



Corporate Valuation

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Olivier Levyne (2012)

The valuation of firms is achieved at least by 3 categories of professionals:

- Financial analysts who determine target prices of listed companies which underlie their recommendations (buy, hold, sell the stock)
- Investment bankers in 2 contexts
 - o Mergers and acquisitions (M&A):
 - If the target company is listed, its valuation has to be presented to the French market authority in order to justify that the offer price is satisfactory for the whole shareholders.
 - Such a valuation is included in a specific document which is bound to obtain a visa from the market authority
 - Several valuation approaches are generally presented. The offer price is therefore the outcome of a multi-criteria analysis
 - If the target company is not listed, an auction bid is generally organized by investment bankers.
 - The potential buyers articulate an indicative price based on their own valuation of the target, the financial information of which o (past accounts and business plan) is provided in an information memorandum.
 - In the next phase of the process, a binding price is provided to the seller, based on additional data which are obtained in the background of a due diligence: data room, meetings with the management, written questions and answers process... The binding offer may be significantly different from the indicative one
 - o Equity capital markets (ECM)
 - Initial public offerings (IPOs)
 - Right issues
- Private equity funds' analysts
 - o Their approach of the firm's valuation is significantly different from the abovementioned ones.
 - o Their transactions are a Leverage Buy Outs. Therefore, they consider that the value of the target the amount of resources (capital and debt) which can be raised by the ad hoc holding which achieves the acquisition
 - The capital is raised from the private equity fund which generally required a 20% IRR over 3 to 5 years
 - The debt is raised from banks which have their own constraints, eg:
 - 80% of the senior debt depreciated on a straight line basis over 7 years
 - 20% of the senior debt fully repaid at the end of the 8th year (bullet)

Professionals and academics consider that there are 3 main ways to value a firm:

- Peers approach ie valuation based on multiples which come from listed company or from M&A transactions.

- Discounted Cash Flows. The discount rate is the weighted average cost of capital (Wacc) the level of which is explained by the modern finance theory (Sharpe and Modigliani & Miller).
- Net assets values and sum of the parts approaches.

For listed firms, the market capitalisation provided a fourth value which can't be ignored whatever the size of the free float.

Black & Scholes [1973] and Galai & Masulis [1977] have also proposed a valuation approach of the firm based on the option pricing model. This approach is not presented in this course. It will be developed in the Real Options elective course

1. Market capitalisation

Market capitalisation = number of shares x market price

If the capital is made of m categories of shares where

n_k = number of shares of the k category;

p_k = price of the shares of the k category

Then: market capitalisation = $\sum_{k=1}^m n_k p_k$

2. Peers approaches

a. Listed peers

i. Sample

This approach relies on a sample of listed companies which have the same business model as the firm to be valued. This means that the sample is generally made of companies belonging to the same sector. In that case, they have the same constraints and, more or less, the same margins.

The company to be valued may not have direct listed peers on its domestic market. For example, Groupe Go Sport is the only listed firm on the French stock exchange which is focussed on the sport and leisure retail. Its valuation based on listed can be based on a sample of British peers such as JJB and [].

But the regulation for retail firms is not as demanding in Great Britain as in France: the Raffarin law prohibits the opening of stores, the size of which is greater than 300m² without obtaining an administrative waiver. In that case, the organic growth is almost impossible for such groups in France which means that the only possible growth is the external one. Therefore, the business model of Bricorama, Mr Bricolage, or even Carrefour or Casino is nearer of Groupe Go Sport than JJB's. From a practical point of view, 2 samples have to be taken into account: one British sport and leisure retail, another one of French retailers (food and non food ones).

ii. Multiples

For each company of the sample, multiples of financial aggregates are calculated:

Sales multiple	= xSales	= EV / sales
EBITDA multiple	= xEBITDA	= EV / EBITDA
EBIT multiple	= xEBIT	= EV / EBIT

Price / Earning ratio = P/E = market cap / restated net profit

Price / Book value = P/BV = market cap / book value¹

In the P/E calculation, the net profit which is taken into account must be a recurring amount. For that reason, the net profit must be restated in order to neutralize the contribution of the after tax the exceptional result. Therefore:

Restated net profit = Achieved net profit – Exceptional result x (1 – τ)

where τ = corporate tax rate.

Furthermore, the use of P/BV means that the minimal economic value of assets (ie before taking latent capital gains into account) is equal to their net book value. But, the goodwill and most of the intangible are worthless from an economic point of view. This is why, in the banking and insurance sectors, a P/TBV is used, the TBV being the tangible book value where:

Tangible Book Value = Book Value – Intangible assets and goodwill.

iii. Valuation

For each series of multiples and each year (generally n-1, n, n+1 and n+2) the average multiple is calculated. This average multiple is looked upon as the sector (or industry) multiple. Then it is applied to the corresponding aggregate of the firm to be valued.

- If the calculation of the multiple is based on the EV of the sample's companies (ie xSales, xEBITDA and xEBIT), such an application enables to get the EV of the firm to be valued. Its equity is equal to the EV reduced by its net financial debt.
- If the calculation of the multiple is only based on the market caps of the sample's companies (ie P/E and P/BV), such an application enables to get directly the equity value of the firm to be valued

The applications of the sales, EBITDA and EBIT multiples to the corresponding sales, EBITDA and EBIT enable to value only the operating assets. In other words, they don't provide the value of the financial assets (investments, shareholdings consolidated by the equity method).

Moreover, the valuation of a firm corresponds to the economic value of its equity group share which means that the "shareholders' equity - minority share" is not taken into account: it corresponds to shareholdings - in controlled companies - which are owned by minority shareholders and which can't be sold to pay the debt of the firm to be valued.

In such a case, the various values of equity are equal to:

- Equity value = Sales multiple x Sales + Financial assets – net debt – minority interests
- Equity value = EBITDA multiple x EBITDA + Financial assets – net debt – minority interests
- Equity value = EBIT multiple x EBIT + Financial assets – net debt – minority interests

¹ Book value = equity group share. This multiple is used for the valuation of firms the business model of which depends on the level of equity, which is the case of banks and insurance companies, given the more and more demanding solvency constraints they have to face: Basel 3 for banks and Solvency 2 for insurance companies.

As the net debt corresponds to the whole financial debt reduced by cash and cash equivalents, the deduction of the net debt means that the financial debt is deducted and the cash and cash equivalents are added. Such an addition is consistent with that of the financial assets.

The abovementioned formulas have to be used for the peers should they own financial assets. In such cases, assuming that the equity value corresponds to the market cap:

- Sales multiple = $(\text{Market cap} - \text{Financial assets} + \text{net debt} + \text{minority interests}) / \text{Sales}$
- EBITDA multiple = $(\text{Market cap} - \text{Financial assets} + \text{net debt} + \text{minority interests}) / \text{EBITDA}$
- EBIT multiple = $(\text{Market cap} - \text{Financial assets} + \text{net debt} + \text{minority interests}) / \text{EBIT}$

The above mentioned net debt includes the retirement provisions.

The earnings which generated by the financial assets are embedded in the net income and therefore in the book value. For that reason the P/E and the P/BV don't include any restatement of the financial assets.

Moreover, the net income and the book value which are respectively embedded in the P/E and P/BV are "group share". For that reason these multiples don't include any restatement of the minority interests.

iv. **Interpretation**

The various equity values enable to compare the firm to be valued to its peers:

- If the equity value based on the xEBITDA is higher than that based on xSales, the valuation is more favourable for the company to be valued if operating margins are taken into account. Therefore, the EBITDA margin of the firm to be valued is higher than its peers. Such a point can be checked by comparing the EBITDA of the firm to be valued with the average EBITDA margin of the sample's companies.
- If the equity value based on the xEBIT is higher than that based on xEBITDA, the valuation is more favourable for the company to be valued if Depreciations and Amortisations (D&A) are taken into account. Then, the firm to be valued has lower D&A than its peers, certainly because its investment policy is less aggressive. The competitiveness of the firm to be valued may be at stake.
- If the equity value based on the P/E is higher than that based on xEBIT, the valuation is more favourable for the company to be valued if net interests (rather than the net financial debt) are taken into account. Then, leverage effect of the firm to be valued has been used more intensively by the firm to be valued than by its peers.

The valuation implied by the xSales may not be significant if it is significantly different from that implied by the other multiples. It's the case when the firm's profitability is higher and lower than the peers. Then, the best thing to do is to exclude the xSales valuation. Such an approach can be replaced by the xSales correlated to EBITDA margin, provided that the correlation, based on the sample, is justified (ie the R^2 , or determination coefficient, is greater than 0,5).

In the same kind of background, a P/BV correlated to ROE can be used to value a bank or an insurance company.

v. Example

Simplified valuation of Pro7 based on 2 listed peers: TF1 and M6 multiples

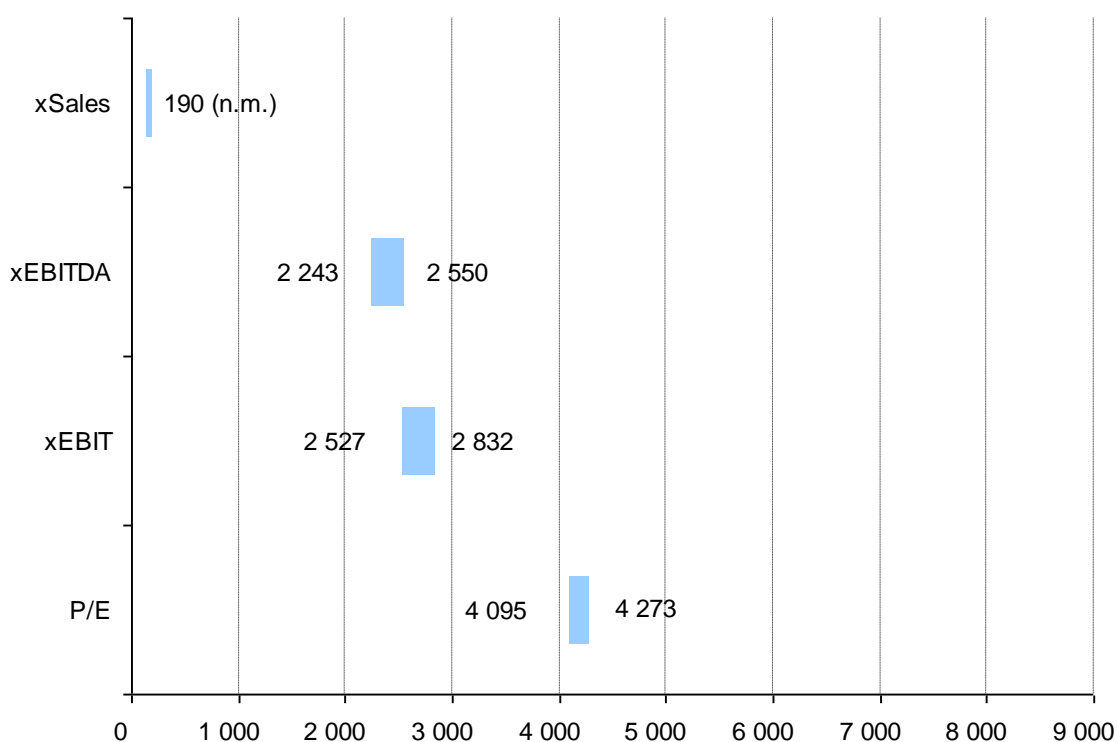
Company	Market cap	Net debt		EV		Sales		EBITDA		EBIT		Net profit	
		2012	2013	2012	2013	2012	2013	2012	2013	2012	2013	2012	2013
TF1	1 592	(18)	(67)	1 574	1 525	2 599	2 536	327	328	235	244	153	166
M6	1 445	(318)	(342)	1 127	1 103	1 414	1 444	216	239	204	225	127	138

	EV/Sales		EV/EBITDA		EV/EBIT		P/E		EBITDA/sales	
	2012	2013	2012	2013	2012	2013	2012	2013	2012	2013
TF1	0,61	0,60	4,8	4,6	6,7	6,3	10,4	9,6	13%	13%
M6	0,80	0,76	5,2	4,6	5,5	4,9	11,4	10,5	15%	17%
Average	0,70	0,68	5,0	4,6	6,1	5,6	10,9	10,0	14%	15%

Aggregates of Pro7	2 863	2 940	871	885	761	786	376	426	30%	30%
EV	2 008	2 007	4 368	4 100	4 650	4 384				
(Net debt)	(1 818)	(1 857)	(1 818)	(1 857)	(1 818)	(1 857)				

Equity value	190	150	2 550	2 243	2 832	2 527	4 095	4 273
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Non meaningful



b. M&A peers

i. Principle

The principle and the calculations to be done are the same as the ones which have been presented in the case of the listed peers approach. But instead of taking market caps of listed peers into account, the equity values of peers correspond to the transaction values in announced M&A deals.

To be fully consistent, the comparison has to be based on the same kind of information as that which was available when the M&A deal was negotiated. As most of the negotiation was based on the fiscal year's financial aggregates just before the deal, the multiples have to be calculated for such a year. In other words, if the deal is announced in the year N, the multiples have to be based on the year N-1. Then for the company to be valued, the average multiples have to be applied to the aggregated of the last year (ie N-1).





If public information on the peers was available regarding N or N+1 aggregates when the deal was negotiated, N and N+1 can be calculated. Then, their means have to be applied to the corresponding N and N+1 forecasted aggregates of the company to be valued.

ii. Example

Buyer	Target	transaction date	Equity value	Target's net debt	Target's EV	Target's aggregates as at 31/12/n-1			
						Sales	EBITDA	EBIT	Net profit
A	D	08/07/2008	1 000	400	1 400	1 500	200	180	120
B	E	03/05/2008	2 000	500	2 500	2 700	300	280	210
C	F	02/07/2007	3 000	600	3 600	3 900	400	380	300

Multiples transactionnels			
xSales	xEBITDA	xEBIT	P/E
0,93	7,0	7,8	8,3
0,93	8,3	8,9	9,5
0,92	9,0	9,5	10,0
Average	0,93	8,1	8,7
			9,3

Aggregates of the company to be valued	1000	200	180	140
EV	927	1 622	1 571	1 300
(Net debt)	(200)	(200)	(200)	
Equity value	727	1 422	1 371	1 300

Target	Ratio		
	EBITDA/Sales		Debt/Equity
D	13,3%		40,0%
E	11,1%		25,0%
F	10,3%		20,0%
	11,6%		28,3%
	<		>
Target	20,0%		16,6%
			
	Higher		Lower
	operating		leverage of the
	profitability of		firm compared
	the firm to be		to its peers
	valued		
	compared		
	to its peers		
			
	Higher value		Lower valuation
	based on		based on the P/E
	xEBITDA		versus xEBITDA

3. DCF

a. Principle of the DCF approach

The value of the operating assets (V) corresponds to the sum of the discounted Free Cash Flows (FCF)

$$V = \sum_{t=1}^{+\infty} \frac{FCF_t}{(1+K)^t}$$

The enterprise value (EV) is the sum of the operating assets and the non operating assets.
The equity value is, as usual: EV – net debt

b. Free cash flow calculation

i. Principle

The free cash flow (FCF) is a change in normative cash based on recurring elements. In that context, the exceptional items are not taken into account in the FCF calculation.

The FCF are bound to be discounted with a discount rate which corresponds to the wacc. As the Wacc includes the cost of net debt, the net interests are not taken into account in the FCF calculation either.

Therefore, the FCF calculation is based on EBIT. And, as net interests and exceptionals are not taken into account, the corporate tax is recalculated, based on EBIT. The after tax EBIT is the Net Operating Profit After Tax (NOPAT)

The NOPAT does not correspond to the change in cash as it has been reduced by the depreciations and amortisations (D&A). Therefore, such charges have to be added to the NOPAT in order to be neutralised.

All these calculations are based on P&L's aggregates which do not include the net capex (ie capital expenditures – divestments), where as the net capex correspond to changes in cash. As net capex are cash outflow, they have to be deducted in a FCF calculation.

Moreover, the sum of NOPAT and D&A is equal to the difference between EBITDA and normative corporate tax. But EBITDA does not necessarily correspond to a pre-tax change in operating cash because of delays of payments:

- part of the sales - which have increased the EBITDA - will be paid later by clients. So, they have to be neutralised in the FCF calculation. As it's impossible to distinguish such sales in the P&L, their counterparts in the balance sheet are taken into account. Therefore, the change in receivables has to be deducted;
- part of the purchases –which have decreased the EBITDA - will be paid later by the firm. So, they have to be neutralised in the FCF calculation. As it's impossible to distinguish such purchases in the P&L, their counterparts in the balance sheet are taken into account. Therefore, the change in payable has to be deducted;
- inventories which are sold yet have increased the EBITDA whereas they have not generated cash inflows. So, they have to be neutralised in the FCF calculation. Their counter part in the balance sheet can also be neutralised. Therefore, the change in inventories has to be deducted.
- Finally, the FCF includes:

=	$-\Delta\text{Receivables} + \Delta\text{Payables} - \Delta\text{Inventories}$
=	$-(\Delta\text{Receivables} + \Delta\text{Inventories} - \Delta\text{Payables})$
=	$-(\Delta\text{Current assets} - \Delta\text{Current liabilities})$
=	$-\Delta\text{WCR}$

Furthermore, as the DCF valuation can not rely on the pay out policy, the dividends paid by the firm to its shareholders are not included in the FCF calculation.

Conclusion:

$\text{FCF} = \text{EBIT} \cdot (1 - \tau) + \text{D\&A} - \text{Net capex} - \Delta\text{WCR}$ where τ = corporate tax rate

ii. Example

The DCF valuation of a firm is generally based on a limited set of information which is provided either by the company or by brokers. The following example illustrates the cash flow calculation based on a company's assumptions (from 2012 to 2015) and on a soft landing of the business plan as prepared by investment bankers. The available data are:

Amounts in € million	Achieved 2011	Budget 2012	Company's BP		
	2013	2014	2015		
Sales	1 000	1 200	1 500	1 800	2 000
EBITDA	300	350	400	420	450
(D&A)	(50)	(60)	(70)	(80)	(85)
(Net capex)	(60)	(65)	(70)	(75)	(80)
WCR	250	300	350	400	450

The DCF valuation can not be exclusively based on the above figures as the growth rate of sales in 2015 is 11%. Such a rate is not sustainable on a long term period. Therefore, a soft landing of the company's business plan has to be achieved. This duration of this soft landing is generally in 3-5 years range (2016-2020 here). Then, a recurring year (2021 here) is added in a order to calculate the terminal value which is presented in the next paragraph. The

- For the 2012-2015 period, the company's forecasts are unchanged
- For the soft landing period (2016-2020) period:
 - o Linear phasing of the sales' growth rate from 11% in 2015 to 3% in 2020:
 - Soft landing over 5 years
 - Consistency of the perpetuity growth rate (3%) with the market risk premium calculation
 - o Sustainability of some of 2015 ratios:
 - EBITDA margin ie EBITDA / Sales: 23%
 - D&A / Sales: -4,3%
 - WCR / Sales: 23%
 - Corporate tax rate: 36,1%
 - o Linear phasing of net capex from € 80 million to the D&A level in 2020 ie € 115 million
- For the recurring year (2021):
 - o Sustainability of the 3% sales' growth rate which is assumed for 2020
 - o Sustainability of the 2015-2020 ratios (EBITDA margin, D&A / Sales, WCR / Sales, corporate tax rate)
 - o Net capex = D&A

Amounts in € million	Achieved 2011	Budget 2012	Company's BP			Soft landing					Recurring 2021
	2013	2014	2015	2016	2017	2018	2019	2020			
Sales	1 000	1 200	1 500	1 800	2 000	2 190	2 362	2 510	2 626	2 704	2 785
Growth rate		20%	25%	20%	11%	9%	8%	6%	5%	3%	3%
EBITDA	300	350	400	420	450	493	531	565	591	608	627
EBITDA margin	30%	29%	27%	23%	23%	23%	23%	23%	23%	23%	23%
(D&A)	(50)	(60)	(70)	(80)	(85)	(93)	(100)	(107)	(112)	(115)	(118)
D&A / sales	-5,0%	-5,0%	-4,7%	-4,4%	-4,3%	-4,3%	-4,3%	-4,3%	-4,3%	-4,3%	-4,3%
EBIT	250	290	330	340	365	400	431	458	479	494	508
(Corporate tax)	(86)	(105)	(119)	(123)	(132)	(144)	(156)	(165)	(173)	(178)	(184)
Corporate tax rate	34,43%	36,10%	36,10%	36,10%	36,10%	36,10%	36,10%	36,10%	36,10%	36,10%	36,10%
NOPAT	164	185	211	217	233	255	275	293	306	315	325
D&A	50	60	70	80	85	93	100	107	112	115	118
(Net capex)	(60)	(65)	(70)	(75)	(80)	(87)	(94)	(101)	(108)	(115)	(118)
(ΔWCR)		(50)	(50)	(50)	(50)	(43)	(39)	(33)	(26)	(18)	(18)
Free Cash Flow		130	161	172	188	219	243	265	284	298	307
WCR	250	300	350	400	450	493	531	565	591	608	627
WCR / sales	25%	25%	23%	22%	23%	23%	23%	23%	23%	23%	23%

c. Terminal value

i. Principle

$$V = \sum_{t=1}^{+\infty} \frac{FCF_t}{(1+K)^t}$$

As the business plan provides forecasts for only n years, the above mentioned formula has to be broken down between 2 components:

$$V = \sum_{t=1}^n \frac{FCF_t}{(1+K)^t} + \sum_{t=n+1}^{+\infty} \frac{FCF_t}{(1+K)^t}$$

Let's assume that the terminal value (TV) is defined as:

$$TV = \sum_{t=n+1}^{+\infty} \frac{FCF_t}{(1+K)^t} = \frac{FCF_{n+1}}{(1+K)^{n+1}} + \frac{FCF_{n+2}}{(1+K)^{n+2}} + \dots$$

And that the FCF have a g yearly growth rate. Then:

$$TV = \frac{(1+g)^1 FCF_n}{(1+K)^{n+1}} + \frac{(1+g)^2 FCF_n}{(1+K)^{n+2}} + \dots = \frac{FCF_n}{(1+K)^n} \left[\frac{(1+g)}{(1+K)} + \frac{(1+g)^2}{(1+K)^2} + \dots \right]$$

$$TV = \frac{FCF_n}{(1+K)^n} \sum_{t=1}^{+\infty} \left(\frac{1+g}{1+K} \right)^t$$

Referring to the geometric series, for $0 < q < 1$: $\sum_{t=1}^{\infty} q^t = \frac{q}{(1-q)}$

Then, if « q » is replaced by: $\frac{1+g}{1+K}$:

$$TV = \frac{FCF_n}{(1+K)^n} \frac{\frac{1+g}{1+K}}{1 - \left(\frac{1+g}{1+K} \right)} = \frac{FCF_n}{(1+K)^n} \frac{\frac{1+g}{1+K}}{\frac{1+K-1-g}{1+K}}$$

This yields to:

$$TV = \frac{FCF_n}{(1+K)^n} \frac{(1+g)}{(K-g)}$$

$$\text{And: } V = \sum_{t=1}^n \frac{FCF_t}{(1+K)^t} + \frac{FCF_n}{(1+K)^n} \frac{(1+g)}{(K-g)}$$

ii. Example

This example is the continuation of the previous ones. The cash flow of the recurring year (2021) is equal to 307 M€ and the valuation is supposed to be achieved on the 30th of September 2012. Then, the discount period for 2012 corresponds to 1 quarter ie 0,25 year. For 2021, ie 9 years after the 31st of December 2012, the discount period will be 9,25.

The wacc (K) is supposed to be 10% and the perpetuity growth rate (g) is 3%, corresponding to the growth rate of the recurring year. Then:

$$TV = \frac{307}{(1+10\%)^{9,25}} \cdot \frac{(1+3\%)}{(10\% - 3\%)} = \text{€ } 1\,868 \text{ million}$$

d. Focus on the discount rate

i. Traditional approach of the relationship between the Wacc and the cost of equity

In the traditional approach of the cost of resources, the cost of equity and the cost of debt are flat as long as gearing ratios (Debt / Equity) are not reached.

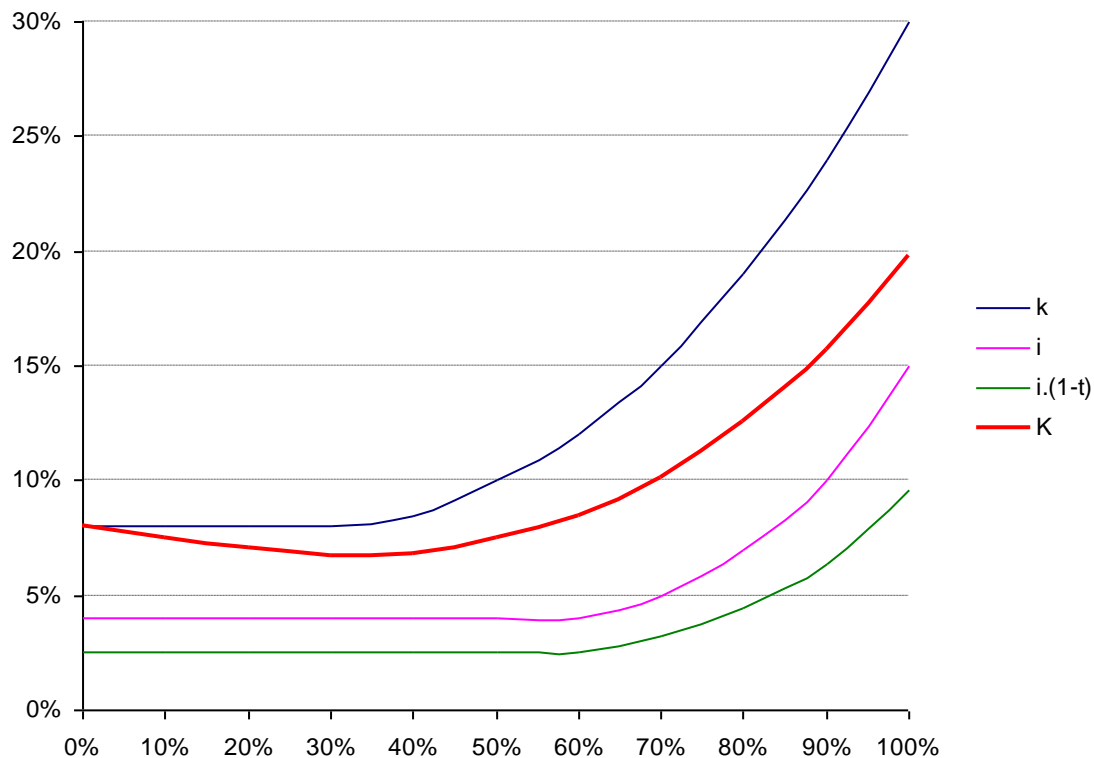
In the following example, the cost of equity is 8% as long as the gearing ratio is below or equal to 30% and the cost of debt is equal to 4% as long as the gearing ratio is below or equal to 60%.

As the cost of debt is lower than the cost of equity, the increase in the weight of debt in the financing of the firm enables to decrease the wacc as long as the cost of equity is flat. Therefore, the lowest level of the wacc (6,7%) is reached when the gearing reaches 30%. Beyond the 30% gearing, the wacc increases.

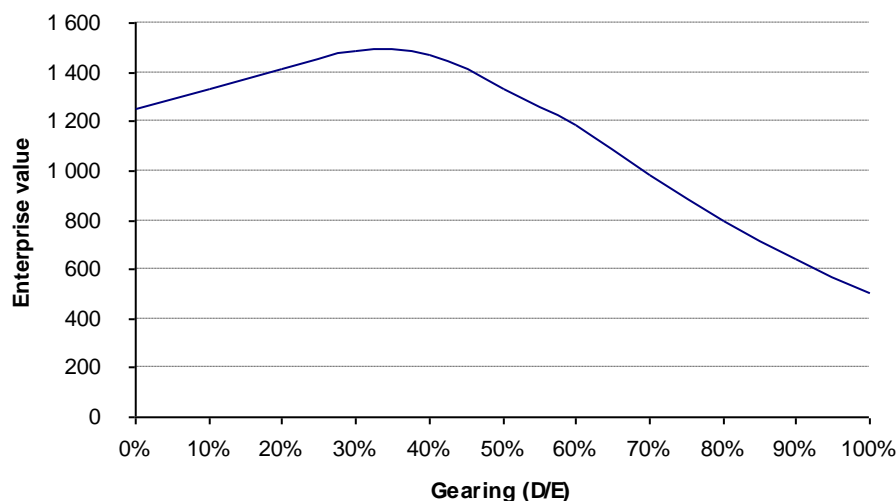
D/E	0,00%	10,00%	20,00%	30,00%	40,00%	50,00%	60,00%	70,00%	80,00%	90,00%	100,00%
E/(E+D)	100,0%	90,9%	83,3%	76,9%	71,4%	66,7%	62,5%	58,8%	55,6%	52,6%	50,0%
D/(E+D)	0,0%	9,1%	16,7%	23,1%	28,6%	33,3%	37,5%	41,2%	44,4%	47,4%	50,0%
k	8,0%	8,0%	8,0%	8,0%	8,5%	10,0%	12,0%	15,0%	19,0%	24,0%	30,0%
i	4,0%	4,0%	4,0%	4,0%	4,0%	4,0%	4,0%	5,0%	7,0%	10,0%	15,0%
i.(1-τ)	2,6%	2,6%	2,6%	2,6%	2,6%	2,6%	2,6%	3,2%	4,5%	6,4%	9,6%
K	8,0%	7,5%	7,1%	6,7%	6,8%	7,5%	8,5%	10,1%	12,5%	15,7%	19,8%

In this context, the 30% gearing corresponds to the optimal structure ie the structure which enables to have the lowest wacc and therefore the highest enterprise value based on a DCF approach.

The graph below illustrates the existence of the optimal structure.



Based on a 100 recurring FCF, the graph below presents the sensitivity of the enterprise value to the gearing ratio and underlines that the maximal EV is reached when the gearing is worth 30%.



ii. Modigliani & Miller approach

1. Without tax [1959]

a. Theorem 1: financial structure and enterprise value

If there is no corporate tax, the cost of capital of a firm is independent from its financial structure. The cost of capital is the discount rate of the sum of the expected discounted EBIT to perpetuity of an unleveraged firm with the same industrial risk. This rate is noted ρ .

In that case, EV being the enterprise value:

$$\boxed{EV = \frac{E(EBIT)}{\rho}} \quad [1]$$

b. Theorem 2: expected return on equity

Notations :

- r_E = Return on Equity
- i = cost of debt
- D = debt
- E = Equity
- NP = net profit

$$r_E = \frac{NP}{E} = \frac{E(EBIT) - iD}{E}.$$

Thanks to the first theorem:

$$r_E = \frac{\rho EV - iD}{E} = \frac{\rho(E + D) - iD}{E}.$$

Finally:

$$r_E = \rho + (\rho - i) \frac{D}{E} \quad [2]$$

2. With corporate tax [1963]

a. Tax shield and value of the leveraged firm

Notations:

- V = EV of a leveraged firm
- V^* = EV of an unleveraged firm with the same industrial risk
- i = cost of debt
- ρ = cost of capital of the unleveraged firm
- X = revenue of the stakeholders of the leveraged firm (shareholders and bondholders)
- X^* = revenue of the stakeholders of the unleveraged firm (shareholders)
- τ = corporate tax rate (36,1% in France since 2012)

Assuming a 100% pay out rate:

$$X^* = \text{EBIT} (1 - \tau)$$

$$X = (\text{EBIT} - iD)(1 - \tau) + i.D = \text{EBIT}(1 - \tau) - i.D(1 - \tau) + iD = \text{EBIT}(1 - \tau) + i.D.\tau.$$

$$\text{Then: } X = X^* + i.D.\tau.$$

Nota: $i.D$ = financial expense hence $i.D.\tau$ = tax shield generated by the deductibility of financial charges.

Assuming that:

- the enterprise value corresponds to a sum of discounted cash flows to perpetuity;
- ρ is the discount rate of the unleveraged firm;
- i is the discount rate of the tax shield generated by the deductible financial charge:

$$V^* = \frac{X^*}{\rho}.$$

$$\text{and: } V = \frac{X^*}{\rho} + \frac{iDt}{i}$$

$$\text{ie : } \boxed{V = V^* + D\tau.} \quad [3]$$

Conclusion

The EV of the leveraged firm is higher than that of the unleveraged firm because the leveraged firm has an additional asset ie the tax shield.

b. Expected return on equity

Notations:

R_i = return of the leveraged firm's share

R_i^* = return of the unleveraged firm's share

V = EV of the leveraged firm

V^* = EV of the unleveraged firm

$$E(R_i^*) = \frac{[E(EBIT)](1-\tau)}{E^*} = \frac{[E(EBIT)](1-\tau)}{V^*} = \rho$$

$$E(R_i) = \frac{[E(EBIT) - iD](1-\tau)}{E} = \frac{E(EBIT).(1-\tau)}{E} - i.(1-\tau) \frac{D}{E}$$

$$k = E(R_i) = \frac{[E(EBIT)](1-\tau)}{V^*} \cdot \frac{V^*}{E} - i.(1-\tau) \frac{D}{E} = \rho \cdot \frac{V^*}{E} - i.(1-\tau) \frac{D}{E}.$$

$$\text{As } V = V^* + D.\tau, \quad V^* = V - D.\tau$$

Then:

$$k = \rho \cdot \frac{V - D.\tau}{E} - i.(1-\tau) \frac{D}{E}.$$

And as $V = E + D$:

$$k = \rho \cdot \frac{E + D - D.\tau}{E} - i.(1-\tau) \frac{D}{E} = \rho \cdot \left[1 + \frac{D(1-\tau)}{E}\right] - i.(1-\tau) \frac{D}{E}$$

$$k = \rho + \rho.(1-\tau) \frac{D}{E} - i.(1-\tau) \frac{D}{E}.$$

Finally:

$$\boxed{k = \rho + (\rho - i)(1-\tau) \frac{D}{E}}$$

[4]

c. Adjusted cost of capital

If k is replaced by $\rho + (\rho - i)(1 - \tau) \frac{D}{E}$ in the wacc formula:

$$K = k \frac{E}{E + D} + i(1 - \tau) \frac{D}{E + D}$$

$$K = \left[\rho + (\rho - i)(1 - \tau) \frac{D}{E} \right] \frac{E}{E + D} + i(1 - \tau) \frac{D}{E + D}$$

$$K = \left[\rho \frac{E}{E + D} \right] + [(\rho - i)(1 - \tau)] \frac{D}{E + D} + i(1 - \tau) \frac{D}{E + D}$$

$$K = \left[\rho \frac{E}{E + D} \right] + [(\rho)(1 - \tau)] \frac{D}{E + D} - [(i)(1 - \tau)] \frac{D}{E + D} + [(i)(1 - \tau)] \frac{D}{E + D}$$

$$K = \left[\rho E \frac{E}{CP + D} \right] + \rho \frac{D}{E + D} - [\rho \tau] \frac{D}{E + D} = \rho \frac{E + D}{E + D} - [\rho \tau] \frac{D}{E + D}$$

Finally:

$$K = \rho \left[1 - \frac{D \cdot \tau}{E + D} \right] \quad [5]$$

The value of K does not include the cost of debt i . It will be seen, in the next paragraph, that it is only linked to the risk free rate, r . This is the reason why, in a DCF approach, a sensitivity analysis to the risk free rate - and not to the cost of debt - is generally presented.

Nota:

If $D = 0$, then (unsurprisingly) : $K = \rho$

When the debt increases significantly (D is narrowing $+\infty$),

$\lim_{D \rightarrow +\infty} \frac{D \cdot \tau}{E + D} = \tau$ when D tend is narrowing $+\infty$. Hence K is converging towards $\rho(1 - \tau)$.

To illustrate the formula [5], the following numerical assumptions can be taken into account as the central case.

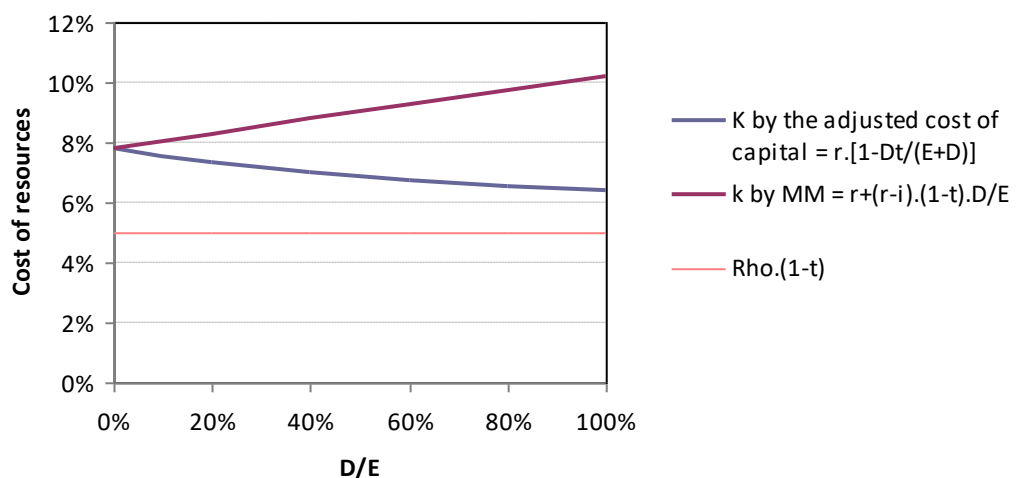
Risk free rate = r	3,00%
Unleveraged beta = β^*	0,8
Market risk premium = $E(R_M) - r$	6,00%
$E(R^*) = \rho = r + \beta^* \cdot [E(R_M) - r]$	7,80%
D	20
E	100
D/E	20%
τ	36,1%

Various values of the D/E ratio are proposed in the table below. For each value of D/E, the adjusted cost of capital and the cost of equity are calculated.

Sensitivity to the financial structure

D/E	0,00%	20,00%	40,00%	60,00%	80,00%	100,00%
D/(E+D)	0,00%	16,67%	28,57%	37,50%	44,44%	50,00%
K by the adjusted cost of capital = $\rho \cdot [1 - D\tau / (E+D)]$	7,80%	7,33%	7,00%	6,74%	6,55%	6,39%
k by MM = $\rho + (\rho - i) \cdot (1 - \tau) \cdot D/E$	7,80%	8,29%	8,77%	9,26%	9,74%	10,23%
$\rho \cdot (1 - \tau)$	4,98%	4,98%	4,98%	4,98%	4,98%	4,98%

The various cases can be summarized by the graph below which shows the decreasing trend of the cost of capital (K) when the gearing (D/E) increases.



iii. Hamada formula [1972]

1. Relationship between the unleveraged and leveraged beta

It has been established (cf.: equation [4]) that:

$$E(R_i) = k = \rho + (\rho - i)(1 - t) \frac{D}{E}$$

Assuming that the company raise debt based on the risk free rate r :

$$E(R_i) = k = \rho + (\rho - r)(1 - t) \frac{D}{E} \quad [A]$$

It is implicit, in the CAPM that investors differentiate assets only according to their expected rates of return and their contribution to the variance of the investors' efficient portfolio. The capital market is in equilibrium if assets are priced so that:

$$E(R_i) = r + \beta_i [E(R_M) - r] = k \quad [B]$$

$$\text{and } E(R_i^*) = \rho = r + \beta_i^* [E(R_M) - r]. \quad [C]$$

Substituting [B] and [C] in [A], the following formula is obtained:

$$r + \beta_i [E(R_M) - r] = r + \beta_i^* [E(R_M) - r] + (r + \beta_i^* [E(R_M) - r] - r)(1 - \tau)D/E$$

After simplification by r :

$$\beta_i [E(R_M) - r] = \beta_i^* [E(R_M) - r] + \beta_i^* [E(R_M) - r](1 - \tau)D/E.$$

After simplification by $[E(R_M) - r]$:

$$\beta_i = \beta_i^* + \beta_i^* (1 - \tau)D/E.$$

Finally:

$$\boxed{\beta_i = \beta_i^* \left[1 + \frac{D(1 - \tau)}{E} \right]} \quad [6]$$

2. Relationship in the case of a risky debt

Thanks to the CAPM:

$$E(R_i) = k = r_f + \beta_i [E(R_M) - r_f] \quad [D]$$

Moreover, it has been established previously that :

$$E(R_i) = k = \rho + (\rho - i)(1 - \tau) \frac{D}{E} \quad [E]$$

In the case of a risky debt, its cost is i which corresponds to the sum of the risk free rate and a risk premium. Then, applying the CAPM to the cost of debt:

$$i = r_f + \beta_D [E(R_M) - r_f] \quad [F]$$

where β_D is the beta of the debt.

$$\text{Furthermore : } \rho = E(R_i^*) = r_f + \beta_i^* [E(R_M) - r_f] \quad [G]$$

Then, combining the two expressions of $E(R_i)$ which are given by the formulas [D] et [E] :

$$E(R_i) = k = r_f + \beta_i [E(R_M) - r_f] = \rho + (\rho - i)(1 - \tau) \frac{D}{E} \quad [H]$$

Finally, combining the values of i et ρ (cf. : equations [F] and [G] in the equation [H]):

$$r_f + \beta_i \cdot [E(R_M) - r_f] = r_f + \beta_i^* \cdot [E(R_M) - r_f] + \{r_f + \beta_i^* \cdot [E(R_M) - r_f] - r_f - \beta_D \cdot [E(R_M) - r_f]\} (1 - \tau) \frac{D}{E}$$

After simplification by r_f and $[E(R_M) - r_f]$:

$$\boxed{\beta_i = \beta_i^* + (\beta_i^* - \beta_D)(1 - \tau) \cdot \frac{D}{E}} \quad [7]$$

Moreover, as: $E(R_i) = k = r_f + \beta_i \cdot [E(R_M) - r_f]$

Then, the replacement of β_i by its value in the formula [7] provides the following expression:

$$k = r_f + \left\{ \beta_i^* + (\beta_i^* - \beta_D)(1 - \tau) \cdot \frac{D}{E} \right\} \cdot [E(R_M) - r_f]$$

And:

$$\boxed{k = \underbrace{r_f}_{\text{Risk_free_rate}} + \underbrace{\beta_i^* \cdot [E(R_M) - r_f]}_{\text{Operating_risk_premium}} + \underbrace{(\beta_i^* - \beta_D) \cdot [E(R_M) - r_f] (1 - \tau) \cdot \frac{D}{E}}_{\text{Financial_risk_premium}}} \quad [8]$$

The example which has been presented previously in order to illustrate the formula [5] is taken into account in order to illustrate the formulas [7] and [8].

The base assumptions are the following ones:

Risk free rate = r	3,00%
Unleveraged beta = β^*	0,8
Market risk premium = $E(R_M) - r$	6,00%
$E(R^*) = \rho = r + \beta^* \cdot [E(R_M) - r]$	7,80%
D	20
E	100
D/E	20%
τ	36,1%

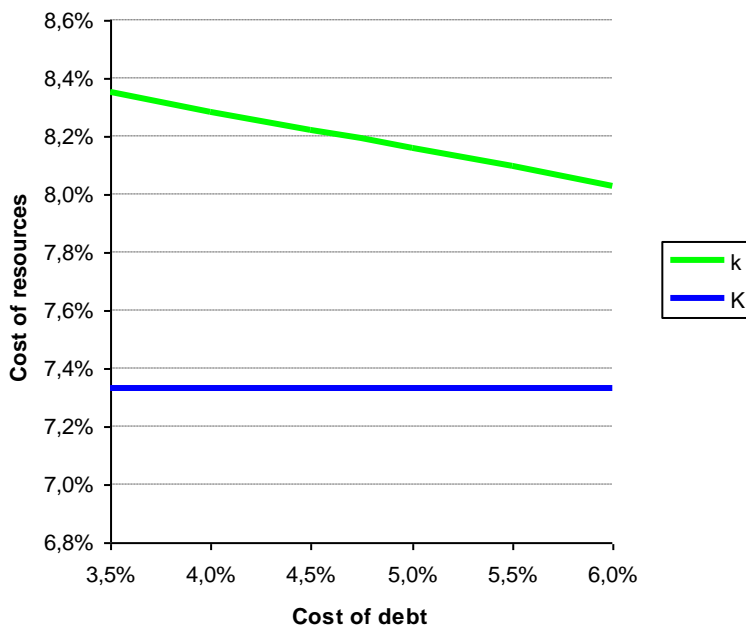
Assuming different costs of debt justified by different levels of the beta of the debt (ie the risk free rate is supposed to be flat), the objective of table below is threefold:

- present the calculation of the cost of equity (k) by the Modigliani Miller (MM) approach (formula [4]) and by the CAPM (formula [B]);
- present the calculation of the cost of capital (K) by the wacc and by the adjusted cost of capital (formula [5]);
- underline that the cost of capital is unchanged. Indeed, the increase in the increase in the cost of debt is compensated by a decrease in the cost of equity. In other words, the risk is supposed to be transferred from the shareholders to the bondholders who demand a higher return which is therefore detrimental to the shareholders' expected return.

Sensitivity to the cost of debt

i	3,5%	4,0%	4,5%	5,0%	5,5%	6,0%
k by MM = $\rho + (\rho - i) \cdot (1 - \tau) \cdot D/E$	8,35%	8,29%	8,22%	8,16%	8,09%	8,03%
K by the wacc = $[k \cdot E + i \cdot (1 - \tau) \cdot D] / (E + D)$	7,33%	7,33%	7,33%	7,33%	7,33%	7,33%
K by the adjusted cost of capital = $\rho \cdot [1 - D\tau / (E + D)]$	7,33%	7,33%	7,33%	7,33%	7,33%	7,33%
Beta of the debt (β_D)	0,08	0,17	0,25	0,33	0,42	0,50
Leveraged beta (β)	0,89	0,88	0,87	0,86	0,85	0,84
k by the CAPM	8,35%	8,29%	8,22%	8,16%	8,09%	8,03%

Such results can be illustrated by the following graph:



Based on the same assumptions regarding the cost of debt, the next table presents the breakdown of the cost of equity between the risk free rate, the operating risk and the financial risk, using formula [8]:

i	3,5%	4,0%	4,5%	5,0%	5,5%	6,0%
k by MM = $\rho + (\rho - i) \cdot (1 - \tau) \cdot D/E$	8,35%	8,29%	8,22%	8,16%	8,09%	8,03%
Breakdown of k						
Risk free rate = r	3,00%	3,00%	3,00%	3,00%	3,00%	3,00%
Operating risk premium	4,80%	4,80%	4,80%	4,80%	4,80%	4,80%
Financial risk premium	0,55%	0,49%	0,42%	0,36%	0,29%	0,23%
Cost of equity (k)	8,35%	8,29%	8,22%	8,16%	8,09%	8,03%

3. Methodology of the Wacc calculation in a DCF

The following example is the continuity of the previous one which had illustrated the FCF and the TV calculations.

Amounts in € million	Achieved 2011	Budget 2012	Company's BP			Soft landing					Recurring 2021
			2013	2014	2015	2016	2017	2018	2019	2020	
Free Cash Flow		130	161	172	188	219	243	265	284	298	307

The discount rate is the wacc (K) the formula of which is:

$$K = k \frac{E}{E + D} + i \cdot (1 - \tau) \frac{D}{E + D}$$

Where:

k = cost of equity

i = cost of net debt

τ = corporate tax rate = 36,1%

E = economic value of equity

D = net debt

The economic value of equity (E) is the value which is the outcome of the DCF valuation. Therefore, the model has to loop on itself.

Moreover, the calculation of the cost of equity (k) is based on the CAPM which includes the company's beta of the shares. The DCF approach is often used to value a company which is not listed and which has no beta. It can also be used to value a listed company. But, in that case, the beta is consistent with the market capitalisation and not with the equity value (E) which is the outcome of the DCF valuation. In that context:

- if the firm is not listed, an unleveraged beta (β^*) has to be obtained from a sample of listed peers. Then, β^* has to be leveraged based on the financial structure which is implied by the DCF valuation. Then, using the Hamada formula: $\beta_i = \beta_i^* + (\beta_i^* - \beta_D) \cdot \frac{D}{E}$ where β_D is the debt's beta and E is the equity value corresponding to the outcome of the DCF valuation;
- if the firm is listed, its beta which can be obtained from a data basis (Datastream, Bloomberg, Facset...) and which is consistent with the company's market cap, has to be unleveraged ie the firm's β^* has to be calculated based on the Hamada formula. Assuming that E is the market cap of the firm to be valued, as $\beta_i = \beta_i^* \left[1 + \frac{D(1-\tau)}{E} \right]$, $\beta_i^* = \frac{\beta_i}{1 + \frac{D(1-\tau)}{E}}$. Then, as

for unlisted firms, β^* has to be leveraged based on the financial structure which is implied by the DCF valuation ie $\beta_i' = \beta_i^* + (\beta_i^* - \beta_D) \cdot \frac{D}{E}$

From a practical point view, the Excel modelling which has to include 2 loops (the first one for E , the second one for β), has to be broken down into 3 steps:

- calculation of the DCF valuation using a whatever wacc, which can be, for example: 10%. This enables to enter formulas for the discounted FCF, for their sum and the terminal value. Assuming that the net debt is worth € 1 000 million:

Risk free rate		Sum of discounted FCF (2012-2021)	1 391
Market risk premium		Terminal value	1 868
Beta from Bloomberg			
Market cap		EV	3 259
Corporate tax rate		Net debt	1 000
Unleveraged beta			
Beta of debt		Equity value	2 259
Cost of debt			
Leveraged beta			
Cost of equity	0,00%		
Perpetuity growth rate	3,00%		
Wacc	10,00%		

Amounts in € million	Achieved 2011	Budget 2012	Company's BP			Soft landing					Recurring 2021
			2013	2014	2015	2016	2017	2018	2019	2020	
Free Cash Flow		130	161	172	188	219	243	265	284	298	307
Discount period		0,25	1,25	2,25	3,25	4,25	5,25	6,25	7,25	8,25	9,25
Discounted FCF		127	143	139	138	146	147	146	142	136	127

- Assuming that the beta from Bloomberg is 0,851 and the market cap of the firm is € 3 000 million, the wacc is based on a proxy of the cost of equity the calculation of which, at this stage, is using the Bloomberg's beta of the firm to be valued. In that case, assuming a 4% risk free rate, a 6% market risk premium and a 5% pretax cost of net debt, the calculations are presented in the table below. This table is the same as the previous one with a few additional data and formulas (k and K as described below). To get rid of the dashes which appear on the Excel sheet because of the loop, it is necessary to use: *Tools, Options, Calculation*, and click on *Iterations*

Risk free rate	4,00%		Sum of discounted FCF (2012-2021)			1 529					
Market risk premium	6,00%		Terminal value			3 224					
Beta from Bloomberg	0,851										
Market cap	3 000		EV			4 754					
Corporate tax rate	36,10%		Net debt			1 000					
Unleveraged beta											
Beta of debt			Equity value			3 754					
Cost of debt	5,00%										
Leveraged beta											
Cost of equity	9,11%										
Perpetuity growth rate	3,00%										
Wacc	7,86%										
Amounts in € million	Achieved 2011	Budget 2012	Company's BP			Soft landing					Recurring 2021
			2013	2014	2015	2016	2017	2018	2019	2020	
Free Cash Flow		130	161	172	188	219	243	265	284	298	307
Discount period		0,25	1,25	2,25	3,25	4,25	5,25	6,25	7,25	8,25	9,25
Discounted FCF		128	146	145	147	159	163	165	164	159	152

Here: $k = 4,00\% + 0,851 \times 6,00\% = 9,11\%$

and $K = 9,11\% \frac{3754}{4754} + 5,00\% \cdot (1 - 36,1\%) \frac{1000}{4754} = 7,86\%$, the value of E (ie 3754) being the outcome of the iterative calculation provided by Excel.

- The last part of the modelling consists in using, in the cost of equity, a beta which is consistent with the DCF valuation. For that reason, the unleveraged beta is calculated:

$$\beta_i^* = \frac{0,851_i}{1 + \frac{1000 \cdot (1 - 36,1)}{3861}} = 0,730, \text{ the new value of E (ie 3861) being the outcome of the}$$

iterative calculation provided by Excel. As the cost of debt is 5,00%, the beta of the debt can be obtained thanks to the CAPM : $i = r_f + \beta_D \cdot [E(R_M) - r_f]$ hence $\beta_D =$

$$\frac{i - r_f}{E(R_M) - r_f} = \frac{5\% - 4\%}{6\%} = 0,167. \text{ Then the beta which has to be used in order to calculate the}$$

$$\text{cost of equity is : } \beta_i' = 0,730 + (0,730 - 0,167) \cdot (1 - 36,1\%) \cdot \frac{1000}{3861} = 0,823 \text{ and}$$

$k = 4\% + 0,823 \times 6\% = 8,94\%$. Finally, the wacc is recalculated and its definitive value is 7,76%.

Risk free rate	4,00%	Sum of discounted FCF (2012-2021)	1 537
Market risk premium	6,00%	Terminal value	3 324
Beta from Bloomberg	0,851		
Market cap	3 000	EV	4 861
Corporate tax rate	36,10%	Net debt	1 000
Unleveraged beta = β^*	0,730		
Beta of debt	0,167	Equity value	3 861
Cost of debt	5,00%		
		Checks	
Leveraged beta	0,823	$k^* = \rho$	8,38%
Cost of equity = k (CAPM)	8,94%	$k = \rho + (\rho - i) \cdot (1 - \tau) \cdot D/E$	8,94%
Perpetuity growth rate	3,00%	$K = \rho \cdot [1 - \tau D / (E + D)]$	7,76%
Wacc = K	7,76%		

Amounts in € million	Achieved 2011	Budget 2012	Company's BP			Soft landing					Recurring 2021
	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Free Cash Flow		130	161	172	188	219	243	265	284	298	307
Discount period		0,25	1,25	2,25	3,25	4,25	5,25	6,25	7,25	8,25	9,25
Discounted FCF		128	147	146	148	159	164	166	165	161	154
WCR	250	300	350	400	450	493	531	565	591	608	627
WCR / sales	25%	25%	23%	22%	23%	23%	23%	23%	23%	23%	23%

The cost of equity (k) and the wacc (K) can be checked thanks to Modigliani & Miller:

As $\rho = k^* = r_f + \beta^* \cdot [E(R_M) - r_f] = 4\% + 0,730 \times 6\% = 8,38\%$:

$$k = 8,38\% + (8,38\% - 5,00\%) \cdot (1 - 36,1\%) \cdot \frac{1000}{3861} = 8,94\%$$

$$K = 0,730 \cdot \left[1 - \frac{1000 \times 36,1\%}{4861} \right] = 7,76\%$$

It can also be checked that if the beta of the debt changes (for example: $\beta_D = 1$), the cost of debt and the cost of equity change ($i = 10,00\%$ and $k = 8,11\%$) but the cost of capital is unchanged ($K = 7,76\%$): the risk and the corresponding return is transferred from shareholders to bond holders. A sensitivity analysis of the equity value to the risk free rate and to the perpetuity growth rate is provided hereafter.

g	Risk free rate		
	3%	4%	5%
2%	4 006	3 332	2 826
3%	4 765	3 861	3 213
4%	5 897	4 599	3 728

4. Net Asset Value and Sum of the Parts

a. Net asset value of a holding company (NAV)

i. Principle

The NAV approach is dedicated to the valuation of holdings companies. Such firms manage securities. The net book values of the securities appear in the holding's balance sheet. In the NAV, the net book values have to be replaced by economic values the calculation of which can be based on the market capitalisations of the subsidiaries and affiliates if they are listed.

It can be noticed that:

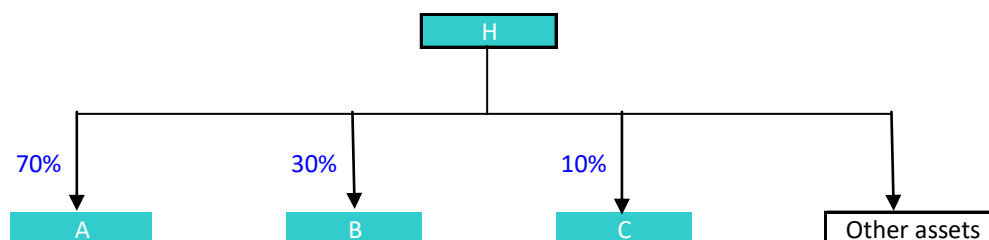
$$\begin{aligned}\text{Pretax NAV} &= \text{economic value of assets} - \text{net debt} \\ &= \text{book value of assets} + \text{latent capital gain} - \text{net debt} \\ &= \text{equity} + \text{latent capital gain}\end{aligned}$$

Then:

$$\text{After tax NAV} = \text{equity} + \text{latent capital gain} - \text{latent corporate tax}$$

ii. Example

The holding H has shareholdings in the companies A, B and C. These 3 companies are listed; H also has other assets in its balance sheet, as indicated in the graph and the table below.



Company balance sheet of H			
A	500	Equity	1 200
B	700	Net debt	1 000
C	200		
Other assets	800		
Total assets	2 200		2 200

Market caps	
A	1 000
B	2 000
C	3 000

The NAV calculation can be based on the following assumptions:

- valuation of the shareholdings in the listed companies based on their market caps;
- valuation of the other assets based on their net book value (ie 800).

The table below presents the detailed calculation. It highlights that the calculation of the pretax NAV (1400) can be based:

- either on the sum of economic values (2 400) from which the net debt of H (1 000) must be deducted;
- or on the sum of the equity of H (1 200) and the latent capital gain (200).

If the shareholdings in A, B and C have been held for more than 2 years, the effective corporate tax rate is 3,61% in France (versus 36,1% if they've been held for less than 2 years).

Asset	Economic value	Book value	Capital gain/loss
A	700	500	200
B	600	700	(100)
C	300	200	100
Other	800	800	0
Total	2 400	2 200	200
Net debt	1 000		
Pretax NAV	1 400		
Equity			1 200
Pretax NAV			1 400
Corporate tax rate on capital gains			3,61%
Tax on capital gain			(7)
Post tax NAV			1 393

b. Sum of the parts of a conglomerate (SotP)

i. Principle

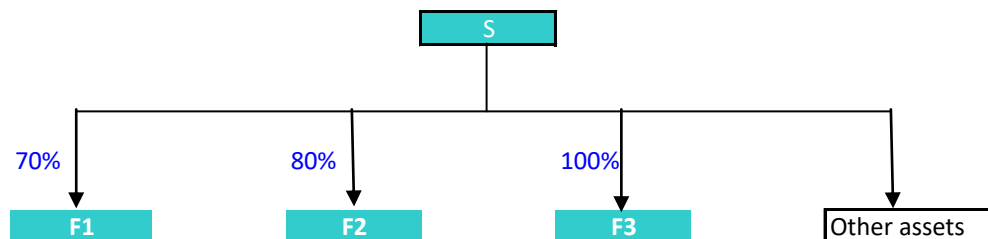
The SotP is used to value conglomerates, ie companies which have various activities. In that case, a valuation of each activity has to be achieved; each valuation can rely on peers (listed or M&A peers) or on a DCF. The sum of the valuations of the various activities is the enterprise value of the conglomerate.

The deduction of the consolidated net debt on the one hand, of the economic value of the minority interests (ie.: on the other hand enables to get the equity value group share).

The calculation of the economic value of the minority interests is based on the economic value of equity of each subsidiary.

ii. Example

The conglomerate H has shareholdings in the companies F1, F2 and F3. These 3 subsidiaries are not listed and that can be valued based on a listed peers approach; H also has other assets in its balance sheet, as indicated in the graph below.



The EBIT and EBIT multiples of the 3 subsidiaries are provided in the table below:

Assets	EBIT 2013	xEBIT 2013
	A	B
F1	80	9,0
F2	150	10,0
F3	500	11,0

The tables below provide the company balance sheet and the consolidated balance sheet of S:

Company balance sheet of S			
Shares F1	300	Equity	1 100
Shares F2	400	Net debt	1 000
Shares F3	800		
Other assets	600		
Total assets	2 100		2 100

Consolidated balance sheet of S			
Assets of F1, F2, F3	8 000	Equity	
Goodwills		Company	1 100
on F1	230	Consolidated reserves	
on F2	240	from F1	420
on F3	300	from F2	800
Other assets	600	from F3	3 500
		Minority interests	
		from F1	210
		from F2	240
		Net debt	3 100
Total assets	9 370		9 370

The balance sheets of the 3 subsidiaries are provided hereafter:

Balance sheet of F1			
Various assets	1 000	Equity	700
		Net debt	300
Total assets	1 000		1 000

Balance sheet of F2			
Various assets	2 000	Equity	1 200
		Net debt	800
Total assets	2 000		2 000

Balance sheet of F3			
Various assets	5 000	Equity	4 000
		Net debt	1 000
Total assets	5 000		5 000

The detailed calculation of the SotP is provided in the table below:

Assets	Equity when bought	EBIT 2013	xEBIT 2013	EV	Net debt	Equity value	% of interest	Minority interests	Equity group share
		A	B	C=A x B	D	E = C - D	F	G = E.(1-F)	H = E - G
F1	100	80	9,0	720	300	420	70%	126	294
F2	200	150	10,0	1 500	800	700	80%	140	560
F3	500	500	11,0	5 500	1 000	4 500	100%	0	4 500
Others				600	1 000	(400)			(400)
Total (1)				8 320	3 100	5 220		266	4 954
Consolidated debt (2)				3 100					
Minority interests (3)				266					
Equity group share : (1) - (2) - (3)				4 954					

c. Consistency of NAV and SotP approaches based on the previous example

The outcome of the SotP valuation approach can be checked with a NAV calculation as the activities which have been valued are located in subsidiaries.

The detailed calculations are provided hereafter:

Assets	Equity when bought	EBIT 2013	xEBIT 2013	EV	Net debt	Equity value	% of interest	Minority interests	Equity group share	Value of the shares
		A	B	C=A x B	D	E = C - D	F	G = E.(1-F)	H = E - G	I = E x F
F1	100	80	9,0	720	300	420	70%	126	294	294
F2	200	150	10,0	1 500	800	700	80%	140	560	560
F3	500	500	11,0	5 500	1 000	4 500	100%	0	4 500	4 500
Others				600	1 000	(400)			(400)	
Total (1)				8 320	3 100	5 220		266	4 954	
Consolidated debt (2)				3 100						
Minority interests (3)				266						
Equity group share : (1) - (2) - (3)				4 954						

Asset	Econ. value	Book value	Capital gain/loss
Shares F1	294	300	(6)
Shares F2	560	400	160
Shares F3	4 500	800	3 700
Others	600	600	0
Total (1)	5 954	2 100	3 854
Net debt (2)	1 000		
NAV: (1) - (2)	4 954		
Equity		1 100	
NAV		4 954	

No tax on latent capital gain is included here as the conglomerate (as a going concern) is not supposed to get rid of its activities/subsidiaries

5. Valuation of financial institutions

a. Banks

i. Principle

The regulation of banks includes the compliance with solvency constraints: based on the Basel 3 rules, the Core Tier 1 (CT1) ratio of each bank has to reach progressively 7% from 2013 to 2019. But most banks have announced that their CT1 ratio would reach 9% or 10% from 2013 onwards.

The CT1 ratio is equal to $CT1 / RWA$

The CT1 or common equity is based on the shareholders equity. In other terms, it does not include any hybrid or debt instrument.

The book value of equity corresponds to the amount the shareholders would receive should the firm be liquidated, assuming the selling prices of its assets correspond to their book values. As the goodwills and intangible assets can't be sold, their book values are deducted from the shareholders equity in the CT1 calculation as a Tangible Book Value calculation.

Moreover, the shareholdings in financial institutions which represent at least 10% of their capital have to be deducted from the CT1:

- If the shareholding is in a 10-19% range, the owned financial institution is not consolidated and its net book value of the corresponding investment is deducted from the CT1
- If the shareholding is in a 20-50% range, the owned financial institution is consolidated by the equity method and its amount in the bank's consolidated balance sheet is deducted from the CT1

The RWA are the risk weighted assets of the banks which are mainly composed of loans granted to clients. They also include off balance sheets elements (guarantees) and other assets

The CT1 and RWA are published by the listed banks.

The Dividend Discount Model (DDM), which is sometimes called DCF of the bank enables to take the medium / long term forecasts (as in a traditional DCF) and the solvency constraints into account.

In this approach, the equity value is the sum of the discounted future theoretical dividends ie the dividends which could be paid so that the CT1 ratio reaches its target level. For example, if the CT1 and RWA are respectively worth 120 and 1000, the CT1 ratio reaches 12%. If the target CT1 ratio is 9%, the bank has an excess equity for a consideration of $120 - 9\% \times 1000 = 30$. In that case, the theoretical dividend amounts to 30. This dividend payment is not included in the business plan of the bank. Then, the after tax cost of its financing has to be included in the net income which has therefore to be restated.

If the excess equity is negative, the bank suffers a CT1 insufficiency and has to be recapitalised. In that the case, the amount of the required capital increase is equal to the negative dividend. The

product of the capital increase can be invested in short term securities. Then it enables to increase the net banking income of the bank and therefore its net income.

As dividends, based on the net income, are discounted, the discount rate is the cost of equity of the bank.

ii. Example

A bank has disclosed the following simplified business plan:

	Achieved 2011	Budget 2012	Company's BP		
	2013	2014	2015	2016	2017
Net profit	10	12	15	16	17
RWA	1 000	1 500	1 800	2 000	2 200

Its target CT1 ratio is 9% and, based on the CAPM, its cost of equity is 10%

With a 6 additional years including a recurring year (2021), the soft landing of the business plan is the following one, assuming for the 2016-2020 period:

- a linear phasing of the growth rate of the RWA from 6,7% in 2015 to 3% in 2020;
- a growth rate of the net income in line with that of the RWA:

	Achieved 2011	Budget 2012	Company's BP			Soft landing					Recurring 2021
	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023
Net profit	10	12	15	16	17	18	20	21	22	23	23
RWA	1 000	1 500	1 800	2 000	2 200	2 389	2 561	2 710	2 829	2 914	3 001
Growth rate		50,0%	20,0%	11,1%	10,0%	8,6%	7,2%	5,8%	4,4%	3,0%	3,0%

The following table provides the detailed DDM calculations assuming that the level of equity as of 31st of December 2011 is worth 120.

	Achieved 2011	Budget 2012	Company's BP			Soft landing					Recurring 2021
	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Net profit	10	12	15	16	17	18	20	21	22	23	23
RWA	1 000	1 500	1 800	2 000	2 200	2 389	2 561	2 710	2 829	2 914	3 001
Growth rate		50,0%	20,0%	11,1%	10,0%	8,6%	7,2%	5,8%	4,4%	3,0%	3,0%
Equity											
1st of Jan		120	135	162	180	198	215	231	244	255	262
Net profit		12	15	16	17	18	20	21	22	23	23
After tax cost of dividends		0	0	0	0	0	0	0	(0)	(0)	(1)
31st of Dec	120	132	150	178	197	217	235	252	266	277	285
RWA		1 500	1 800	2 000	2 200	2 389	2 561	2 710	2 829	2 914	3 001
Target CT1 ratio		9%	9%	9%	9%	9%	9%	9%	9%	9%	9%
Required equity		135	162	180	198	215	231	244	255	262	270
Excess equity		(3)	(12)	(2)	(1)	2	5	8	11	14	15
Dividend (+) ; capital increase (-)		(3)	(12)	(2)	(1)	2	5	8	11	14	15
Discount rate		10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
Discount period		0,25	1,25	2,25	3,25	4,25	5,25	6,25	7,25	8,25	9,25
Discounted dividend		(3)	(10)	(1)	(0)	1	3	4	6	7	6
Dividend of the year		(3)	(12)	(2)	(1)	2	5	8	11	14	15
Cumulative dividend	0	(3)	(15)	(16)	(17)	(15)	(10)	(3)	8	23	37
Average cumulative dividend		(1)	(9)	(16)	(17)	(16)	(13)	(7)	3	16	30
Pretax cost of debt		4,0%	4,0%	4,0%	4,0%	4,0%	4,0%	4,0%	4,0%	4,0%	4,0%
Corporate tax rate		36,1%	36,1%	36,1%	36,1%	36,1%	36,1%	36,1%	36,1%	36,1%	36,1%
After tax cost of debt		2,6%	2,6%	2,6%	2,6%	2,6%	2,6%	2,6%	2,6%	2,6%	2,6%
After tax financing cost		(0)	(0)	(0)	(0)	(0)	(0)	(0)	0	0	1
Sum of discounted dividends	11										
Terminal value	89										
Equity value	100										

Sensitivity analysis of the equity value to the discount rate and to the perpetuity growth rate

Perpetuity growth rate	Discount rate			
	9%	10%	11%	12%
1%	123	102	85	72
2%	124	101	84	70
3%	126	100	81	67
4%	128	99	78	63

Sensitivity analysis of the equity value to the discount rate and to the target CT1 ratio

Target CT1 ratio	Discount rate			
	9%	10%	11%	12%
8%	149	123	104	89
9%	126	100	81	67
10%	102	77	59	45
11%	78	54	36	22

The equity as of 1st of January N is equal to the equity as of 31st of December N-1 less the theoretical dividend paid as of 31st of December N-1. In that case, the equity as of 1st of January N is equal to the required equity as of 31st of December N-1.

As of 31st of December N, the level of equity is equal to:

Equity as of 1st of January N

+ Net profit as of 31st of December N

- After tax financial cost of the theoretical dividend distribution.

The last table above shows that the increase in the target CT1 ratio implies a decrease in the bank's equity value. Indeed, an increase in the CT1 ratio reduces the amount of the theoretical dividend payments and therefore the sum of the discounted corresponding amounts.

b. Insurance companies

i. Principle

As banks, insurance companies have solvency constraints: their solvency capital must be equal as least to 100% of their risks.

In the Solvency 1 environment, only underwriting risks are taken into account. They can be easily calculated as they correspond to the sum of the 3 following elements:

- 14% of premium of non life business
- For the life business:
 - o 4% of technical reserves related to € contracts. In that kind of contract, the insurance company guarantees its clients' savings
 - o 1% of technical provisions related to unit linked contracts. In that case, the insurance company provides no guarantee to its clients. Its risk and the weighting of its corresponding reserves are therefore reduced from 4% to 1%.

The DDM model is similar as that of a bank.

ii. Example

An insurance company has disclosed the following simplified business plan:

	Achieved 2011	Budget 2012	Company's BP		
	2013	2014	2015		
Net profit	10	12	15	16	17
Premium in non life	500	600	700	800	900
Growth rate		20,0%	16,7%	14,3%	12,5%
Technical reserves in life					
€ contracts	80	90	100	110	120
Growth rate		12,5%	11,1%	10,0%	9,1%
Unit linked contracts	200	180	160	170	180
Growth rate		-10,0%	-11,1%	6,3%	5,9%

Its target solvency ratio is 100% and, based on the CAPM, its cost of equity is 10%

The following table provides the detailed DDM calculations assuming that the level of equity as of 31st of December 2011 is worth 120. It includes 6 additional years with a recurring year (2021), the soft landing of the business plan is the following one, assuming for the 2016-2020 period:

- a linear phasing of the growth rate of the risks to 3% in 2020;
- a growth rate of the net income in line with that of the risks.

	Achieved 2011	Budget 2012	Company's BP			Soft landing					Recurring 2021
	10	12	2013 15	2014 16	2015 17	2016 19	2017 20	2018 22	2019 23	2020 23	24
Net profit											
Premium in non life	500	600	700	800	900	995	1 082	1 156	1 212	1 249	1 286
Growth rate		20,0%	16,7%	14,3%	12,5%	10,6%	8,7%	6,8%	4,9%	3,0%	3,0%
Technical reserves in life											
€ contracts	80	90	100	110	120	129	138	146	152	156	161
Growth rate		12,5%	11,1%	10,0%	9,1%	7,9%	6,7%	5,4%	4,2%	3,0%	3,0%
Unit linked contracts	200	180	160	170	180	190	199	207	214	221	227
Growth rate		-10,0%	-11,1%	6,3%	5,9%	5,3%	4,7%	4,2%	3,6%	3,0%	3,0%
Equity											
1st of Jan		120	89	104	118	133	146	159	170	178	183
Net profit		12	15	16	17	19	20	22	23	23	24
After tax cost of dividends		(1)	(1)	(1)	(1)	(1)	(1)	(2)	(2)	(2)	(3)
31st of Dec	120	131	103	119	134	150	166	179	191	199	205
Risks											
Non life	14%	84	98	112	126	139	151	162	170	175	180
Life: € contracts	4%	4	4	4	5	5	6	6	6	6	6
Life: unit linked contracts	1%	2	2	2	2	2	2	2	2	2	2
Total risks		89	104	118	133	146	159	170	178	183	189
Growth rate			15,9%	14,0%	12,3%	10,4%	8,6%	6,7%	4,9%	3,0%	3,0%
Target solvency ratio		100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Required equity		89	104	118	133	146	159	170	178	183	189
Excess equity		42	(0)	0	1	4	7	10	13	16	16
Dividend (+) ; capital increase (-)		42	(0)	0	1	4	7	10	13	16	16
Discount rate		10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
Discount period		0,25	1,25	2,25	3,25	4,25	5,25	6,25	7,25	8,25	9,25
Discounted dividend		41	(0)	0	1	3	4	5	6	7	7
Dividend of the year		42	(0)	0	1	4	7	10	13	16	16
Cumulative dividend	0	42	42	42	44	47	54	63	76	92	108
Average cumulative dividend		21	42	42	43	46	51	59	70	84	100
Pretax cost of debt		4,0%	4,0%	4,0%	4,0%	4,0%	4,0%	4,0%	4,0%	4,0%	4,0%
Corporate tax rate		36,1%	36,1%	36,1%	36,1%	36,1%	36,1%	36,1%	36,1%	36,1%	36,1%
After tax cost of debt		2,6%	2,6%	2,6%	2,6%	2,6%	2,6%	2,6%	2,6%	2,6%	2,6%
After tax financing cost		1	1	1	1	1	1	2	2	2	3
Sum of discounted dividends	74										
Terminal value	98										
Equity value	173										

Sensitivity analysis of the equity value to the discount rate and to the perpetuity growth rate

Perpetuity growth rate	Discount rate			
	9%	10%	11%	12%
1%	165	147	132	120
2%	181	158	141	127
3%	201	173	151	135
4%	230	192	165	145

Sensitivity analysis of the equity value to the discount rate and to the solvency ratio

Target CT1 ratio	Discount rate			
	9%	10%	11%	12%
100%	201	173	151	135
120%	172	144	123	107
140%	142	115	95	80
160%	113	87	67	53

As for banks, the last table above shows that the increase in the target solvency ratio implies a decrease in the insurance company's equity value. Indeed, an increase in the solvency ratio reduces the amount of the theoretical dividend payments and therefore the sum of the discounted corresponding amounts.

Appendix : peer groups valuation

1. Comparison of multiples

Reminder of the Gordon and Shapiro formula: $V_0 = \frac{D_1}{k - g}$

where:

D_1 = dividend to be paid next year

k = cost of equity

g = perpetuity growth rate

Assuming that

d = pay out rate;

EPS = earning per share for next year;

NP = net profit for next year;

$BVPS$ = book value per share;

NMR = net margin rate.

$$- \quad P/E = \frac{V_0}{EPS} = \frac{D_1}{EPS.(k - g)} = \frac{d.EPS}{EPS.(k - g)} = \frac{d}{k - g}$$

A high P/E is therefore consistent with a high perpetuity growth rate (g).

$$- \quad P/PV = \frac{V_0}{BVPS} = \frac{D_1}{BVPS.(k - g)} = \frac{d.EPS}{BVPS.(k - g)} = \frac{d.ROE}{k - g}$$

A high P/BV is therefore consistent with a high ROE

$$- \quad P/Sales = \frac{V_0}{Sales} = \frac{D_1}{Sales.(k - g)} = \frac{d.NP}{Sales.(k - g)} = \frac{d.NMR}{k - g}$$

A high P/Sales is therefore consistent with a high net margin rate

$$- \quad EV/Sales = \frac{V_0 + ND}{Sales} = P/Sales + \frac{ND}{Sales} = \frac{d.NMR}{k - g} + \frac{ND}{Sales}$$

A high EV/Sales is therefore consistent with a high net margin rate

$$\begin{aligned} - \quad EV/EBITDA &= \frac{FCF/EBITDA}{k - g} \\ &= \frac{EBIT(1 - \tau)/EBITDA}{k - g} + \frac{D \& A/EBITDA}{k - g} - \frac{Capex/EBITDA}{k - g} - \frac{\Delta WCR/EBITDA}{k - g} \\ &= \frac{(EBITDA - D \& A)(1 - \tau)/EBITDA}{k - g} + \frac{D \& A/EBITDA}{k - g} - \frac{Capex/EBITDA}{k - g} - \frac{\Delta WCR/EBITDA}{k - g} \\ &= \frac{1 - \tau}{k - g} + \frac{\tau.D \& A/EBITDA}{k - g} - \frac{Capex/EBITDA}{k - g} - \frac{\Delta WCR/EBITDA}{k - g} \end{aligned}$$

A high EV/EBITDA is therefore consistent with low Capex requirements

$$\begin{aligned}
- \quad EV / EBITDA &= \frac{FCF / EBIT}{k - g} \\
&= \frac{EBIT(1 - \tau) / EBIT}{k - g} + \frac{D \& A / EBIT}{k - g} - \frac{Capex / EBIT}{k - g} - \frac{\Delta WCR / EBIT}{k - g} \\
&= \frac{1 - \tau}{k - g} + \frac{D \& A / EBIT}{k - g} - \frac{Capex / EBIT}{k - g} - \frac{\Delta WCR / EBIT}{k - g}
\end{aligned}$$

A high EV/EBIT is therefore consistent with low *Capex* requirements

2. Comparison of implied valuations

$$- \quad EV = xEBITDA \cdot EBITDA = xEBITDA \cdot \frac{EBITDA}{Sales} \cdot sales = xEBITDA \cdot \frac{EBITDA}{Sales} \cdot sales$$

where $xEBITDA$ is the industry's EBITDA multiple. Then, an increase in the EBITDA margin is consistent with an increase in the EV

$$- \quad EV = xEBIT \cdot EBIT = xEBIT \cdot \frac{EBITDA - D \& A}{EBITDA} \cdot EBITDA = xEBIT \cdot \left(1 - \frac{D \& A}{EBITDA}\right) \cdot EBITDA$$

Then, an increase in the depreciation and therefore in the *Capex* requirements is consistent with a decrease in the EV