

# Custom Representations of Inductive Families

## (Extended Abstract, Research Paper)

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**Abstract.** Inductive families provide a convenient way of programming with dependent types. Yet, when it comes to compilation, both their default linked-tree runtime representations, as well as the need to convert between different indexed views of the same data, can lead to unsatisfactory runtime performance. In this paper, we aim to introduce a language with dependent types, and inductive families with custom representations. Representations are a version of Wadler’s views [14], refined to inductive families like in Epigram [12]. However, representations come with compilation guarantees: a represented inductive family will not leave any runtime traces behind, without having to rely on automated optimisations such as deforestation [15]. This way, we can build a library of convenient inductive families based on a minimal set of primitives, whose re-indexing and conversion functions are erased at compile-time. In addition, we show how we can express inductive data optimisation techniques, such as representing `Nat`-like types as GMP-style big integers, without special casing in the compiler. With dependent types, we can indeed reason about data representations internally; in this spirit, we are awarded isomorphisms between the original and represented data.

## 1 Introduction

Inductive families are a broad generalisation of inductive data types found in most functional programming languages. Every inductive definition is equipped with an eliminator that captures the notion of mathematical induction over the data, and in particular, enables structural recursion over the data. This is a powerful tool for programming as well as theorem proving.

However, this abstraction has a cost when it comes to compilation: the runtime representation of inductive types is a linked tree structure. A list as an inductively defined data structure is stored as a linked list on the heap, and a natural number is stored as a unary number where each digit is an empty heap cell. This representation is not always the most efficient for all operations, and often forces users to rely on more efficient machine primitives to achieve desirable performance. The ‘Nat-hack’ in languages with dependent types is a prime example of this, where natural numbers are represented as GMP-style big integers for efficient arithmetic operations [CITE?].

Despite advances in the erasure of irrelevant indices in inductive families [5] and the use of theories with irrelevant fragments [3,13,2], there is still a need to convert between different indexed views of the same data. For example, the function to convert from `List T` to `Vec T n` by forgetting the length index  $n$  is *not* erased by any current language with dependent types, unless vectors are defined as a refinement of lists with an erased length field (which hinders dependent pattern matching due to the presence of non-structural witnesses), or a Church encoding is used in a Curry-style context [9] (which restricts the flexibility of data representation).

Wadler’s views [14] provide a way to abstract over inductive interfaces, so that different views of the same data can be defined and converted between seamlessly. In the context of inductive families, views have been used in Epigram [12] that utilise the index refinement machinery of dependent pattern matching to avoid certain proof obligations with eliminator-like constructs. While views exhibit a nice way to transport across a bijection between the original data and the viewed data, they do not utilise this bijection to erase the view from the program. Despite deforestation being able to handle this erasure to some extent, it is not guaranteed to erase all traces of the view from the program, and the optimisation might be difficult to predict.

In this paper, we propose an extension  $\lambda_{\text{REP}}$  to a core language with dependent types and inductive families  $\lambda_{\text{IND}}$ , which allows programmers to define custom, correct-by-construction data representations. This is done through user-defined translations of the constructors and eliminators of an inductive type to a concrete implementation, which form a bijective view of the original data called a ‘representation’. Representations are defined internally to the language, and require coherence properties that ensure a representation defines an initial algebra for the endofunctor associated with the original inductive family. We define a type- and equality-preserving translation from the language with representations to a language without them, which provably erases all inductive types with defined representations from the program.

The in the final version of the paper, we plan to contribute the following:

- A dependent type system with inductive families  $\lambda_{\text{IND}}$ , and its extension by representations  $\lambda_{\text{REP}}$ .
- A formulation of common optimisations such as the ‘Nat-hack’, and similarly for other inductive types, as representations.
- A demonstration of zero-cost data reuse when reindexing by using representations.
- A translation from  $\lambda_{\text{REP}}$  to  $\lambda_{\text{IND}}$  that erases all inductive types with representations from the program.
- An implementation of this system and accompanying examples in SUPERFLUID, a programming language with inductive types and dependent pattern matching.

## 2 A tour of data representations

A common optimisation done by programming languages with dependent types such as Idris 2 and Lean is to represent natural numbers as GMP-style [1] big integers. The core definition of natural numbers looks like

$$\text{data Nat} \left\{ \begin{array}{l} 0 : \text{Nat} \\ 1+ : \text{Nat} \rightarrow \text{Nat} \end{array} \right\} \quad (1)$$

This definition is convenient because it generates a Peano-style induction principle

$$\begin{aligned} \text{elim}_{\text{Nat}} : & (P : \text{Nat} \rightarrow \mathcal{U}) \\ & \rightarrow (m_0 : P \ 0) \rightarrow (m_{1+} : (n : \text{Nat}) \rightarrow P \ n \rightarrow P \ (1+ \ n)) \\ & \rightarrow (s : \text{Nat}) \rightarrow P \ s. \end{aligned}$$

Moreover, dependent pattern matching with structural recursion on `Nat` can be elaborated into invocations of `elimNat` [10,7,6], and lemmas such as constructor injectivity, disjointness and acyclicity [11] can be substituted automatically to simplify the context and goal. This way, if addition is defined using recursion and pattern matching, proofs like the commutativity of addition or the additive identity of `0` are easy to write.

When it comes to compilation, without further intervention, the `Nat` type is represented in unary form, where each digit is an empty heap cell. This is inefficient for a lot of the basic operations on natural numbers, especially since computers are particularly well-equipped to deal with numbers natively. For this reason, most real-world implementations will treat `Nat` specially, swapping the default inductive type representation with one based on GMP integers. This is done by performing the replacements

$$|0| = 0 \quad (2)$$

$$|1+| = 1 + \quad (3)$$

$$|\text{elim}_{\text{Nat}} P \ m_0 \ m_{1+} \ s| = \text{ubig-rec} \ |s| \ |m_0| \ |m_{1+}| \quad (4)$$

where  $|\cdot|$  denotes a source translation into an untyped compilation target language with primitives `ubig-*`.<sup>1</sup>

In addition to the constructors and eliminators, the compiler might also define translations for commonly used definitions which have a more efficient counterpart in the untyped target, such as recursively defined addition, multiplication, etc. The recursive versions are well-suited to proofs and reasoning, while the untyped versions based on GMP primitives are more efficient for computation.

<sup>1</sup> Idris 2 will in fact look for any ‘Nat-like’ types and apply this optimisation. A Nat-like type is any type with two constructors, one with arity zero and the other with arity one. A similar optimisation is also done with list-like and boolean-like types because they have a canonical representation in the target runtime, Chez Scheme.

The issue with this approach is that it only works for the data types which the compiler can recognise as special. Particularly in the presence of dependent types, other data types might end up being equivalent to `Nat` or another ‘nicely-representable’ type, but in a non-trivial way that the compiler cannot recognise. Hence, our first goal is to extend this optimisation to work for any data type. Additionally, the optimisation is defined as a compilation step, which means that if non-trivial representations are provided by the user, their correctness is not guaranteed. To address this, our framework requires that representations are fully typed in a way that ensures the behaviour of the representation of a data type matches the behaviour of the data type itself.

## 2.1 The well-typed Nat-hack

A representation definition looks like

$$\text{repr Nat as UBig} \left\{ \begin{array}{l} \text{0 as 0} \\ \text{1+ } n \text{ as } 1 + n \\ \text{elim}_{\text{Nat}} \text{ as ubig-elim} \\ \text{by ubig-elim-zero-id,} \\ \text{ubig-elim-add-one-id} \end{array} \right\}$$

We choose to represent the `Nat` type as the type `UBig` of GMP-style unlimited-size unsigned integers, and provide translations for the constructors `0` and `1+`, and the eliminator `elimNat`. Additionally, we have to show that the eliminator satisfies the expected computation rules of the `Nat` eliminator, which are postulated as propositional equalities. All in all, we assume access to a signature

$$\begin{aligned} 0 &: \text{UBig} \\ 1 &: \text{UBig} \\ + &: \text{UBig} \rightarrow \text{UBig} \rightarrow \text{UBig} \\ \times &: \text{UBig} \rightarrow \text{UBig} \rightarrow \text{UBig} \\ \text{ubig-elim} &: (P : \text{UBig} \rightarrow \mathcal{U}) \rightarrow P \ 0 \\ &\rightarrow ((n : \text{UBig}) \rightarrow P \ (1 + n)) \\ &\rightarrow (s : \text{UBig}) \rightarrow P \ s \end{aligned}$$

which also contains propositional equalities

$$\begin{aligned} \text{ubig-elim-zero-id} &:_{\forall P b r} \text{ubig-elim } P \ b \ r \ 0 = b \\ \text{ubig-elim-add-one-id} &:_{\forall P b r n} \text{ubig-elim } P \ b \ r \ (1 + n) = r \ n \ (\text{ubig-elim } P \ b \ r \ n). \end{aligned}$$

This will effectively erase the `Nat` type from the compiled program, replacing all occurrences with the `UBig` type and its primitives. In a way, the hard work is done by the postulates above; we expect that the underlying implementation of `UBig` indeed satisfies them, which is a separate concern from the correctness

of the representation itself. However, postulates are only needed when the representation target is a primitive; the next examples use defined types as targets, so that the coherence of the target eliminator holds by the coherence of other eliminators used in its implementation.

## 2.2 Vectors are just certain lists

In addition to representing inductive types as primitives, we can use representations to share the underlying data when converting between indexed views of the same data. For example, we can define a representation of `Vec` in terms of `List`, so that the conversion from one to the other is ‘compiled away’. We can do this by representing the more indexed type as a refinement of the less indexed type by an appropriate relation. For the case of `Vec`, we know intuitively that

$$\text{Vec } T \ n \simeq \{l : \text{List } T \mid \text{length } l = n\}$$

so we can start by choosing the type

$$\text{List}' T \ n := \{l : \text{List } T \mid \text{length } l = n\}$$

as the representation of `Vec`  $T \ n$ . We will take the subset  $\{x : A \mid P \ x\}$  to mean a  $\Sigma$ -type  $(x : A) \times P \ x$  where the right component is irrelevant and erased at runtime.<sup>2</sup> We are then tasked with providing terms that correspond to the constructors of `Vec` but for `List'`. These can be defined as

$$\begin{aligned} \text{nil} &: \text{List}' T \ 0 & \text{cons} &: T \rightarrow \text{List}' T \ n \rightarrow \text{List}' T \ (1+ \ n) \\ \text{nil} &= (\text{nil}, \text{refl}) & \text{cons } x \ (xs, p) &= (\text{cons } x \ xs, \text{cong } (1+) \ p) \end{aligned}$$

Next we need to define the eliminator for `List'`, which should have the form

$$\begin{aligned} \text{elim-List}' &: (E : (n : \text{Nat}) \rightarrow \text{List}' T \ n \rightarrow \text{Type}) \\ &\rightarrow E \ 0 \ \text{nil} \\ &\rightarrow ((x : T) \rightarrow (n : \text{Nat}) \rightarrow (xs : \text{List}' T \ n) \rightarrow E \ (1+ \ n) \ (\text{cons } x \ xs)) \\ &\rightarrow (n : \text{Nat}) \rightarrow (v : \text{List}' T \ n) \rightarrow E \ n \ v \end{aligned}$$

Dependent pattern matching does a lot of the heavy lifting by refining the length index and equality proof by matching on the underlying list. However we still need to substitute the lemma  $\text{cong } (1+) \ (1+-\text{inj } p) = p$  in the recursive case.

$$\begin{aligned} \text{elim-List}' \ P \ b \ r \ 0 \ (\text{nil}, \text{refl}) &= b \\ \text{elim-List}' \ P \ b \ r \ (1+ \ m) \ (\text{cons } x \ xs, e) &= \text{subst } (\lambda p. P \ (1+ \ m) \ (\text{cons } x \ xs, p)) \\ &\quad (1+-\text{cong-id } e) \ (r \ x \ (xs, 1+-\text{inj } e)) \end{aligned}$$

<sup>2</sup> In practice, a theory with erasure can be used to achieve this. In SUPERFLUID we use Quantitative Type Theory [3].

Finally, we need to prove that the eliminator satisfies the expected computation rules propositionally. These are

$$\begin{aligned} \text{elim-List'-nil-id} &: \text{elim-List}' P b r \mathbf{0} (\text{nil}, \text{refl}) = b \\ \text{elim-List'-cons-id} &: \text{elim-List}' P b r (\mathbf{1+} m) (\text{cons } x \text{ } xs, \text{cong } (\mathbf{1+}) p) = r x (xs, p) \end{aligned}$$

which we leave as an exercise, though they are evident from the definition of  $\text{elim-List}'$ . This completes the definition of the representation of  $\text{Vec}$  as  $\text{List}'$ , which would be written as

$$\text{repr } \text{Vec } T n \text{ as } \text{List}' T n \left\{ \begin{array}{l} \text{nil as nil} \\ \text{cons as cons} \\ \text{elim}_{\text{Vec}} \text{ as elim-List}' \\ \quad \text{by elim-List'-nil-id,} \\ \quad \quad \text{elim-List'-cons-id} \end{array} \right\}$$

Now the hard work is done; Every time we are working with a  $v : \text{Vec } T n$ , its real form will be  $(l, p) : \text{List}' T n$  at runtime, where  $l$  is the underlying list and  $p$  is the proof that the length of  $l$  is  $n$ . Under the assumption that the  $\Sigma$ -type's right component is irrelevant and erased at runtime, every vector is simply a list at runtime, where the length proof has been erased. In the full paper we will show how this erasure is achieved in practice in SUPERFLUID using Quantitative Type Theory [3].

We can utilise this representation to convert between  $\text{Vec}$  and  $\text{List}$  at zero runtime cost, by using the **repr** and **unrepr** operators of the language. Specifically, we can define the functions

$$\begin{aligned} \text{forget-length} &: \text{Vec } T n \rightarrow \text{List } T \\ \text{forget-length } v &= \text{let } (l, \_) = \text{repr } v \text{ in } l \\ \text{recall-length} &: (l : \text{List } T) \rightarrow \text{Vec } T (\text{length } l) \\ \text{recall-length } l &= \text{unrepr } (l, \text{refl}) \end{aligned}$$

and it holds by reflexivity that  $\text{forget-length}$  is a left inverse of  $\text{recall-length}$ .

### 2.3 General reindexing

The idea from the previous example can be generalised to any data type. In general, suppose that we have two inductive families

$$\begin{aligned} \mathbf{F} &: \Xi \rightarrow \mathcal{U} \\ \mathbf{G} &: \Xi \rightarrow X \xi \rightarrow \mathcal{U} \end{aligned}$$

for some index family  $X : \Xi \rightarrow \mathcal{U}$ . If we hope to represent  $\mathbf{G}$  as some refinement of  $\mathbf{F}$  then we must be able to provide a way to compute  $\mathbf{G}$ 's extra indices  $X$  from

$\mathbf{F}$ , like we computed  $\mathbf{Vec}$ 's extra  $\mathbf{Nat}$  index from  $\mathbf{List}$  with `length` in the previous example. This means that we need to provide a function

$$\mathbf{comp} : \forall \xi. \mathbf{F} \ \xi \rightarrow X \ \xi$$

which can then be used to form the family

$$\mathbf{F}^{\mathbf{comp}} \ \xi \ x := \{f : \mathbf{F} \ \xi \mid \mathbf{comp} \ f = x\}.$$

If  $\mathbf{G}$  is ‘equivalent’ to the algebraic ornament of  $\mathbf{F}$  by the algebra defining `comp` (given by a cartesian isomorphism between the underlying polynomial functors), then it is also equivalent to the  $\Sigma$ -type above. The ‘recomputation lemma’ of algebraic ornaments [8] then arises from its projections. Our system allows us to *set* the representation of  $\mathbf{G}$  as  $\mathbf{F}^{\mathbf{comp}}$ , so that the forgetful map from  $\mathbf{G}$  to  $\mathbf{F}$  is the identity at runtime.

## 2.4 Zero-copy deserialisation

The machinery of representations can be used to implement zero-copy deserialisation of data formats into inductive types. For example, consider the following record for a player in a game:

$$\mathbf{data} \ \mathbf{Player} \left\{ \begin{array}{l} \mathbf{player} : (\mathbf{position} : \mathbf{Position}) \\ \quad \rightarrow (\mathbf{direction} : \mathbf{Direction}) \\ \quad \rightarrow (\mathbf{items} : \mathbf{Fin} \ \mathbf{MAX\_INVENTORY}) \\ \quad \rightarrow (\mathbf{inventory} : \mathbf{Inventory} \ \mathbf{items}) \\ \quad \rightarrow \mathbf{Player} \end{array} \right\}$$

We can use the  $\mathbf{Fin}$  type to maintain the invariant that the inventory has a maximum size. Additionally, we can index the  $\mathbf{Inventory}$  type by the number of items it contains, which might be defined similarly to  $\mathbf{Vec}$ :

$$\mathbf{data} \ \mathbf{Inventory} \ (n : \mathbf{Nat}) \left\{ \begin{array}{l} \mathbf{empty} : \mathbf{Inventory} \ 0 \\ \mathbf{add} : \mathbf{Item} \rightarrow \mathbf{Inventory} \ n \rightarrow \mathbf{Inventory} \ (1 + n) \end{array} \right\}$$

We can use the full power of inductive families to model the domain of our problem in the way that is most convenient for us. If we were writing this in a low-level language like C, we might choose to use the serialised format directly when manipulating the data, relying on the appropriate pointer arithmetic to access the fields of the serialised data. Doing this can result in much better performance, as we avoid the overhead of copying the data into a new format. Representations allow us to do this while still being able to work with the high-level inductive type.

We can define a representation for  $\mathbf{Player}$  as a pair of a byte buffer and a proof that the byte buffer contents correspond to a player record. Similarly, we can define a representation for  $\mathbf{Inventory}$  as a pair of a byte buffer and a proof

that the byte buffer contents correspond to an inventory record of a certain size. The projection

$$\text{inventory} : (p : \text{Player}) \rightarrow \text{Inventory } p.\text{items}$$

is compiled into some code to slice into the inventory part of the player's byte buffer. We assume that the standard library already represents `Fin` in the same way as `Nat`, so that reading the `items` field is a constant-time operation (we do not need to build a unary numeral). We can thus define the representation of `Player` as

$$\text{repr Player as } \{\text{Buf} \mid \text{IsPlayer}\} \left\{ \begin{array}{l} \text{player as buf-is-player} \\ \text{elim}_{\text{Player}} \text{ as elim-buf-is-player} \\ \text{by elim-buf-is-player-id} \end{array} \right\}$$

with an appropriate definition of `IsPlayer` which refines a byte buffer. We will provide the full details of this construction in the final paper.

## 2.5 Transitivity

Representations are transitive, so in the previous example, the ‘terminal’ representation of `Vec` also depends on the representation of `List`. It is possible to define a custom representation for `List` itself, for example a heap-backed array or a finger tree, and `Vec` would inherit this representation. However it will still be the case that  $\text{Repr } (\text{Vec } T \ n) \equiv \text{List } T$ , which means the `repr/Repr` operators only look at the immediate representation of a term, not its terminal representation. Regardless, we can construct predicates that find terms which satisfy a certain ‘eventual’ representation. For example, given a `Buf` type of byte buffers, we can consider the set of all types which are eventually represented as a `Buf`:

$$\text{data ReprBuf } (T : \mathcal{U}) \left\{ \begin{array}{l} \text{buf} : \text{ReprBuf Buf} \\ \text{from} : \text{ReprBuf } (\text{Repr } T) \rightarrow \text{ReprBuf } T \\ \text{refined} : \text{ReprBuf } T \rightarrow \text{ReprBuf } \{t : T \mid P \ t\} \end{array} \right\}$$

Every such type comes with a projection function to the `Buf` type

$$\begin{aligned} \text{as-buf} &: \forall T. \text{ReprBuf } T \rightarrow \text{Buf} \\ \text{as-buf buf} &= x \\ \text{as-buf (from } t) &= \text{as-buf } t \ (\text{repr } x) \\ \text{as-buf (refined } t) &(x, \_) = \text{as-buf } t \ x \end{aligned}$$

which eventually computes to the identity function after applying `repr` the appropriate amount of times. Upon compilation, every type is converted to its terminal representation, and all `repr` calls are erased, so the `as-buf` function is effectively the identity function at runtime.<sup>3</sup>

<sup>3</sup> We do not guarantee that an invocation of `as-buf` will be entirely erased, but rather that any invocation will eventually produce the identity function without having to perform a case analysis on its `T` subject.



### 3 A type system for data representations

In this section, we state the basics of the type system of  $\lambda_{\text{REP}}$ , a core language with dependent types and representations. We start with a core language with dependent types and inductive constructions  $\lambda_{\text{IND}}$ . We then extend this language with data representations to form  $\lambda_{\text{REP}}$ , which allow users to define custom representations for inductive types and other global symbols.

#### 3.1 A core language with inductive types, $\lambda_{\text{IND}}$

The core language we start with is  $\lambda_{\text{IND}}$ . It contains  $\Pi$ -types and a hierarchy of universe  $\mathcal{U}$  with  $\mathcal{U} : \mathcal{U}$ . We are not concerned with universe polymorphism or a sound logical interpretation, but all the results should be readily extensible to a sound language with a universe hierarchy. We follow a similar approach to [6] by packaging named inductive constructions and global function definitions into a signature  $\Sigma$ , and indexing contexts by signatures.

#### 3.2 Extending $\lambda_{\text{IND}}$ with representations

We extend the language  $\lambda_{\text{IND}}$  to form  $\lambda_{\text{REP}}$ , which allows users to define custom representations for inductive types and global functions. First, we add a modal type former

$$\mathbf{Repr} : \text{Ty } (\Sigma \mid \Gamma) \rightarrow \text{Ty } (\Sigma \mid \Gamma) \quad (5)$$

along with an isomorphism

$$\mathbf{repr} : \text{Tm } (\Sigma \mid \Gamma) \ T \simeq \text{Tm } (\Sigma \mid \Gamma) \ (\mathbf{Repr} \ T) : \mathbf{unrepr}. \quad (6)$$

which preserves  $\Pi$ -types and universes. The type  $\mathbf{Repr} \ T$  is the defined representation of the type  $T$ . The term  $\mathbf{repr}$  takes a term of type  $T$  to its representation of type  $\mathbf{Repr} \ T$ , and  $\mathbf{unrepr}$  undoes the effect of  $\mathbf{repr}$ , treating a represented term as an inhabitant of its original type. These new constructs satisfy certain additional definitional equalities given in fig. 1, defining how these constructs can be used to rewrite type families and constructors as their defined representations.

$\frac{\text{REPR-CTOR-ID} \quad \mathbf{repr} \ \mathbf{c} \ \Pi \ \mathbf{as} \ \kappa \in \Sigma}{\Sigma \mid \Gamma \vdash \mathbf{repr} \ (\mathbf{c} \ \delta \ \pi) \equiv \kappa[\delta, \pi] : A[\delta, \xi[\pi]]}$	$\frac{\text{REPR-DATA-ID} \quad \mathbf{repr} \ \mathbf{D} \ \Delta \ \Xi \ \mathbf{as} \ A \in \Sigma}{\Sigma \mid \Gamma \vdash \mathbf{Repr} \ (\mathbf{D} \ \delta \ \psi) \equiv A[\delta, \psi] \ \text{type}}$
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**Fig. 1.** Definitional equalities for  $\mathbf{Repr}$  and  $\mathbf{repr}$  relating to data types and constructors with defined representations. Similar equalities hold for representations of global function definitions and eliminators, albeit propositionally.

We state some basic lemmas about the term formers  $\mathbf{repr}$  and  $\mathbf{unrepr}$  below, whose proofs we omit for the full version of the paper.

**Lemma 1.** *The term formers **repr** and **unrepr** are injective, i.e.*

$$\frac{\Sigma \mid \Gamma \vdash \mathbf{repr} \, t \equiv \mathbf{repr} \, t' : \mathbf{Repr} \, T}{\Sigma \mid \Gamma \vdash t \equiv t' : T} \quad \frac{\Sigma \mid \Gamma \vdash \mathbf{unrepr} \, t \equiv \mathbf{unrepr} \, t' : T}{\Sigma \mid \Gamma \vdash t \equiv t' : \mathbf{Repr} \, T}$$

**Lemma 2.** *The type former **Repr** is injective up to internal isomorphism, i.e.*

$$\frac{\Sigma \mid \Gamma \vdash \mathbf{Repr} \, T \equiv \mathbf{Repr} \, T' \text{ type}}{\Sigma \mid \Gamma \vdash p : T \simeq T'}$$

Moreover, this isomorphism is computationally irrelevant, i.e. it compiles to the identity function.

## 4 Translating from $\lambda_{\text{REP}}$ to $\lambda_{\text{IND}}^{\text{EXT}}$

We can define a translation step  $\mathcal{R}$  from  $\lambda_{\text{REP}}$  to  $\lambda_{\text{IND}}^{\text{EXT}}$ , meant to be applied during the compilation process. More specifically, the translation target is the extensional flavour of  $\lambda_{\text{IND}}$  by adding the equality reflection rule. We do this by translating well-formed contexts, substitutions, types, and terms in a mutual manner such that definitional equality is preserved. In other words,  $\mathcal{R}$  is strict CWF morphism comprising of setoid homomorphisms on all syntactical categories. Informally,  $\mathcal{R}$  preserves the structure of  $\lambda_{\text{IND}}$ , but maps constructs to their eventual representations. Eliminator coherence rules are preserved by reflecting the propositional coherence rules provided by the defined representations. We prove some desired properties of  $\mathcal{R}$  [4] such as typing and computational soundness, and preservation of consistency. The final program can then be converted into an untyped language which erases irrelevant data.

## 5 Implementation

SUPERFLUID is a programming language with dependent  $\Pi$  types, quantities, inductive families and dependent pattern matching. Its compiler is written in Haskell and the compilation target is JavaScript. Dependent pattern matching in SUPERFLUID is elaborated to a core language with internal eliminators. The  $\mathcal{R}$  transformation is then applied to the core program, which erases all inductive constructs with defined representations. This is finally translated to JavaScript, erasing all irrelevant data. As a result, we are able to represent **Nat** as JavaScript’s **BigInt**, and **List**  $T$ /**SnocList**  $T$ /**Vec**  $T$   $n$  as JavaScript’s arrays with the appropriate index refinement, such that we can convert between them without any runtime overhead.

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