183499
Price_GOLD	1,040	1721.421	181.8543	1273.2	2054.6
Return_Bonds	1,038	1.376755	.4813111	.499	2.4555
return_BTC	1,040	.0017885	.0404676	-.4647302	.1718206
return_ETH	1,040	.0026175	.0511092	-.5507317	.2306952
return_SP500	1,040	.0003604	.0149175	-.1424414	.1458994
return_Gold	1,040	.0004103	.0219758	-.4201244	.4205053

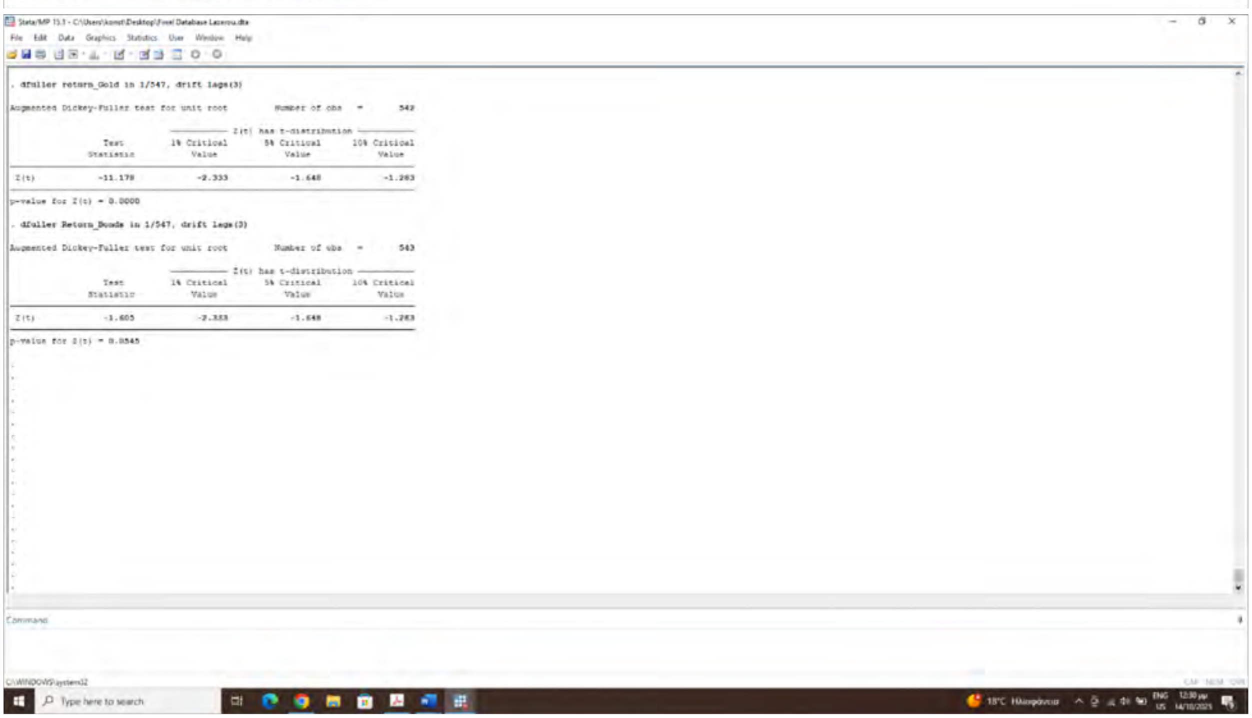
Unit Root Tests-ADF (Pre-Halving)

Figure 2 ADF Unit Root Test-BTC ETH-SP500



```
. dfuller return_BTC in 1/547, drift lags(3)

Augmented Dickey-Fuller test for unit root         Number of obs   =       542

                               ---------- Z(t) has t-distribution ----------
                  Test          1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -11.576            -2.333            -1.648            -1.283
------------------------------------------------------------------------------
p-value for Z(t) = 0.0000

. dfuller return_ETH in 1/547, drift lags(3)

Augmented Dickey-Fuller test for unit root         Number of obs   =       542

                               ---------- Z(t) has t-distribution ----------
                  Test          1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -11.107            -2.333            -1.648            -1.283
------------------------------------------------------------------------------
p-value for Z(t) = 0.0000

. dfuller return_SP500 in 1/547, drift lags(3)

Augmented Dickey-Fuller test for unit root         Number of obs   =       542

                               ---------- Z(t) has t-distribution ----------
                  Test          1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -11.088            -2.333            -1.648            -1.283
------------------------------------------------------------------------------
p-value for Z(t) = 0.0000
```

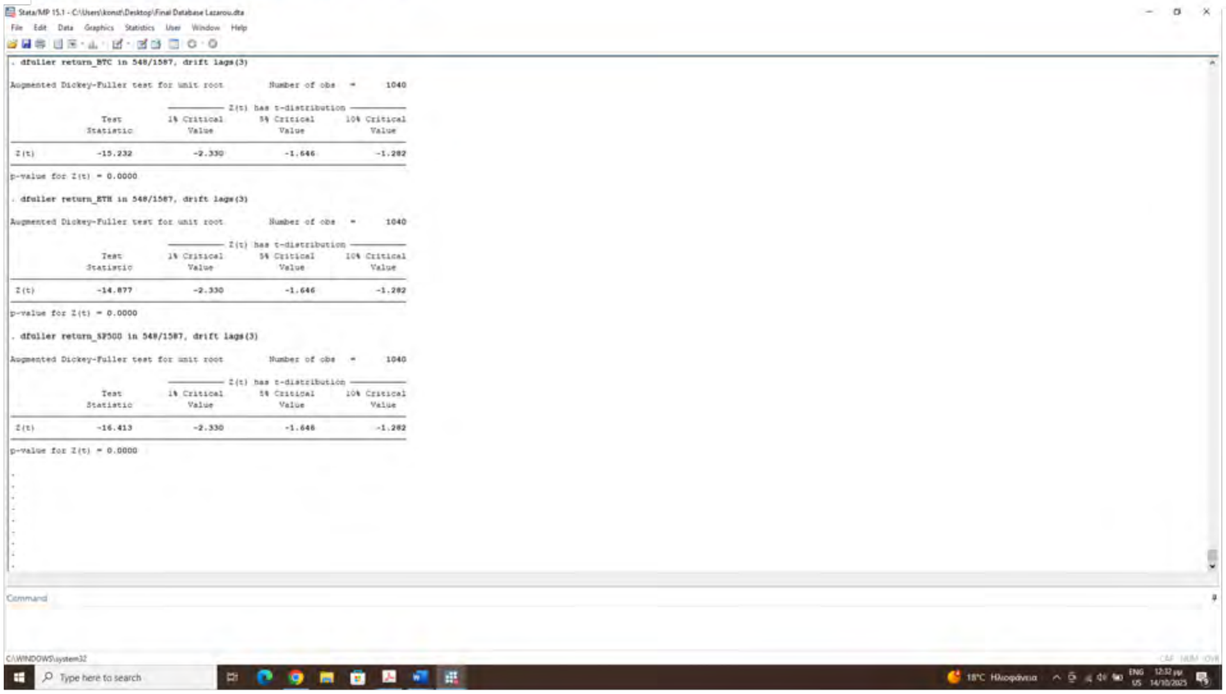
Figure 3 ADF Unit Root Test-Bonds Gold



```
. dfuller return_Gold in 1/547, drift lags(3)

Augmented Dickey-Fuller test for unit root         Number of obs   =       542

                               ---------- Z(t) has t-distribution ----------
                  Test          1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -11.178            -2.333            -1.648            -1.283
------------------------------------------------------------------------------
p-value for Z(t) = 0.0000

. dfuller Return_Bonds in 1/547, drift lags(3)

Augmented Dickey-Fuller test for unit root         Number of obs   =       543

                               ---------- Z(t) has t-distribution ----------
                  Test          1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -1.605            -2.333            -1.648            -1.283
------------------------------------------------------------------------------
p-value for Z(t) = 0.0545
```

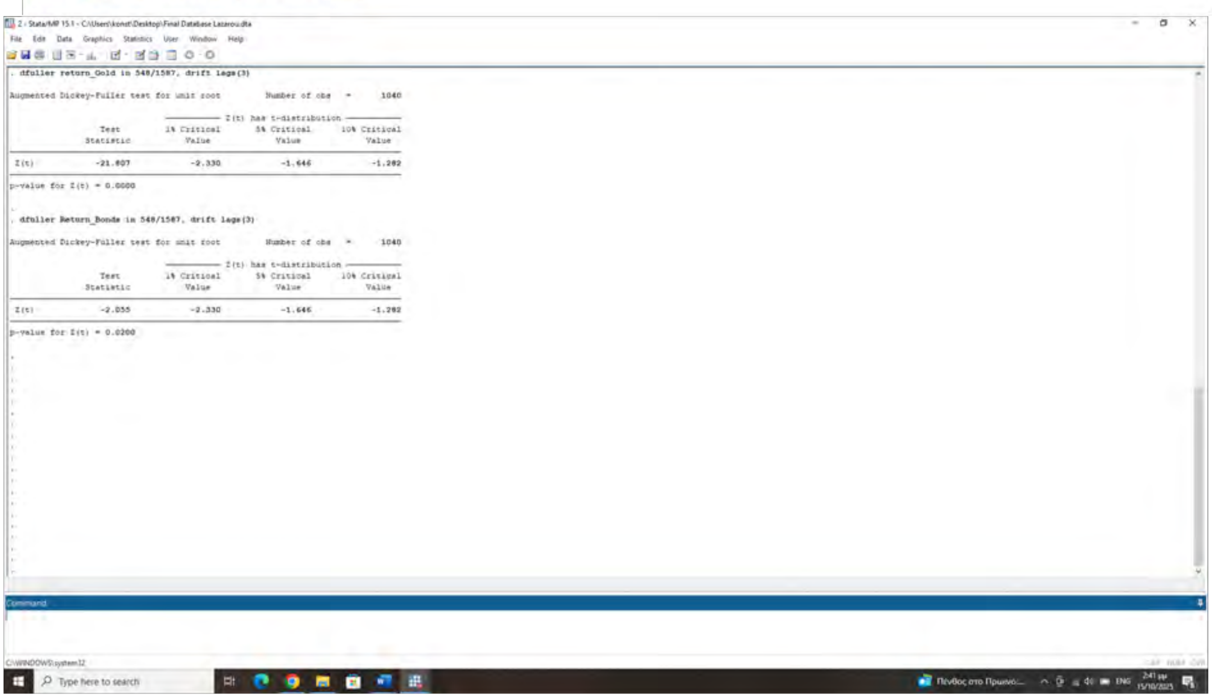
Unit Root Tests-ADF (Post-Halving)

Figure 4 ADF Unit Root Test SP500 BTC ETH



```
. dfuller return_BTC in 548/1587, drift lags(3)

Augmented Dickey-Fuller test for unit root         Number of obs   =      1040

                               ---------- Z(t) has t-distribution ----------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)           -15.232           -2.330            -1.646            -1.282
------------------------------------------------------------------------------
p-value for Z(t) = 0.0000

. dfuller return_ETH in 548/1587, drift lags(3)

Augmented Dickey-Fuller test for unit root         Number of obs   =      1040

                               ---------- Z(t) has t-distribution ----------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)           -14.877           -2.330            -1.646            -1.282
------------------------------------------------------------------------------
p-value for Z(t) = 0.0000

. dfuller return_SP500 in 548/1587, drift lags(3)

Augmented Dickey-Fuller test for unit root         Number of obs   =      1040

                               ---------- Z(t) has t-distribution ----------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)           -16.413           -2.330            -1.646            -1.282
------------------------------------------------------------------------------
p-value for Z(t) = 0.0000
```

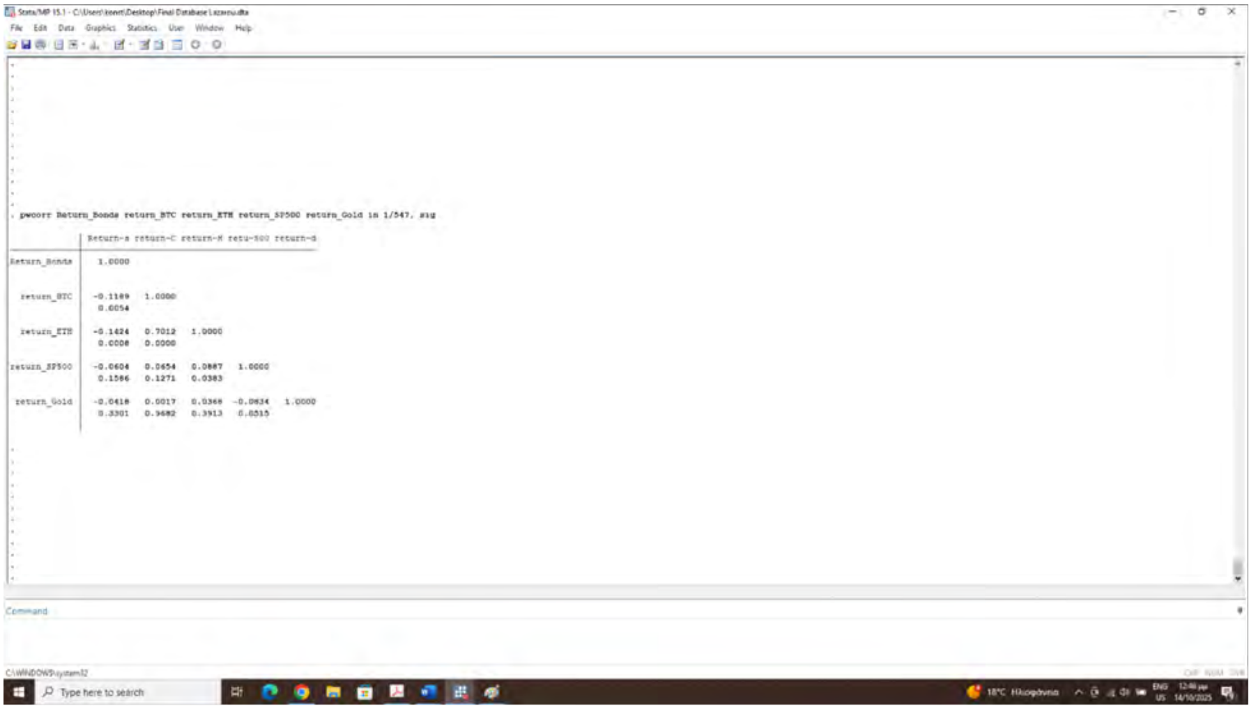
Figure 5 ADF Unit root Test Gold Bonds



```
. dfuller return_Gold in 548/1587, drift lags(3)

Augmented Dickey-Fuller test for unit root         Number of obs   =      1040

                               ---------- Z(t) has t-distribution ----------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)           -21.807           -2.330            -1.646            -1.282
------------------------------------------------------------------------------
p-value for Z(t) = 0.0000

. dfuller Return_Bonds in 548/1587, drift lags(3)

Augmented Dickey-Fuller test for unit root         Number of obs   =      1040

                               ---------- Z(t) has t-distribution ----------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)            -2.055           -2.330            -1.646            -1.282
------------------------------------------------------------------------------
p-value for Z(t) = 0.0200
```

Correlation Matrices Pre- and Post- Halving

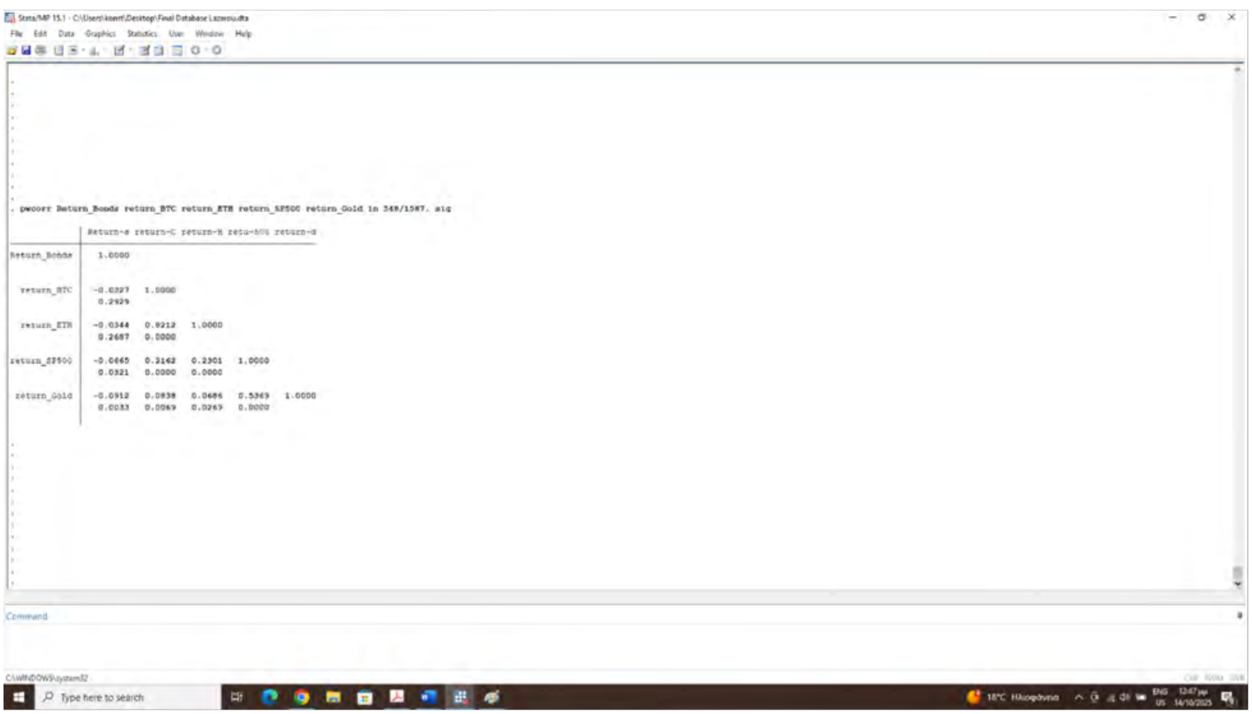
Figure 6 Correlation Among Returns Pre-Halving



. pwcorr Return_Bonds return_BTC return_ETH return_SP500 return_Gold in 1/547, sig

	Return~s	return~C	return~H	retu~500	return~d
Return_Bonds	1.0000				
return_BTC	-0.1189	1.0000			
	0.0054				
return_ETH	-0.1424	0.7012	1.0000		
	0.0008	0.0000			
return_SP500	-0.0604	0.0654	0.0887	1.0000	
	0.1586	0.1271	0.0383		
return_Gold	-0.0418	0.0017	0.0368	-0.0834	1.0000
	0.3301	0.9682	0.3913	0.0515	

Figure 7 Correlation Among Returns Post Halving



. pwcorr Return_Bonds return_BTC return_ETH return_SP500 return_Gold in 548/1587, sig

	Return~s	return~C	return~H	retu~500	return~d
Return_Bonds	1.0000				
return_BTC	-0.0327	1.0000			
	0.2929				
return_ETH	-0.0344	0.9212	1.0000		
	0.2687	0.0000			
return_SP500	-0.0665	0.3162	0.2301	1.0000	
	0.0321	0.0000	0.0000		
return_Gold	-0.0912	0.0838	0.0686	0.5369	1.0000
	0.0033	0.0069	0.0263	0.0000	

Spectral Causality Graphs (Pre-Halving)

Figure 8 Spectral Causality returns _ETH to returns_BTC

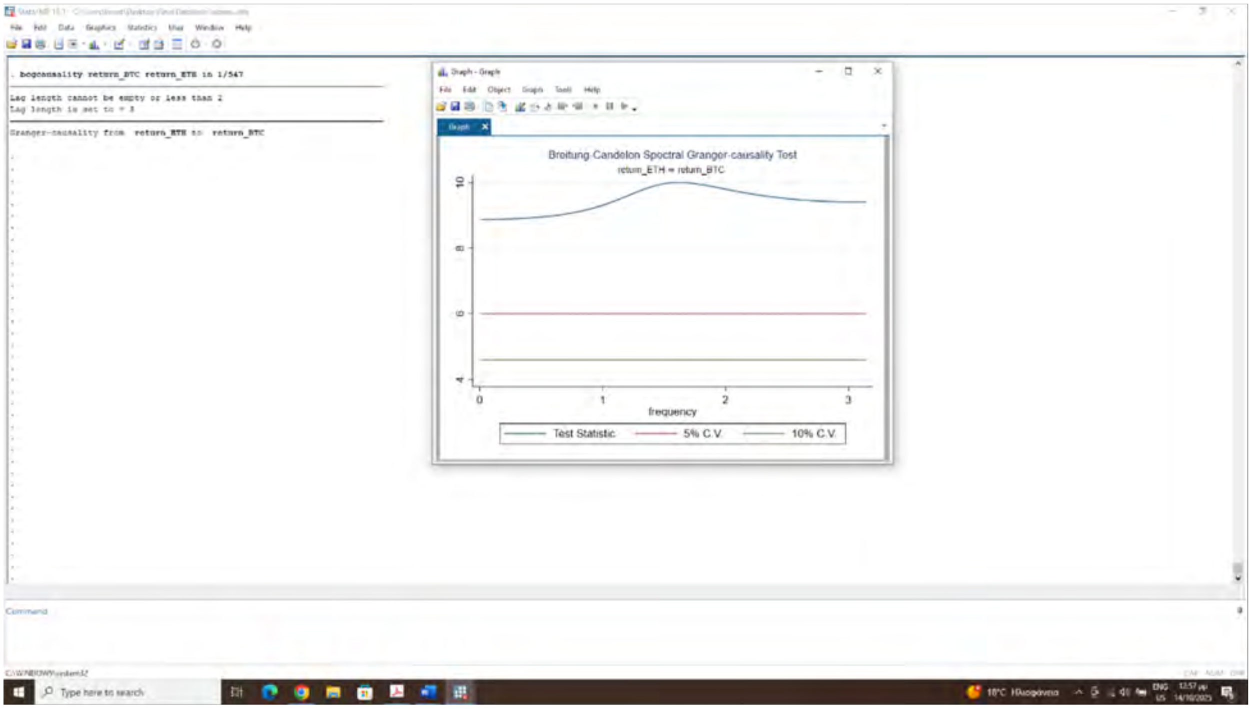


Figure 9 Spectral Causality returns _SP500 to returns_BTC

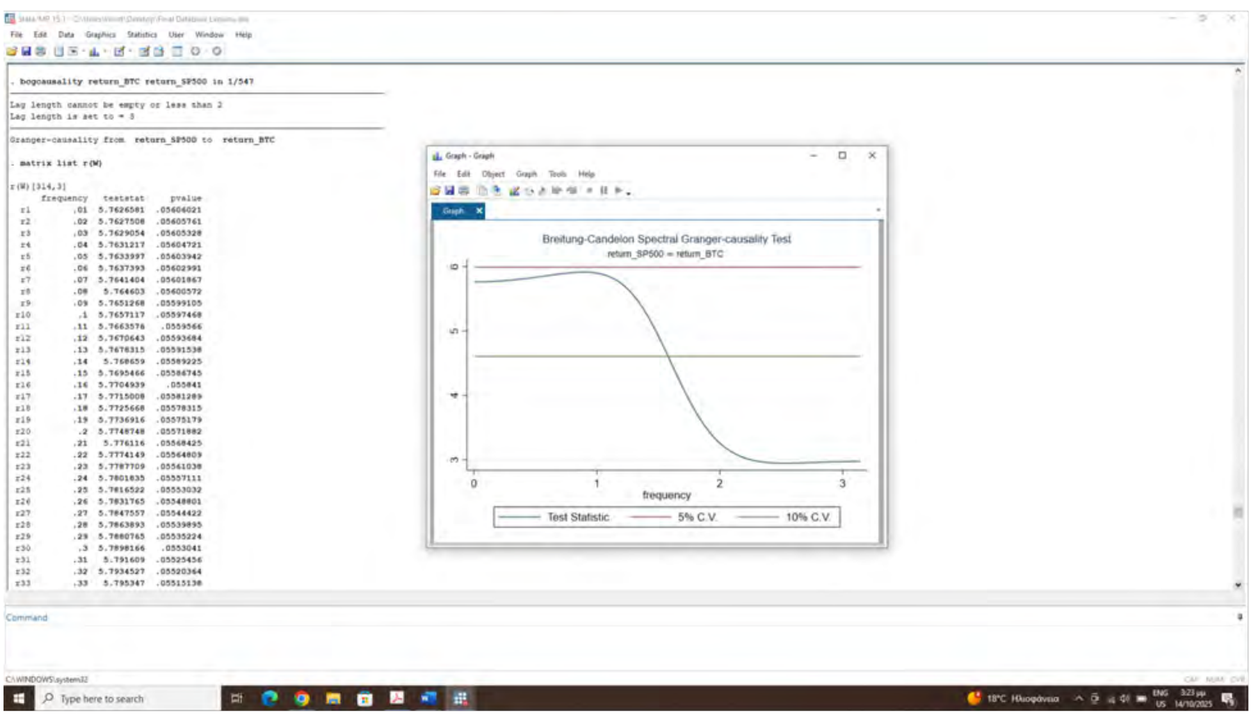
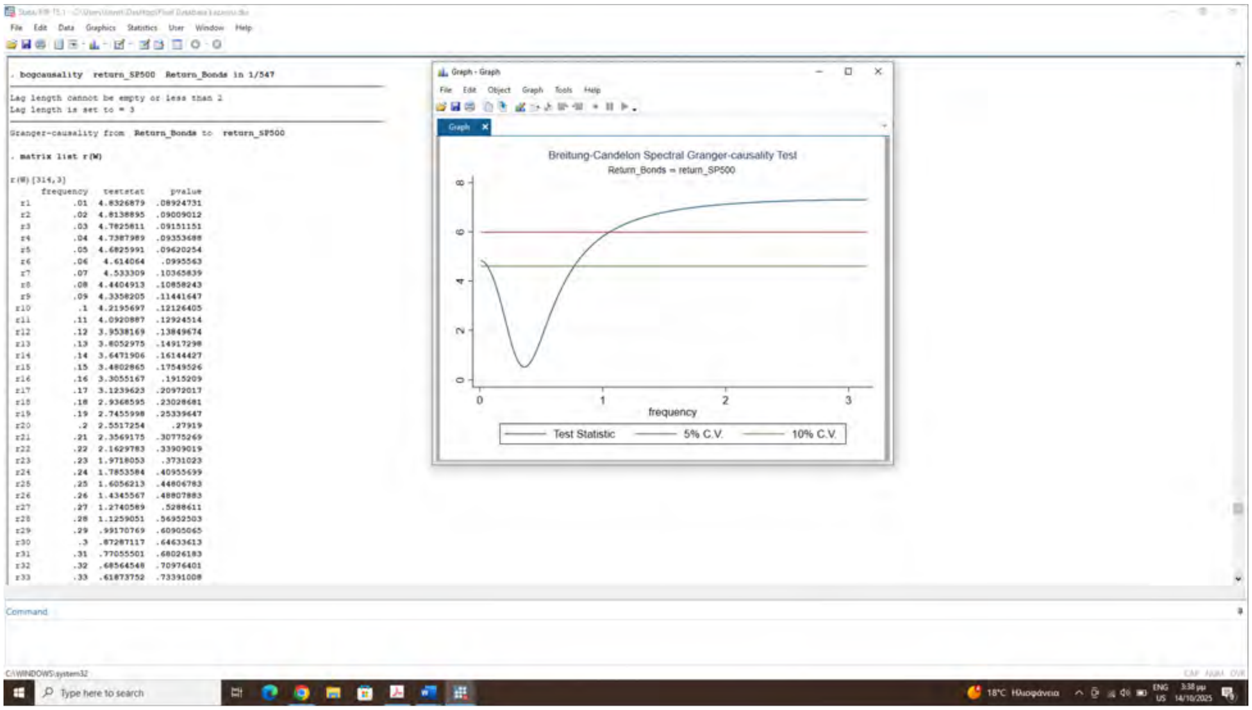


Figure 10 Spectral Causality returns _Bonds to returns_SP500



Spectral Causality Graphs (Post-Halving)

Figure 11 Spectral Causality returns_Gold to returns_SP500

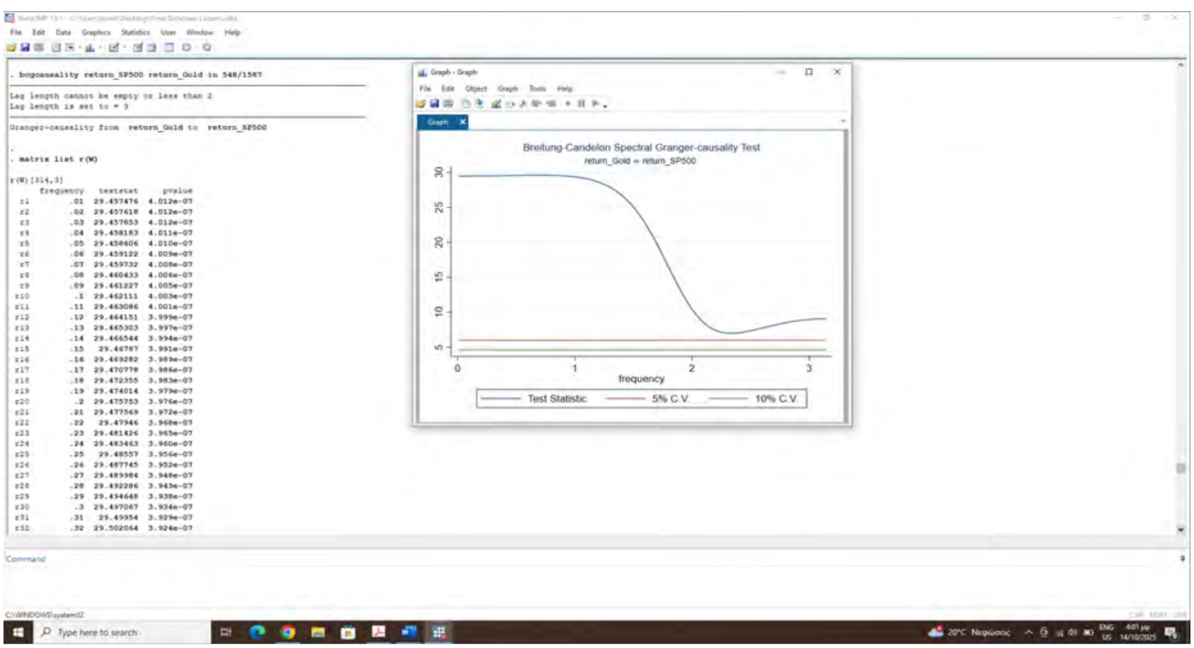


Figure 12 Spectral Causality returns_ETH to returns_BTC

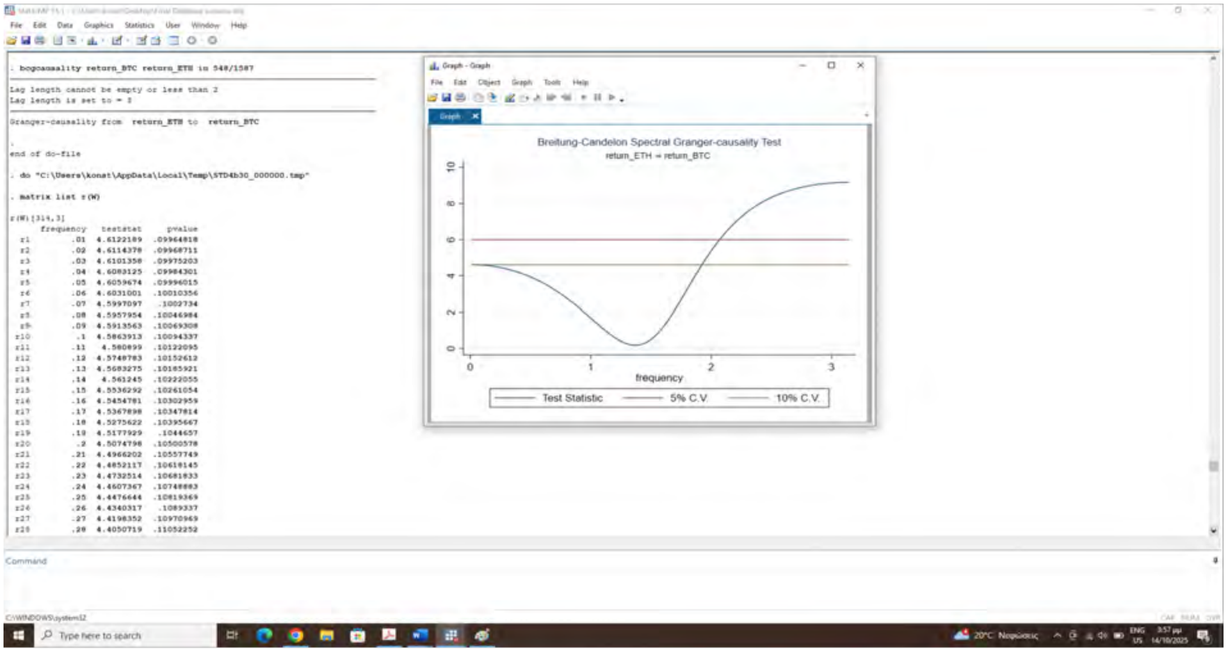
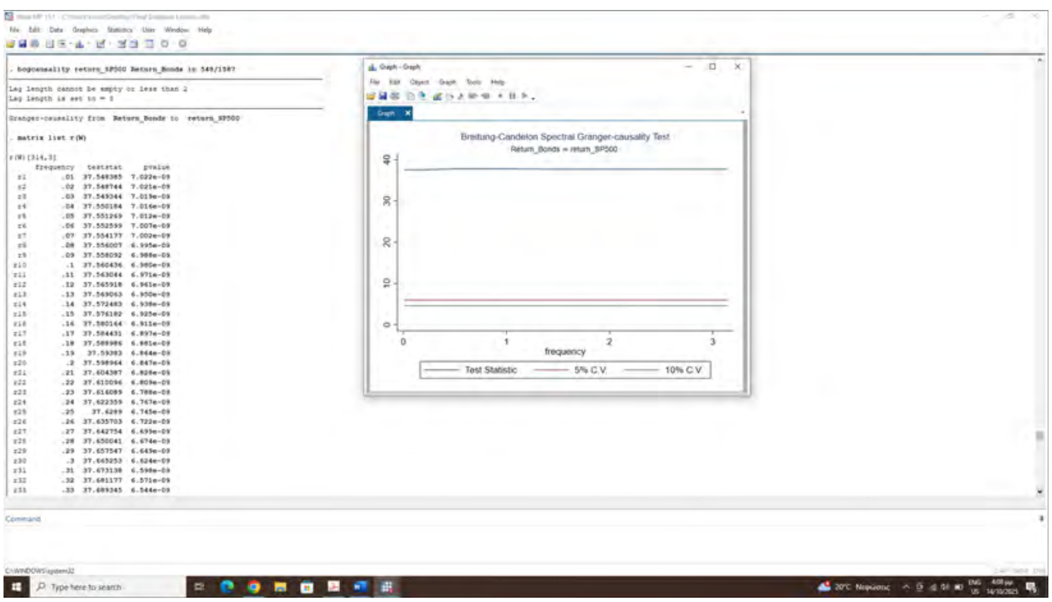


Figure 13 Spectral Causality returns_Bonds to returns_SP500



VIII. Conclusion

We re-audited the full data pipeline, and confirm that all estimations are conducted on stationary variables, specifically per-period log returns, sampled on a regular daily grid. Non-trading spans (e.g. Friday→Monday) are handled by price averaging, which preserves the exact close-to-close move and avoids reopen discontinuities.

To ensure full transparency and reproducibility, we provide a replication package containing: (i) the calendar-day data used in the estimations; (ii) **STATA** code that rebuilds Table 1, and reproduces the frequency-domain (Breitung–Candelon) tests with the relevant mapping; and (iv) time-stamped outputs that match the figures and tables in the paper. As a result, the current Supplement ($\sim$ 3,100 words) is longer than the article ($\sim$ 2,000 words).

Our aim has been to enable an independent researcher to follow every step and replicate the results end-to-end. Of course, as in all science, our findings are conditional on (i) the maintained assumptions, (ii) data-construction and sampling choices, and (iii) model specification. Reasonable alternative choices may yield different quantitative estimates.

Appendix I: STATA Code

```
//Creation of Returns
gen return_BTC=ln(Price_BTC)-ln(L.Price_BTC)
gen return_ETH=ln(Price_ETH)-ln(L.Price_ETH)
gen return_SP500=ln(Price_SP500)-ln(L.Price_SP500)
gen return_Gold=ln(Price_GOLD)-ln(L.Price_GOLD)

///Descriptive
summarize Price_BTC MarketCap_BTC Price_ETH MarketCap_ETH Price_SP500
Return_SP500 Price_GOLD Return_Bonds return_BTC return_ETH return_SP500
return_Gold in 1/547// Pre-Halving Period
summarize Price_BTC MarketCap_BTC Price_ETH MarketCap_ETH Price_SP500
Return_SP500 Price_GOLD Return_Bonds return_BTC return_ETH return_SP500
return_Gold in 548/1587//Post-Halving Period

///Unit root Tests Pre-Halving
dfuller return_BTC in 1/547, drift lags(3)
dfuller return_ETH in 1/547, drift lags(3)
dfuller return_SP500 in 1/547, drift lags(3)
dfuller return_Gold in 1/547, drift lags(3)
dfuller Return_Bonds in 1/547, drift lags(3)

///Unit Root Tests Post Halving
dfuller return_BTC in 548/1587, drift lags(3)
dfuller return_ETH in 548/1587, drift lags(3)
dfuller return_SP500 in 548/1587, drift lags(3)
dfuller return_Gold in 548/1587, drift lags(3)
dfuller Return_Bonds in 548/1587, drift lags(3)
```

```
///Correlation Pre-Halving

pwcorr Return_Bonds return_BTC return_ETH return_SP500 return_Gold in 1/547, sig

///Correlation Post-Halving

pwcorr Return_Bonds return_BTC return_ETH return_SP500 return_Gold in 548/1587, sig

///Spectral Causalities Pre-Halving
bcgcausality return_BTC return_ETH in 1/547
matrix list r(W)
bcgcausality return_BTC return_SP500 in 1/547
matrix list r(W)
bcgcausality return_BTC return_Gold in 1/547
matrix list r(W)
bcgcausality return_BTC Return_Bonds in 1/547
matrix list r(W)
bcgcausality return_ETH return_BTC in 1/547
matrix list r(W)
bcgcausality return_ETH return_SP500 in 1/547
matrix list r(W)
bcgcausality return_ETH return_Gold in 1/547
matrix list r(W)
bcgcausality return_ETH Return_Bonds in 1/547
matrix list r(W)
bcgcausality return_SP500 return_BTC in 1/547
matrix list r(W)
bcgcausality return_SP500 return_ETH in 1/547
matrix list r(W)
bcgcausality return_SP500 return_Gold in 1/547
```

```
matrix list r(W)
bcgcausality return_SP500 Return_Bonds in 1/547
matrix list r(W)
bcgcausality return_Gold return_BTC in 1/547
matrix list r(W)
bcgcausality return_Gold return_ETH in 1/547
matrix list r(W)
bcgcausality return_Gold return_SP500 in 1/547
matrix list r(W)
bcgcausality return_Gold Return_Bonds in 1/547
matrix list r(W)
bcgcausality Return_Bonds return_BTC in 1/547
matrix list r(W)
bcgcausality Return_Bonds return_ETH in 1/547
matrix list r(W)
bcgcausality Return_Bonds return_SP500 in 1/547
matrix list r(W)
bcgcausality Return_Bonds return_Gold in 1/547
matrix list r(W)
//Spectral Causalities Post-Halving
bcgcausality return_BTC return_ETH in 548/1587
matrix list r(W)
bcgcausality return_BTC return_SP500 in 548/1587
matrix list r(W)
bcgcausality return_BTC return_Gold in 548/1587
matrix list r(W)
bcgcausality return_BTC Return_Bonds in 548/1587
matrix list r(W)
bcgcausality return_ETH return_BTC in 548/1587
matrix list r(W)
```

```
bcgcausality return_ETH return_SP500 in 548/1587

matrix list r(W)

bcgcausality return_ETH return_Gold in 548/1587

matrix list r(W)

bcgcausality return_ETH Return_Bonds in 548/1587

matrix list r(W)

bcgcausality return_SP500 return_BTC in 548/1587

matrix list r(W)

bcgcausality return_SP500 return_ETH in 548/1587

matrix list r(W)

bcgcausality return_SP500 return_Gold in 548/1587

matrix list r(W)

bcgcausality return_SP500 Return_Bonds in 548/1587

matrix list r(W)

bcgcausality return_Gold return_BTC in 548/1587

matrix list r(W)

bcgcausality return_Gold return_ETH in 548/1587

matrix list r(W)

bcgcausality return_Gold return_SP500 in 548/1587

matrix list r(W)

bcgcausality return_Gold Return_Bonds in 548/1587

matrix list r(W)

bcgcausality Return_Bonds return_BTC in 548/1587

matrix list r(W)

bcgcausality Return_Bonds return_ETH in 548/1587

matrix list r(W)

bcgcausality Return_Bonds return_SP500 in 548/1587

matrix list r(W)

bcgcausality Return_Bonds return_Gold in 548/1587

matrix list r(W)
```

Appendix II: "Read me"

- Step 1: Upload the dataset into **STATA** (double-click the filename.dta).

- Step 2: Run each part of the code separately so you can transform the **STATA** output into Tables or Figures. Note that each command should be submitted separately, i.e. do not run multiple commands on the same line.

- Step 3: When running spectral causality (bcgcausality), after matrix list r(W) inspect the frequencies and identify the ranges where the p-value is below 0.10 to determine the designated radians (ω) intervals.