

In general, we have the following formula.

The Binomial Theorem

If k is a positive integer, then

$$\begin{aligned}(a+b)^k &= a^k + ka^{k-1}b + \frac{k(k-1)}{1 \cdot 2}a^{k-2}b^2 \\ &\quad + \frac{k(k-1)(k-2)}{1 \cdot 2 \cdot 3}a^{k-3}b^3 + \dots \\ &\quad + \frac{k(k-1)\dots(k-n+1)}{1 \cdot 2 \cdot 3 \dots n}a^{k-n}b^n \\ &\quad + \dots + kab^{k-1} + b^k\end{aligned}$$

Or Summarized

$$(a+b)^k = \sum_{n=0}^k \binom{k}{n} a^{k-n} \cdot b^n$$

Example 13 Expand $(x-2)^5$

Solution Using the Binomial Theorem with $a = x, b = -2, k = 5$, we have

$$\begin{aligned}(x-2)^5 &= x^5 + 5x^4(-2) + \frac{5 \cdot 4}{1 \cdot 2}x^3(-2)^2 + \frac{5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3}x^2(-2)^3 + 5x(-2)^4 + (-2)^5 \\ &= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32\end{aligned}$$