

I wonder By Sam

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Introduction

```
print("hello")
```

```
## [1] "hello"
```

Chapter 1

Section 1.1

```
v1 <- matrix(c(1,-1), nrow=2)
print(v1)
```

```
##      [,1]
## [1,]    1
## [2,]   -1
```

```
v1 <- matrix(c(3,4), nrow=2)
print(v1)
```

```
##      [,1]
## [1,]    3
## [2,]    4
```

```
r1 <- matrix(c(1,-1,3), nrow=1)
print(r1)
```

```
##      [,1] [,2] [,3]
## [1,]    1  -1    3
```

```
v1 <- matrix(c(3,4),nrow=2)
print(v1)
```

```
##      [,1]
## [1,]    3
## [2,]    4
```

```
v2<-matrix(c(-1.5,2),nrow=2)
print(-2*v2)
```

```
##      [,1]
## [1,]    3
## [2,]   -4
```

Section 1.2

```
v1 <- matrix(c(8,6),nrow=2)
v2 <- matrix(c(4,0),nrow=2)

print(t(v1)%*%v2)
```

```
##      [,1]
## [1,]   32
```

Section 1.3

```
v1 <- matrix(c(8,6),nrow=2)
print(v1)
```

```
##      [,1]
## [1,]    8
## [2,]    6
```

```
Norm(v1)
```

```
## [1] 10
```

```
v2 <- matrix(c(4,0),nrow=2)
print(v2)
```

```
##      [,1]
## [1,]    4
## [2,]    0
```

```
print(t(v1)%*%v2/(t(v1)%*%v1))
```

```
##      [,1]
## [1,] 0.32
```

```
print(t(v1)%*%v2/Norm(v1)^2)
```

```
##      [,1]
## [1,] 0.32
```

```
print(t(v1)%*%v2/(t(v2)%*%v2))
```

```
##      [,1]  
## [1,]    2
```

```
print(t(v1)%*%v2/Norm(v2)^2)
```

```
##      [,1]  
## [1,]    2
```

Chapter 2

Section 2.1

```
v1 <- matrix(c(1,-1), nrow =2)
v2 <- matrix(c(2,1), nrow =2)
A <- cbind(v1,v2)
print(A)
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]   -1    1
```

```
dim(A)
```

```
## [1] 2 2
```

```
b <- matrix(c(1,1), nrow =2)
A%*%b
```

```
##      [,1]
## [1,]    3
## [2,]    0
```

```
v1 <- matrix(c(1,1), nrow =2)
v2 <- matrix(c(1,-1), nrow =2)
v3 <- matrix(c(1,3), nrow =2)

A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    1
## [2,]    1   -1    3
```

```
x <- matrix(c(2,3,1.5),nrow=3)
print(x)
```

```
##      [,1]
## [1,]  2.0
## [2,]  3.0
## [3,]  1.5
```

```
A%*%x
```

```
##      [,1]
## [1,]  6.5
## [2,]  3.5
```

Section 2.2

```
v1 <- matrix(c(1,-1), nrow =2)
v2 <- matrix(c(2,-1), nrow =2)
v3 <- matrix(c(-2,-3), nrow =2)
```

```
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -2
## [2,]   -1   -1   -3
```

```
Rank(A)
```

```
## [1] 2
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    8
## [2,]    0    1   -5
```

problem 1

```
r1 <- matrix(c(-3,1,-1,1,4), nrow =1)
r2 <- matrix(c(1,-2,2,3,1), nrow =1)
r3 <- matrix(c(-3,1,-1,1,4), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   -3    1   -1    1    4
## [2,]    1   -2    2    3    1
## [3,]   -3    1   -1    1    4
```

```
rref(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0   -1 -1.8
## [2,]    0    1   -1   -2 -1.4
## [3,]    0    0    0    0  0.0
```

problem 2

```

r1 <- matrix(c(-3,-6,-1,-2,4), nrow =1)
r2 <- matrix(c(1,2,2,4,1), nrow =1)
r3 <- matrix(c(2,4,5,10,7), nrow =1)
A <- rbind(r1,r2,r3)
print(A)

```

```

##      [,1] [,2] [,3] [,4] [,5]
## [1,]  -3  -6  -1  -2   4
## [2,]   1   2   2   4   1
## [3,]   2   4   5  10   7

```

```
rref(A)
```

```

##      [,1] [,2] [,3] [,4] [,5]
## [1,]   1   2   0   0   0
## [2,]   0   0   1   2   0
## [3,]   0   0   0   0   1

```

problem 3

```

r1 <- matrix(c(-3,-1,4), nrow =1)
r2 <- matrix(c(1,2,1), nrow =1)
r3 <- matrix(c(2,5,7), nrow =1)
A <- rbind(r1,r2,r3)
print(A)

```

```

##      [,1] [,2] [,3]
## [1,]  -3  -1   4
## [2,]   1   2   1
## [3,]   2   5   7

```

```
rref(A)
```

```

##      [,1] [,2] [,3]
## [1,]   1   0   0
## [2,]   0   1   0
## [3,]   0   0   1

```

problem 4

```

r1 <- matrix(c(-3,-1), nrow =1)
r2 <- matrix(c(1,2), nrow =1)
r3 <- matrix(c(2,5), nrow =1)
A <- rbind(r1,r2,r3)
print(A)

```

```
##      [,1] [,2]
## [1,]  -3  -1
## [2,]   1   2
## [3,]   2   5
```

```
rref(A)
```

```
##      [,1] [,2]
## [1,]   1   0
## [2,]   0   1
## [3,]   0   0
```

problem 5

```
r1 <- matrix(c(-3), nrow =1)
r2 <- matrix(c(1), nrow =1)
r3 <- matrix(c(2), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1]
## [1,]  -3
## [2,]   1
## [3,]   2
```

```
rref(A)
```

```
##      [,1]
## [1,]   1
## [2,]   0
## [3,]   0
```

Section 2.3

```
v1 <- matrix(c(1,-1), nrow =2)
v2 <- matrix(c(2,-1), nrow =2)
v3 <- matrix(c(-2,-3), nrow =2)

A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]   1   2  -2
## [2,]  -1  -1  -3
```

```
rref(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    8
## [2,]    0    1   -5
```

```
B <- A[,c(1,2)]
print(B)
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]   -1   -1
```

```
D <- A[,-c(1,2)]
print(D)
```

```
## [1] -2 -3
```

```
x <- matrix(c(2,3,7), nrow=3)
print(x)
```

```
##      [,1]
## [1,]    2
## [2,]    3
## [3,]    7
```

```
xB <- matrix(c(2,3), nrow=2)
print(xB)
```

```
##      [,1]
## [1,]    2
## [2,]    3
```

```
xD <- matrix(c(7), nrow=1)
print(xD)
```

```
##      [,1]
## [1,]    7
```

```
print(B%*%xB + D%*%xD)
```

```
##      [,1]
## [1,]   -6
## [2,]  -26
```

```
b <- A%*%x
print(b)
```

```
##      [,1]
## [1,]   -6
## [2,]  -26
```


problem 1

```
v1 <- matrix(c(1,-1), nrow =2)
v2 <- matrix(c(2,-1), nrow =2)
v3 <- matrix(c(-2,-3), nrow =2)
```

```
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -2
## [2,]   -1   -1   -3
```

```
x<- matrix(c(2,3,7), nrow =3)
print(x)
```

```
##      [,1]
## [1,]    2
## [2,]    3
## [3,]    7
```

```
b <- A%%x
print(b)
```

```
##      [,1]
## [1,]   -6
## [2,]  -26
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    8
## [2,]    0    1   -5
```

```
B<- A[,c(1,2)]
print(B)
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]   -1   -1
```

```
D<- A[, -c(1,2)]
print(D)
```

```
## [1] -2 -3
```

problem 2

```

r1 <- matrix(c(-3,1,-1,1,4), nrow =1)
r2 <- matrix(c(1,-2,2,3,1), nrow =1)
r3 <- matrix(c(-2,-4,5,8,7), nrow =1)
A <- rbind(r1,r2,r3)
print(A)

```

```

##      [,1] [,2] [,3] [,4] [,5]
## [1,]  -3   1  -1   1   4
## [2,]   1  -2   2   3   1
## [3,]  -2  -4   5   8   7

```

```

x<- matrix(c(2,3,7,-1,4),nrow=5)

b <- A%*%x
print(b)

```

```

##      [,1]
## [1,]    5
## [2,]   11
## [3,]   39

```

```

rref(A)

```

```

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0  -1 -1.8
## [2,]    0    1    0  -4 -3.6
## [3,]    0    0    1  -2 -2.2

```

```

B <- A[,c(1,2,3)]
D <- A[,-c(1,2,3)]
xB <- matrix(c(2,3,7),nrow=3)
xD <- matrix(c(-1,4),nrow=2)
print(B%*%xB + D%*%xD)

```

```

##      [,1]
## [1,]    5
## [2,]   11
## [3,]   39

```

problem 3

```

r1 <- matrix(c(-3,-6,-1,-2,4), nrow =1)
r2 <- matrix(c(1,2,2,4,1), nrow =1)
r3 <- matrix(c(2,4,5,10,7), nrow =1)
A <- rbind(r1,r2,r3)
print(A)

```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  -3   -6   -1   -2    4
## [2,]   1    2    2    4    1
## [3,]   2    4    5   10    7
```

```
x<- matrix(c(2,4,7,3,1),nrow=5)
```

```
b <- A%*%x
print(b)
```

```
##      [,1]
## [1,] -39
## [2,]  37
## [3,]  92
```

```
rref(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   1    2    0    0    0
## [2,]   0    0    1    2    0
## [3,]   0    0    0    0    1
```

```
B <- A[,c(1,3,5)]
D <- A[,-c(1,3,5)]
xB <- matrix(c(2,7,1),nrow=3)
xD <- matrix(c(4,3),nrow=2)
print(B%*%xB + D%*%xD)
```

```
##      [,1]
## [1,] -39
## [2,]  37
## [3,]  92
```

Section 2.4

```
r1 <- matrix(c(-1,1,1,3,-1), nrow =1)
r2 <- matrix(c(3,-2,-1,-7,5), nrow =1)
r3 <- matrix(c(2,1,4,0,8), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  -1    1    1    3   -1
## [2,]   3   -2   -1   -7    5
## [3,]   2    1    4    0    8
```

```
r1 <- matrix(c(1,3,2,-4,3), nrow =1)
r2 <- matrix(c(1,-2,3,-1,2), nrow =1)
F <- rbind(r1,r2)
print(F)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    3    2   -4    3
## [2,]    1   -2    3   -1    2
```

```
A%*%t(F)
```

```
##      [,1] [,2]
## [1,]  -11   -5
## [2,]   38   21
## [3,]   37   28
```

Problem 1

```
r1 <- matrix(c(-1,1,1,3,-1), nrow =1)
r2 <- matrix(c(3,-2,-1,-7,5), nrow =1)
r3 <- matrix(c(2,1,4,0,8), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   -1    1    1    3   -1
## [2,]    3   -2   -1   -7    5
## [3,]    2    1    4    0    8
```

```
Rank(A)
```

```
## [1] 2
```

```
GA <- t(A)%*%A
print(GA)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   14   -5    4  -24   32
## [2,]   -5    6    7   17   -3
## [3,]    4    7   18   10   26
## [4,]  -24   17   10   58  -38
## [5,]   32   -3   26  -38   90
```

```
inv(GA)
```

```
## Warning in inv(GA): Matrix appears to be singular.
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  Inf  Inf  Inf  Inf  Inf
## [2,]  Inf  Inf  Inf  Inf  Inf
## [3,]  Inf  Inf  Inf  Inf  Inf
## [4,]  Inf  Inf  Inf  Inf  Inf
## [5,]  Inf  Inf  Inf  Inf  Inf
```

Problem 2

```
r1 <- matrix(c(-1,1), nrow =1)
r2 <- matrix(c(3,-2), nrow =1)
r3 <- matrix(c(2,1), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2]
## [1,]  -1   1
## [2,]   3  -2
## [3,]   2   1
```

```
Rank(A)
```

```
## [1] 2
```

```
GA <- t(A)%*%A
print(GA)
```

```
##      [,1] [,2]
## [1,]  14  -5
## [2,]  -5   6
```

```
print(inv(GA))
```

```
##      [,1] [,2]
## [1,] 0.10169492 0.08474576
## [2,] 0.08474576 0.23728814
```

```
print(round(inv(GA),2))
```

```
##      [,1] [,2]
## [1,] 0.10 0.08
## [2,] 0.08 0.24
```

Problem 3

```
r1 <- matrix(c(-1,1,1), nrow =1)
r2 <- matrix(c(3,-2,-1), nrow =1)
r3 <- matrix(c(2,1,4), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]  -1   1   1
## [2,]   3  -2  -1
## [3,]   2   1   4
```

```
Rank(A)
```

```
## [1] 2
```

```
GA <- t(A)%*%A  
print(GA)
```

```
##      [,1] [,2] [,3]  
## [1,]  14  -5   4  
## [2,]  -5   6   7  
## [3,]   4   7  18
```

```
print(GA)
```

```
##      [,1] [,2] [,3]  
## [1,]  14  -5   4  
## [2,]  -5   6   7  
## [3,]   4   7  18
```

```
inv(GA)
```

```
## Warning in inv(GA): Matrix appears to be singular.
```

```
##      [,1] [,2] [,3]  
## [1,] Inf Inf Inf  
## [2,] Inf Inf Inf  
## [3,] Inf Inf Inf
```

Section 2.5

```
diag(c(-4,0,1))
```

```
##      [,1] [,2] [,3]  
## [1,]  -4   0   0  
## [2,]   0   0   0  
## [3,]   0   0   1
```

```
r1 <- matrix(c(2,4,2), nrow =1)  
r2 <- matrix(c(0,3,1), nrow =1)  
r3 <- matrix(c(2,5,2), nrow =1)  
P <- rbind(r1,r2,r3)  
print(P)
```

```
##      [,1] [,2] [,3]  
## [1,]   2   4   2  
## [2,]   0   3   1  
## [3,]   2   5   2
```

```
print(inv(P))
```

```
##      [,1] [,2] [,3]
## [1,] -0.5  -1   1
## [2,] -1.0   0   1
## [3,]  3.0   1  -3
```

```
print(inv(P)%*%P)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

```
r1 <- matrix(c(2,0), nrow =1)
r2 <- matrix(c(0,1), nrow =1)
r3 <- matrix(c(2,1), nrow =1)
B <- rbind(r1,r2,r3)
print(B)
```

```
##      [,1] [,2]
## [1,]    2    0
## [2,]    0    1
## [3,]    2    1
```

```
GB <- t(B)%*%B
print(GB)
```

```
##      [,1] [,2]
## [1,]    8    2
## [2,]    2    2
```

```
print(round(inv(GB),2))
```

```
##      [,1] [,2]
## [1,]  0.17 -0.17
## [2,] -0.17  0.67
```

```
print(inv(GB)%*%GB)
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

Problem 1

```

r1 <- matrix(c(2,0,4,4,2), nrow =1)
r2 <- matrix(c(0,1,2,3,1), nrow =1)
r3 <- matrix(c(2,1,3,5,2), nrow =1)
A <- rbind(r1,r2,r3)
print(A)

```

```

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    2    0    4    4    2
## [2,]    0    1    2    3    1
## [3,]    2    1    3    5    2

```

```

print(round(rref(A),2))

```

```

##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0 0.67 0.33
## [2,]    0    1    0 1.67 0.33
## [3,]    0    0    1 0.67 0.33

```

```

B <- A[,c(1,2,3)]
print(B)

```

```

##      [,1] [,2] [,3]
## [1,]    2    0    4
## [2,]    0    1    2
## [3,]    2    1    3

```

```

print(round(inv(B)%*%B),2)

```

```

##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1

```

Problem 2

```

v1 <- matrix(c(2,0,2,4,5), nrow =5)
v2 <- matrix(c(0,1,1,3,4), nrow =5)
v3 <- matrix(c(4,2,4,7,7), nrow =5)
A <- cbind(v1,v2,v3)
print(A)

```

```

##      [,1] [,2] [,3]
## [1,]    2    0    4
## [2,]    0    1    2
## [3,]    2    1    4
## [4,]    4    3    7
## [5,]    5    4    7

```



```
Rank(A)
```

```
## [1] 3
```

```
GA <- t(A)%*%A  
print(GA)
```

```
##      [,1] [,2] [,3]  
## [1,]  49  34  79  
## [2,]  34  27  55  
## [3,]  79  55 134
```

```
print(round(inv(GA)%*%GA),2)
```

```
##      [,1] [,2] [,3]  
## [1,]    1    0    0  
## [2,]    0    1    0  
## [3,]    0    0    1
```

Problem 3

```
v1 <- matrix(c(2,0,2), nrow =3)  
v2 <- matrix(c(0,1,1), nrow =3)  
v3 <- matrix(c(4,1,5), nrow =3)  
A <- cbind(v1,v2,v3)  
print(A)
```

```
##      [,1] [,2] [,3]  
## [1,]    2    0    4  
## [2,]    0    1    1  
## [3,]    2    1    5
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]  
## [1,]    1    0    2  
## [2,]    0    1    1  
## [3,]    0    0    0
```

```
B <- A[,c(1,2)]  
print(B)
```

```
##      [,1] [,2]  
## [1,]    2    0  
## [2,]    0    1  
## [3,]    2    1
```

```
GB <- t(B)%*%B
print(GB)
```

```
##      [,1] [,2]
## [1,]    8    2
## [2,]    2    2
```

```
print(inv(GB)%*%GB)
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

Problem 4

```
v1 <- matrix(c(2,0,2), nrow =3)
v2 <- matrix(c(0,1,1), nrow =3)
v3 <- matrix(c(4,2,4), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    2    0    4
## [2,]    0    1    2
## [3,]    2    1    4
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

```
I <- inv(A)%*%A
print(I)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

Problem 5

```
v1 <- matrix(c(2,0,2), nrow =3)
A <- v1

GA <- t(A)%*%A
print(GA)
```

```
##      [,1]  
## [1,]    8
```

```
print(round(inv(GA)%*%GA),2)
```

```
##      [,1]  
## [1,]    1
```

Chapter 3

Section 3.1

```
A <- matrix(c(1,1,1,1,-1,-3), nrow = 2, byrow=T)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    1
## [2,]    1   -1   -3
```

```
dim(A)
```

```
## [1] 2 3
```

```
dim(A)[1]
```

```
## [1] 2
```

```
dim(A)[2]
```

```
## [1] 3
```

Section 3.3

```
r1 <- matrix(c(1.1,1.1,3.3,1.1), nrow =1)
r2 <- matrix(c(-1.2,3.3,5.4,7.8), nrow =1)
r3 <- matrix(c(1.3,2.2,5.7,3.1), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  1.1  1.1  3.3  1.1
## [2,] -1.2  3.3  5.4  7.8
## [3,]  1.3  2.2  5.7  3.1
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    1   -1
## [2,]    0    1    2    2
## [3,]    0    0    0    0
```

```
Rank(A)
```

```
## [1] 2
```

```
B <- A[,c(1,2)]
print(B)
```

```
##      [,1] [,2]
## [1,]  1.1  1.1
## [2,] -1.2  3.3
## [3,]  1.3  2.2
```

```
D <- A[,-c(1,2)]
print(D)
```

```
##      [,1] [,2]
## [1,]  3.3  1.1
## [2,]  5.4  7.8
## [3,]  5.7  3.1
```

```
GB <- t(B)%*%B
print(GB)
```

```
##      [,1] [,2]
## [1,]  4.34  0.11
## [2,]  0.11 16.94
```

```
print(-inv(GB)%*%t(B)%*%D)
```

```
##      [,1] [,2]
## [1,]   -1   1
## [2,]   -2  -2
```

```
I <- diag(2)
print(I)
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

```
print(rbind(-inv(GB)%*%(t(B)%*%D),I))
```

```
##      [,1] [,2]
## [1,]   -1   1
## [2,]   -2  -2
## [3,]    1    0
## [4,]    0    1
```

```
N <- rbind(-inv(GB)%*%(t(B)%*%D),I)
print(round(A%*%N,2))
```

```
##      [,1] [,2]
## [1,]    0    0
## [2,]    0    0
## [3,]    0    0
```

```
print(A)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  1.1  1.1  3.3  1.1
## [2,] -1.2  3.3  5.4  7.8
## [3,]  1.3  2.2  5.7  3.1
```

```
AT <- t(A)
print(AT)
```

```
##      [,1] [,2] [,3]
## [1,]  1.1 -1.2  1.3
## [2,]  1.1  3.3  2.2
## [3,]  3.3  5.4  5.7
## [4,]  1.1  7.8  3.1
```

```
print(rref(t(A)))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0  1.4
## [2,]    0    1  0.2
## [3,]    0    0  0.0
## [4,]    0    0  0.0
```

```
RA <- AT[,c(1,2)]
print(RA)
```

```
##      [,1] [,2]
## [1,]  1.1 -1.2
## [2,]  1.1  3.3
## [3,]  3.3  5.4
## [4,]  1.1  7.8
```

Section 3.5

```
library(far)
```

```
v1 <- matrix(c(1,2,3), nrow =3)
v2 <- matrix(c(4,5,6), nrow =3)
v3 <- matrix(c(13,2,-4), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    4   13
## [2,]    2    5    2
## [3,]    3    6   -4
```

```
Rank(A)
```

```
## [1] 3
```

```
OA <- orthonormalization(A)
print(round(OA,2))
```

```
##      [,1] [,2] [,3]
## [1,] 0.27 0.87 0.41
## [2,] 0.53 0.22 -0.82
## [3,] 0.80 -0.44 0.41
```

```
Rank(OA)
```

```
## [1] 3
```

```
Norm(OA[,1])
```

```
## [1] 1
```

```
#between column vectors
print(round(t(OA[,1])%*%OA[,2],2))
```

```
##      [,1]
## [1,]    0
```

```
print(round(t(OA[,1])%*%OA[,3],2))
```

```
##      [,1]
## [1,]    0
```

```
#between row vectors
print(round(t(OA[1,])%*%OA[2,],2))
```

```
##      [,1]
## [1,]    0
```

```
print(round(t(OA[1,])%*%OA[3,],2))
```

```
##      [,1]
## [1,]    0
```

```
v1 <- matrix(c(1,2,-3), nrow =3)
v2 <- matrix(c(-4,2,1), nrow =3)
v3 <- matrix(c(11,2,-11), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1  -4   11
## [2,]    2    2    2
## [3,]   -3    1  -11
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    3
## [2,]    0    1   -2
## [3,]    0    0    0
```

```
B <- A[,c(1,2)]
print(B)
```

```
##      [,1] [,2]
## [1,]    1  -4
## [2,]    2    2
## [3,]   -3    1
```

```
OB <- orthonormalization(B)
print(round(OB,2))
```

```
##      [,1] [,2] [,3]
## [1,]  0.27 -0.84  0.47
## [2,]  0.53  0.54  0.65
## [3,] -0.80  0.08  0.59
```

Problem 1

```
v1 <- matrix(c(1,2,-3), nrow =3)
v2 <- matrix(c(-4,2,1), nrow =3)
v3 <- matrix(c(-3,4,-2), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1  -4   -3
## [2,]    2    2    4
## [3,]   -3    1   -2
```



```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    1
## [2,]    0    1    1
## [3,]    0    0    0
```

```
B <- A[,c(1,2)]
```

```
OB <- orthonormalization(B)
print(round(OB,2))
```

```
##      [,1] [,2] [,3]
## [1,] 0.27 -0.84 0.47
## [2,] 0.53 0.54 0.65
## [3,] -0.80 0.08 0.59
```

```
#basis spanning column space of A
CA <- OB[,c(1,2)]
print(round(CA,2))
```

```
##      [,1] [,2]
## [1,] 0.27 -0.84
## [2,] 0.53 0.54
## [3,] -0.80 0.08
```

```
#basis spanning left nullspace of A
NAT <- OB[,c(3)]
print(round(NAT,2))
```

```
## [1] 0.47 0.65 0.59
```

Problem 2

```
r1 <- matrix(c(1,-4,-3,6,23), nrow =1)
r2 <- matrix(c(2,2,4,4,-1), nrow =1)
r3 <- matrix(c(-3,1,-2,-7,-14), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1   -4   -3    6   23
## [2,]    2    2    4    4   -1
## [3,]   -3    1   -2   -7  -14
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    1    0  0.0
## [2,]    0    1    1    0 -3.5
## [3,]    0    0    0    1  1.5
```

```
B <- A[,c(1,2)]
```

```
OB <- orthonormalization(B)
print(round(OB,2))
```

```
##      [,1] [,2] [,3]
## [1,]  0.27 -0.84 0.47
## [2,]  0.53  0.54 0.65
## [3,] -0.80  0.08 0.59
```

```
#basis spanning column space of A
CA <- OB[,c(1,2)]
print(round(CA,2))
```

```
##      [,1] [,2]
## [1,]  0.27 -0.84
## [2,]  0.53  0.54
## [3,] -0.80  0.08
```

```
#basis spanning left nullspace of A
NAT <- OB[, -c(1,2)]
print(round(NAT,2))
```

```
## [1] 0.47 0.65 0.59
```

Chapter 4

Section 4.1

```
v1 <- matrix(c(1,-1,1), nrow =3)
v2 <- matrix(c(1,3,2), nrow =3)
v3 <- matrix(c(2,3,4), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    2
## [2,]   -1    3    3
## [3,]    1    2    4
```

```
Rank(A)
```

```
## [1] 3
```

```
b <- matrix(c(1,4,-4))
print(b)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```

```
Rank(b)
```

```
## [1] 1
```

```
x <- inv(A)%*%b
print(round(x,2))
```

```
##      [,1]
## [1,]  6.00
## [2,] 11.67
## [3,] -8.33
```

```
print(A%%x)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```

Section 4.2

```
v1 <- matrix(c(1,-1,1), nrow =3)
v2 <- matrix(c(1,3,2), nrow =3)
v3 <- matrix(c(3,5,5), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]   -1    3    5
## [3,]    1    2    5
```

```
b <- matrix(c(1,4,-4))
print(b)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```

```
Ab <- cbind(A,b)
```

```
Rank(A)
```

```
## [1] 2
```

```
Rank(Ab)
```

```
## [1] 3
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    1
## [2,]    0    1    2
## [3,]    0    0    0
```

```
B <- A[,c(1,2)]
GB <- t(B)%*%B
xB <- inv(GB)%*%t(B)%*%b
print(round(xB,2))
```

```
##      [,1]
## [1,] -2.33
## [2,]  0.36
```

```
b_hat <- B%*%xB
print(round(b_hat,2))
```

```
##      [,1]
## [1,] -1.98
## [2,]  3.40
## [3,] -1.62
```

```
x <- rbind(xB,0)
print(round(x,2))
```

```
##      [,1]
## [1,] -2.33
## [2,]  0.36
## [3,]  0.00
```

```
print(round(A%*%x,2))
```

```
##      [,1]
## [1,] -1.98
## [2,]  3.40
## [3,] -1.62
```

Section 4.3

Problem 1

```
r1 <- matrix(c(1,1,2,2,3), nrow =1)
r2 <- matrix(c(-1,3,3,-1,5), nrow =1)
r3 <- matrix(c(1,2,4,3,5), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    1    2    2    3
## [2,]   -1    3    3   -1    5
## [3,]    1    2    4    3    5
```

```
b <- matrix(c(1,4,-4))
print(b)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```

```
#####
#STEP 1
#####
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0    1    1
## [2,]    0    1    0   -1    2
## [3,]    0    0    1    1    0
```

```
Ab <- cbind(A,b)
print(round(rref(Ab),2))
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6]
## [1,]    1    0    0    1    1 6.00
## [2,]    0    1    0   -1    2 11.67
## [3,]    0    0    1    1    0 -8.33
```

```
B<- A[,c(1,2,3)]
D<- A[,-c(1,2,3)]
print(B)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    2
## [2,]   -1    3    3
## [3,]    1    2    4
```

```
print(D)
```

```
##      [,1] [,2]
## [1,]    2    3
## [2,]   -1    5
## [3,]    3    5
```

```
GB <- t(B)%*%B
print(GB)
```

```
##      [,1] [,2] [,3]
## [1,]    3    0    3
## [2,]    0   14   19
## [3,]    3   19   29
```

```
xB <- inv(GB)%*%t(B)%*%b
print(round(xB,2))
```

```
##      [,1]
## [1,] 6.00
## [2,] 11.67
## [3,] -8.33
```

```
x_zero <- rbind(xB[1],xB[2],xB[3],0,0)
print(round(x_zero,2))
```

```
##      [,1]
## [1,]  6.00
## [2,] 11.67
## [3,] -8.33
## [4,]  0.00
## [5,]  0.00
```

```
#####
# STEP 2
#####
N <- rbind(-inv(GB)%*%t(B)%*%D, diag(2))
print(round(N,0))
```

```
##      [,1] [,2]
## [1,]  -1  -1
## [2,]   1  -2
## [3,]  -1   0
## [4,]   1   0
## [5,]   0   1
```

```
# point 1
x_D <- matrix(c(-3,2),nrow=2)
print(x_D)
```

```
##      [,1]
## [1,]  -3
## [2,]   2
```

```
x_1 <- x_zero - N%*%x_D
print(round(x_1,2))
```

```
##      [,1]
## [1,]  5.00
## [2,] 18.67
## [3,] -11.33
## [4,]  3.00
## [5,] -2.00
```

```
print(A%*%x_1)
```

```
##      [,1]
## [1,]   1
## [2,]   4
## [3,]  -4
```

```
# point 2
x_D <- matrix(c(1,5),nrow=2)
print(x_D)
```

```
##      [,1]
## [1,]   1
## [2,]   5
```

```
x_2 <- x_zero - N*%x_D
print(round(x_2,2))
```

```
##      [,1]
## [1,] 12.00
## [2,] 20.67
## [3,] -7.33
## [4,] -1.00
## [5,] -5.00
```

```
print(A*%x_2)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```


Chapter 5

Section 5.1

```
r1 <- matrix(c(-2,4,2), nrow =1)
r2 <- matrix(c(-4.8,8.4,1.6), nrow =1)
r3 <- matrix(c(-4.8,2.4,5.6), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,] -2.0  4.0  2.0
## [2,] -4.8  8.4  1.6
## [3,] -4.8  2.4  5.6
```

```
eigen(A)
```

```
## eigen() decomposition
## $values
## [1] 6 4 2
##
## $vectors
##      [,1]      [,2]      [,3]
## [1,] 4.472136e-01 3.162278e-01 -0.6666667
## [2,] 8.944272e-01 3.830179e-15 -0.3333333
## [3,] -2.673771e-15 9.486833e-01 -0.6666667
```

```
eigen(A)$values
```

```
## [1] 6 4 2
```

```
round(eigen(A)$vectors,2)
```

```
##      [,1] [,2] [,3]
## [1,] 0.45 0.32 -0.67
## [2,] 0.89 0.00 -0.33
## [3,] 0.00 0.95 -0.67
```

Problem 1

```
r1 <- matrix(c(1,4,7), nrow =1)
r2 <- matrix(c(2,5,8), nrow =1)
r3 <- matrix(c(3,6,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    4    7
## [2,]    2    5    8
## [3,]    3    6    9
```

```
Rank(A)
```

```
## [1] 2
```

```
L <- diag(eigen(A)$values)
print(round(L,2))
```

```
##      [,1] [,2] [,3]
## [1,] 16.12  0.00    0
## [2,]  0.00 -1.12    0
## [3,]  0.00  0.00    0
```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##      [,1] [,2] [,3]
## [1,] -0.46 -0.88  0.41
## [2,] -0.57 -0.24 -0.82
## [3,] -0.68  0.40  0.41
```

```
Rank(E)
```

```
## [1] 3
```

Problem 2

```
r1 <- matrix(c(1,4,2), nrow =1)
r2 <- matrix(c(5,5,8), nrow =1)
r3 <- matrix(c(-3,69,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    4    2
## [2,]    5    5    8
## [3,]   -3   69    9
```

```
Rank(A)
```

```
## [1] 3
```

```
L <- diag(eigen(A)$values)
print(round(L,2))
```

```
##      [,1] [,2] [,3]
## [1,] 31.19  0.00  0.00
## [2,]  0.00 -16.31  0.00
## [3,]  0.00  0.00  0.12
```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##      [,1] [,2] [,3]
## [1,] 0.10 -0.03 -0.81
## [2,] 0.31 -0.35 -0.11
## [3,] 0.95  0.94  0.57
```

```
Rank(E)
```

```
## [1] 3
```

Problem 3

```
r1 <- matrix(c(1,2,2), nrow =1)
r2 <- matrix(c(2,4,8), nrow =1)
r3 <- matrix(c(3,6,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]  1   2   2
## [2,]  2   4   8
## [3,]  3   6   9
```

```
Rank(A)
```

```
## [1] 2
```

```
L <- diag(eigen(A)$values)
print(round(L,2))
```

```
##      [,1] [,2] [,3]
## [1,] 14.62  0.00  0
## [2,]  0.00 -0.62  0
## [3,]  0.00  0.00  0
```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##      [,1] [,2] [,3]
## [1,] -0.20  0.58 -0.89
## [2,] -0.61 -0.76  0.45
## [3,] -0.76  0.29  0.00
```

```
Rank(E)
```

```
## [1] 3
```

Problem 4

```
r1 <- matrix(c(0.8,-0.6,0), nrow =1)
r2 <- matrix(c(0.6,0.8,0), nrow =1)
r3 <- matrix(c(3,3,1), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]  0.8 -0.6  0
## [2,]  0.6  0.8  0
## [3,]  3.0  3.0  1
```

```
Rank(A)
```

```
## [1] 3
```

```
L <- diag(eigen(A)$values)
print(round(L,2))
```

```
##      [,1]      [,2]      [,3]
## [1,] 1+0i 0.0+0.0i 0.0+0.0i
## [2,] 0+0i 0.8+0.6i 0.0+0.0i
## [3,] 0+0i 0.0+0.0i 0.8-0.6i
```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##      [,1]      [,2]      [,3]
## [1,] 0+0i -0.13+0.07i -0.13-0.07i
## [2,] 0+0i  0.07+0.13i  0.07-0.13i
## [3,] 1+0i  0.98+0.00i  0.98+0.00i
```

Section 5.2

```

r1 <- matrix(c(-2,4,2), nrow =1)
r2 <- matrix(c(-4.8,8.4,1.6), nrow =1)
r3 <- matrix(c(-4.8,2.4,5.6), nrow =1)
A <- rbind(r1,r2,r3)
print(A)

```

```

##      [,1] [,2] [,3]
## [1,] -2.0  4.0  2.0
## [2,] -4.8  8.4  1.6
## [3,] -4.8  2.4  5.6

```

```
Rank(A)
```

```
## [1] 3
```

```

Q<- A - diag(eigen(A)$values[1],3)
print(Q)

```

```

##      [,1] [,2] [,3]
## [1,] -8.0  4.0  2.0
## [2,] -4.8  2.4  1.6
## [3,] -4.8  2.4 -0.4

```

```
Rank(Q)
```

```
## [1] 2
```

Section 5.3

```

r1 <- matrix(c(1,4,2), nrow =1)
r2 <- matrix(c(5,5,8), nrow =1)
r3 <- matrix(c(-3,69,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)

```

```

##      [,1] [,2] [,3]
## [1,]    1    4    2
## [2,]    5    5    8
## [3,]   -3   69    9

```

```

L <- diag(eigen(A)$values)
print(round(L,2))

```

```

##      [,1] [,2] [,3]
## [1,] 31.19  0.00  0.00
## [2,]  0.00 -16.31  0.00
## [3,]  0.00  0.00  0.12

```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##      [,1] [,2] [,3]
## [1,] 0.10 -0.03 -0.81
## [2,] 0.31 -0.35 -0.11
## [3,] 0.95  0.94  0.57
```

Section 5.4

Problem 1

```
r1 <- matrix(c(1,2,2), nrow =1)
r2 <- matrix(c(2,4,8), nrow =1)
r3 <- matrix(c(3,6,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    2
## [2,]    2    4    8
## [3,]    3    6    9
```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##      [,1] [,2] [,3]
## [1,] -0.20  0.58 -0.89
## [2,] -0.61 -0.76  0.45
## [3,] -0.76  0.29  0.00
```

```
x<- matrix(c(2,4,8),nrow=3)
print(x)
```

```
##      [,1]
## [1,]    2
## [2,]    4
## [3,]    8
```

```
h<- inv(E)%*%x
```

```
h[1]%*%E[,1] + h[2]%*%E[,2] + h[3]%*%E[,3]
```

```
##      [,1] [,2] [,3]
## [1,]    2    4    8
```

Problem 2

```
L <- diag(c(1.2,0.7))
e1 <- c(2,1)
e2 <- c(1,0)
E <- cbind(e1,e2)
A <- E%*%L%*%inv(E)
print(A)
```

```
##      [,1] [,2]
## [1,]  0.7  1.0
## [2,]  0.0  1.2
```

Section 5.6

Problem 1

```
r1 <- matrix(c(1,2,2), nrow =1)
r2 <- matrix(c(2,4,8), nrow =1)
r3 <- matrix(c(3,6,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    2
## [2,]    2    4    8
## [3,]    3    6    9
```

```
Rank(A)
```

```
## [1] 2
```

```
GA <- t(A)%*%A
print(GA)
```

```
##      [,1] [,2] [,3]
## [1,]   14   28   45
## [2,]   28   56   90
## [3,]   45   90  149
```

```
L <- round(diag(eigen(GA)$values,3),1)
print(L)
```

```
##      [,1] [,2] [,3]
## [1,] 217.6  0.0   0
## [2,]   0.0  1.4   0
## [3,]   0.0  0.0   0
```

```
E <- round(eigen(GA)$vectors,3)
E
```

```
##      [,1] [,2] [,3]
## [1,] -0.252 -0.370 0.894
## [2,] -0.504 -0.739 -0.447
## [3,] -0.826 0.563 0.000
```

```
t(E)
```

```
##      [,1] [,2] [,3]
## [1,] -0.252 -0.504 -0.826
## [2,] -0.370 -0.739 0.563
## [3,] 0.894 -0.447 0.000
```

```
round(inv(E),3)
```

```
##      [,1] [,2] [,3]
## [1,] -0.252 -0.504 -0.827
## [2,] -0.369 -0.739 0.564
## [3,] 0.895 -0.448 0.000
```

Problem 2

```
r1 <- matrix(c(1,2,-1), nrow =1)
r2 <- matrix(c(2,2,3), nrow =1)
r3 <- matrix(c(0,3,2), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,] 1 2 -1
## [2,] 2 2 3
## [3,] 0 3 2
```

```
Rank(A)
```

```
## [1] 3
```

```
GA <- t(A)%*%A
print(GA)
```

```
##      [,1] [,2] [,3]
## [1,] 5 6 5
## [2,] 6 17 10
## [3,] 5 10 14
```



```
L <- round(diag(eigen(GA)$values,3),1)
print(L)
```

```
##      [,1] [,2] [,3]
## [1,] 28.2  0.0  0.0
## [2,]  0.0  5.4  0.0
## [3,]  0.0  0.0  2.4
```

```
E <- round(eigen(GA)$vectors,3)
E
```

```
##      [,1] [,2] [,3]
## [1,] -0.319 -0.037  0.947
## [2,] -0.719 -0.641 -0.267
## [3,] -0.617  0.766 -0.178
```

```
t(E)
```

```
##      [,1] [,2] [,3]
## [1,] -0.319 -0.719 -0.617
## [2,] -0.037 -0.641  0.766
## [3,]  0.947 -0.267 -0.178
```

```
round(inv(E),3)
```

```
##      [,1] [,2] [,3]
## [1,] -0.319 -0.719 -0.617
## [2,] -0.037 -0.642  0.767
## [3,]  0.947 -0.267 -0.178
```

Section 5.6

Problem 1

```
M <- matrix(c(0.7,0.25,0.3,0.75), nrow=2, byrow = T)
```

```
print(M)
```

```
##      [,1] [,2]
## [1,]  0.7 0.25
## [2,]  0.3 0.75
```

```
E <- eigen(M)$vectors
print(round(eigen(M)$vectors,2))
```

```
##      [,1] [,2]
## [1,] -0.64 -0.71
## [2,] -0.77  0.71
```

```
print(eigen(M)$values)
```

```
## [1] 1.00 0.45
```

```
x0 <- matrix(c(0.6,0.4),nrow=2)
print(x0)
```

```
##      [,1]
## [1,]  0.6
## [2,]  0.4
```

```
p1 <- E[,1]/sum(E[,1])
print(round(p1,2))
```

```
## [1] 0.45 0.55
```

Section 5.7

```
Q <- matrix(c(0,0.5,0,0.5,0,0.5,0,0.5,0), nrow = 3, byrow =T)
print(Q)
```

```
##      [,1] [,2] [,3]
## [1,]  0.0  0.5  0.0
## [2,]  0.5  0.0  0.5
## [3,]  0.0  0.5  0.0
```

```
N <- inv(diag(3)-Q)
print(N)
```

```
##      [,1] [,2] [,3]
## [1,]  1.5   1  0.5
## [2,]  1.0   2  1.0
## [3,]  0.5   1  1.5
```

```
R <- matrix(c(0.5,0,0,0,0,0.5), nrow=3, byrow = T)
print(R)
```

```
##      [,1] [,2]
## [1,]  0.5  0.0
## [2,]  0.0  0.0
## [3,]  0.0  0.5
```

Problem 1

```
N <- inv(diag(3)-Q)
print(N)
```

```
##      [,1] [,2] [,3]
## [1,]  1.5   1  0.5
## [2,]  1.0   2  1.0
## [3,]  0.5   1  1.5
```

Problem 2

```
c1 <- matrix(c(1,1,1), nrow=3)
print(N%*%c1)
```

```
##      [,1]
## [1,]    3
## [2,]    4
## [3,]    3
```

Problem 3

```
print(N%*%R)
```

```
##      [,1] [,2]
## [1,] 0.75 0.25
## [2,] 0.50 0.50
## [3,] 0.25 0.75
```

Chapter 6

Section 6.1

```
A <- matrix(c(1,2,-2,-1,-1,-3),  
            nrow=2, byrow = TRUE)
```

```
print(A)
```

```
##      [,1] [,2] [,3]  
## [1,]    1    2   -2  
## [2,]   -1   -1   -3
```

```
Rank(A)
```

```
## [1] 2
```

```
S <- diag(svd(A)$d)  
print(round(S,2))
```

```
##      [,1] [,2]  
## [1,] 3.63 0.00  
## [2,] 0.00 2.61
```

```
U <- svd(A)$u  
print(round(U,2))
```

```
##      [,1] [,2]  
## [1,] -0.58 -0.81  
## [2,] -0.81  0.58
```

```
print(round((svd(A)$u)%*%S),2)
```

```
##      [,1] [,2]  
## [1,]   -2   -2  
## [2,]   -3    2
```

```
V <- svd(A)$v  
print(round(V,2))
```

```
##      [,1] [,2]  
## [1,]  0.06 -0.53  
## [2,] -0.10 -0.84  
## [3,]  0.99 -0.05
```

```
print(round(A%*%(svd(A)$v),2))
```

```
##      [,1] [,2]
## [1,]   -2  -2
## [2,]   -3   2
```

```
U%*%S%*%t(V)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2  -2
## [2,]   -1   -1  -3
```

Problem 1

```
A <- matrix(c(1,1,2,1,-1,3),
             nrow=2, byrow = TRUE)

print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    2
## [2,]    1   -1    3
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##      [,1] [,2]
## [1,] 3.87 0.00
## [2,] 0.00 1.41
```

```
U <- svd(A)$u
print(round(U,2))
```

```
##      [,1] [,2]
## [1,] -0.55 -0.83
## [2,] -0.83  0.55
```

```
V <- svd(A)$v
print(round(V,2))
```

```
##      [,1] [,2]
## [1,] -0.36 -0.20
## [2,]  0.07 -0.98
## [3,] -0.93  0.00
```

```
print(round(A%*(svd(A)$v),0))
```

```
##      [,1] [,2]
## [1,]   -2  -1
## [2,]   -3   1
```

```
print(round(svd(A)$u*%S,0))
```

```
##      [,1] [,2]
## [1,]   -2  -1
## [2,]   -3   1
```

```
print(U*%S*%t(V))
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    2
## [2,]    1   -1    3
```

Problem 2

```
A <- matrix(c(1,4,7,1,1,9),
             nrow=3, byrow = TRUE)
```

```
print(A)
```

```
##      [,1] [,2]
## [1,]    1    4
## [2,]    7    1
## [3,]    1    9
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##      [,1] [,2]
## [1,] 10.26 0.00
## [2,]  0.00 6.61
```

```
U <- svd(A)$u
print(round(U,2))
```

```
##      [,1] [,2]
## [1,] 0.40 -0.07
## [2,] 0.33  0.94
## [3,] 0.86 -0.33
```

```
V <- svd(A)$v
print(round(V,2))
```

```
##      [,1] [,2]
## [1,] 0.35  0.94
## [2,] 0.94 -0.35
```

```
print(U%%S%%t(V))
```

```
##      [,1] [,2]
## [1,]    1    4
## [2,]    7    1
## [3,]    1    9
```

```
print(U%%S%%inv(V))
```

```
##      [,1] [,2]
## [1,]    1    4
## [2,]    7    1
## [3,]    1    9
```

Section 6.2

Problem 1

```
A <- matrix(c(1,2,-2,-1,-1,-3),
             nrow=2, byrow = TRUE)
```

```
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -2
## [2,]   -1   -1   -3
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##      [,1] [,2]
## [1,] 3.63 0.00
## [2,] 0.00 2.61
```

```
U <- svd(A)$u
print(round(U,2))
```

```
##      [,1] [,2]
## [1,] -0.58 -0.81
## [2,] -0.81  0.58
```

```
V <- svd(A)$v
print(round(V,2))
```

```
##      [,1] [,2]
## [1,]  0.06 -0.53
## [2,] -0.10 -0.84
## [3,]  0.99 -0.05
```

```
A_inv <- V%*%inv(S)%*%inv(U)
print(round(A_inv,2))
```

```
##      [,1] [,2]
## [1,]  0.16 -0.13
## [2,]  0.28 -0.17
## [3,] -0.14 -0.23
```

```
result <- A%*%A_inv
print(round(result,2))
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

Problem 2

```
A <- matrix(c(1,1,2,1,-1,3),
             nrow=2, byrow = TRUE)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    2
## [2,]    1   -1    3
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##      [,1] [,2]
## [1,] 3.87 0.00
## [2,] 0.00 1.41
```

```
U <- svd(A)$u
print(round(U,2))
```

```
##      [,1] [,2]
## [1,] -0.55 -0.83
## [2,] -0.83  0.55
```

```
V <- svd(A)$v
print(round(V,2))
```

```
##      [,1] [,2]
## [1,] -0.36 -0.20
## [2,]  0.07 -0.98
## [3,] -0.93  0.00
```



```
A_inv <- V%*%inv(S)%*%inv(U)
print(round(A_inv,2))
```

```
##      [,1] [,2]
## [1,] 0.17  0.0
## [2,] 0.57 -0.4
## [3,] 0.13  0.2
```

```
result <- A%*%A_inv
print(round(result,2))
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

Section 6.3

```
v1 <- matrix(c(5,2),nrow=2)
v2 <- matrix(c(10,0),nrow=2)
v3 <- matrix(c(0,1),nrow=2)

#project onto v2
print(t(v1)%*%v2/(t(v2)%*%v2))
```

```
##      [,1]
## [1,]  0.5
```

```
#project onto v2
print(t(v1)%*%v3/(t(v3)%*%v3))
```

```
##      [,1]
## [1,]    2
```

- How to normalize a vector

```
#how to normalize v2
nv2 <- v2 / sqrt(sum(v2^2))
print(nv2)
```

```
##      [,1]
## [1,]    1
## [2,]    0
```

```
print(Norm(nv2))
```

```
## [1] 1
```

```
#how to normalize v2
nv3 <- v3 / sqrt(sum(v3^2))
print(nv3)
```

```
##      [,1]
## [1,]    0
## [2,]    1
```

```
print(Norm(nv3))
```

```
## [1] 1
```

- Projecting onto normalized vector

```
#project onto v2
print(t(v1)%*%nv2/(t(nv2)%*%nv2))
```

```
##      [,1]
## [1,]    5
```

```
#project onto v3
print(t(v1)%*%nv3/(t(nv3)%*%nv3))
```

```
##      [,1]
## [1,]    2
```

- orthogonal matrix and a vector multiplication

```
A <- matrix(c(1,2,3,-1,3,2), nrow = 3)
A
```

```
##      [,1] [,2]
## [1,]    1   -1
## [2,]    2    3
## [3,]    3    2
```

```
Rank(A)
```

```
## [1] 2
```

```
OA <- orthonormalization(A)
OA
```

```
##      [,1] [,2] [,3]
## [1,] 0.2672612 -0.7715167 0.5773503
## [2,] 0.5345225 0.6172134 0.5773503
## [3,] 0.8017837 -0.1543033 -0.5773503
```

```
x <- matrix(c(2,1,5),nrow = 3)
x
```

```
##      [,1]
## [1,]    2
## [2,]    1
## [3,]    5
```

```
b <- OA%*%x
b
```

```
##      [,1]
## [1,] 2.649757
## [2,] 4.573010
## [3,] -1.437487
```

```
OA%*%x
```

```
##      [,1]
## [1,] 2.649757
## [2,] 4.573010
## [3,] -1.437487
```

```
t(OA)%*%b
```

```
##      [,1]
## [1,]    2
## [2,]    1
## [3,]    5
```

```
t(b)%*%OA
```

```
##      [,1] [,2] [,3]
## [1,]    2    1    5
```

- principal component analysis

```
A <- matrix(c(1,2,3,4,5,
              0.2,0.8,0.7,1.1,1.7),
            nrow=2, byrow = TRUE)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  1.0  2.0  3.0  4.0  5.0
## [2,]  0.2  0.8  0.7  1.1  1.7
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##      [,1] [,2]
## [1,] 7.76 0.00
## [2,] 0.00 0.36
```

```
U <- svd(A)$u
print(round(U,2))
```

```
##      [,1] [,2]
## [1,] -0.96 -0.29
## [2,] -0.29 0.96
```

```
V <- svd(A)$v
print(round(V,2))
```

```
##      [,1] [,2]
## [1,] -0.13 -0.29
## [2,] -0.28 0.50
## [3,] -0.40 -0.59
## [4,] -0.53 -0.34
## [5,] -0.68 0.45
```

```
T1 <- t(A)%*%U
print(round(T1,2))
```

```
##      [,1] [,2]
## [1,] -1.01 -0.10
## [2,] -2.15 0.18
## [3,] -3.07 -0.21
## [4,] -4.15 -0.12
## [5,] -5.28 0.16
```