# wk2

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## Review

- Let $\vec{y}$, $\vec{v_1}$ and $\vec{v_2} \in R^m$ (this refers to m dimensional space where each elements is real number)

- Suppose you can express $\vec{y}$ as linear combination of $\vec{v_1}$ and $\vec{v_2}$

$$\vec{y} = c_1\vec{v_1} + c_2\vec{v_2}$$

- $c_1$ and $c_2$ can be viewed as weight of $\vec{v_1}$ and $\vec{v_2}$
- Above equation is also saying that $\vec{y}$ can be expressed as linear combination of $\vec{v_1}$ and $\vec{v_2}$
- Above equation is also saying that $\vec{y} \in \text{Span}\{\vec{v_1}, \vec{v_2}\}$

### Basis

- Basis refers to minimum set of vectors to span a subspace or space.
  - Basis are independent of each other
  - For a given subspace or space, there are $\infty$ number of set of basis
  - Basis are not unique.

### Orthogonal basis

- Refers to basis that are orthogonal to each other
- $\vec{v_1}$ and $\vec{v_2}$ are orthogonal, if $\vec{v_1}\vec{v_2} = 0$
- $\vec{v}\vec{v}^{\perp} = 0$

### Orthonormal basis

- basis that are
  - orthogonal
  - $< \vec{v_i}, \vec{v_i} > = 1$ for all basis

# Conditional Probablity example

In each week of a class, you are either caught up or behind.

- The probability that you are caught up in Week 1 is 0.7.
- If you are caught up in a given week, the probability that you will be caught up in the next week is 0.7.
- If you are behind in a given week, the probability that you will be caught up in the next week is 0.4.
- **What is the probability that you are caught up in week 3?**
- `Identify as many ways to improve this proof as you can`:

## Conditional probability with not so good notation

- If you are caught up in a week, there are two possibilities for the previous week: caught up and behind.
- Let $P(X)$ be the probability of being caught up.
  - In week 1, the probability of being caught up $P(X) = .7$.
  - In week 1, the probability of being behind is $P(Y) = 1 - .7 = .3$.
- We first break down the probability for week 2:

$$P(X) = .7 \cdot .7 + .3 \cdot .4 = .61$$

Now we can repeat the process for week 3:

$$P(X) = .61 * .7 + .39 * .4 = .583$$

- Let $C_i$ be the event that you are caught up in week $i$.
  - Given:
    - $P(C_1) = 0.7$
    - $P(C_{i+1}|C_i) = 0.7$
- Let $C_i^C$ be the event that you are behind in week $i$
  - $P(C_{i+1}|C_i^C) = 0.4$.
- `For week 2`, we can partition the sample space into $\{C_1, B_1\}$ and apply the law of total probability:

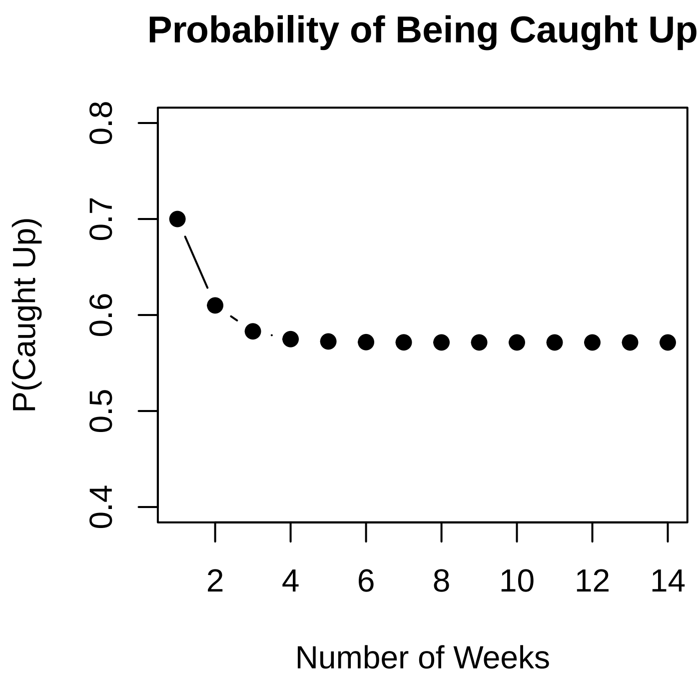
$$\begin{aligned} P(C_2) &= P(C_1)P(C_2|C_1) + P(B_1)P(C_2|B_1) \\ &= 0.7 \cdot 0.7 + 0.3 \cdot 0.4 = 0.61 \end{aligned}$$

- Next, repeat the process for `week 3`:

$$\begin{aligned} P(C_3) &= P(C_2)P(C_3|C_2) + P(B_2)P(C_3|B_2) \\ &= 0.7 \cdot 0.61 + 0.39 \cdot 0.4 = 0.58 \end{aligned}$$

### Solving it using R

- You can write a function in R and solve it




### Solving it using matrix

Given: - The probability of getting caught up with homework in this week only depends on the the outcome of the previous period.

- The transition matrix, $\mathbb{P}$, has nonzero values such that it is `regular`
- Since $\mathbb{P}$ is regular, it has limiting matrix

| | $C_i$ | $C_i^C$ |
|---|---|---|
| $C_{i+1}$ | 0.7 | 0.4 |
| $C_{i+1}^C$ | | |

- Above matrix contains the given information:
- Let $C_i$ be the event that you are caught up in week $i$.
  - $P(C_{i+1}|C_i) = 0.7$
- Let $C_i^C$ be the event that you are behind in week $i$
  - $P(C_{i+1}|C_i^C) = 0.4$.
- Then, we can fill in the blank:

|  | $C_i$ | $C_i^C$ |
|---|---|---|
| $C_{i+1}$ | 0.7 | 0.4 |
| $C_{i+1}^C$ | 0.3 | 0.6 |

And if we multiply the above matrix by the initial state vector, see what you get

$$[0.7, 0.3]^T$$

```
P <- matrix(c(0.7,0.4,0.3,0.6), nrow=2, byrow =T)
print(P)
```

```
##      [,1] [,2]
## [1,]  0.7  0.4
## [2,]  0.3  0.6
```

```
print(P%^%2)
```

```
##      [,1] [,2]
## [1,] 0.61 0.52
## [2,] 0.39 0.48
```

```
print(P%^%1000)
```

```
##           [,1]      [,2]
## [1,] 0.5714286 0.5714286
## [2,] 0.4285714 0.4285714
```

## Solving it using eigenvalue

- Will talk about this more later in the class

```
######################
# Using eignevalues
######################
myeigen <- eigen(P)    #gets you the eigenvalues and eigenvectors

## getting the eigenvalues and eigenvectors into vector and matrix.

lambda <- myeigen$values   #eigenvalues

E <- myeigen$vectors     #corresponding eigenvectors

print(lambda)
```

```
## [1] 1.0 0.3
```

```
print(E)
```

```
##      [,1]       [,2]
## [1,]  0.8 -0.7071068
## [2,]  0.6  0.7071068
```

```
p_vector <- function(x){
y <- sum(abs(x))
x <- abs(x)/y
return(x)
}

#converting the eigenvector corresponding to eigenvalue = 1
p_vector(E[,1])
```

```
## [1] 0.5714286 0.4285714
```

# More about linear combination

## Definiations

### Linear combination

$$\mathbb{A}\vec{x} = \vec{b}$$

### Subspace

- If $\vec{v}_1, ..\vec{v}_p \in R^n$, then Span$\{\vec{v}_1, ..\vec{v}_p\}$ is called the subset of $R^n$ by these vectors.

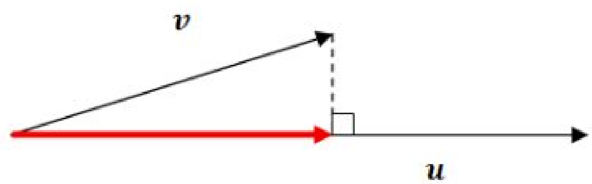
### Linear combination, Projection and transformation

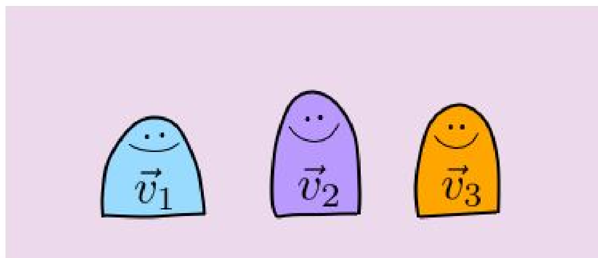
$$\mathbb{A}\vec{x} = \vec{b}$$

# Projection

$$\text{Proj}_{\vec{u}}\vec{v} = \frac{\vec{v}\vec{u}}{||\vec{u}||}$$

Given two vectors $u, v$, what is $proj_u v$?




## example 1

```
v <- c(3,4)
u <- c(5,-12)

p <- v%*%u/Norm(u)

p
```

```
##           [,1]
## [1,] -2.538462
```