

ARTIFICIAL INTELLIGENCE 2

LE QUANG MINH

THE CONCEPT OF RATIONALITY

Fully observable vs. **partially observable**: If an agent's sensors give it access to the complete state of the environment at each point in time, then we say that the task environment is fully observable. A task environment is effectively fully observable if the sensors detect all aspects that are *relevant* to the choice of action; relevance, in turn, depends on the performance measure. Fully observable environments are convenient because the agent need not maintain any internal state to keep track of the world. An environment might be partially observable because of noisy and inaccurate sensors or because parts of the state are simply missing from the sensor data—for example, a vacuum agent with only a local dirt sensor cannot tell whether there is dirt in other squares, and an automated taxi cannot see what other drivers are thinking. If the agent has no sensors at all then the environment is **unobservable**. One might think that in such cases the agent's plight is hopeless, but, as we discuss in Chapter 4, the agent's goals may still be achievable, sometimes with certainty.

THE CONCEPT OF RATIONALITY

Deterministic vs. **stochastic**. If the next state of the environment is completely determined by the current state and the action executed by the agent, then we say the environment is deterministic; otherwise, it is stochastic. In principle, an agent need not worry about uncertainty in a fully observable, deterministic environment. (In our definition, we ignore uncertainty that arises purely from the actions of other agents in a multiagent environment; thus, a game can be deterministic even though each agent may be unable to predict the actions of the others.) If the environment is partially observable, however, then it could *appear* to be stochastic. Most real situations are so complex that it is impossible to keep track of all the unobserved aspects; for practical purposes, they must be treated as stochastic. Taxi driving is clearly stochastic in this sense, because one can never predict the behavior of traffic exactly; moreover, one's tires blow out and one's engine seizes up without warning. The vacuum world as we described it is deterministic, but variations can include stochastic elements such as randomly appearing dirt and an unreliable suction mechanism (Exercise 2.13). We say an environment is **uncertain** if it is not fully observable or not deterministic. One final note: our use of the word “stochastic” generally implies that uncertainty about outcomes is quantified in terms of probabilities; a **nondeterministic** environment is one in which actions are characterized by their *possible* outcomes, but no probabilities are attached to them. Nondeterministic environment descriptions are usually associated with performance measures that require the agent to succeed for *all possible* outcomes of its actions.

THE CONCEPT OF RATIONALITY

Episodic vs. **sequential**: In an episodic task environment, the agent's experience is divided into atomic episodes. In each episode the agent receives a percept and then performs a single action. Crucially, the next episode does not depend on the actions taken in previous episodes. Many classification tasks are episodic. For example, an agent that has to spot defective parts on an assembly line bases each decision on the current part, regardless of previous decisions; moreover, the current decision doesn't affect whether the next part is defective. In sequential environments, on the other hand, the current decision could affect all future decisions.³ Chess and taxi driving are sequential: in both cases, short-term actions can have long-term consequences. Episodic environments are much simpler than sequential environments because the agent does not need to think ahead.

THE CONCEPT OF RATIONALITY

Static vs. dynamic: If the environment can change while an agent is deliberating, then we say the environment is dynamic for that agent; otherwise, it is static. Static environments are easy to deal with because the agent need not keep looking at the world while it is deciding on an action, nor need it worry about the passage of time. Dynamic environments, on the other hand, are continuously asking the agent what it wants to do; if it hasn't decided yet, that counts as deciding to do nothing. If the environment itself does not change with the passage of time but the agent's performance score does, then we say the environment is **semidynamic**. Taxi driving is clearly dynamic: the other cars and the taxi itself keep moving while the driving algorithm dithers about what to do next. Chess, when played with a clock, is semidynamic. Crossword puzzles are static.

THE CONCEPT OF RATIONALITY

Discrete vs. continuous: The discrete/continuous distinction applies to the *state* of the environment, to the way *time* is handled, and to the *percepts* and *actions* of the agent. For example, the chess environment has a finite number of distinct states (excluding the clock). Chess also has a discrete set of percepts and actions. Taxi driving is a continuous-state and continuous-time problem: the speed and location of the taxi and of the other vehicles sweep through a range of continuous values and do so smoothly over time. Taxi-driving actions are also continuous (steering angles, etc.). Input from digital cameras is discrete, strictly speaking, but is typically treated as representing continuously varying intensities and locations.

THE CONCEPT OF RATIONALITY

Known vs. **unknown**: Strictly speaking, this distinction refers not to the environment itself but to the agent’s (or designer’s) state of knowledge about the “laws of physics” of the environment. In a known environment, the outcomes (or outcome probabilities if the environment is stochastic) for all actions are given. Obviously, if the environment is unknown, the agent will have to learn how it works in order to make good decisions. Note that the distinction between known and unknown environments is not the same as the one between fully and partially observable environments. It is quite possible for a *known* environment to be *partially* observable—for example, in solitaire card games, I know the rules but am still unable to see the cards that have not yet been turned over. Conversely, an *unknown* environment can be *fully* observable—in a new video game, the screen may show the entire game state but I still don’t know what the buttons do until I try them.

As one might expect, the hardest case is *partially observable, multiagent, stochastic, sequential, dynamic, continuous*, and *unknown*. Taxi driving is hard in all these senses, except that for the most part the driver’s environment is known. Driving a rented car in a new country with unfamiliar geography and traffic laws is a lot more exciting.

THE STRUCTURE OF AGENTS

So far we have talked about agents by describing *behavior*—the action that is performed after any given sequence of percepts. Now we must bite the bullet and talk about how the insides work. The job of AI is to design an **agent program** that implements the agent function—the mapping from percepts to actions. We assume this program will run on some sort of computing device with physical sensors and actuators—we call this the **architecture**:

$$\textit{agent} = \textit{architecture} + \textit{program}$$
.

Obviously, the program we choose has to be one that is appropriate for the architecture. If the program is going to recommend actions like *Walk*, the architecture had better have legs. The architecture might be just an ordinary PC, or it might be a robotic car with several onboard computers, cameras, and other sensors. In general, the architecture makes the percepts from the sensors available to the program, runs the program, and feeds the program's action choices to the actuators as they are generated. Most of this book is about designing agent programs, although Chapters 24 and 25 deal directly with the sensors and actuators.

THE STRUCTURE OF AGENTS

The agent programs that we design in this book all have the same skeleton: they take the current percept as input from the sensors and return an action to the actuators.⁴ Notice the difference between the agent program, which takes the current percept as input, and the agent function, which takes the entire percept history. The agent program takes just the current percept as input because nothing more is available from the environment; if the agent's actions need to depend on the entire percept sequence, the agent will have to remember the percepts.

We describe the agent programs in the simple pseudocode language that is defined in Appendix B. (The online code repository contains implementations in real programming

THE STRUCTURE OF AGENTS

It is instructive to consider why the table-driven approach to agent construction is doomed to failure. Let \mathcal{P} be the set of possible percepts and let T be the lifetime of the agent (the total number of percepts it will receive). The lookup table will contain $\sum_{t=1}^T |\mathcal{P}|^t$ entries. Consider the automated taxi: the visual input from a single camera comes in at the rate of roughly 27 megabytes per second (30 frames per second, 640×480 pixels with 24 bits of color information). This gives a lookup table with over $10^{250,000,000,000}$ entries for an hour's driving. Even the lookup table for chess—a tiny, well-behaved fragment of the real world—would have at least 10^{150} entries. The daunting size of these tables (the number of atoms in the observable universe is less than 10^{80}) means that (a) no physical agent in this universe will have the space to store the table, (b) the designer would not have time to create the table, (c) no agent could ever learn all the right table entries from its experience, and (d) even if the environment is simple enough to yield a feasible table size, the designer still has no guidance about how to fill in the table entries.

THE STRUCTURE OF AGENTS

Despite all this, TABLE-DRIVEN-AGENT *does* do what we want: it implements the desired agent function. The key challenge for AI is to find out how to write programs that, to the extent possible, produce rational behavior from a smallish program rather than from a vast table. We have many examples showing that this can be done successfully in other areas: for example, the huge tables of square roots used by engineers and schoolchildren prior to the 1970s have now been replaced by a five-line program for Newton's method running on electronic calculators. The question is, can AI do for general intelligent behavior what Newton did for square roots? We believe the answer is yes.

In the remainder of this section, we outline four basic kinds of agent programs that embody the principles underlying almost all intelligent systems:

- Simple reflex agents;
- Model-based reflex agents;
- Goal-based agents; and
- Utility-based agents.

SIMPLE REFLEX AGENT

Simple reflex behaviors occur even in more complex environments. Imagine yourself as the driver of the automated taxi. If the car in front brakes and its brake lights come on, then you should notice this and initiate braking. In other words, some processing is done on the visual input to establish the condition we call “The car in front is braking.” Then, this triggers some established connection in the agent program to the action “initiate braking.” We call such a connection a **condition–action rule**,⁵ written as

if *car-in-front-is-braking* **then** *initiate-braking*.

Humans also have many such connections, some of which are learned responses (as for driving) and some of which are innate reflexes (such as blinking when something approaches the eye). In the course of the book, we show several different ways in which such connections can be learned and implemented.

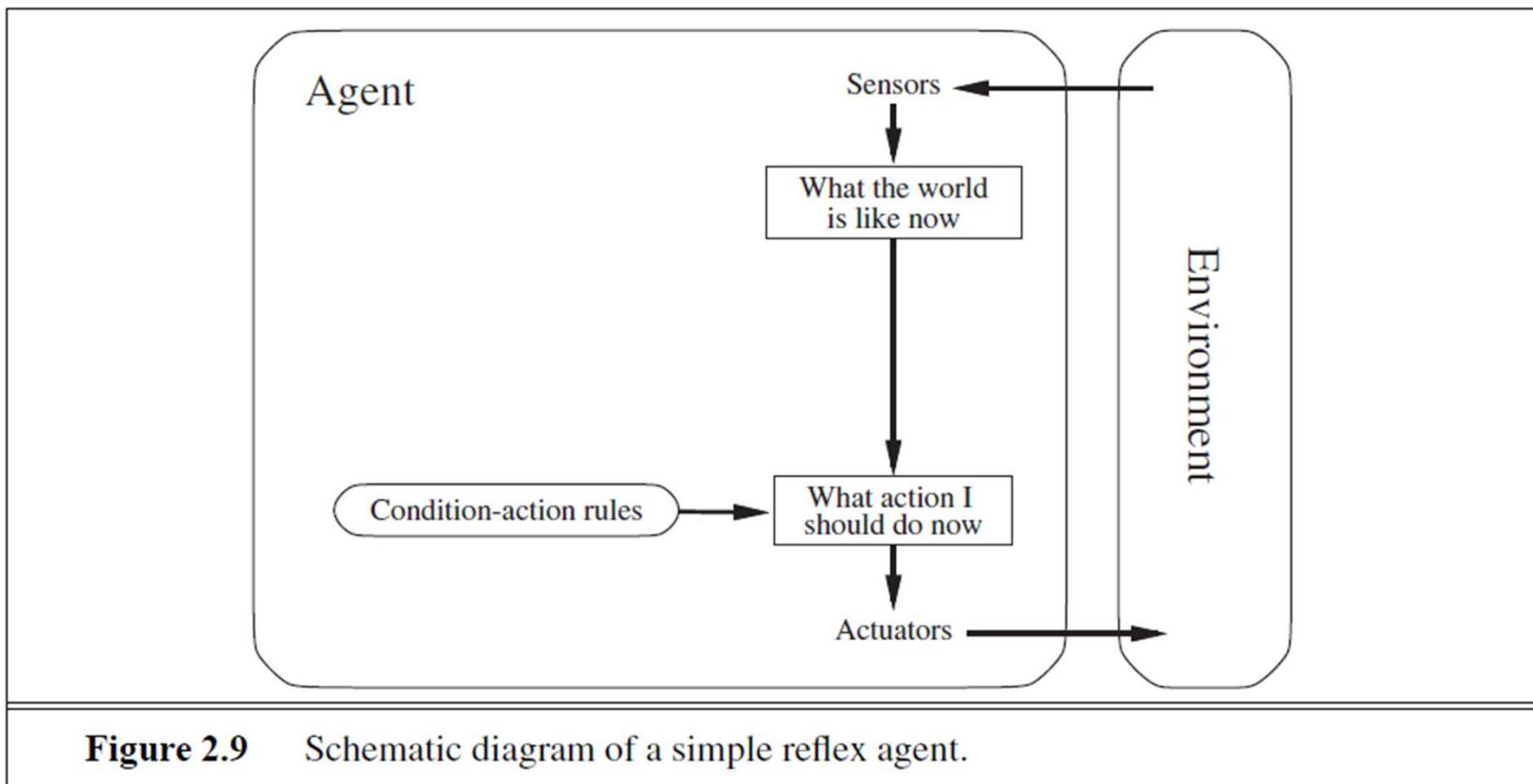
The program in Figure 2.8 is specific to one particular vacuum environment. A more general and flexible approach is first to build a general-purpose interpreter for condition–action rules and then to create rule sets for specific task environments. Figure 2.9 gives the structure of this general program in schematic form, showing how the condition–action rules

SIMPLE REFLEX AGENT

```
function REFLEX-VACUUM-AGENT([location,status]) returns an action
  if status = Dirty then return Suck
  else if location = A then return Right
  else if location = B then return Left
```

Figure 2.8 The agent program for a simple reflex agent in the two-state vacuum environment. This program implements the agent function tabulated in Figure 2.3.

SIMPLE REFLEX AGENT



SIMPLE REFLEX AGENT

```
function SIMPLE-REFLEX-AGENT(percept) returns an action
  persistent: rules, a set of condition-action rules

  state  $\leftarrow$  INTERPRET-INPUT(percept)
  rule  $\leftarrow$  RULE-MATCH(state, rules)
  action  $\leftarrow$  rule.ACTION
  return action
```

Figure 2.10 A simple reflex agent. It acts according to a rule whose condition matches the current state, as defined by the percept.

SIMPLE REFLEX AGENT

We can see a similar problem arising in the vacuum world. Suppose that a simple reflex vacuum agent is deprived of its location sensor and has only a dirt sensor. Such an agent has just two possible percepts: [*Dirty*] and [*Clean*]. It can *Suck* in response to [*Dirty*]; what should it do in response to [*Clean*]? Moving *Left* fails (forever) if it happens to start in square *A*, and moving *Right* fails (forever) if it happens to start in square *B*. Infinite loops are often unavoidable for simple reflex agents operating in partially observable environments.

Escape from infinite loops is possible if the agent can **randomize** its actions. For example, if the vacuum agent perceives [*Clean*], it might flip a coin to choose between *Left* and *Right*. It is easy to show that the agent will reach the other square in an average of two steps. Then, if that square is dirty, the agent will clean it and the task will be complete. Hence, a randomized simple reflex agent might outperform a deterministic simple reflex agent.

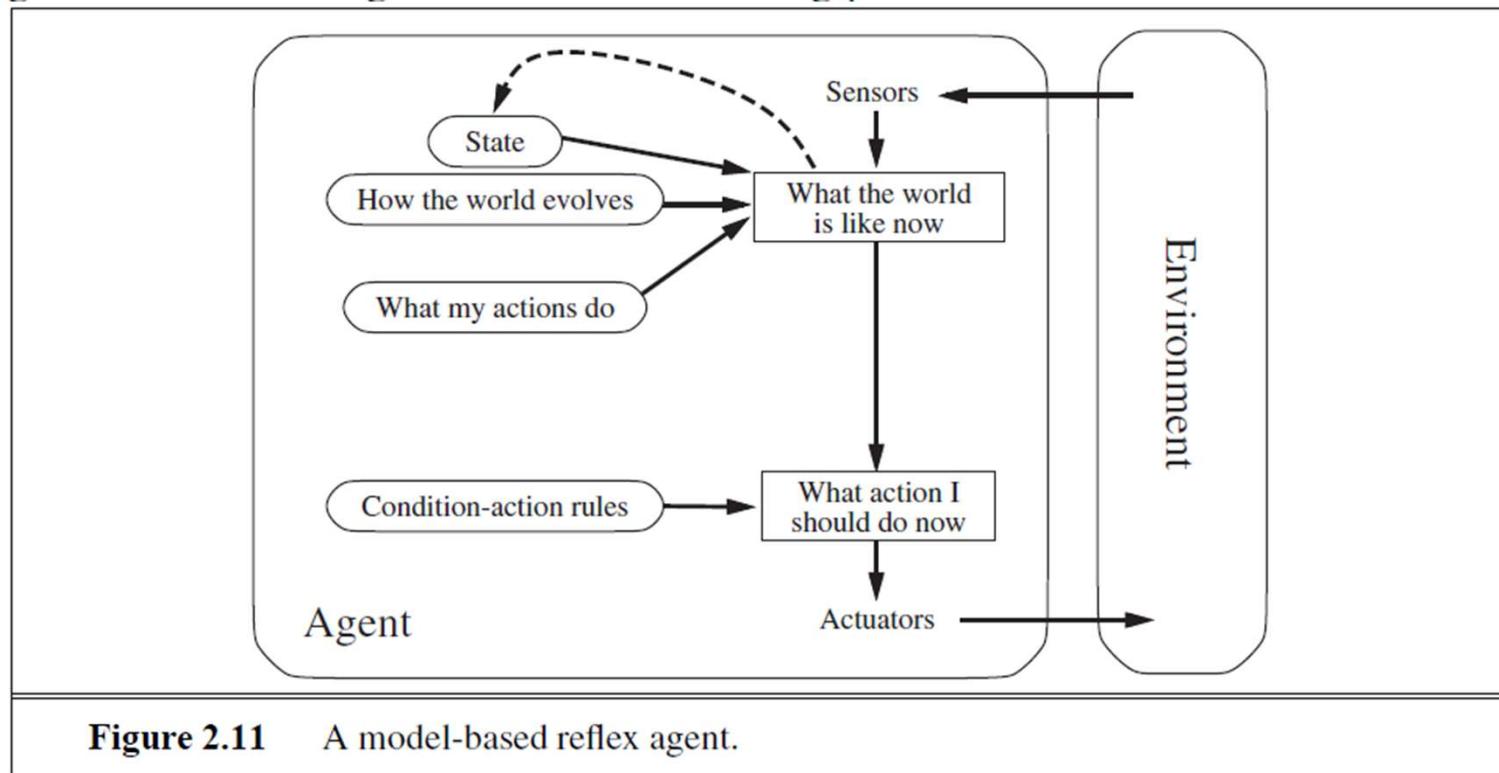
MODEL-BASED REFLEX AGENTS

The most effective way to handle partial observability is for the agent to *keep track of the part of the world it can't see now*. That is, the agent should maintain some sort of **internal state** that depends on the percept history and thereby reflects at least some of the unobserved aspects of the current state. For the braking problem, the internal state is not too extensive—just the previous frame from the camera, allowing the agent to detect when two red lights at the edge of the vehicle go on or off simultaneously. For other driving tasks such as changing lanes, the agent needs to keep track of where the other cars are if it can't see them all at once. And for any driving to be possible at all, the agent needs to keep track of where its keys are.

Updating this internal state information as time goes by requires two kinds of knowledge to be encoded in the agent program. First, we need some information about how the world evolves independently of the agent—for example, that an overtaking car generally will be closer behind than it was a moment ago. Second, we need some information about how the agent's own actions affect the world—for example, that when the agent turns the steering wheel clockwise, the car turns to the right, or that after driving for five minutes northbound on the freeway, one is usually about five miles north of where one was five minutes ago. This knowledge about “how the world works”—whether implemented in simple Boolean circuits or in complete scientific theories—is called a **model** of the world. An agent that uses such a model is called a **model-based agent**.

MODEL-BASED REFLEX AGENTS

Figure 2.11 gives the structure of the model-based reflex agent with internal state, showing how the current percept is combined with the old internal state to generate the updated description of the current state, based on the agent's model of how the world works. The agent program is shown in Figure 2.12. The interesting part is the function UPDATE-STATE, which



MODEL-BASED REFLEX AGENTS

```
function MODEL-BASED-REFLEX-AGENT(percept) returns an action
    persistent: state, the agent's current conception of the world state
                model, a description of how the next state depends on current state and action
                rules, a set of condition-action rules
                action, the most recent action, initially none

    state  $\leftarrow$  UPDATE-STATE(state, action, percept, model)
    rule  $\leftarrow$  RULE-MATCH(state, rules)
    action  $\leftarrow$  rule.ACTION
    return action
```

Figure 2.12 A model-based reflex agent. It keeps track of the current state of the world, using an internal model. It then chooses an action in the same way as the reflex agent.

is responsible for creating the new internal state description. The details of how models and states are represented vary widely depending on the type of environment and the particular technology used in the agent design. Detailed examples of models and updating algorithms appear in Chapters 4, 12, 11, 15, 17, and 25.

SOLVING PROBLEM BY SEARCHING

Imagine an agent in the city of Arad, Romania, enjoying a touring holiday. The agent's performance measure contains many factors: it wants to improve its suntan, improve its Romanian, take in the sights, enjoy the nightlife (such as it is), avoid hangovers, and so on. The decision problem is a complex one involving many tradeoffs and careful reading of guidebooks. Now, suppose the agent has a nonrefundable ticket to fly out of Bucharest the following day. In that case, it makes sense for the agent to adopt the **goal** of getting to Bucharest. Courses of action that don't reach Bucharest on time can be rejected without further consideration and the agent's decision problem is greatly simplified. Goals help organize behavior by limiting the objectives that the agent is trying to achieve and hence the actions it needs to consider. **Goal formulation**, based on the current situation and the agent's performance measure, is the first step in problem solving.

SOLVING PROBLEM BY SEARCHING

We will consider a goal to be a set of world states—exactly those states in which the goal is satisfied. The agent’s task is to find out how to act, now and in the future, so that it reaches a goal state. Before it can do this, it needs to decide (or we need to decide on its behalf) what sorts of actions and states it should consider. If it were to consider actions at the level of “move the left foot forward an inch” or “turn the steering wheel one degree left,” the agent would probably never find its way out of the parking lot, let alone to Bucharest, because at that level of detail there is too much uncertainty in the world and there would be too many steps in a solution. **Problem formulation** is the process of deciding what actions and states to consider, given a goal. We discuss this process in more detail later. For now, let us assume that the agent will consider actions at the level of driving from one major town to another. Each state therefore corresponds to being in a particular town.

SOLVING PROBLEM BY SEARCHING

Our agent has now adopted the goal of driving to Bucharest and is considering where to go from Arad. Three roads lead out of Arad, one toward Sibiu, one to Timisoara, and one to Zerind. None of these achieves the goal, so unless the agent is familiar with the geography of Romania, it will not know which road to follow.¹ In other words, the agent will not know which of its possible actions is best, because it does not yet know enough about the state that results from taking each action. If the agent has no additional information—i.e., if the environment is **unknown** in the sense defined in Section 2.3—then it has no choice but to try one of the actions at random. This sad situation is discussed in Chapter 4.

But suppose the agent has a map of Romania. The point of a map is to provide the agent with information about the states it might get itself into and the actions it can take. The agent can use this information to consider *subsequent* stages of a hypothetical journey via each of the three towns, trying to find a journey that eventually gets to Bucharest. Once it has found a path on the map from Arad to Bucharest, it can achieve its goal by carrying out the driving actions that correspond to the legs of the journey. In general, *an agent with several immediate options of unknown value can decide what to do by first examining future actions that eventually lead to states of known value.*

SOLVING PROBLEM BY SEARCHING

we assume that the environment is **observable**, so the agent always knows the current state. For the agent driving in Romania, it's reasonable to suppose that each city on the map has a sign indicating its presence to arriving drivers. We also assume the environment is **discrete**, so at any given state there are only finitely many actions to choose from. This is true for navigating in Romania because each city is connected to a small number of other cities. We will assume the environment is **known**, so the agent knows which states are reached by each action. (Having an accurate map suffices to meet this condition for navigation problems.) Finally, we assume that the environment is **deterministic**, so each action has exactly one outcome. Under ideal conditions, this is true for the agent in Romania—it means that if it chooses to drive from Arad to Sibiu, it does end up in Sibiu. Of course, conditions are not always ideal, as we show in Chapter 4.

SOLVING PROBLEM BY SEARCHING

The process of looking for a sequence of actions that reaches the goal is called **search**. A search algorithm takes a problem as input and returns a **solution** in the form of an action sequence. Once a solution is found, the actions it recommends can be carried out. This is called the **execution** phase. Thus, we have a simple “formulate, search, execute” design for the agent, as shown in Figure 3.1. After formulating a goal and a problem to solve, the agent calls a search procedure to solve it. It then uses the solution to guide its actions, doing whatever the solution recommends as the next thing to do—typically, the first action of the sequence—and then removing that step from the sequence. Once the solution has been executed, the agent will formulate a new goal.

SOLVING PROBLEM BY SEARCHING

Notice that while the agent is executing the solution sequence it *ignores its percepts* when choosing an action because it knows in advance what they will be. An agent that carries out its plans with its eyes closed, so to speak, must be quite certain of what is going on. Control theorists call this an **open-loop** system, because ignoring the percepts breaks the loop between agent and environment.

We first describe the process of problem formulation, and then devote the bulk of the chapter to various algorithms for the SEARCH function. We do not discuss the workings of the UPDATE-STATE and FORMULATE-GOAL functions further in this chapter.

3.1.1 Well-defined problems and solutions

A **problem** can be defined formally by five components:

- The **initial state** that the agent starts in. For example, the initial state for our agent in Romania might be described as *In(Arad)*.

SOLVING PROBLEM BY SEARCHING

- A description of the possible **actions** available to the agent. Given a particular state s , $\text{ACTIONS}(s)$ returns the set of actions that can be executed in s . We say that each of these actions is **applicable** in s . For example, from the state $In(Arad)$, the applicable actions are $\{Go(Sibiu), Go(Timisoara), Go(Zerind)\}$.
- A description of what each action does; the formal name for this is the **transition model**, specified by a function $\text{RESULT}(s, a)$ that returns the state that results from doing action a in state s . We also use the term **successor** to refer to any state reachable from a given state by a single action.² For example, we have

$$\text{RESULT}(In(Arad), Go(Zerind)) = In(Zerind) .$$

SOLVING PROBLEM BY SEARCHING

Together, the initial state, actions, and transition model implicitly define the **state space** of the problem—the set of all states reachable from the initial state by any sequence of actions. The state space forms a directed network or **graph** in which the nodes are states and the links between nodes are actions. (The map of Romania shown in Figure 3.2 can be interpreted as a state-space graph if we view each road as standing for two driving actions, one in each direction.) A **path** in the state space is a sequence of states connected by a sequence of actions.

- The **goal test**, which determines whether a given state is a goal state. Sometimes there is an explicit set of possible goal states, and the test simply checks whether the given state is one of them. The agent’s goal in Romania is the singleton set $\{In(Bucharest)\}$.

SOLVING PROBLEM BY SEARCHING

- A **path cost** function that assigns a numeric cost to each path. The problem-solving agent chooses a cost function that reflects its own performance measure. For the agent trying to get to Bucharest, time is of the essence, so the cost of a path might be its length in kilometers. In this chapter, we assume that the cost of a path can be described as the *sum* of the costs of the individual actions along the path.³ The **step cost** of taking action a in state s to reach state s' is denoted by $c(s, a, s')$. The step costs for Romania are shown in Figure 3.2 as route distances. We assume that step costs are nonnegative.⁴

The preceding elements define a problem and can be gathered into a single data structure that is given as input to a problem-solving algorithm. A **solution** to a problem is an action sequence that leads from the initial state to a goal state. Solution quality is measured by the path cost function, and an **optimal solution** has the lowest path cost among all solutions.

EXAMPLE PROBLEMS

3.2.1 Toy problems

The first example we examine is the **vacuum world** first introduced in Chapter 2. (See Figure 2.2.) This can be formulated as a problem as follows:

- **States:** The state is determined by both the agent location and the dirt locations. The agent is in one of two locations, each of which might or might not contain dirt. Thus, there are $2 \times 2^2 = 8$ possible world states. A larger environment with n locations has $n \cdot 2^n$ states.
- **Initial state:** Any state can be designated as the initial state.
- **Actions:** In this simple environment, each state has just three actions: *Left*, *Right*, and *Suck*. Larger environments might also include *Up* and *Down*.
- **Transition model:** The actions have their expected effects, except that moving *Left* in the leftmost square, moving *Right* in the rightmost square, and *Sucking* in a clean square have no effect. The complete state space is shown in Figure 3.3.

EXAMPLE PROBLEMS

- **Goal test:** This checks whether all the squares are clean.
- **Path cost:** Each step costs 1, so the path cost is the number of steps in the path.

Compared with the real world, this toy problem has discrete locations, discrete dirt, reliable cleaning, and it never gets any dirtier. Chapter 4 relaxes some of these assumptions.

The **8-puzzle**, an instance of which is shown in Figure 3.4, consists of a 3×3 board with eight numbered tiles and a blank space. A tile adjacent to the blank space can slide into the space. The object is to reach a specified goal state, such as the one shown on the right of the figure. The standard formulation is as follows:

EXAMPLE PROBLEMS

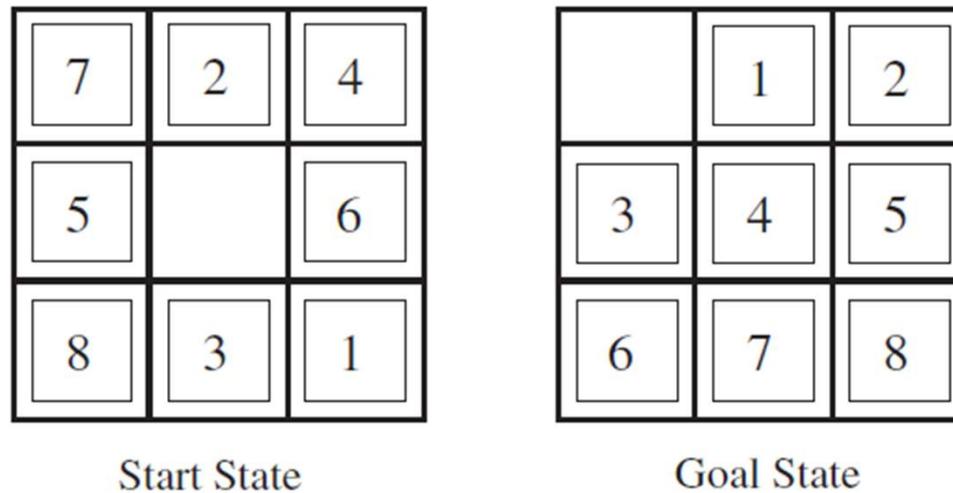


Figure 3.4 A typical instance of the 8-puzzle.

EXAMPLE PROBLEMS

- **States:** A state description specifies the location of each of the eight tiles and the blank in one of the nine squares.
- **Initial state:** Any state can be designated as the initial state. Note that any given goal can be reached from exactly half of the possible initial states (Exercise 3.5).
- **Actions:** The simplest formulation defines the actions as movements of the blank space *Left*, *Right*, *Up*, or *Down*. Different subsets of these are possible depending on where the blank is.
- **Transition model:** Given a state and action, this returns the resulting state; for example, if we apply *Left* to the start state in Figure 3.4, the resulting state has the 5 and the blank switched.
- **Goal test:** This checks whether the state matches the goal configuration shown in Fig-

EXAMPLE PROBLEMS

- **Path cost:** Each step costs 1, so the path cost is the number of steps in the path.

What abstractions have we included here? The actions are abstracted to their beginning and final states, ignoring the intermediate locations where the block is sliding. We have abstracted away actions such as shaking the board when pieces get stuck and ruled out extracting the pieces with a knife and putting them back again. We are left with a description of the rules of the puzzle, avoiding all the details of physical manipulations.

The 8-puzzle belongs to the family of **sliding-block puzzles**, which are often used as test problems for new search algorithms in AI. This family is known to be NP-complete, so one does not expect to find methods significantly better in the worst case than the search algorithms described in this chapter and the next. The 8-puzzle has $9!/2 = 181,440$ reachable states and is easily solved. The 15-puzzle (on a 4×4 board) has around 1.3 trillion states, and random instances can be solved optimally in a few milliseconds by the best search algorithms. The 24-puzzle (on a 5×5 board) has around 10^{25} states, and random instances take several hours to solve optimally.

The goal of the **8-queens problem** is to place eight queens on a chessboard such that no queen attacks any other. (A queen attacks any piece in the same row, column or diagonal.) Figure 3.5 shows an attempted solution that fails: the queen in the rightmost column is attacked by the queen at the top left.

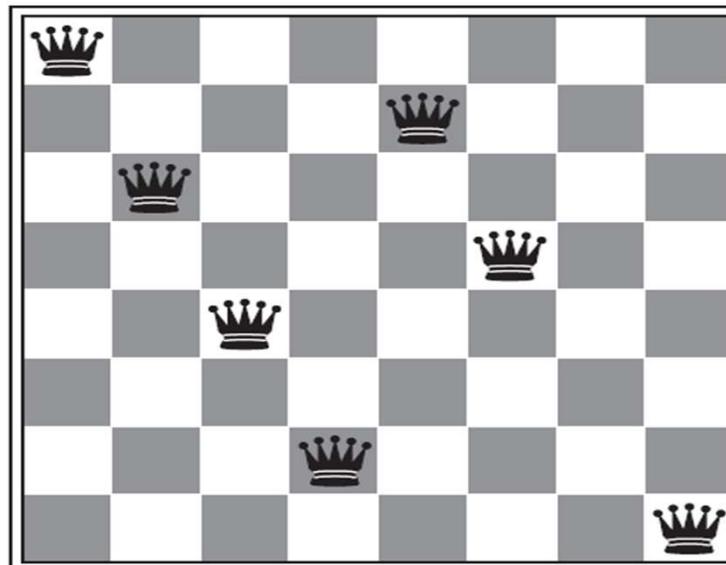


Figure 3.5 Almost a solution to the 8-queens problem. (Solution is left as an exercise.)

EXAMPLE PROBLEMS

Although efficient special-purpose algorithms exist for this problem and for the whole n -queens family, it remains a useful test problem for search algorithms. There are two main kinds of formulation. An **incremental formulation** involves operators that *augment* the state description, starting with an empty state; for the 8-queens problem, this means that each action adds a queen to the state. A **complete-state formulation** starts with all 8 queens on the board and moves them around. In either case, the path cost is of no interest because only the final state counts. The first incremental formulation one might try is the following:

- **States:** Any arrangement of 0 to 8 queens on the board is a state.
- **Initial state:** No queens on the board.
- **Actions:** Add a queen to any empty square.
- **Transition model:** Returns the board with a queen added to the specified square.
- **Goal test:** 8 queens are on the board, none attacked.

EXAMPLE PROBLEMS

In this formulation, we have $64 \cdot 63 \cdots 57 \approx 1.8 \times 10^{14}$ possible sequences to investigate. A better formulation would prohibit placing a queen in any square that is already attacked:

- **States:** All possible arrangements of n queens ($0 \leq n \leq 8$), one per column in the leftmost n columns, with no queen attacking another.
- **Actions:** Add a queen to any square in the leftmost empty column such that it is not attacked by any other queen.

This formulation reduces the 8-queens state space from 1.8×10^{14} to just 2,057, and solutions are easy to find. On the other hand, for 100 queens the reduction is from roughly 10^{400} states to about 10^{52} states (Exercise 3.6)—a big improvement, but not enough to make the problem tractable. Section 4.1 describes the complete-state formulation, and Chapter 6 gives a simple algorithm that solves even the million-queens problem with ease.

EXAMPLE PROBLEMS

Our final toy problem was devised by Donald Knuth (1964) and illustrates how infinite state spaces can arise. Knuth conjectured that, starting with the number 4, a sequence of factorial, square root, and floor operations will reach any desired positive integer. For example, we can reach 5 from 4 as follows:

$$\lfloor \sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}} \rfloor = 5 .$$

The problem definition is very simple:

- **States:** Positive numbers.
- **Initial state:** 4.
- **Actions:** Apply factorial, square root, or floor operation (factorial for integers only).
- **Transition model:** As given by the mathematical definitions of the operations.
- **Goal test:** State is the desired positive integer.

To our knowledge there is no bound on how large a number might be constructed in the process of reaching a given target—for example, the number 620,448,401,733,239,439,360,000 is generated in the expression for 5—so the state space for this problem is infinite. Such state spaces arise frequently in tasks involving the generation of mathematical expressions, circuits, proofs, programs, and other recursively defined objects.

REAL WORLD PROBLEM

We have already seen how the **route-finding problem** is defined in terms of specified locations and transitions along links between them. Route-finding algorithms are used in a variety of applications. Some, such as Web sites and in-car systems that provide driving directions, are relatively straightforward extensions of the Romania example. Others, such as routing video streams in computer networks, military operations planning, and airline travel-planning systems, involve much more complex specifications. Consider the airline travel problems that must be solved by a travel-planning Web site:

- **States:** Each state obviously includes a location (e.g., an airport) and the current time. Furthermore, because the cost of an action (a flight segment) may depend on previous segments, their fare bases, and their status as domestic or international, the state must record extra information about these “historical” aspects.

REAL WORLD PROBLEM

- **Initial state:** This is specified by the user's query.
- **Actions:** Take any flight from the current location, in any seat class, leaving after the current time, leaving enough time for within-airport transfer if needed.
- **Transition model:** The state resulting from taking a flight will have the flight's destination as the current location and the flight's arrival time as the current time.
- **Goal test:** Are we at the final destination specified by the user?
- **Path cost:** This depends on monetary cost, waiting time, flight time, customs and immigration procedures, seat quality, time of day, type of airplane, frequent-flyer mileage awards, and so on.

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Commercial travel advice systems use a problem formulation of this kind, with many additional complications to handle the byzantine fare structures that airlines impose. Any seasoned traveler knows, however, that not all air travel goes according to plan. A really good system should include contingency plans—such as backup reservations on alternate flights—to the extent that these are justified by the cost and likelihood of failure of the original plan.

Touring problems are closely related to route-finding problems, but with an important difference. Consider, for example, the problem “Visit every city in Figure 3.2 at least once, starting and ending in Bucharest.” As with route finding, the actions correspond to trips between adjacent cities. The state space, however, is quite different. Each state must include not just the current location but also the *set of cities the agent has visited*. So the initial state would be $In(Bucharest), Visited(\{Bucharest\})$, a typical intermediate state would be $In(Vaslui), Visited(\{Bucharest, Urziceni, Vaslui\})$, and the goal test would check whether the agent is in Bucharest and all 20 cities have been visited.

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The **traveling salesperson problem** (TSP) is a touring problem in which each city must be visited exactly once. The aim is to find the *shortest* tour. The problem is known to be NP-hard, but an enormous amount of effort has been expended to improve the capabilities of TSP algorithms. In addition to planning trips for traveling salespersons, these algorithms have been used for tasks such as planning movements of automatic circuit-board drills and of stocking machines on shop floors.

A **VLSI layout** problem requires positioning millions of components and connections on a chip to minimize area, minimize circuit delays, minimize stray capacitances, and maximize manufacturing yield. The layout problem comes after the logical design phase and is usually split into two parts: **cell layout** and **channel routing**. In cell layout, the primitive components of the circuit are grouped into cells, each of which performs some recognized function. Each cell has a fixed footprint (size and shape) and requires a certain number of connections to each of the other cells. The aim is to place the cells on the chip so that they do not overlap and so that there is room for the connecting wires to be placed between the cells.

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Channel routing finds a specific route for each wire through the gaps between the cells. These search problems are extremely complex, but definitely worth solving. Later in this chapter, we present some algorithms capable of solving them.

Robot navigation is a generalization of the route-finding problem described earlier. Rather than following a discrete set of routes, a robot can move in a continuous space with (in principle) an infinite set of possible actions and states. For a circular robot moving on a flat surface, the space is essentially two-dimensional. When the robot has arms and legs or wheels that must also be controlled, the search space becomes many-dimensional. Advanced techniques are required just to make the search space finite. We examine some of these methods in Chapter 25. In addition to the complexity of the problem, real robots must also deal with errors in their sensor readings and motor controls.

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Having formulated some problems, we now need to solve them. A solution is an action sequence, so search algorithms work by considering various possible action sequences. The possible action sequences starting at the initial state form a **search tree** with the initial state at the root; the branches are actions and the **nodes** correspond to states in the state space of the problem. Figure 3.6 shows the first few steps in growing the search tree for finding a route from Arad to Bucharest. The root node of the tree corresponds to the initial state, *In(Arad)*. The first step is to test whether this is a goal state. (Clearly it is not, but it is important to check so that we can solve trick problems like “starting in Arad, get to Arad.”) Then we need to consider taking various actions. We do this by **expanding** the current state; that is, applying each legal action to the current state, thereby **generating** a new set of states. In this case, we add three branches from the **parent node** *In(Arad)* leading to three new **child nodes**: *In(Sibiu)*, *In(Timisoara)*, and *In(Zerind)*. Now we must choose which of these three possibilities to consider further.

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This is the essence of search—following up one option now and putting the others aside for later, in case the first choice does not lead to a solution. Suppose we choose Sibiu first. We check to see whether it is a goal state (it is not) and then expand it to get *In(Arad)*, *In(Fagaras)*, *In(Oradea)*, and *In(Rimnicu Vilcea)*. We can then choose any of these four or go back and choose Timisoara or Zerind. Each of these six nodes is a **leaf node**, that is, a node with no children in the tree. The set of all leaf nodes available for expansion at any given point is called the **frontier**. (Many authors call it the **open list**, which is both geographically less evocative and less accurate, because other data structures are better suited than a list.) In Figure 3.6, the frontier of each tree consists of those nodes with bold outlines.