

Investigation of Flash Crash via Topological Data Analysis

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Abstract

Topological data analysis has been acknowledged as one of the most successful mathematical data analytic methodologies in various fields including medicine, genetics, and image analysis. In this paper, we explore the potential of this methodology in finance by applying persistence landscape and dynamic time series analysis to analyze an extreme event in the stock market, known as *Flash Crash*. We will provide results of our empirical investigation to confirm the effectiveness of our new method not only for the characterization of this extreme event but also for its prediction purposes.

Keywords: Persistence landscape, Flash Crash, Time series analysis

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1. Introduction

Topological data analysis (TDA) is a relatively new, large, and growing field in mathematics using a wide variety of techniques based on homology theory and statistics. TDA extracts hidden intelligence from the *shape* of data that

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previous data analytic methods could not reveal, and various fields including medicine, image analysis, and genetics [7, 9] have tremendously benefited from this intriguing methodology. Nevertheless, in the field of finance, there has been only a small number of applications so far [5]. The purpose of this paper is to apply persistence landscape to detect extreme anomalies in financial market, and demonstrate how TDA can provide not only global characterization of data but also dynamic local features for prediction purposes.

In May 6, 2010, there was an unprecedented sudden intraday stock market crash in U.S. financial markets known as *Flash Crash*, and that event has recently emerged as an interesting research topic in finance [3, 4, 6, 8]. In this paper, we combine the methods of TDA including persistent landscape proposed by [5] and techniques of time series analysis, and create a new method for detecting intraday stock market crashes based on L^1 -norm of persistence landscape. We then empirically show that our method can characterize and predict the event of flash crash as well.

The rest of the paper is organized as follows: Section 2 introduces background and techniques of topological data analysis which are used in the paper, and provide a summary of the flash crash event. Section 3 describes our data constructed from major stock market indexes, and explains our new method based on TDA and statistical analysis. Section 4 reports empirical results to validate effectiveness of our methods for prediction purposes. Section 5 concludes the paper with further practical implications of our results.

2. Background

2.1. Topological Data Analysis

In this section, we gather key concepts of topological data analysis that will be used later in the paper. Consider a data set $\mathbb{X} = \{x_1, x_2, \dots, x_n\}$, which is a finite subset of a Euclidean space \mathbb{R}^d . Let $R(\mathbb{X}, \epsilon)$ denote the *Vietoris-Rips complex* for the data set \mathbb{X} and a distance $\epsilon > 0$, i.e., $R(\mathbb{X}, \epsilon)$ is the simplicial

complex on the vertex set \mathbb{X} such that

a subset σ of \mathbb{X} is a simplex in $R(\mathbb{X}, \sigma)$ if and only if $d(x, y) < \epsilon$ for all $x, y \in \sigma$.

(1)

Here $d(x, y)$ is the Euclidean distance between x and y .

In what follows, let us denote by $H_i(K)$ the i -th homology group of a simplicial complex K . Throughout this paper we only use homology with real coefficients. It follows from the very definition of Vietoris-Rips complex (1) that we have $R(\mathbb{X}, \epsilon) \subset R(\mathbb{X}, \epsilon')$ whenever $\epsilon < \epsilon'$. Using this we see that the Vietoris-Rips complexes $R(\mathbb{X}, \epsilon)$, $\epsilon > 0$, form a filtration, which is called the *Vietoris-Rips filtration*. This filtration induces homomorphisms between homology groups. That is, for each i , we get a canonical linear map,

$$H_i(R(\mathbb{X}, \epsilon)) \longrightarrow H_i(R(\mathbb{X}, \epsilon')) \quad \text{whenever } \epsilon < \epsilon'. \quad (2)$$

The set of homology groups $\{H_i(R(\mathbb{X}, \epsilon))\}_{\epsilon > 0}$ and the linear maps (2) form a *persistence module* in the sense of [2], which enables us to track the *birth* and *death* of homology classes as the parameter $\epsilon > 0$ increases. More precisely, given $\epsilon > 0$ and i , let $0 \neq \alpha \in H_i(R(\mathbb{X}, \epsilon))$. By using the Vietoris-Rips filtration we see that there exist positive real numbers b_α and d_α satisfying $b_\alpha \leq \epsilon \leq d_\alpha$ and

- α is not the image of the map $H_i(R(\mathbb{X}, \delta)) \rightarrow H_i(R(\mathbb{X}, \epsilon))$ if $\delta < b_\alpha$.
- For $b_\alpha \leq \delta \leq \epsilon$, there is $0 \neq \beta \in H_i(R(\mathbb{X}, \delta))$ whose image by the map $H_i(R(\mathbb{X}, \delta)) \rightarrow H_i(R(\mathbb{X}, \epsilon))$ is α .
- For $\epsilon \leq \delta \leq d_\alpha$, the image of α under the map $H_i(R(\mathbb{X}, \epsilon)) \rightarrow H_i(R(\mathbb{X}, \delta))$ is non-zero.
- The map $H_i(R(\mathbb{X}, \epsilon)) \rightarrow H_i(R(\mathbb{X}, \delta))$ sends α to the zero in $H_i(R(\mathbb{X}, \delta))$ if $\delta > d_\alpha$.

Let (b_α, d_α) denote the open interval corresponding to the birth and death of a given homology class α . In what follows let \mathfrak{T}_i denote the set of open intervals,

$$\{(b_\alpha, d_\alpha) \mid \alpha \text{ is a non-zero } i\text{-dimensional homology class}\}.$$

Here the elements of \mathfrak{T}_i are counted with multiplicity, i.e., if α and β are distinct i -dimensional homology classes, then (b_α, d_α) and (b_β, d_β) are regarded as different elements of \mathfrak{T}_i even though these two open intervals may coincide.

Given a pair of real numbers $b < d$, set

$$f_{(b,d)}(x) = \begin{cases} x - b & \text{for } b \leq x \leq \frac{b+d}{2} \\ -x + d & \text{for } \frac{b+d}{2} \leq x \leq d \\ 0 & \text{otherwise.} \end{cases}$$

The *persistence landscape* associated with the data set \mathbb{X} is a function $\lambda(\mathbb{X}) : \mathbb{N} \times \mathbb{R} \rightarrow \mathbb{R}$ which encodes the information about the birth and death of homology classes [1]. It is defined by

$$\lambda(\mathbb{X})(k, x) = k\text{-max}\{f_{(b,d)}(x) \mid (b, d) \in \mathfrak{T}_i\}, \quad (k, x) \in \mathbb{N} \times \mathbb{R}.$$

Here k -max denotes the k -th largest value counted with multiplicity. Note that the persistence landscape $\lambda(\mathbb{X})$ can be seen as a sequence of functions $\{\lambda(\mathbb{X})_k\}_{k \geq 1}$.

For $1 \leq p < \infty$, the L^p -norm of the persistence landscape $\lambda(\mathbb{X}) = \{\lambda(\mathbb{X})_k\}_{k \geq 1}$ is defined by

$$\|\lambda(\mathbb{X})\|_p = \sum_{k=1}^{\infty} \left(\int_{-\infty}^{\infty} |\lambda(\mathbb{X})_k(x)|^p dx \right)^{\frac{1}{p}}. \quad (3)$$

As \mathbb{X} is a finite set, all simplicial complexes $R(\mathbb{X}, \epsilon)$, $\epsilon > 0$, are finite. This shows that for k large enough, we have $\lambda(\mathbb{X})_k = 0$, and hence there is no issue of convergence of the series given in (3).

2.2. Flash Crash

In May 6, 2010, there was a sudden intraday stock market crash in U.S. financial markets known as *Flash Crash*. The crash event, started at 2:32 p.m. EDT, lasted for approximately 36 minutes, during which major U.S. stock indexes such as the S&P 500, Dow Jones Industrial Average, and Nasdaq Composite dropped larger than 6% and rebounded very rapidly. Figure 1 shows intraday price time series of S&P 500 futures, Dow Jones futures, and NASDAQ futures

in May 6, 2010. Since such an extreme price-swing event lasting only for a very short time had never been reported before in financial markets, many financial researchers have been trying to understand the *Flash Crash* [3, 4, 6, 8].

Intraday prices in May 6, 2010

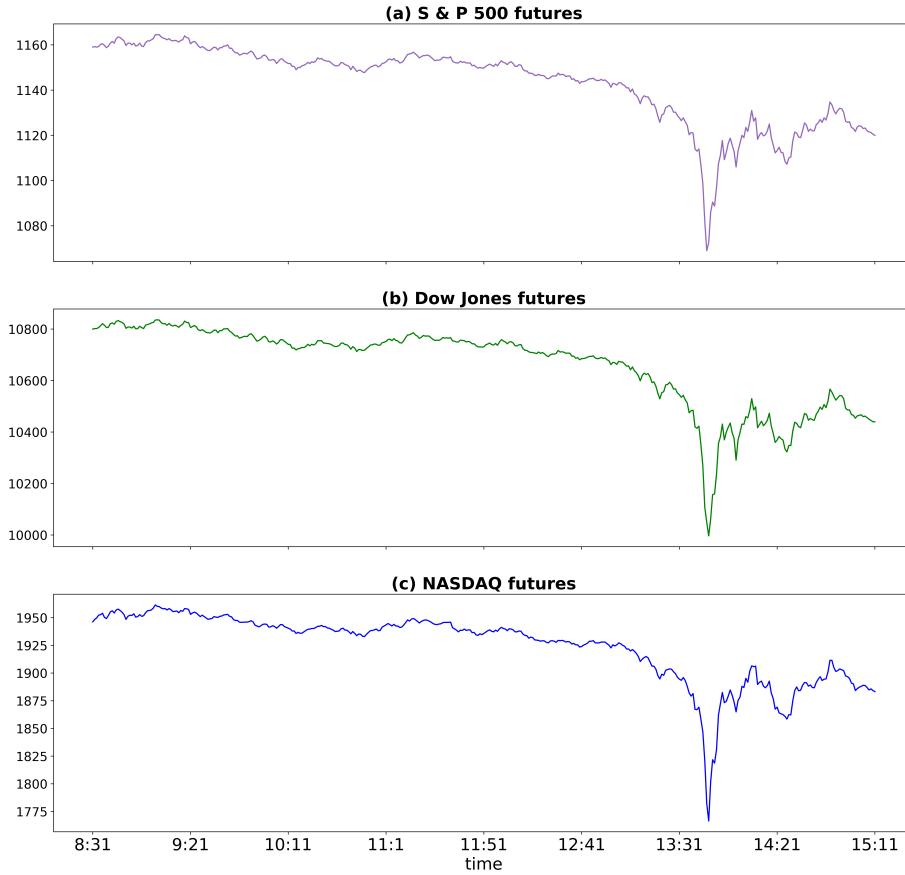


Figure 1: Intraday price time series of S&P 500 futures, Dow Jones futures, and NASDAQ futures on May 6, 2010.

3. Data and Methods

We purchased one-minute price data for three futures based on three major U.S. stock market indexes: (a) S&P 500 futures, (b) Dow Jones futures, and

(c) NASDAQ futures from April 1, 2010 to May 28, 2010 (42 trading days) from BacktestMarket (<https://www.backtestmarket.com>). For each future i ($i = a, b, c$) and for each minute j , we calculate one-minute log-return as $r_{i,j} = \log(\frac{P_{i,j}}{P_{i,j-1}})$, where $P_{i,j}$ represents the closing price of the futures i at the minute j . Thus, for each minute j , we have a 3-dimensional vector defined by

$$X_j = (r_{(a),j}, r_{(b),j}, r_{(c),j}).$$

Given a window size $w = 50$, and for each $t \geq w$, we then construct the 3-dimensional time series defined by

$$\mathbb{X}_t = \{X_{t-49}, X_{t-48}, \dots, X_{t-1}, X_t\}.$$

For each 3-dimensional time series \mathbb{X}_t , we compute L^1 -norm of persistence landscape, $\|\lambda(\mathbb{X}_t)\|_1$, so we get a time series of L^1 -norm of persistence landscape,

$$\mathbb{Y} = (Y_1, Y_2, \dots, Y_n) = (\|\lambda(\mathbb{X}_w)\|_1, \|\lambda(\mathbb{X}_{w+1})\|_1, \dots, \|\lambda(\mathbb{X}_{w+(n-1)})\|_1),$$

as in [5]. In order to detect an abnormality of the time series \mathbb{Y} , we develop a new abnormality measure based on the notions of Exponential Moving Average (EMA) and Exponential Moving Variance (EMVar) processes: Given the initial values,

$$\text{EMA}_1 = Y_1, \text{EMVar}_1 = 0,$$

the subsequent values, EMA_i and EMVar_i , are computed using the following recursive formulae,

$$\begin{aligned} \delta_i &= Y_i - \text{EMA}_{i-1} \\ \text{EMA}_i &= \text{EMA}_{i-1} + \alpha \cdot \delta_i \\ \text{EMVar}_i &= (1 - \alpha) \cdot (\text{EMVar}_{i-1} + \alpha \cdot \delta_i^2). \end{aligned}$$

Then, our new abnormality measure Z_t is defined by

$$Z_t = \frac{Y_t - \text{EMA}_{t-1}}{\sqrt{\text{EMVar}_{t-1}}}.$$

The new measure Z_t represents the extent to which the current value Y_t is deviated from the previous values.

4. Empirical Results

Figure 2 displays the intraday time series plots of our new abnormality measure Z_t and the prices of S&P 500 futures on May 6, 2010, the day of Flash Crash. As Figure 2 shows, just before the extreme price-swing of Flash Crash begins, the abnormality measure Z_t has the value of 28,034.92. The value is exceptionally huge compared to the values of Z_t at other times of the day. Figure 3 (resp., Figure 4) also presents the intraday time series plots of Z_t and S&P 500 futures in four randomly selected days before (resp., after) the day of Flash Crash out of our sample period. Figure 3 and 4 show that the largest value of Z_t in Flash Crash day, 28,034.92, is also extremely large compared to the values of Z_t in other days.

5. Conclusion

In this paper, we present a new method of detecting an abnormal phenomenon based on TDA and time series analysis, and investigate the event of Flash Crash via the new method. The empirical results from our study reveal that via the new method based on TDA, we can predict the event of Flash Crash. Therefore, our result shows that TDA not only can be used in forecasting long-term market crash events [5], but also can be used in predicting intraday market crash events.

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May 6, 2010 (the day of Flash Crash)

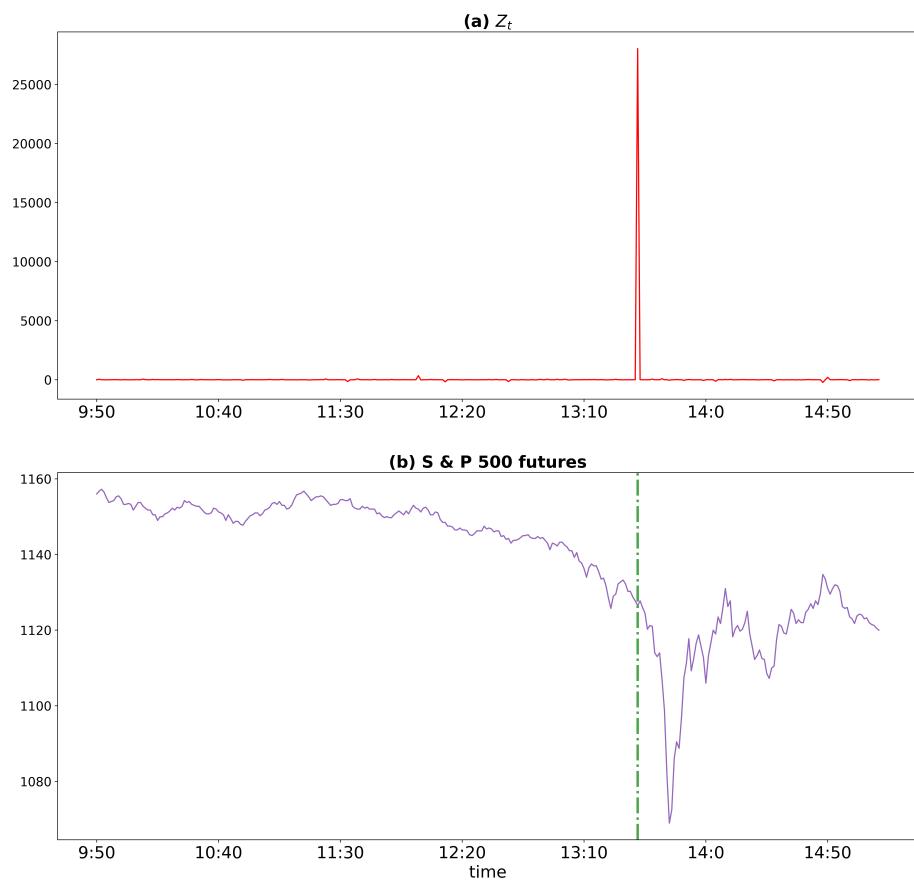


Figure 2: Intraday time series plots of our new abnormality measure, Z_t , and prices of S&P 500 futures on May 6, 2010, the day of Flash Crash.

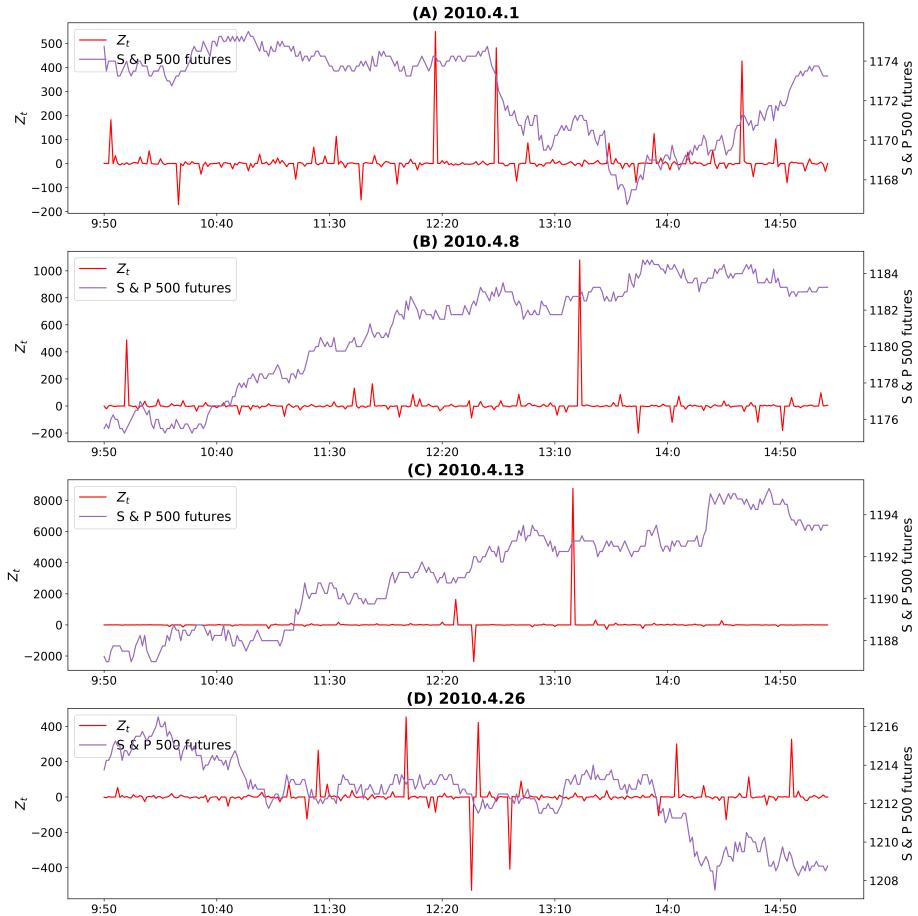


Figure 3: Intraday time series plots of our new abnormality measure, Z_t , and prices of S&P 500 futures on April 1, 8, 13, and 26, 2010.

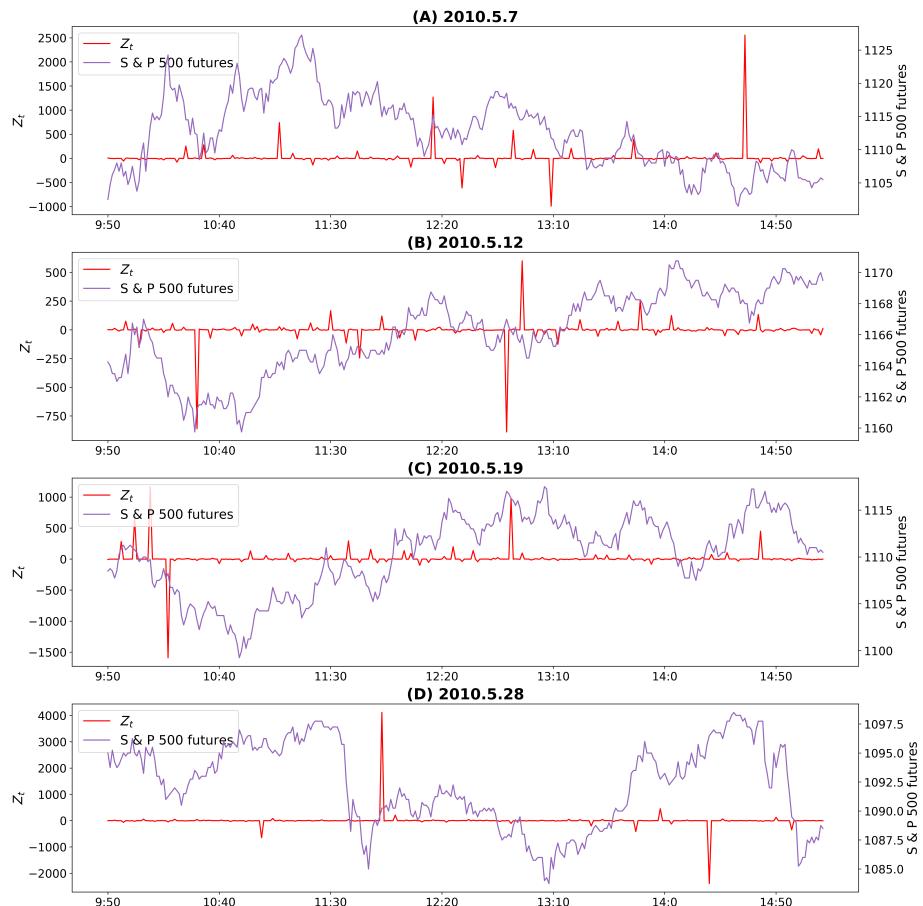


Figure 4: Intraday time series plots of our new abnormality measure, Z_t , and prices of S&P 500 futures on May 7, 12, 19, and 28, 2010.