Optimization - Project 1

Minimization of concave function of one variable in a given interval

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Contents

1 Introduction

In this work it is requested to implement the algorithms:

- Without using a derivative:
  - Dichotomous Method,
  - Golden section Method,
  - Fibonacci method.
- Using derivatives:
  - Dichotomous method using derivative.

The functions to which we will apply the algorithms are:

- $f_1(x) = (x-2)^2 + xln(x+3)$
- $f_2(x) = 5^x + (2 - cos(x))^2$
- $f_3(x) = e^x(x^3 - 1) + (x-1)sin(x)$

, and the interval of interest is $[-1, 3]$.

Before applying the algorithms, we draw and observe the graphs of the functions to confirm graphically that they are convex (stated in the speech) and to see where the bottlenecks are so that we can understand whether the algorithms are working correctly in continuity.

The implementation has been done in matlab and algorithms can be found respectively in the files dichotomous.m, golden_section.m, fib_min.m, dich_diff.m. The requested tests and graphs were performed in the task{x}.m files, while the fib.m file is auxiliary, and calculates the terms of the Fibonacci sequence.

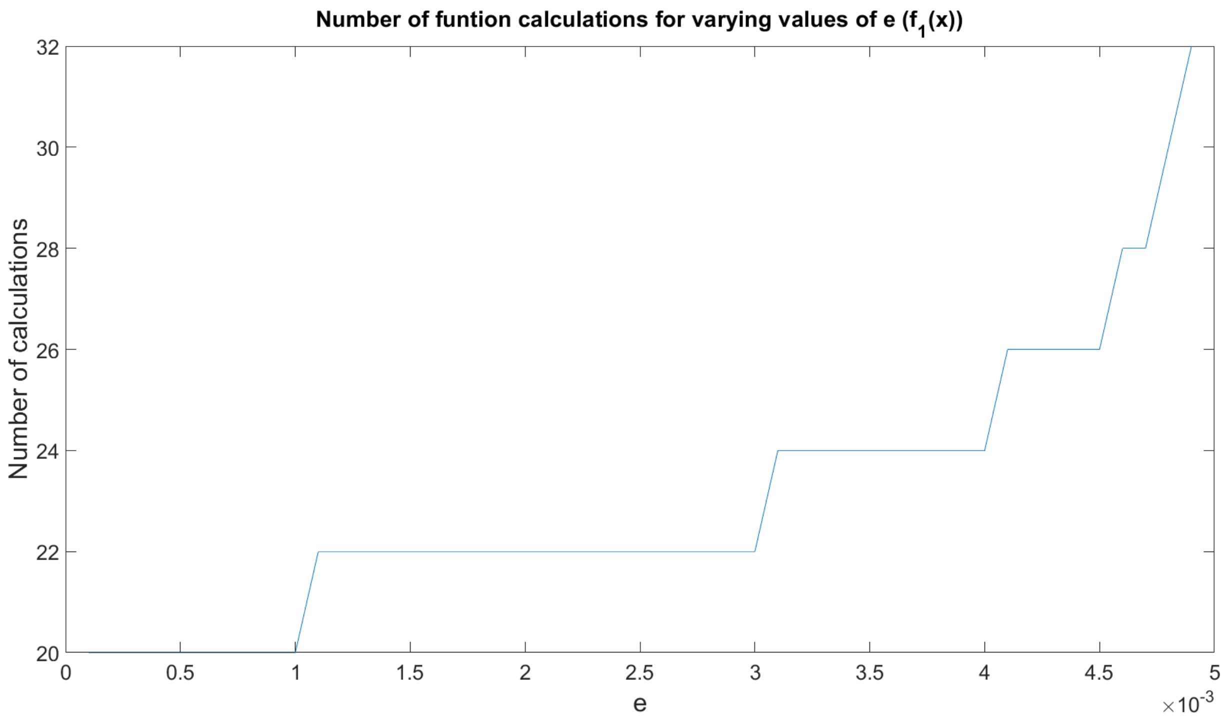
2 Topic 1 - Bisection Method

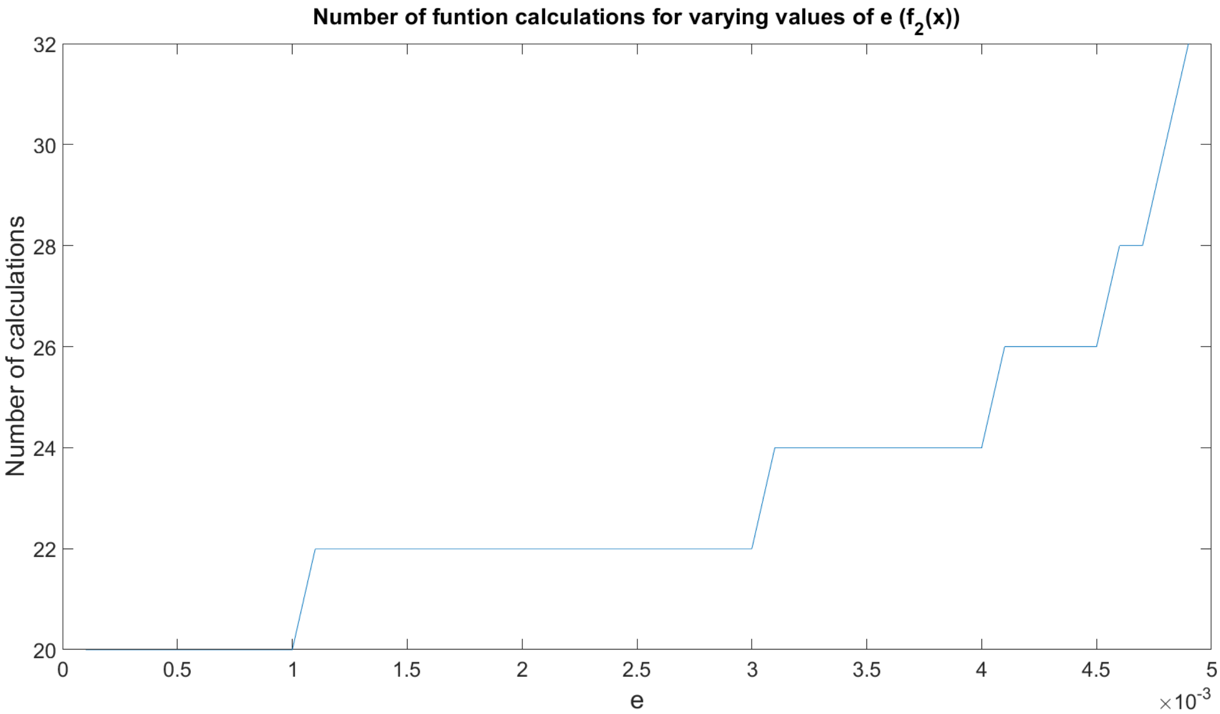
First, having implemented the bisection algorithm in matlab I run it for the 3 functions. The parameters I use are $l = 0.01$ & $\varepsilon = 0.001$, for which the necessary condition $\varepsilon > 2l$ is satisfied. The final intervals reached by the algorithm, to which the minimum of each function belongs, are the following:

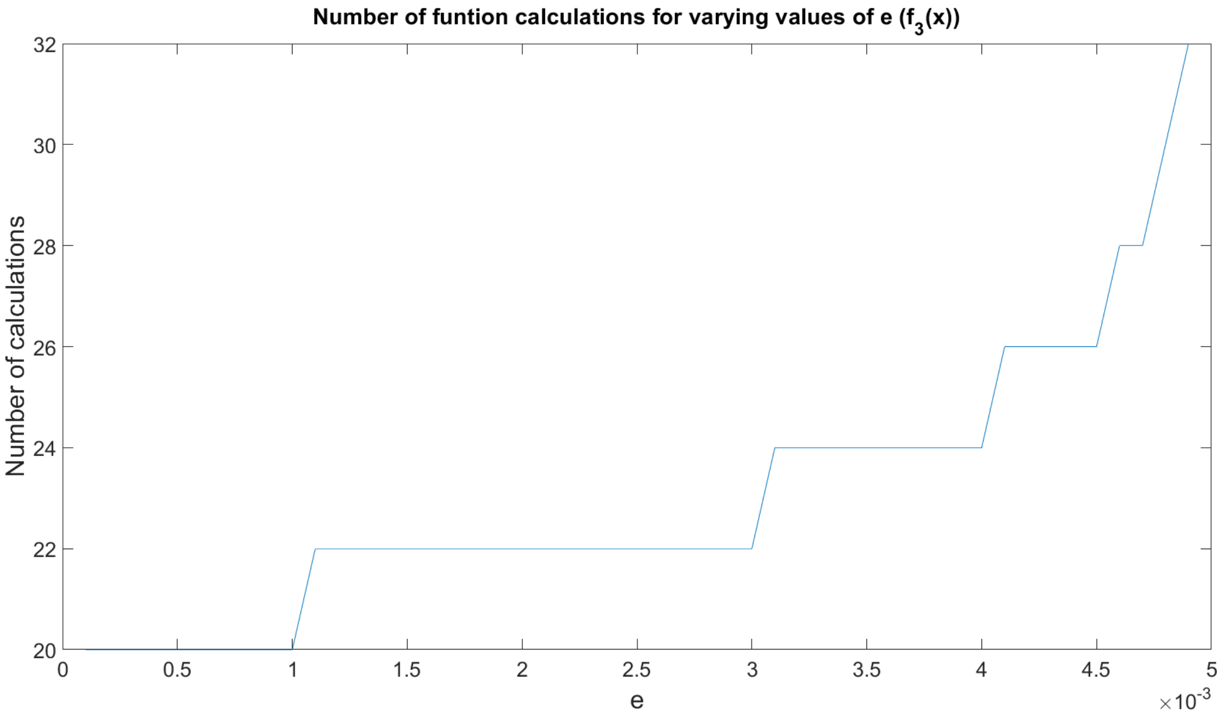
- $f_1(x): \ [1.1474, 1.1572]$
- $f_2(x): \ [-0.4065, -0.3967]$
- $f_3(x): \ [0.5149, 0.5247]$

Then, by keeping the interval range $l = 0.01$ constant and varying ε, we are asked to record the corresponding change in the total number of calculations of the objective function $f_i(x)$. We achieve the recording of the specific size by taking the number of repetitions of the algorithm, and in this particular case by doubling it. For the purposes of this experiment, values in the interval $[0.0001, 0.0049]$ (with a step of 0.0001) were used, so that the condition $\varepsilon > 2l$ is always respected.

For all 3 functions, identical behavior was observed, with increasing ε leading to a gradual increase in the objective function computations as well. The resulting graphs are presented below.

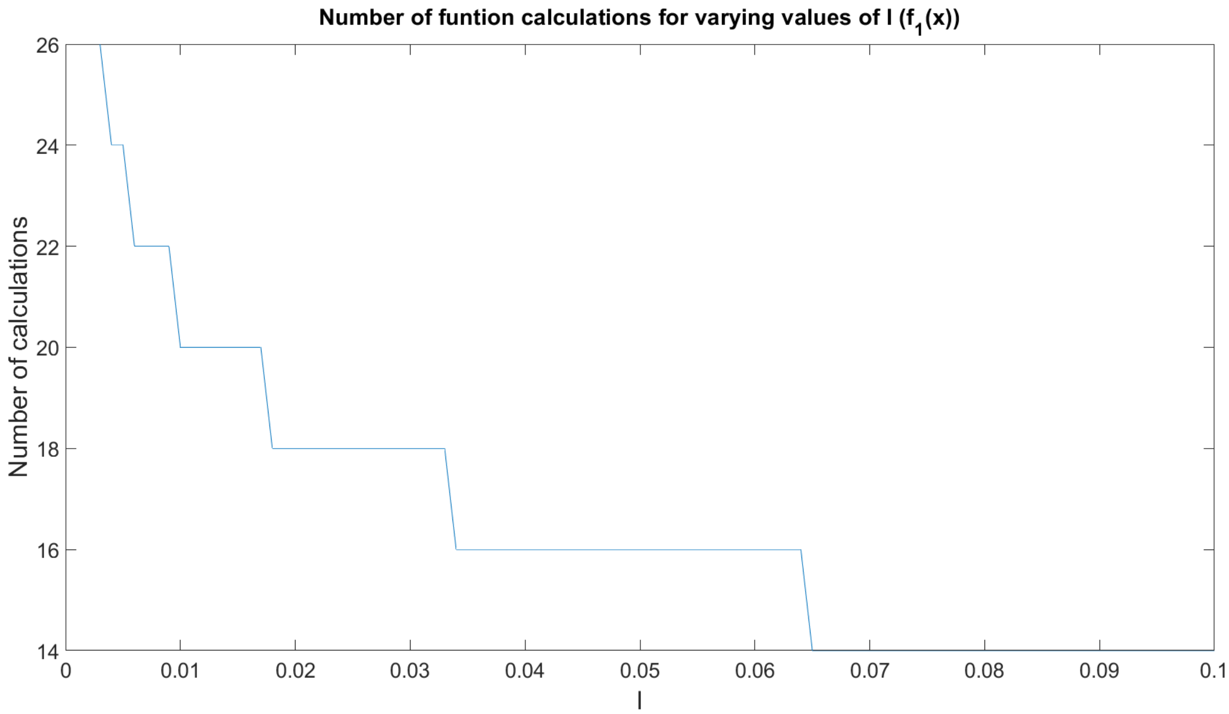


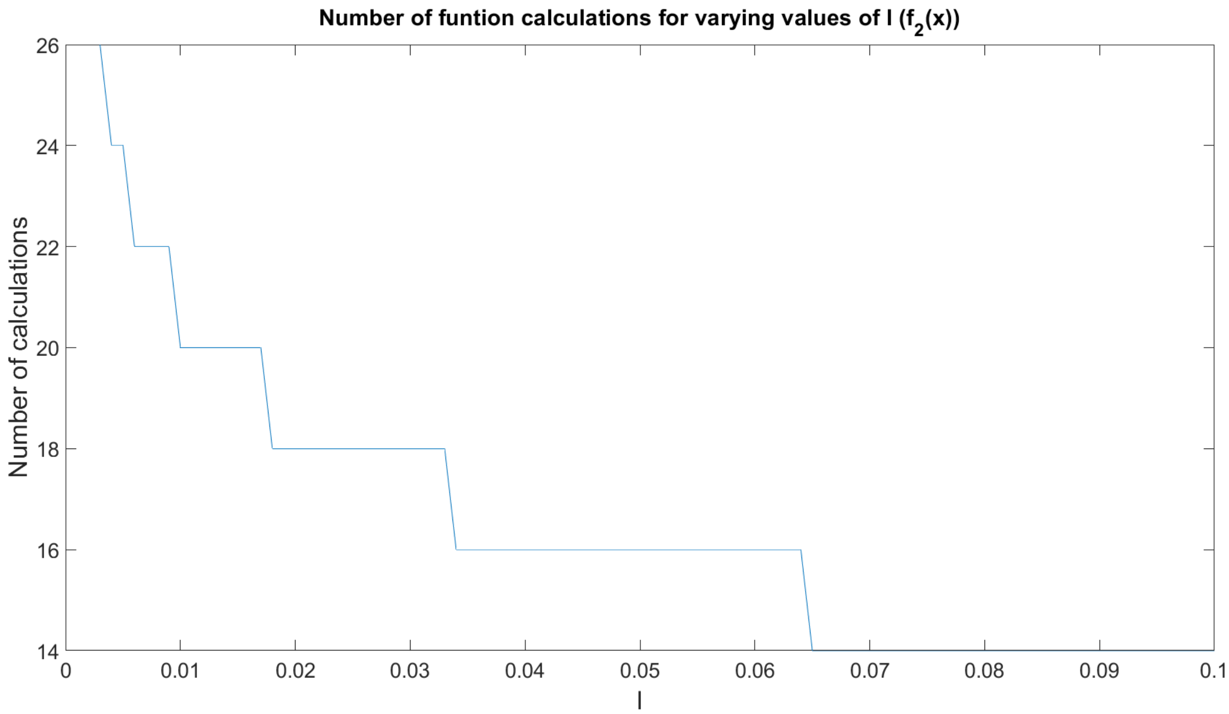


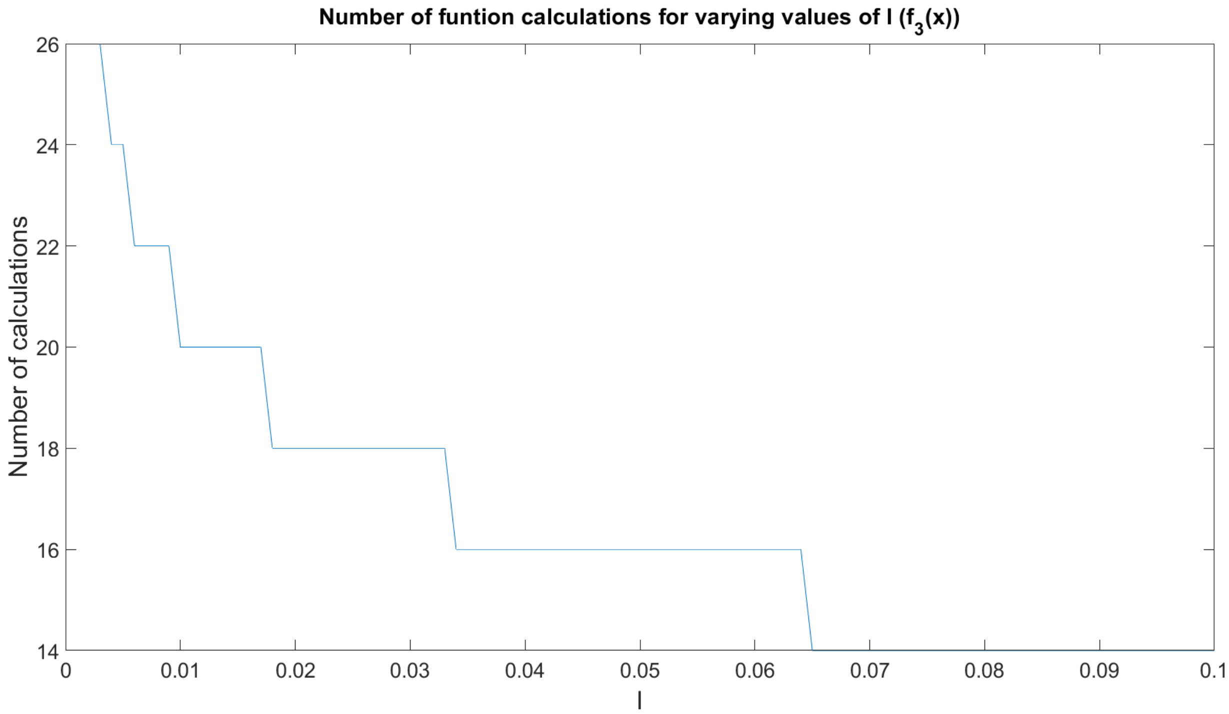


The next test we are asked to do is the reverse of the one above. This time having fixed $\varepsilon = 0.001$ we vary l and create the corresponding graphs of the objective function calculations. We choose for l the interval $[0.003, 0.1]$ (with a step of 0.0001), so that, as above, the condition $\varepsilon > 2l$ is always respected.

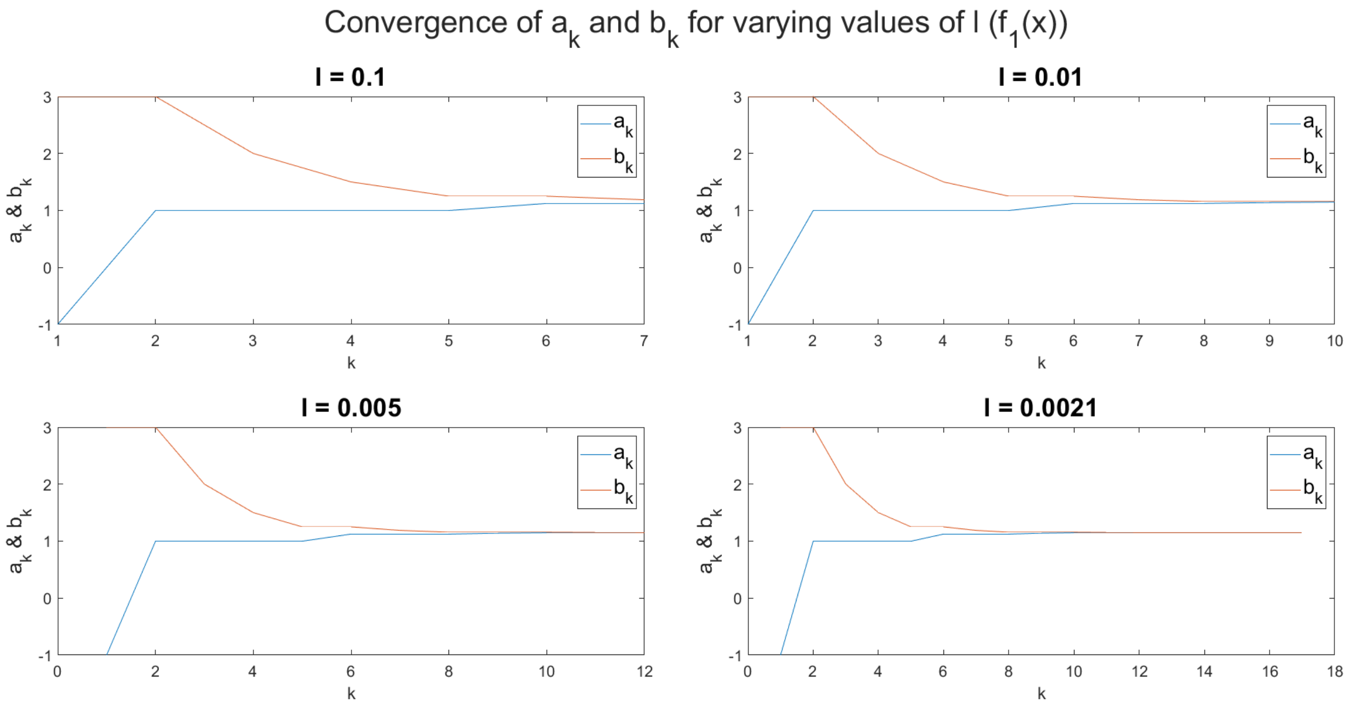
Again the behavior for all 3 functions is identical, but, as expected, for smaller values of l, more iterations of the algorithm are needed and therefore more calculations of $f_i(x)$. Here are the graphs that emerged.

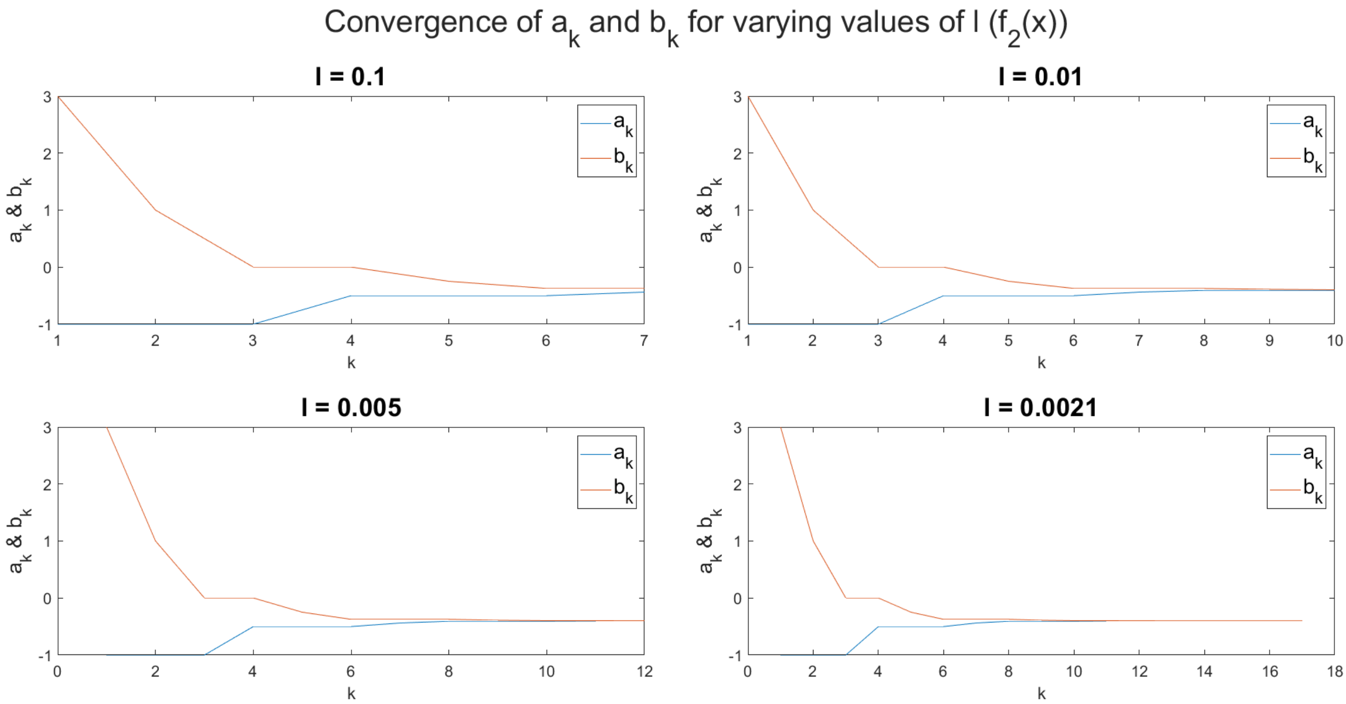


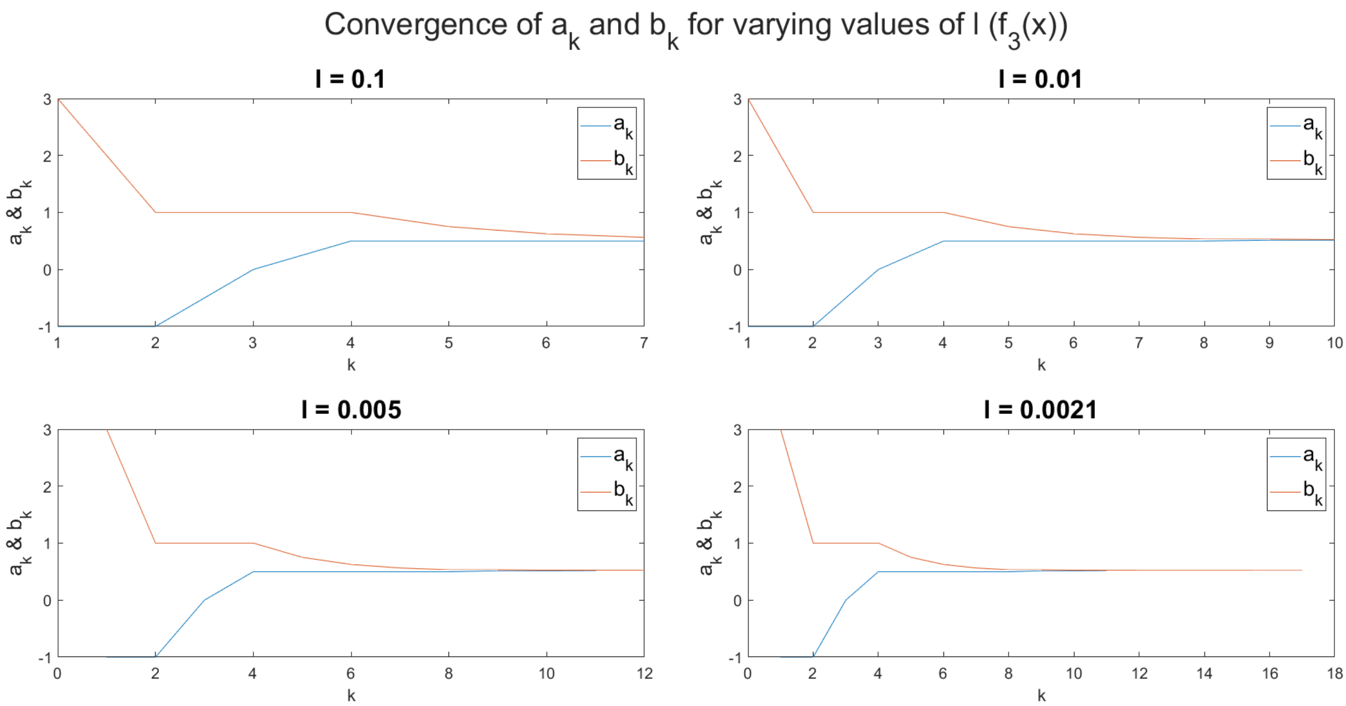




Finally, below we show the behavior/convergence of the edges $[a_k, b_k]$ of the interval of the minimum for different values of l. Obviously, the only resulting difference, for a given function $f_i(x)$, is the number of iterations k to achieve a smaller interval l.




Convergence of a_k and b_k for varying values of l ($f_2(x)$)
l = 0.1
l = 0.01
l = 0.005
l = 0.0021
a_k & b_k
k
a_k
b_k


Convergence of a_k and b_k for varying values of l ($f_3(x)$)
l = 0.1
l = 0.01
l = 0.005
l = 0.0021
a_k & b_k
k
a_k
b_k

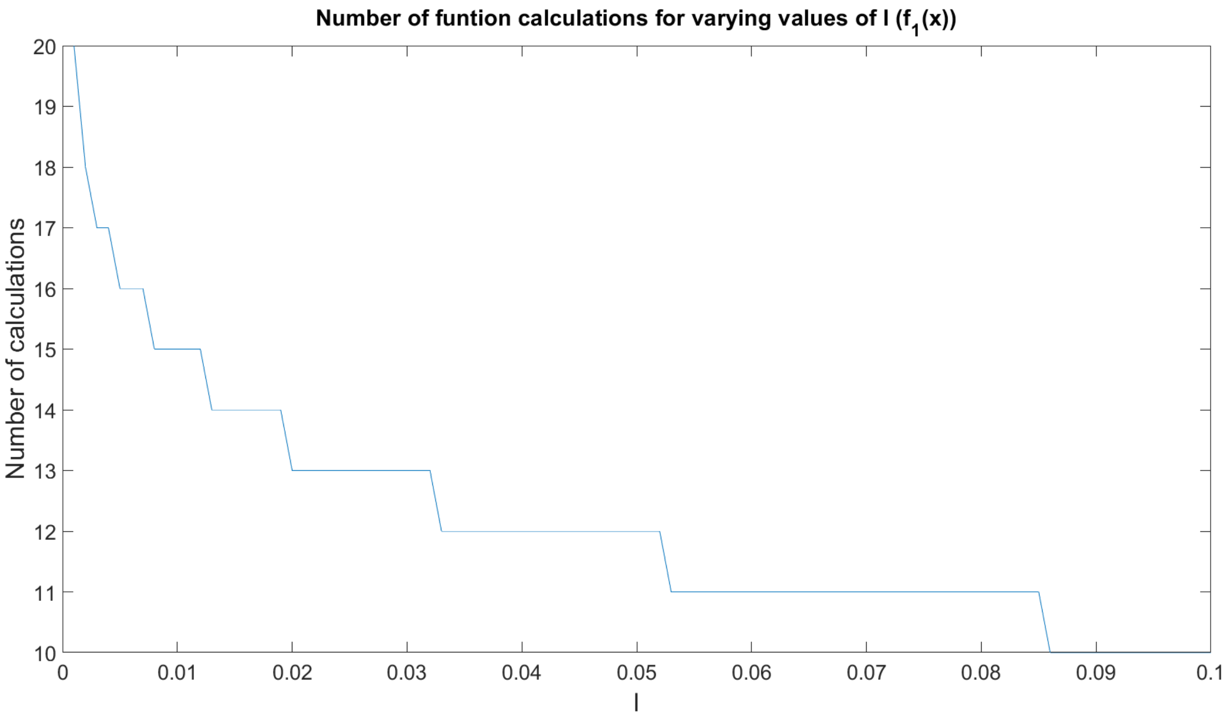
3 Topic 2 - Golden Section Method

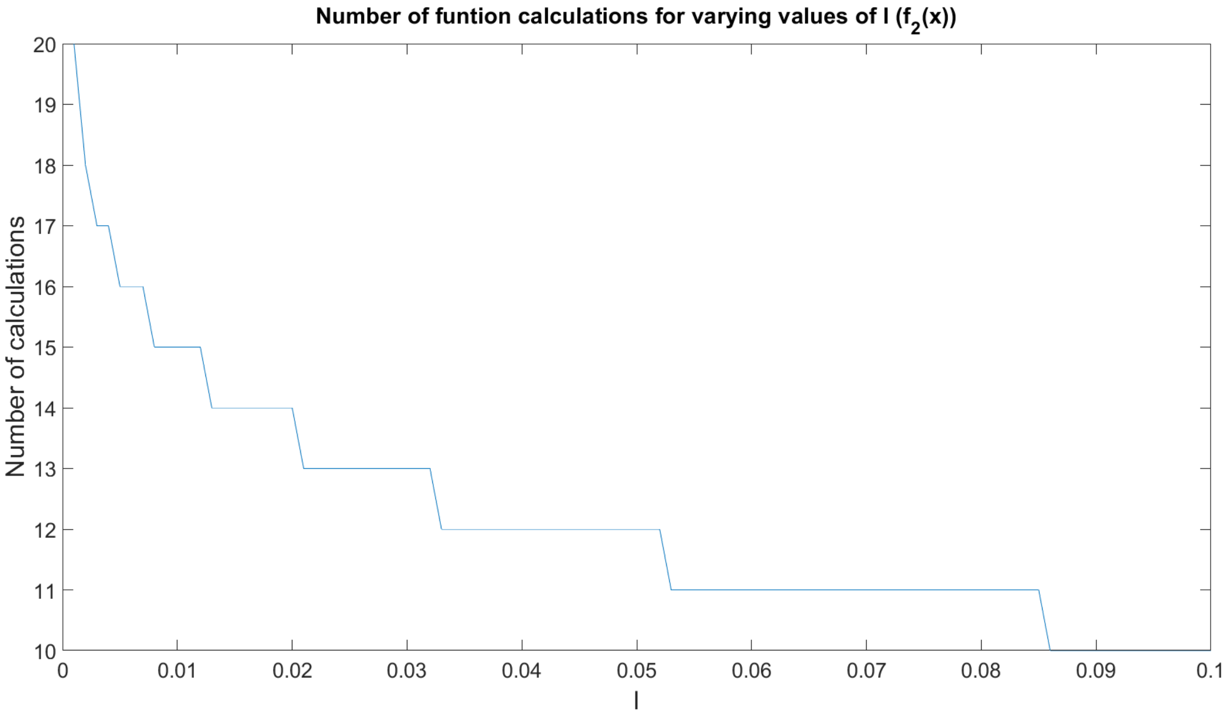
In the first phase, we apply the algorithm with $l = 0.01$ to calculate the edges of the interval to which the minimum of each function belongs.

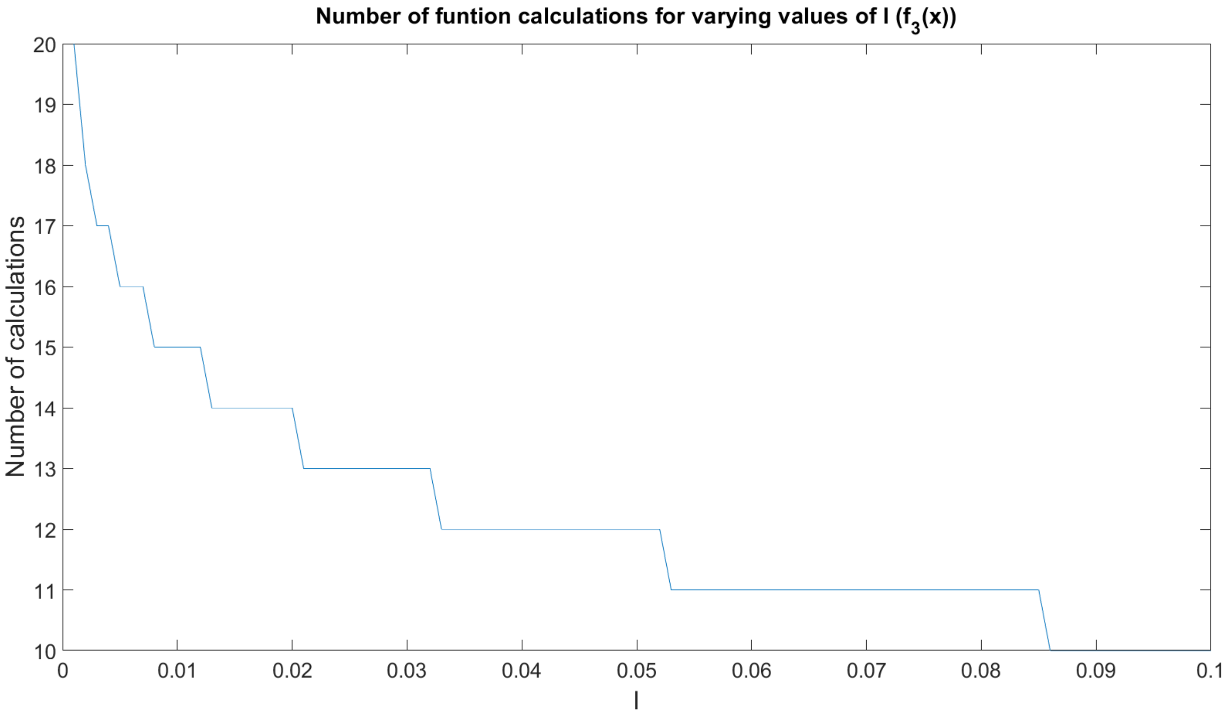
- $f_1(x):\ [1.1441, 1.1516]$
- $f_2(x):\ [-0.4039, -0.3963]$
- $f_3(x):\ [0.5154, 0.5232]$

Then we vary l by giving it values in the interval $[0.001, 0.1]$ (with a step of 0.001), in order to record the number of calculations of the objective function. This is obtained by adding the unit to the number of iterations ($n = k + 1$), since after the first iteration, calculation of $f_i(x)$ is needed at just one point.

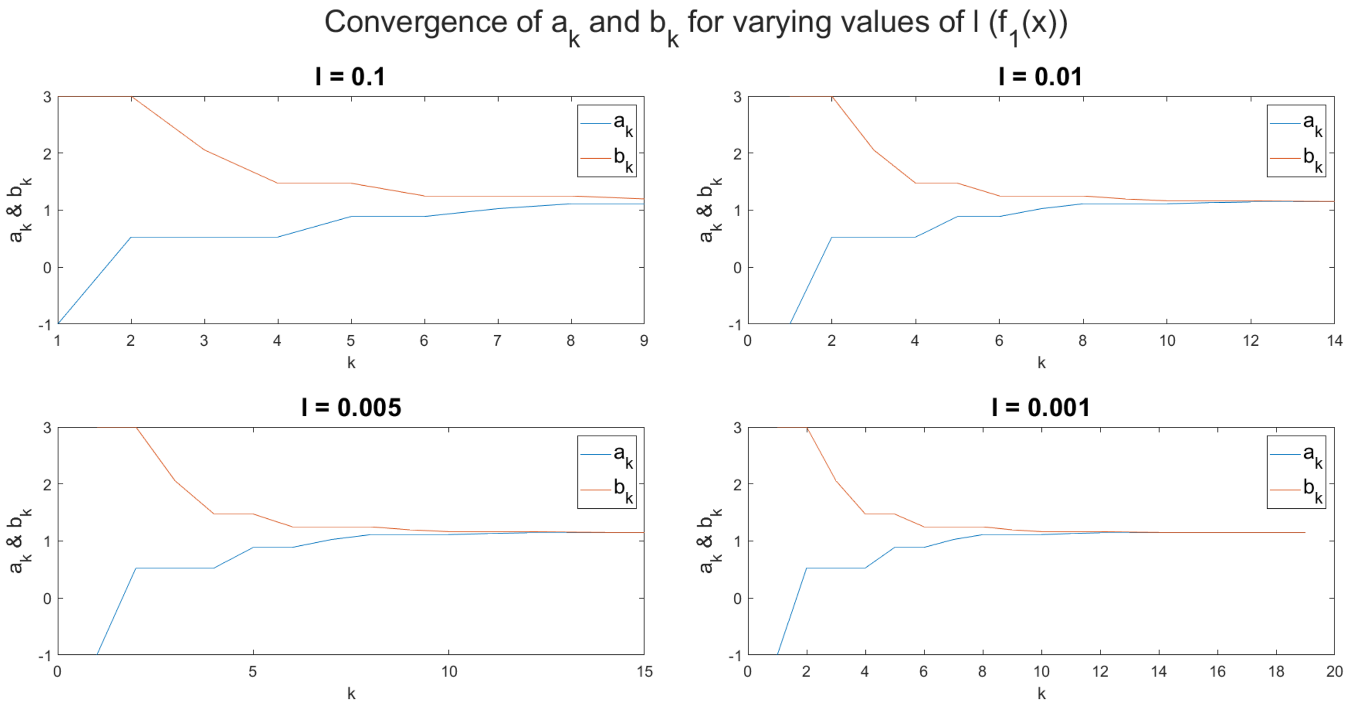
For all 3 functions, identical behavior was observed, with increasing l leading to a gradual decrease in the required objective function computations. The resulting graphs are presented below.

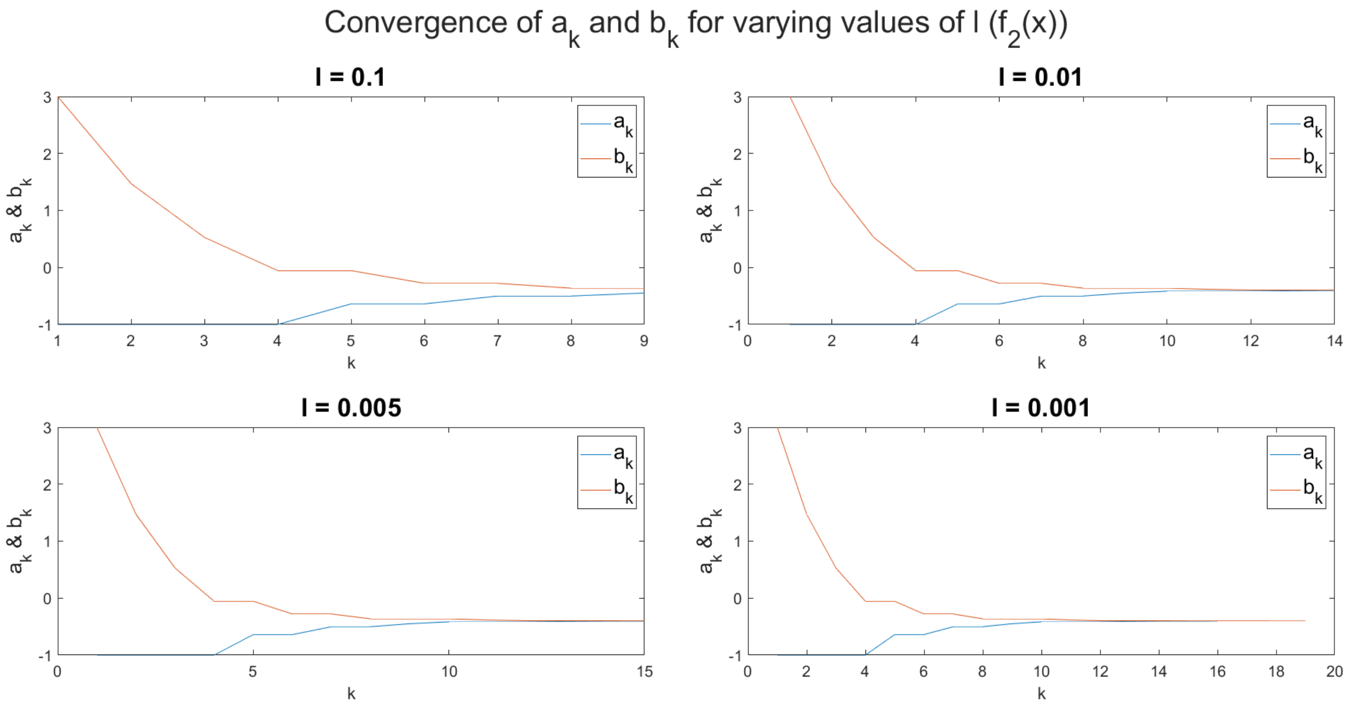


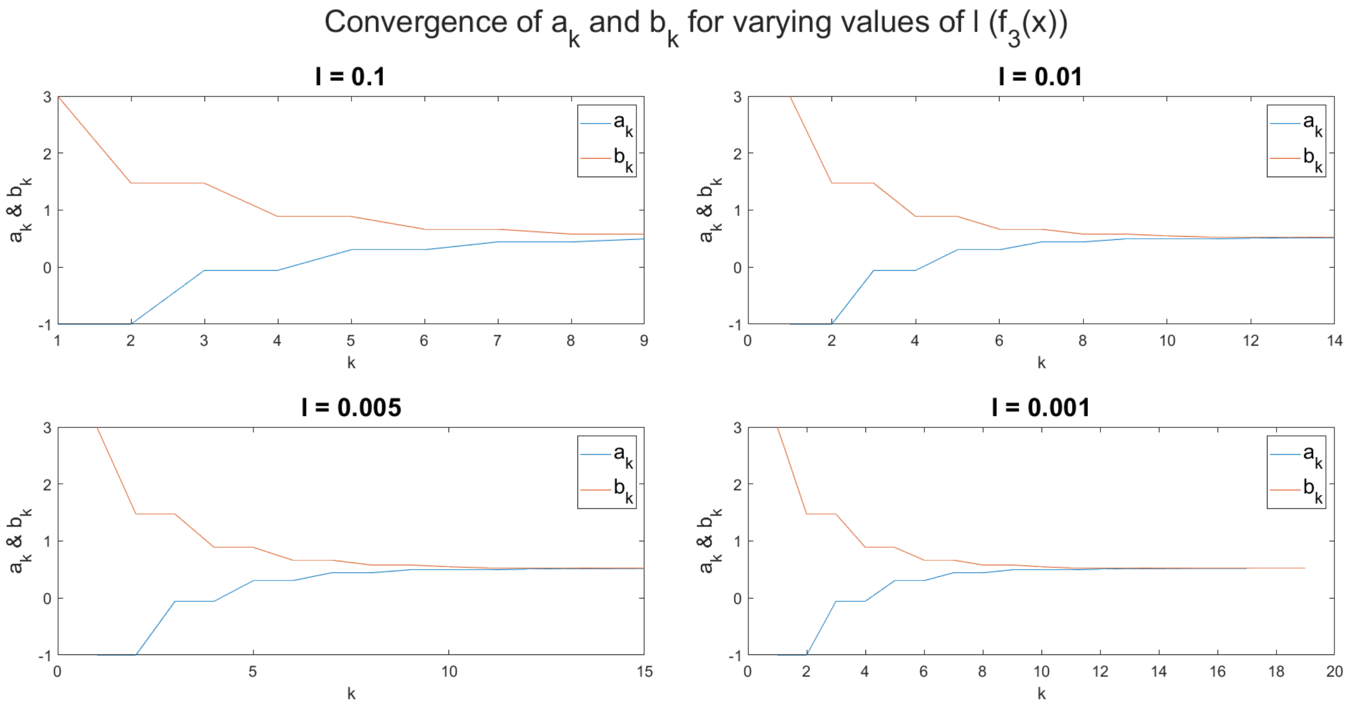




Finally, below we show the behavior/convergence of the edges $[a_k, b_k]$ of the interval of the minimum for different values of l. As above, the only difference that arises, for a given function $f_i(x)$, is the number of iterations k to achieve a smaller interval l, while in the first iterations an absolute identity is observed.


Convergence of a_k and b_k for varying values of l ($f_1(x)$)
l = 0.1
l = 0.01
l = 0.005
l = 0.001
a_k & b_k
k
a_k
b_k


Convergence of a_k and b_k for varying values of l ($f_2(x)$)
l = 0.1
l = 0.01
l = 0.005
l = 0.001
a_k & b_k
k
a_k
b_k


Convergence of a_k and b_k for varying values of l ($f_3(x)$)
l = 0.1
l = 0.01
l = 0.005
l = 0.001
a_k
b_k
a_k & b_k
k

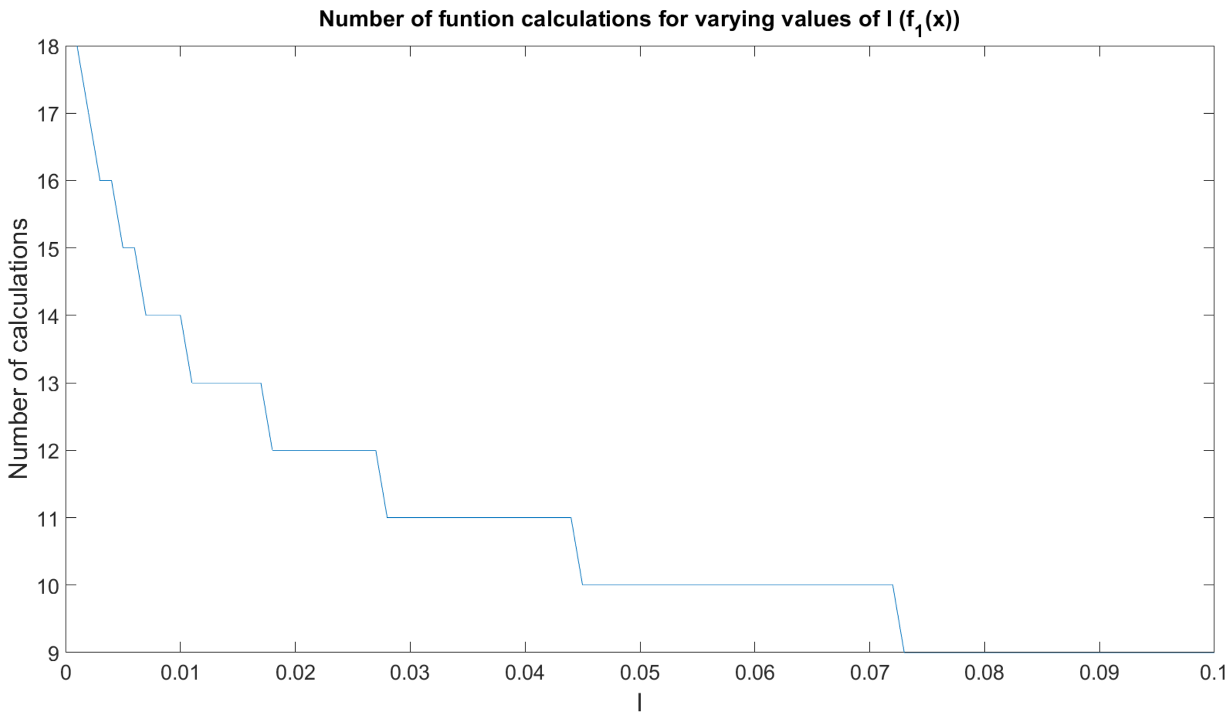
4 Topic 3 - Fibonacci Method

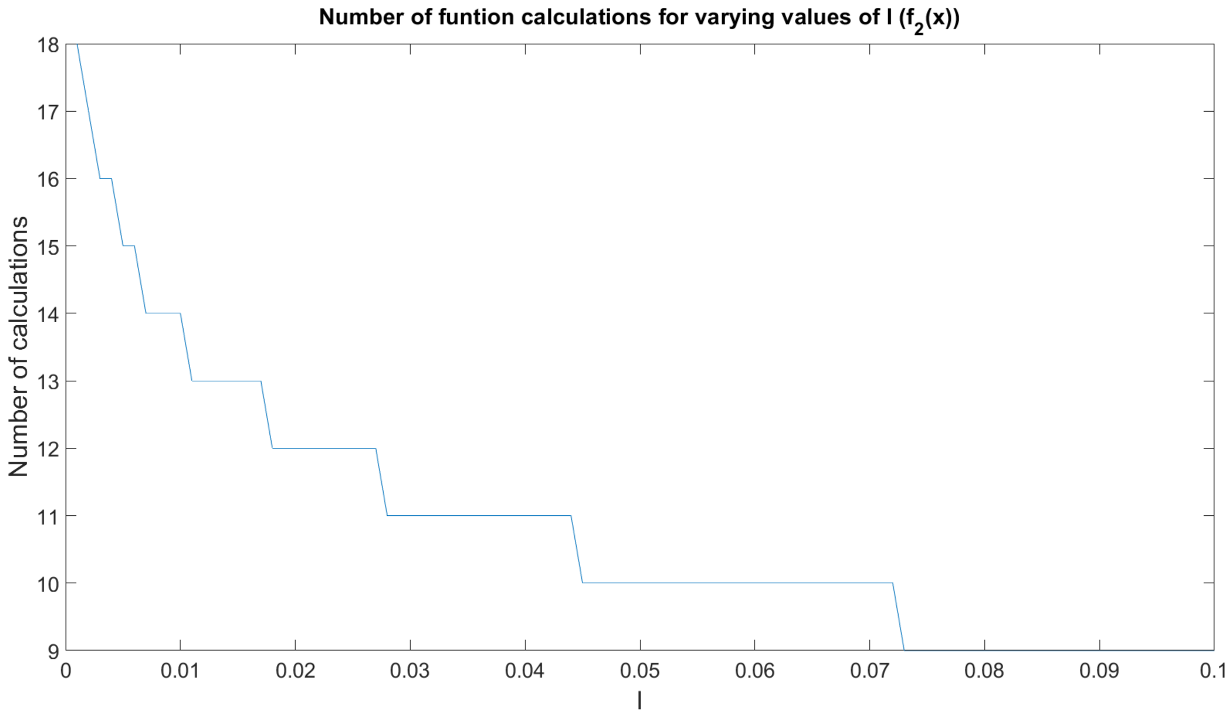
First, having implemented the Fibonacci algorithm in matlab I run it for the 3 functions. The parameters I use are $l = 0.01 \,\&\, \varepsilon = 0.001$. The final intervals reached by the algorithm, to which the minimum of each function belongs, are the following:

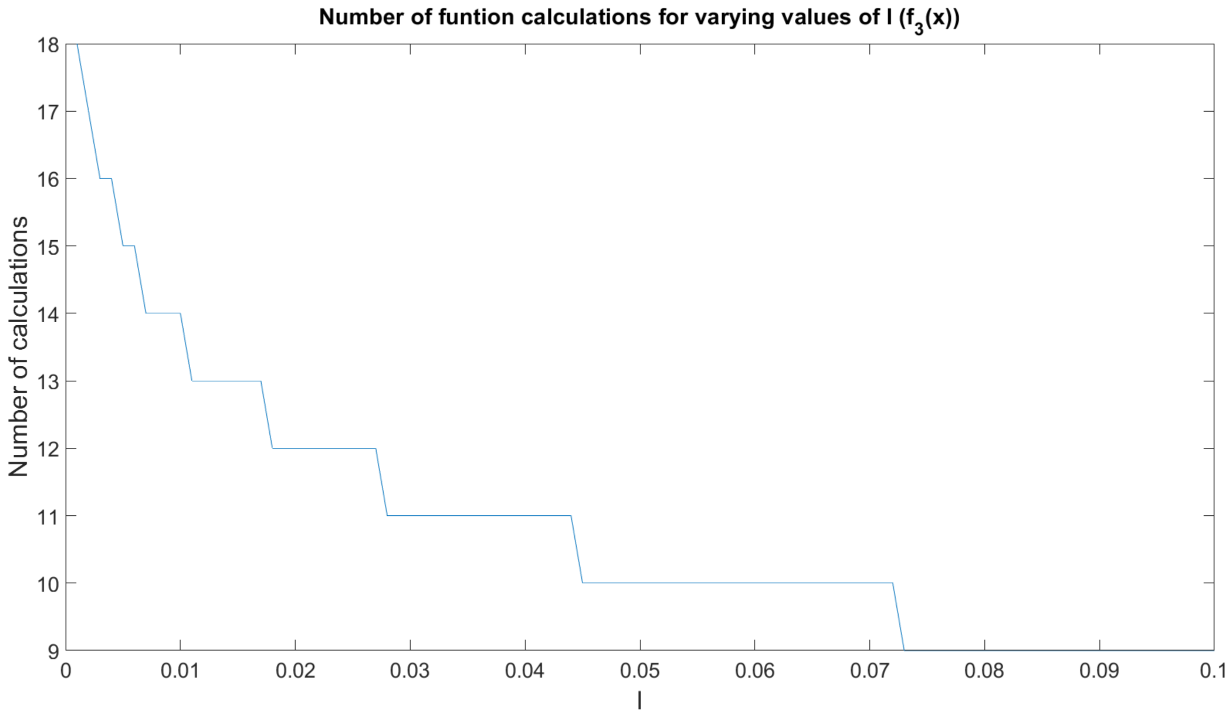
- $f_1(x): \ [1.1443, 1.1518]$
- $f_2(x): \ [-0.4033, -0.3967]$
- $f_3(x): \ [0.5148, 0.5223]$

Then we vary l by giving it values in the interval $[0.001, 0.1]$ (with a step of 0.001), keeping constant $\varepsilon = 0.0001$, in order to record the number of calculations of the objective function. This corresponds to the number of iterations ($n = k$).

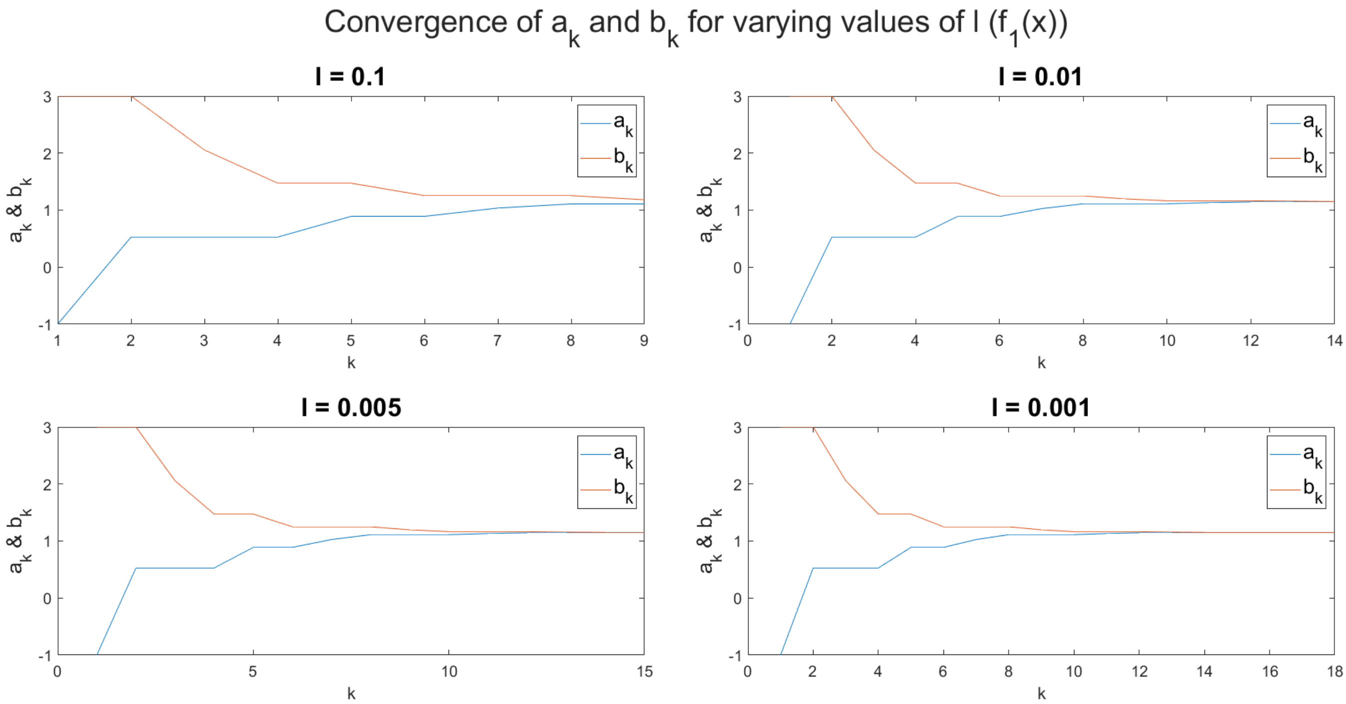
For all 3 functions, identical behavior was observed, with increasing l leading to a gradual decrease in the required objective function computations. The resulting graphs are presented below.

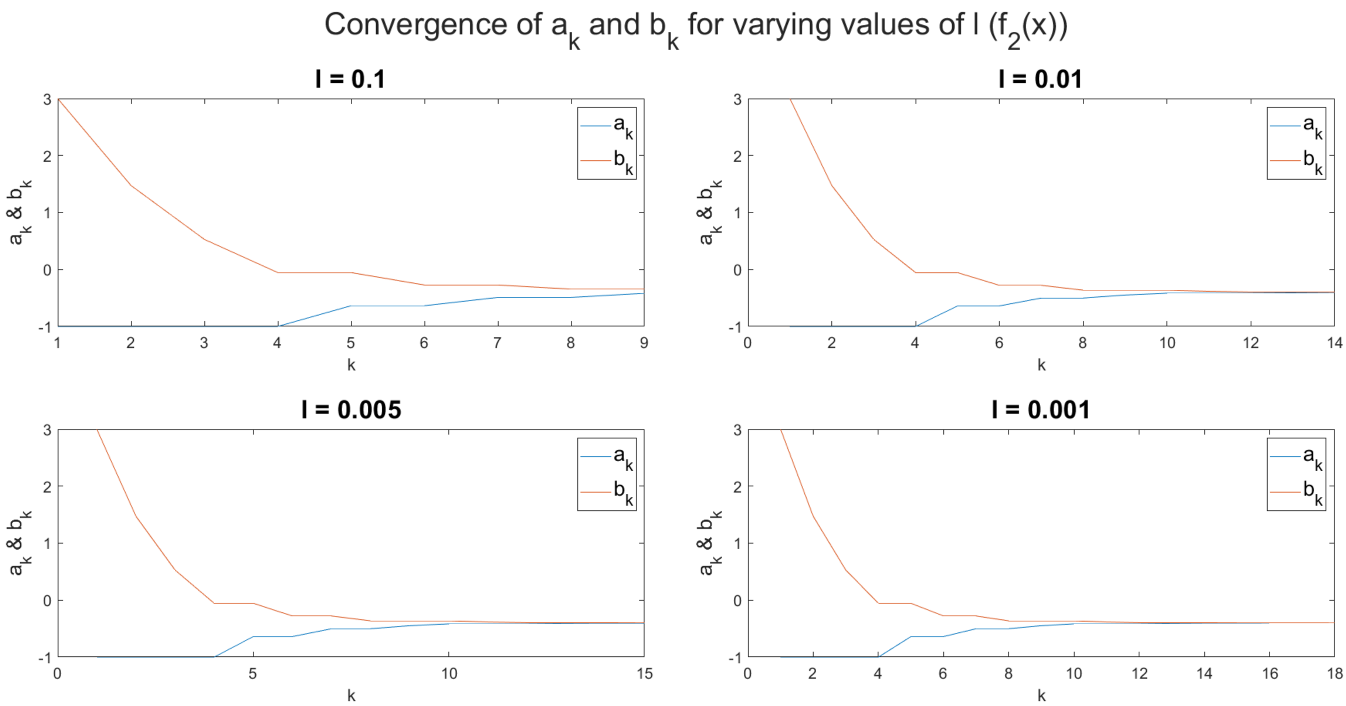


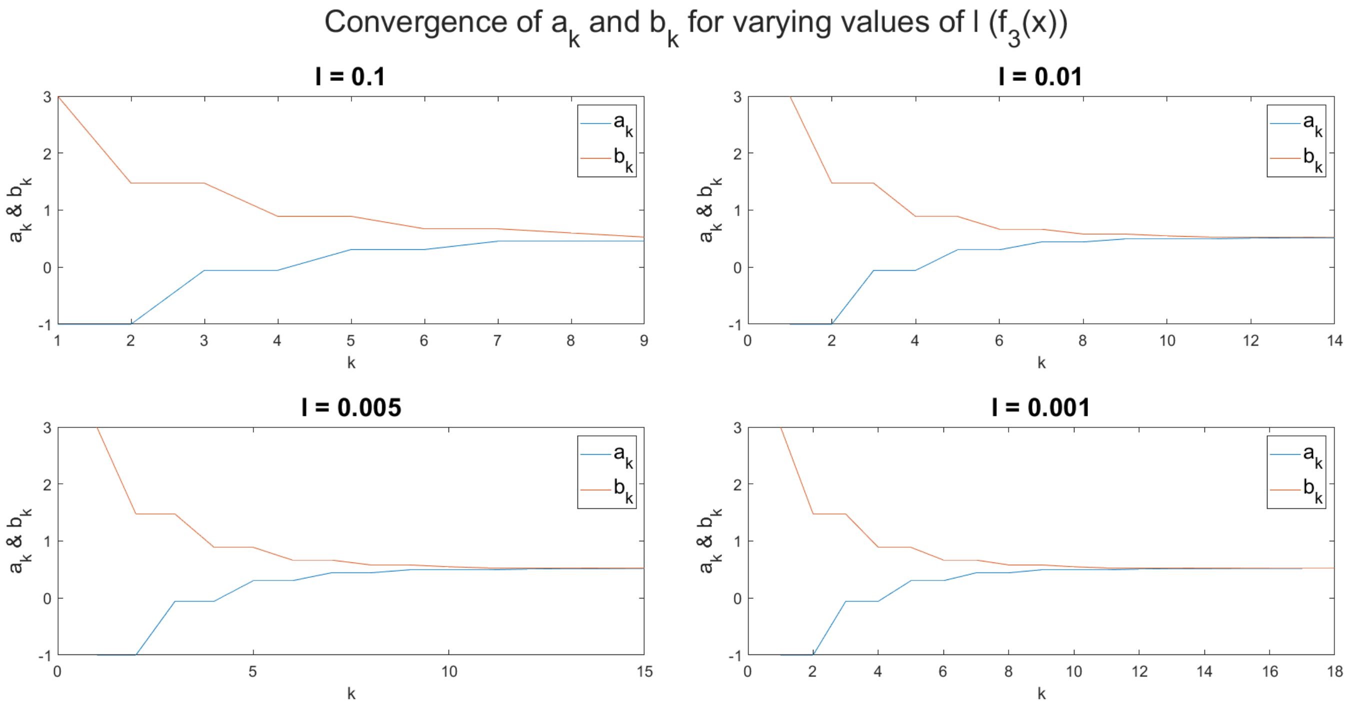




Finally, below we show the behavior/convergence of the edges $[a_k, b_k]$ of the interval of the minimum for different values of l. Once again, the only difference that arises, for a given function $f_i(x)$, is the number of iterations k to achieve a smaller interval l, while in the first iterations an absolute identity is observed.


Convergence of a_k and b_k for varying values of l ($f_1(x)$)
l = 0.1
l = 0.01
l = 0.005
l = 0.001
a_k
b_k
a_k & b_k
k


Convergence of a_k and b_k for varying values of l ($f_2(x)$)
l = 0.1
l = 0.01
l = 0.005
l = 0.001
a_k
b_k
a_k & b_k
k



Convergence of a_k and b_k for varying values of l (f_3(x))
l = 0.1
l = 0.01
l = 0.005
l = 0.001
a_k & b_k
k
a_k
b_k

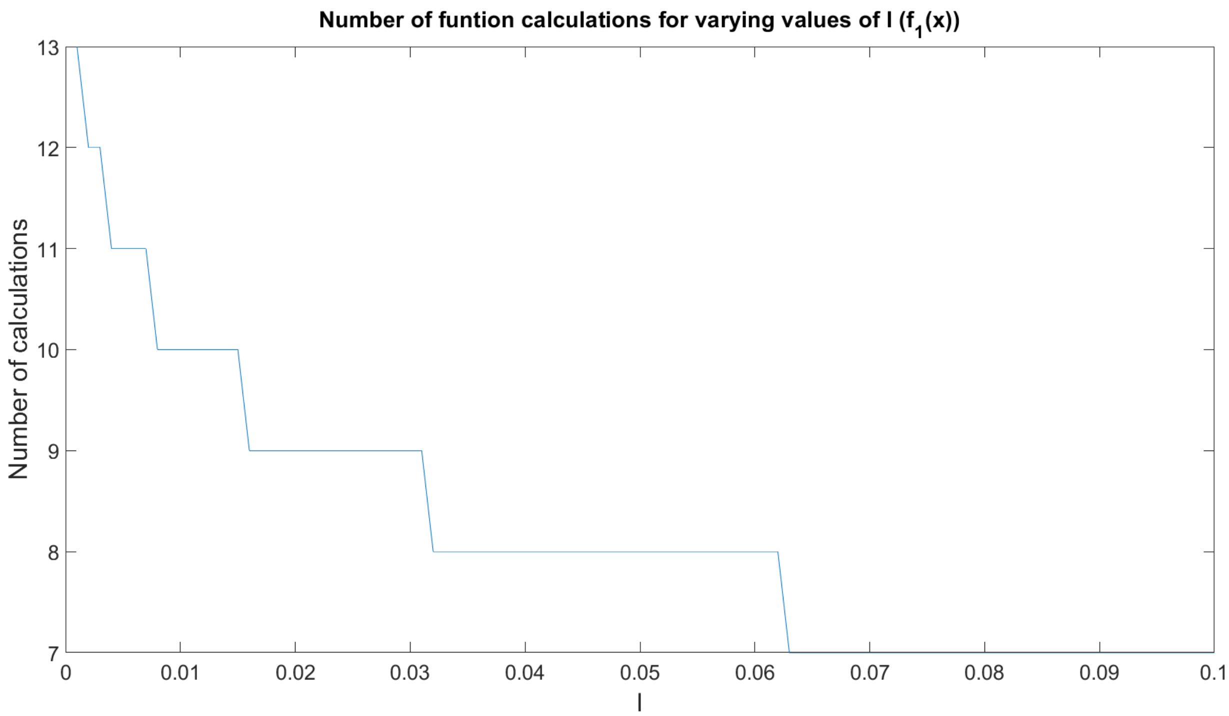
5 Topic 4 - Bisection method using derivative

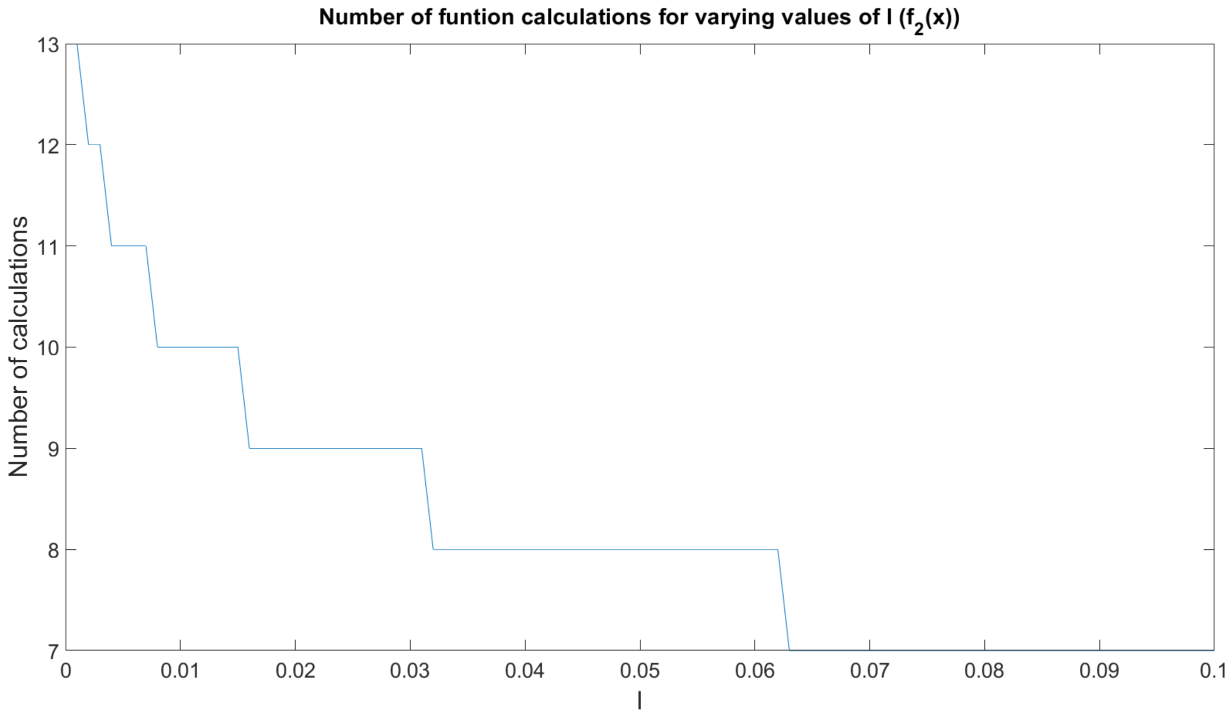
We apply the above procedure one last time. First, we apply the bisection algorithm using a derivative to the 3 functions to find the intervals to which the minima belong. For this purpose we set $l = 0.01$, and we have:

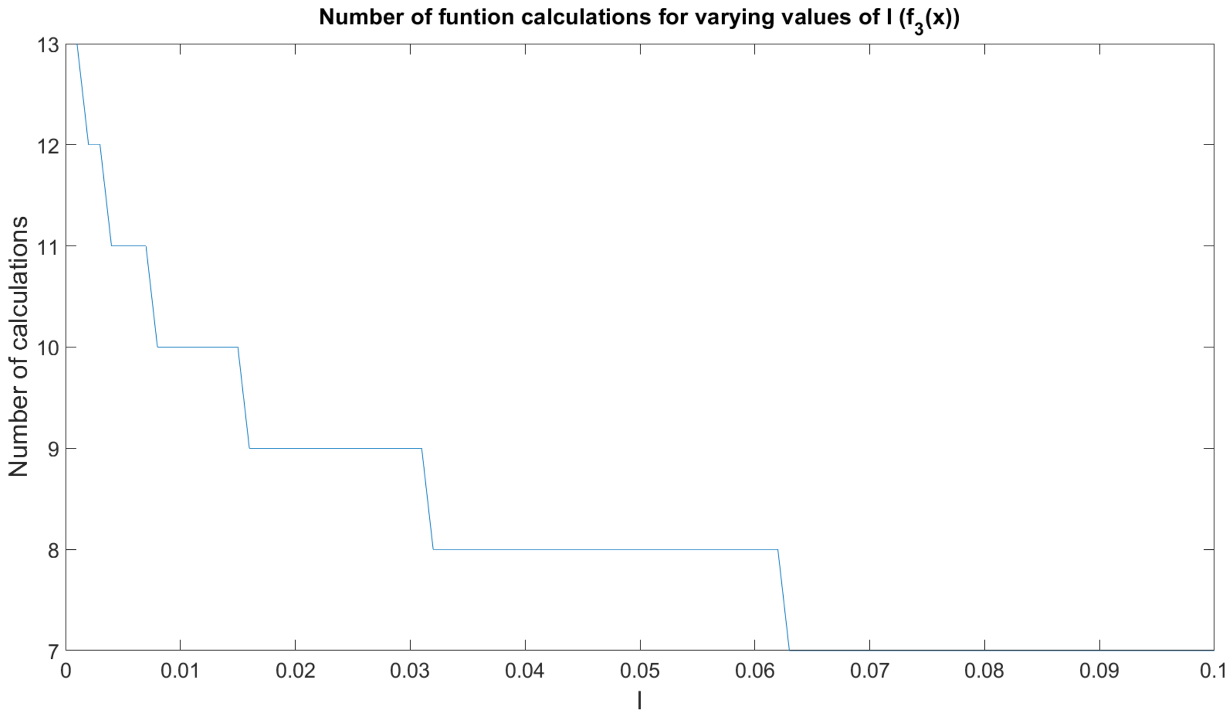
- $f_1(x): \ [1.1441, 1.1516]$
- $f_2(x): \ [-0.4039, -0.3963]$
- $f_3(x): \ [0.5154, 0.5232]$

Continuing, we try values of l in the interval $[0.001, 0.1]$ (with a step of 0.001), in order to study the variation of the objective function calculations required. In this particular algorithm, the calculations are identical to the repetitions since a calculation is made in each repetition.

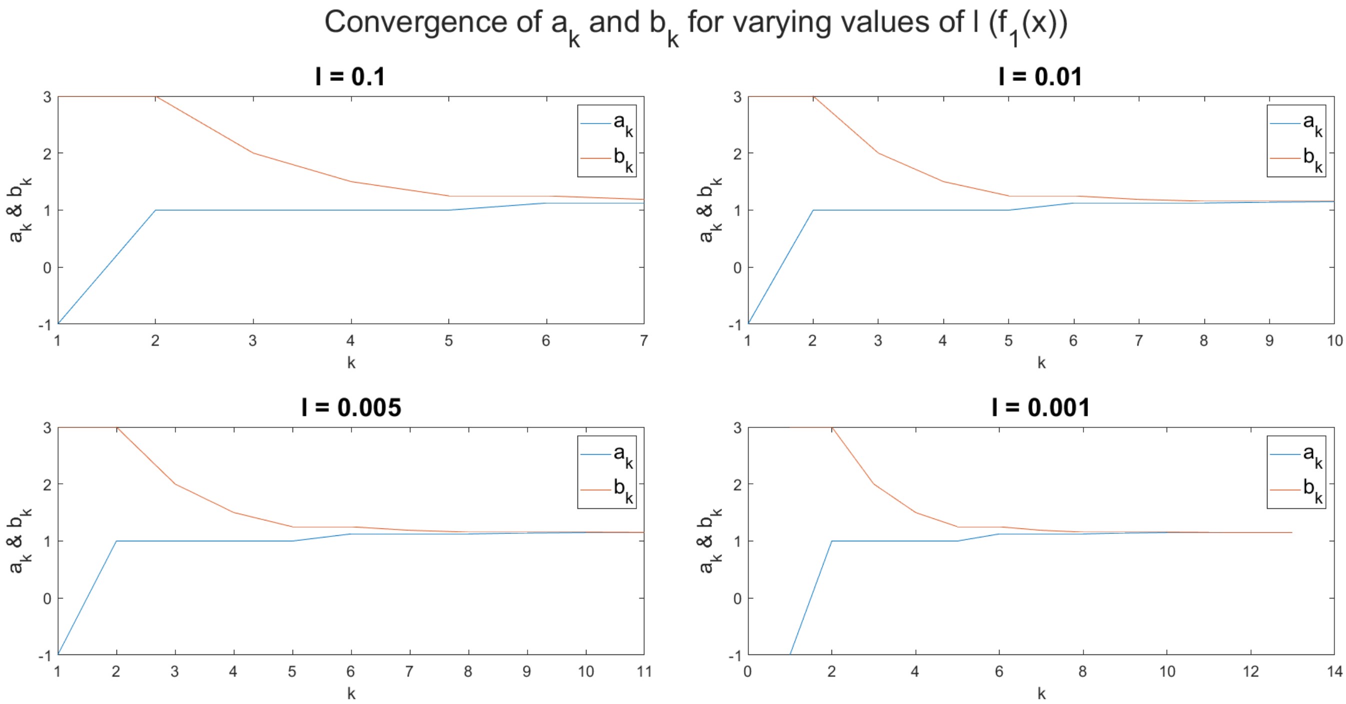
We don't notice any difference in behavior for the 3 functions, but the larger l becomes, i.e. the "error" we allow, the less computations are required.

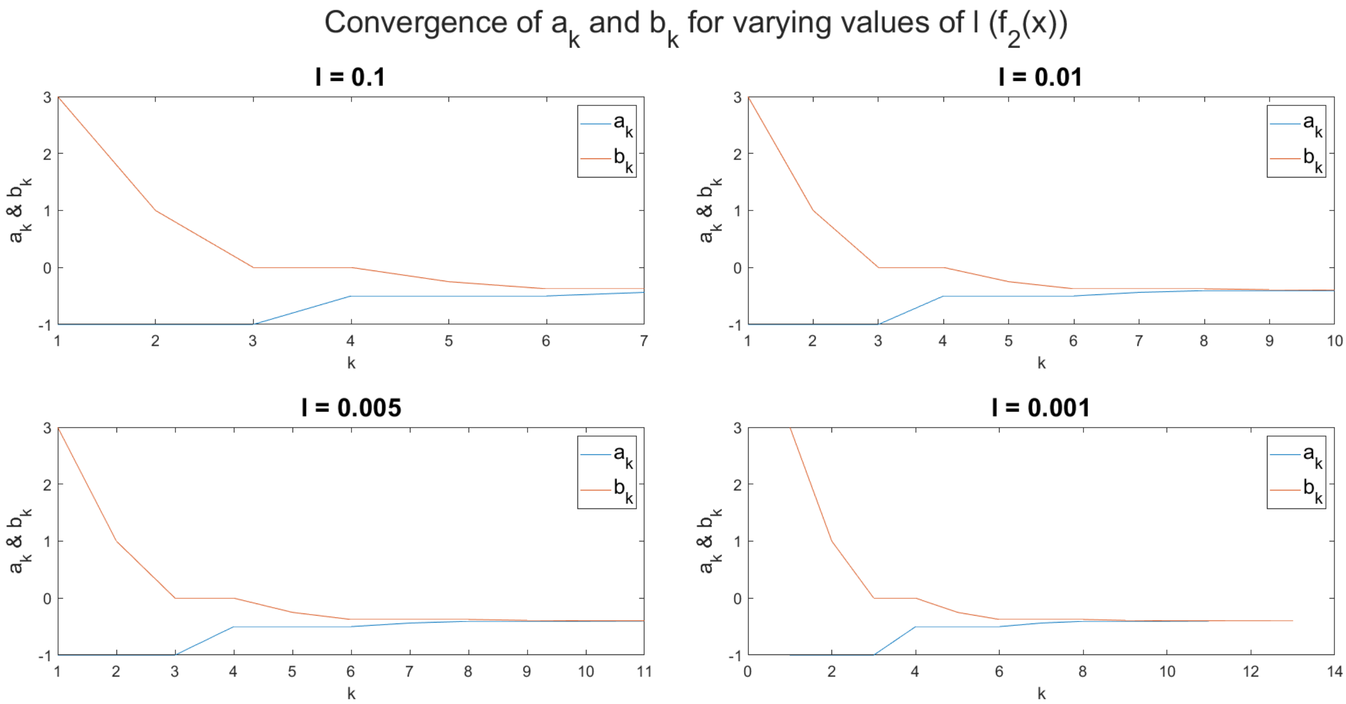


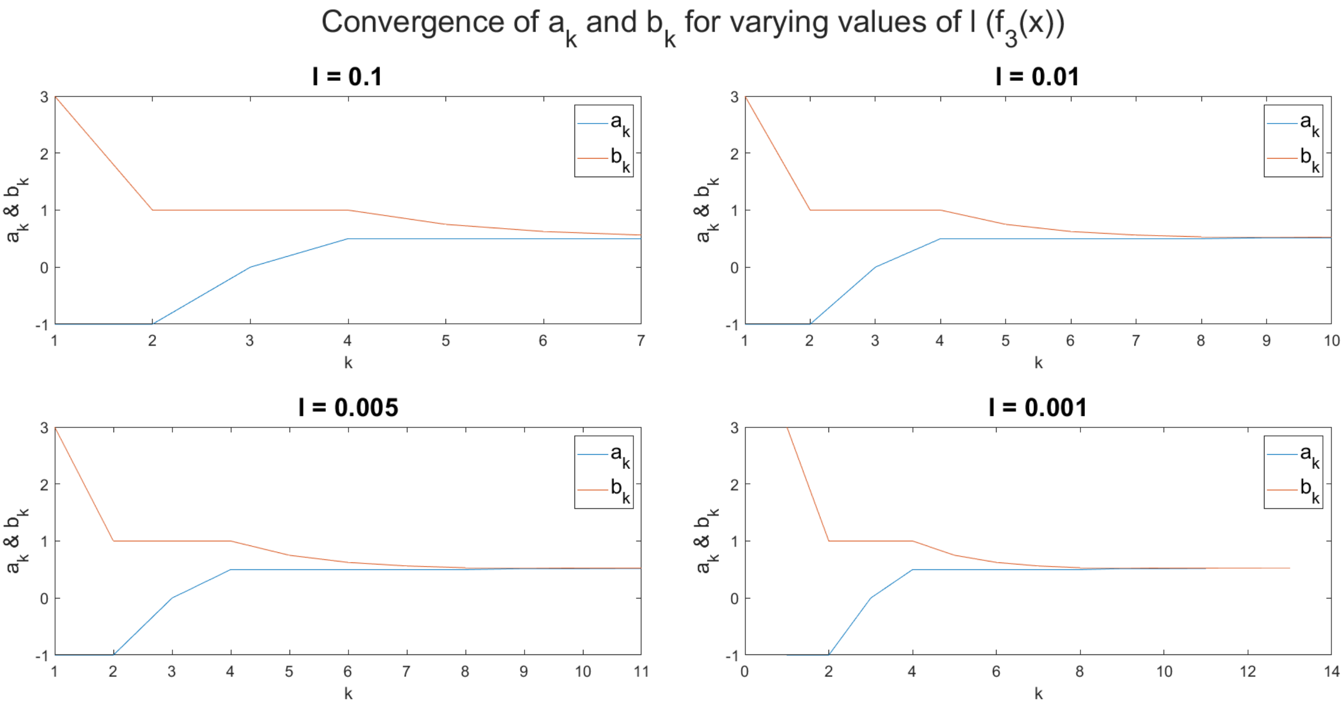




Finally, we again try some different values for l and plot the variation of the edges of the minimum interval $[a_k, b_k]$:


Convergence of a$_k$ and b$_k$ for varying values of l (f$_1$(x))
l = 0.1
l = 0.01
l = 0.005
l = 0.001
a$_k$ & b$_k$
k
a$_k$
b$_k$


Convergence of a$_k$ and b$_k$ for varying values of l (f$_2$(x))
l = 0.1
l = 0.01
l = 0.005
l = 0.001
a$_k$ & b$_k$
k
a$_k$
b$_k$



Convergence of a_k and b_k for varying values of l ($f_3(x)$)
l = 0.1
l = 0.01
l = 0.005
l = 0.001
a_k & b_k
k
a_k
b_k

6 Conclusions

Having implemented and applied the 4 algorithms we can draw some basic conclusions. First, regarding the algorithms without the use of a derivative, the bisection algorithm is the simplest in terms of implementation, but it is the least efficient of all those studied, requiring by a significant margin the most calculations of the objective function. In terms of both complexity and efficiency, the golden section algorithm follows. The Fibonacci algorithm is very close to the golden section algorithm, although requiring some less calculations. Finally, adding the differentiability property (due to pseudoconvexity), we can use the bisection algorithm using derivative, which is quite simpler than the last 2, and achieves finding the desired interval with fewer iterations and calculations. In conclusion, in some future unconstrained (almost convex) function minimization problem, we would prefer to use the Fibonacci algorithm, unless we can guarantee the function's derivability, where the derivative-using bisection algorithm is, clearly, the best choice.