I wonder By Sam

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Introduction

```
print("hello")
```

```
## [1] "hello"
```

Chapter 1

Section 1.1

```
v1 <- matrix(c(1,-1), nrow=2)
print(v1)
```

```
##      [,1]
## [1,]    1
## [2,]   -1
```

```
v1 <- matrix(c(3,4), nrow=2)
print(v1)
```

```
##      [,1]
## [1,]    3
## [2,]    4
```

```
r1 <- matrix(c(1,-1,3), nrow=1)
print(r1)
```

```
##      [,1] [,2] [,3]
## [1,]    1   -1    3
```

```
v1 <- matrix(c(3,4),nrow=2)
print(v1)
```

```
##      [,1]
## [1,]    3
## [2,]    4
```

```r
v2<-matrix(c(-1.5,2),nrow=2)
print(v2)
```

```
##      [,1]
## [1,] -1.5
## [2,]  2.0
```

```r
print(-2*v2)
```

```
##      [,1]
## [1,]    3
## [2,]   -4
```

Section 1.2

```r
v1 <- matrix(c(8,6),nrow=2)
print(v1)
```

```
##      [,1]
## [1,]    8
## [2,]    6
```

```r
print(t(v1))
```

```
##      [,1] [,2]
## [1,]    8    6
```

```r
v2 <- matrix(c(4,0),nrow=2)

print(t(v1)%*%v2)
```

```
##      [,1]
## [1,]   32
```

Section 1.3

```r
v1 <- matrix(c(8,6),nrow=2)
print(v1)
```

```
##      [,1]
## [1,]    8
## [2,]    6
```

```r
Norm(v1)
```

```
## [1] 10
```

```r
v2 <- matrix(c(4,0),nrow=2)
print(v2)
```

```
##      [,1]
## [1,]    4
## [2,]    0
```

```r
print(t(v1)%*%v2/(t(v1)%*%v1))
```

```
##      [,1]
## [1,] 0.32
```

```r
print(t(v1)%*%v2/Norm(v1)^2)
```

```
##      [,1]
## [1,] 0.32
```

Chapter 2

Section 2.1

```
v1 <- matrix(c(1,-1), nrow =2)
v2 <- matrix(c(2,1), nrow =2)
A <- cbind(v1,v2)
print(A)
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]   -1    1
```

```
dim(A)
```

```
## [1] 2 2
```

Section 2.2

```
v1 <- matrix(c(1,1), nrow =2)
v2 <- matrix(c(1,-1), nrow =2)
v3 <- matrix(c(1,3), nrow =2)

A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    1
## [2,]    1   -1    3
```

```
x <- matrix(c(2,3,1.5),nrow=3)
print(x)
```

```
##      [,1]
## [1,]  2.0
## [2,]  3.0
## [3,]  1.5
```

```
A%*%x
```

```
##      [,1]
## [1,]  6.5
## [2,]  3.5
```

Section 2.3

```
v1 <- matrix(c(1,-1), nrow =2)
v2 <- matrix(c(2,-1), nrow =2)
v3 <- matrix(c(-2,-1), nrow =2)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -2
## [2,]   -1   -1   -1
```

```
Rank(A)
```

```
## [1] 2
```

```
v1 <- matrix(c(1,-1), nrow =2)
v2 <- matrix(c(2,-1), nrow =2)
v3 <- matrix(c(-2,-3), nrow =2)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -2
## [2,]   -1   -1   -3
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    8
## [2,]    0    1   -5
```

problem 1

```
r1 <- matrix(c(-3,1,-1,1,4), nrow =1)
r2 <- matrix(c(1,-2,2,3,1), nrow =1)
r3 <- matrix(c(-3,1,-1,1,4), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   -3    1   -1    1    4
## [2,]    1   -2    2    3    1
## [3,]   -3    1   -1    1    4
```

```
rref(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0   -1 -1.8
## [2,]    0    1   -1   -2 -1.4
## [3,]    0    0    0    0  0.0
```

problem 2

```
r1 <- matrix(c(-3,-6,-1,-2,4), nrow =1)
r2 <- matrix(c(1,2,2,4,1), nrow =1)
r3 <- matrix(c(2,4,5,10,7), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   -3   -6   -1   -2    4
## [2,]    1    2    2    4    1
## [3,]    2    4    5   10    7
```

```
rref(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    2    0    0    0
## [2,]    0    0    1    2    0
## [3,]    0    0    0    0    1
```

problem 3

```
r1 <- matrix(c(-3,-1,4), nrow =1)
r2 <- matrix(c(1,2,1), nrow =1)
r3 <- matrix(c(2,5,7), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]   -3   -1    4
## [2,]    1    2    1
## [3,]    2    5    7
```

```
rref(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

problem 4

```
r1 <- matrix(c(-3,-1), nrow =1)
r2 <- matrix(c(1,2), nrow =1)
r3 <- matrix(c(2,5), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2]
## [1,]   -3   -1
## [2,]    1    2
## [3,]    2    5
```

```
print(rref(A))
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
## [3,]    0    0
```

problem 5

```
r1 <- matrix(c(-3), nrow =1)
r2 <- matrix(c(1), nrow =1)
r3 <- matrix(c(2), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1]
## [1,]   -3
## [2,]    1
## [3,]    2
```

```
rref(A)
```

```
##      [,1]
## [1,]    1
## [2,]    0
## [3,]    0
```

Section 2.4

```
v1 <- matrix(c(1,-1), nrow =2)
v2 <- matrix(c(2,-1), nrow =2)
v3 <- matrix(c(-2,-3), nrow =2)

A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -2
## [2,]   -1   -1   -3
```

```
rref(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    8
## [2,]    0    1   -5
```

```
B <- A[,c(1,2)]
print(B)
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]   -1   -1
```

```
D <- A[,-c(1,2)]
print(D)
```

```
## [1] -2 -3
```

```
x <- matrix(c(2,3,7), nrow=3)
print(x)
```

```
##      [,1]
## [1,]    2
## [2,]    3
## [3,]    7
```

```
xB <- matrix(c(2,3), nrow=2)
print(xB)
```

```
##      [,1]
## [1,]    2
## [2,]    3
```

```
xD <- matrix(c(7), nrow=1)
print(xD)
```

```
##      [,1]
## [1,]    7
```

```
b <- A%*%x
print(b)
```

```
##      [,1]
## [1,]   -6
## [2,]  -26
```

```
print(B%*%xB + D%*%xD)
```

```
##      [,1]
## [1,]   -6
## [2,]  -26
```

problem 1

```
v1 <- matrix(c(1,-1), nrow =2)
v2 <- matrix(c(2,-1), nrow =2)
v3 <- matrix(c(-2,-3), nrow =2)

A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -2
## [2,]   -1   -1   -3
```

```
x<- matrix(c(2,3,7), nrow =3)
print(x)
```

```
##      [,1]
## [1,]    2
## [2,]    3
## [3,]    7
```

```
b <- A%*%x
print(b)
```

```
##      [,1]
## [1,]   -6
## [2,]  -26
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    8
## [2,]    0    1   -5
```

```
B<- A[,c(1,2)]
print(B)
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]   -1   -1
```

```
D<- A[,-c(1,2)]
print(D)
```

```
## [1] -2 -3
```

problem 2

```
r1 <- matrix(c(-3,1,-1,1,4), nrow =1)
r2 <- matrix(c(1,-2,2,3,1), nrow =1)
r3 <- matrix(c(2,-4,5,8,7), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   -3    1   -1    1    4
## [2,]    1   -2    2    3    1
## [3,]    2   -4    5    8    7
```

```
x<- matrix(c(2,3,7,-1,4),nrow=5)

b <- A%*%x
print(b)
```

```
##      [,1]
## [1,]    5
## [2,]   11
## [3,]   47
```

```
rref(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0   -1 -1.8
## [2,]    0    1    0    0  3.6
## [3,]    0    0    1    2  5.0
```

```
B <- A[,c(1,2,3)]
D <- A[,-c(1,2,3)]
xB <- matrix(c(2,3,7),nrow=3)
xD <- matrix(c(-1,4),nrow=2)
print(B%*%xB + D%*%xD)
```

```
##      [,1]
## [1,]    5
## [2,]   11
## [3,]   47
```

problem 3

```
r1 <- matrix(c(-3,-6,-1,-2,4), nrow =1)
r2 <- matrix(c(1,2,2,4,1), nrow =1)
r3 <- matrix(c(2,4,5,10,7), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   -3   -6   -1   -2    4
## [2,]    1    2    2    4    1
## [3,]    2    4    5   10    7
```

```
x<- matrix(c(2,4,7,3,1),nrow=5)

b <- A%*%x
print(b)
```

```
##      [,1]
## [1,]  -39
## [2,]   37
## [3,]   92
```

```
rref(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    2    0    0    0
## [2,]    0    0    1    2    0
## [3,]    0    0    0    0    1
```

```
B <- A[,c(1,3,5)]
D <- A[,-c(1,3,5)]
xB <- matrix(c(2,7,1),nrow=3)
xD <- matrix(c(4,3),nrow=2)
print(B%*%xB + D%*%xD)
```

```
##      [,1]
## [1,]  -39
## [2,]   37
## [3,]   92
```

Section 2.5

```
r1 <- matrix(c(-1,1,1,3,-1), nrow =1)
r2 <- matrix(c(3,-2,-1,-7,5), nrow =1)
r3 <- matrix(c(2,1,4,0,8), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   -1    1    1    3   -1
## [2,]    3   -2   -1   -7    5
## [3,]    2    1    4    0    8
```

```
r1 <- matrix(c(1,3,2,-4,3), nrow =1)
r2 <- matrix(c(1,-2,3,-1,2), nrow =1)
F <- rbind(r1,r2)
print(F)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    3    2   -4    3
## [2,]    1   -2    3   -1    2
```

```
A%*%t(F)
```

```
##      [,1] [,2]
## [1,]  -11   -5
## [2,]   38   21
## [3,]   37   28
```

Problem 1

```
r1 <- matrix(c(-1,1,1,3,-1), nrow =1)
r2 <- matrix(c(3,-2,-1,-7,5), nrow =1)
r3 <- matrix(c(2,1,4,0,8), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   -1    1    1    3   -1
## [2,]    3   -2   -1   -7    5
## [3,]    2    1    4    0    8
```

```
Rank(A)
```

```
## [1] 2
```

```
GA <- t(A)%*%A
print(GA)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]   14   -5    4  -24   32
## [2,]   -5    6    7   17   -3
## [3,]    4    7   18   10   26
## [4,]  -24   17   10   58  -38
## [5,]   32   -3   26  -38   90
```

```
inv(GA)
```

```
## Warning in inv(GA): Matrix appears to be singular.
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  Inf  Inf  Inf  Inf  Inf
## [2,]  Inf  Inf  Inf  Inf  Inf
## [3,]  Inf  Inf  Inf  Inf  Inf
## [4,]  Inf  Inf  Inf  Inf  Inf
## [5,]  Inf  Inf  Inf  Inf  Inf
```

Problem 2

```
r1 <- matrix(c(-1,1), nrow =1)
r2 <- matrix(c(3,-2), nrow =1)
r3 <- matrix(c(2,1), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2]
## [1,]   -1    1
## [2,]    3   -2
## [3,]    2    1
```

```
Rank(A)
```

```
## [1] 2
```

```
GA <- t(A)%*%A
print(GA)
```

```
##      [,1] [,2]
## [1,]   14   -5
## [2,]   -5    6
```

```
print(round(inv(GA),2))
```

```
##      [,1] [,2]
## [1,] 0.10 0.08
## [2,] 0.08 0.24
```

Problem 3

```
r1 <- matrix(c(-1,1,1), nrow =1)
r2 <- matrix(c(3,-2,-1), nrow =1)
r3 <- matrix(c(2,1,4), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]   -1    1    1
## [2,]    3   -2   -1
## [3,]    2    1    4
```

```
Rank(A)
```

```
## [1] 2
```

```r
GA <- t(A)%*%A
print(GA)
```

```
##      [,1] [,2] [,3]
## [1,]   14   -5    4
## [2,]   -5    6    7
## [3,]    4    7   18
```

```r
inv(GA)
```

```
## Warning in inv(GA): Matrix appears to be singular.
```

```
##      [,1] [,2] [,3]
## [1,]  Inf  Inf  Inf
## [2,]  Inf  Inf  Inf
## [3,]  Inf  Inf  Inf
```

Section 2.6

```r
diag(c(-4,0,1))
```

```
##      [,1] [,2] [,3]
## [1,]   -4    0    0
## [2,]    0    0    0
## [3,]    0    0    1
```

```r
diag(c(5,3,1))
```

```
##      [,1] [,2] [,3]
## [1,]    5    0    0
## [2,]    0    3    0
## [3,]    0    0    1
```

```r
diag(c(6,2,1))
```

```
##      [,1] [,2] [,3]
## [1,]    6    0    0
## [2,]    0    2    0
## [3,]    0    0    1
```

```r
r1 <- matrix(c(2,4,2), nrow =1)
r2 <- matrix(c(0,3,1), nrow =1)
r3 <- matrix(c(2,5,2), nrow =1)
P <- rbind(r1,r2,r3)
print(P)
```

```
##      [,1] [,2] [,3]
## [1,]    2    4    2
## [2,]    0    3    1
## [3,]    2    5    2
```

```
print(inv(P))
```

```
##      [,1] [,2] [,3]
## [1,] -0.5   -1    1
## [2,] -1.0    0    1
## [3,]  3.0    1   -3
```

```
print(inv(P)%*%P)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

```
r1 <- matrix(c(2,0), nrow =1)
r2 <- matrix(c(0,1), nrow =1)
r3 <- matrix(c(2,1), nrow =1)
B <- rbind(r1,r2,r3)
print(B)
```

```
##      [,1] [,2]
## [1,]    2    0
## [2,]    0    1
## [3,]    2    1
```

```
GB <- t(B)%*%B
print(GB)
```

```
##      [,1] [,2]
## [1,]    8    2
## [2,]    2    2
```

```
print(round(inv(GB),2))
```

```
##       [,1]  [,2]
## [1,]  0.17 -0.17
## [2,] -0.17  0.67
```

```
print(inv(GB)%*%GB)
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

Problem 1

```
r1 <- matrix(c(2,0,4,4,2), nrow =1)
r2 <- matrix(c(0,1,2,3,1), nrow =1)
r3 <- matrix(c(2,1,3,5,2), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    2    0    4    4    2
## [2,]    0    1    2    3    1
## [3,]    2    1    3    5    2
```

```
print(round(rref(A),2))
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0 0.67 0.33
## [2,]    0    1    0 1.67 0.33
## [3,]    0    0    1 0.67 0.33
```

```
B <- A[,c(1,2,3)]
print(B)
```

```
##      [,1] [,2] [,3]
## [1,]    2    0    4
## [2,]    0    1    2
## [3,]    2    1    3
```

```
print(round(inv(B)%*%B),2)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

Problem 2

```
v1 <- matrix(c(2,0,2,4,5), nrow =5)
v2 <- matrix(c(0,1,1,3,4), nrow =5)
v3 <- matrix(c(4,2,4,7,7), nrow =5)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    2    0    4
## [2,]    0    1    2
## [3,]    2    1    4
## [4,]    4    3    7
## [5,]    5    4    7
```

```
Rank(A)
```

```
## [1] 3
```

```
GA <- t(A)%*%A
print(GA)
```

```
##      [,1] [,2] [,3]
## [1,]   49   34   79
## [2,]   34   27   55
## [3,]   79   55  134
```

```
print(round(inv(GA)%*%GA),2)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

Problem 3

```
v1 <- matrix(c(2,0,2), nrow =3)
v2 <- matrix(c(0,1,1), nrow =3)
v3 <- matrix(c(4,1,5), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    2    0    4
## [2,]    0    1    1
## [3,]    2    1    5
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    2
## [2,]    0    1    1
## [3,]    0    0    0
```

```
B <- A[,c(1,2)]
print(B)
```

```
##      [,1] [,2]
## [1,]    2    0
## [2,]    0    1
## [3,]    2    1
```

```
GB <- t(B)%*%B
print(GB)
```

```
##      [,1] [,2]
## [1,]    8    2
## [2,]    2    2
```

```
print(inv(GB)%*%GB)
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

Problem 4

```
v1 <- matrix(c(2,0,2), nrow =3)
v2 <- matrix(c(0,1,1), nrow =3)
v3 <- matrix(c(4,2,4), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    2    0    4
## [2,]    0    1    2
## [3,]    2    1    4
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

```
I <- inv(A)%*%A
print(I)
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    0
## [2,]    0    1    0
## [3,]    0    0    1
```

Problem 5

```
v1 <- matrix(c(2,0,2), nrow =3)
A <- v1

GA <- t(A)%*%A
print(GA)
```

```
##      [,1]
## [1,]    8
```

```
print(round(inv(GA)%*%GA),2)
```

```
##      [,1]
## [1,]    1
```

Chapter 3

Section 3.1

```
A <- matrix(c(1,1,1,1,-1,-3), nrow = 2, byrow=T)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    1
## [2,]    1   -1   -3
```

```
dim(A)
```

```
## [1] 2 3
```

```
dim(A)[1]
```

```
## [1] 2
```

```
dim(A)[2]
```

```
## [1] 3
```

Section 3.3

```
r1 <- matrix(c(1.1,1.1,3.3,1.1), nrow =1)
r2 <- matrix(c(-1.2,3.3,5.4,7.8), nrow =1)
r3 <- matrix(c(1.3,2.2,5.7,3.1), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  1.1  1.1  3.3  1.1
## [2,] -1.2  3.3  5.4  7.8
## [3,]  1.3  2.2  5.7  3.1
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    1    0    1   -1
## [2,]    0    1    2    2
## [3,]    0    0    0    0
```

```
Rank(A)
```

```
## [1] 2
```

```r
B <- A[,c(1,2)]
print(B)
```

```
##      [,1] [,2]
## [1,]  1.1  1.1
## [2,] -1.2  3.3
## [3,]  1.3  2.2
```

```r
D <- A[,-c(1,2)]
print(D)
```

```
##      [,1] [,2]
## [1,]  3.3  1.1
## [2,]  5.4  7.8
## [3,]  5.7  3.1
```

```r
GB <- t(B)%*%B
print(GB)
```

```
##      [,1]  [,2]
## [1,] 4.34  0.11
## [2,] 0.11 16.94
```

```r
print(-inv(GB)%*%t(B)%*%D)
```

```
##      [,1] [,2]
## [1,]   -1    1
## [2,]   -2   -2
```

```r
I <- diag(2)
print(I)
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

```r
print(rbind(-inv(GB)%*%(t(B)%*%D),I))
```

```
##      [,1] [,2]
## [1,]   -1    1
## [2,]   -2   -2
## [3,]    1    0
## [4,]    0    1
```

```r
N <- rbind(-inv(GB)%*%(t(B)%*%D),I)
print(round(A%*%N,2))
```

```
##      [,1] [,2]
## [1,]    0    0
## [2,]    0    0
## [3,]    0    0
```

```
print(A)
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  1.1  1.1  3.3  1.1
## [2,] -1.2  3.3  5.4  7.8
## [3,]  1.3  2.2  5.7  3.1
```

```
AT <- t(A)
print(AT)
```

```
##      [,1] [,2] [,3]
## [1,]  1.1 -1.2  1.3
## [2,]  1.1  3.3  2.2
## [3,]  3.3  5.4  5.7
## [4,]  1.1  7.8  3.1
```

```
print(rref(t(A)))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0  1.4
## [2,]    0    1  0.2
## [3,]    0    0  0.0
## [4,]    0    0  0.0
```

```
RA <- AT[,c(1,2)]
print(RA)
```

```
##      [,1] [,2]
## [1,]  1.1 -1.2
## [2,]  1.1  3.3
## [3,]  3.3  5.4
## [4,]  1.1  7.8
```

Section 3.5

```
library(far)

v1 <- matrix(c(1,2,3), nrow =3)
v2 <- matrix(c(4,5,6), nrow =3)
v3 <- matrix(c(13,2,-4), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    4   13
## [2,]    2    5    2
## [3,]    3    6   -4
```

```
Rank(A)
```

```
## [1] 3
```

```
OA <- orthonormalization(A)
print(round(OA,2))
```

```
##      [,1]  [,2]  [,3]
## [1,] 0.27  0.87  0.41
## [2,] 0.53  0.22 -0.82
## [3,] 0.80 -0.44  0.41
```

```
Rank(OA)
```

```
## [1] 3
```

```
Norm(OA[,1])
```

```
## [1] 1
```

```
#between column vectors
print(round(t(OA[,1])%*%OA[,2],2))
```

```
##      [,1]
## [1,]    0
```

```
print(round(t(OA[,1])%*%OA[,3],2))
```

```
##      [,1]
## [1,]    0
```

```
#between row vectors
print(round(t(OA[1,])%*%OA[2,],2))
```

```
##      [,1]
## [1,]    0
```

```
print(round(t(OA[1,])%*%OA[3,],2))
```

```
##      [,1]
## [1,]    0
```

```
v1 <- matrix(c(1,2,-3), nrow =3)
v2 <- matrix(c(-4,2,1), nrow =3)
v3 <- matrix(c(11,2,-11), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1   -4   11
## [2,]    2    2    2
## [3,]   -3    1  -11
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    3
## [2,]    0    1   -2
## [3,]    0    0    0
```

```
B <- A[,c(1,2)]
print(B)
```

```
##      [,1] [,2]
## [1,]    1   -4
## [2,]    2    2
## [3,]   -3    1
```

```
OB <- orthonormalization(B)
print(round(OB,2))
```

```
##       [,1]  [,2] [,3]
## [1,]  0.27 -0.84 0.47
## [2,]  0.53  0.54 0.65
## [3,] -0.80  0.08 0.59
```

Problem 1

```
v1 <- matrix(c(1,2,-3), nrow =3)
v2 <- matrix(c(-4,2,1), nrow =3)
v3 <- matrix(c(-3,4,-2), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1   -4   -3
## [2,]    2    2    4
## [3,]   -3    1   -2
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    1
## [2,]    0    1    1
## [3,]    0    0    0
```

```
B <- A[,c(1,2)]

OB <- orthonormalization(B)
print(round(OB,2))
```

```
##       [,1]  [,2] [,3]
## [1,]  0.27 -0.84 0.47
## [2,]  0.53  0.54 0.65
## [3,] -0.80  0.08 0.59
```

```
#basis spanning column space of A
CA <- OB[,c(1,2)]
print(round(CA,2))
```

```
##       [,1]  [,2]
## [1,]  0.27 -0.84
## [2,]  0.53  0.54
## [3,] -0.80  0.08
```

```
#basis spanning left nullspace of A
NAT <- OB[,c(3)]
print(round(NAT,2))
```

```
## [1] 0.47 0.65 0.59
```

Problem 2

```
r1 <- matrix(c(1,-4,-3,-3,23), nrow =1)
r2 <- matrix(c(2,2,4,4,-1), nrow =1)
r3 <- matrix(c(-3,1,-2,-2,-14), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1   -4   -3   -3   23
## [2,]    2    2    4    4   -1
## [3,]   -3    1   -2   -2  -14
```

```
print(rref(A))
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    1    1    0
## [2,]    0    1    1    1    0
## [3,]    0    0    0    0    1
```

```
B <- A[,c(1,2,5)]

OB <- orthonormalization(B)
print(round(OB,2))
```

```
##       [,1]  [,2] [,3]
## [1,]  0.27 -0.84 0.47
## [2,]  0.53  0.54 0.65
## [3,] -0.80  0.08 0.59
```

```
#basis spanning column space of A
CA <- OB[,c(1,2,3)]
print(round(CA,2))
```

```
##       [,1]  [,2] [,3]
## [1,]  0.27 -0.84 0.47
## [2,]  0.53  0.54 0.65
## [3,] -0.80  0.08 0.59
```

```
#left nullspace contains only zero
```

Chapter 4

Section 4.1

```
v1 <- matrix(c(1,-1,1), nrow =3)
v2 <- matrix(c(1,3,2), nrow =3)
v3 <- matrix(c(2,3,4), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    2
## [2,]   -1    3    3
## [3,]    1    2    4
```

```
Rank(A)
```

```
## [1] 3
```

```
b <- matrix(c(1,4,-4))
print(b)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```

```
Rank(b)
```

```
## [1] 1
```

```
x <-inv(A)%*%b
print(round(x,2))
```

```
##       [,1]
## [1,]  6.00
## [2,] 11.67
## [3,] -8.33
```

```
print(A%*%x)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```

Section 4.2

```r
v1 <- matrix(c(1,-1,1), nrow =3)
v2 <- matrix(c(1,3,2), nrow =3)
v3 <- matrix(c(3,5,5), nrow =3)
A <- cbind(v1,v2,v3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    3
## [2,]   -1    3    5
## [3,]    1    2    5
```

```r
b <- matrix(c(1,4,-4))
print(b)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```

```r
Ab <- cbind(A,b)

Rank(A)
```

```
## [1] 2
```

```r
Rank(Ab)
```

```
## [1] 3
```

```r
print(rref(A))
```

```
##      [,1] [,2] [,3]
## [1,]    1    0    1
## [2,]    0    1    2
## [3,]    0    0    0
```

```r
B <- A[,c(1,2)]
GB <- t(B)%*%B
xB <- inv(GB)%*%t(B)%*%b
print(round(xB,2))
```

```
##       [,1]
## [1,] -2.33
## [2,]  0.36
```

```r
b_hat <- B%*%xB
print(round(b_hat,2))
```

```
##       [,1]
## [1,] -1.98
## [2,]  3.40
## [3,] -1.62
```

```
x <- rbind(xB,0)
print(round(x,2))
```

```
##       [,1]
## [1,] -2.33
## [2,]  0.36
## [3,]  0.00
```

```
print(round(A%*%x,2))
```

```
##       [,1]
## [1,] -1.98
## [2,]  3.40
## [3,] -1.62
```

Section 4.3

Problem 1

```
r1 <- matrix(c(1,1,2,2,3), nrow =1)
r2 <- matrix(c(-1,3,3,-1,5), nrow =1)
r3 <- matrix(c(1,2,4,3,5), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    1    2    2    3
## [2,]   -1    3    3   -1    5
## [3,]    1    2    4    3    5
```

```
b <- matrix(c(1,4,-4))
print(b)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```

```
#######################################
#STEP 1
#######################################

print(rref(A))
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1    0    0    1    1
## [2,]    0    1    0   -1    2
## [3,]    0    0    1    1    0
```

```
Ab <- cbind(A,b)
print(round(rref(Ab),2))
```

```
##      [,1] [,2] [,3] [,4] [,5]  [,6]
## [1,]    1    0    0    1    1  6.00
## [2,]    0    1    0   -1    2 11.67
## [3,]    0    0    1    1    0 -8.33
```

```
B<- A[,c(1,2,3)]
D<- A[,-c(1,2,3)]
print(B)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    2
## [2,]   -1    3    3
## [3,]    1    2    4
```

```
print(D)
```

```
##      [,1] [,2]
## [1,]    2    3
## [2,]   -1    5
## [3,]    3    5
```

```
GB <- t(B)%*%B
print(GB)
```

```
##      [,1] [,2] [,3]
## [1,]    3    0    3
## [2,]    0   14   19
## [3,]    3   19   29
```

```
xB <- inv(GB)%*%t(B)%*%b
print(round(xB,2))
```

```
##       [,1]
## [1,]  6.00
## [2,] 11.67
## [3,] -8.33
```

```
x_zero <- rbind(xB[1],xB[2],xB[3],0,0)
print(round(x_zero,2))
```

```
##        [,1]
## [1,]  6.00
## [2,] 11.67
## [3,] -8.33
## [4,]  0.00
## [5,]  0.00
```

```
##########################################
# STEP 2
##########################################
N <- rbind(-inv(GB)%*%t(B)%*%D, diag(2))
print(round(N,0))
```

```
##      [,1] [,2]
## [1,]   -1   -1
## [2,]    1   -2
## [3,]   -1    0
## [4,]    1    0
## [5,]    0    1
```

```
# point 1
x_D <- matrix(c(-3,2),nrow=2)
print(x_D)
```

```
##      [,1]
## [1,]   -3
## [2,]    2
```

```
x_1 <- x_zero - N%*%x_D
print(round(x_1,2))
```

```
##         [,1]
## [1,]   5.00
## [2,]  18.67
## [3,] -11.33
## [4,]   3.00
## [5,]  -2.00
```

```
print(A%*%x_1)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```

```
# point 2
x_D <- matrix(c(1,5),nrow=2)
print(x_D)
```

```
##      [,1]
## [1,]    1
## [2,]    5
```

```r
x_2 <- x_zero - N%*%x_D
print(round(x_2,2))
```

```
##       [,1]
## [1,] 12.00
## [2,] 20.67
## [3,] -7.33
## [4,] -1.00
## [5,] -5.00
```

```r
print(A%*%x_2)
```

```
##      [,1]
## [1,]    1
## [2,]    4
## [3,]   -4
```

Chapter 5

Section 5.1

```
r1 <- matrix(c(-2,4,2), nrow =1)
r2 <- matrix(c(-4.8,8.4,1.6), nrow =1)
r3 <- matrix(c(-4.8,2.4,5.6), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,] -2.0  4.0  2.0
## [2,] -4.8  8.4  1.6
## [3,] -4.8  2.4  5.6
```

```
eigen(A)
```

```
## eigen() decomposition
## $values
## [1] 6 4 2
##
## $vectors
##               [,1]          [,2]       [,3]
## [1,]  4.472136e-01 3.162278e-01 -0.6666667
## [2,]  8.944272e-01 3.830179e-15 -0.3333333
## [3,] -2.673771e-15 9.486833e-01 -0.6666667
```

```
eigen(A)$values
```

```
## [1] 6 4 2
```

```
round(eigen(A)$vectors,2)
```

```
##      [,1] [,2]  [,3]
## [1,] 0.45 0.32 -0.67
## [2,] 0.89 0.00 -0.33
## [3,] 0.00 0.95 -0.67
```

Problem 1

```
r1 <- matrix(c(1,4,7), nrow =1)
r2 <- matrix(c(2,5,8), nrow =1)
r3 <- matrix(c(3,6,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    4    7
## [2,]    2    5    8
## [3,]    3    6    9
```

```
Rank(A)
```

```
## [1] 2
```

```
L <- diag(eigen(A)$values)
print(round(L,2))
```

```
##       [,1]  [,2] [,3]
## [1,] 16.12  0.00    0
## [2,]  0.00 -1.12    0
## [3,]  0.00  0.00    0
```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##       [,1]  [,2]  [,3]
## [1,] -0.46 -0.88  0.41
## [2,] -0.57 -0.24 -0.82
## [3,] -0.68  0.40  0.41
```

```
Rank(E)
```

```
## [1] 3
```

Problem 2

```
r1 <- matrix(c(1,4,2), nrow =1)
r2 <- matrix(c(5,5,8), nrow =1)
r3 <- matrix(c(-3,69,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    4    2
## [2,]    5    5    8
## [3,]   -3   69    9
```

```
Rank(A)
```

```
## [1] 3
```

```
L <- diag(eigen(A)$values)
print(round(L,2))
```

```
##       [,1]   [,2] [,3]
## [1,] 31.19   0.00 0.00
## [2,]  0.00 -16.31 0.00
## [3,]  0.00   0.00 0.12
```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##      [,1]  [,2]  [,3]
## [1,] 0.10 -0.03 -0.81
## [2,] 0.31 -0.35 -0.11
## [3,] 0.95  0.94  0.57
```

```
Rank(E)
```

```
## [1] 3
```

Problem 3

```
r1 <- matrix(c(1,2,2), nrow =1)
r2 <- matrix(c(2,4,8), nrow =1)
r3 <- matrix(c(3,6,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    2
## [2,]    2    4    8
## [3,]    3    6    9
```

```
Rank(A)
```

```
## [1] 2
```

```
L <- diag(eigen(A)$values)
print(round(L,2))
```

```
##       [,1]  [,2] [,3]
## [1,] 14.62  0.00    0
## [2,]  0.00 -0.62    0
## [3,]  0.00  0.00    0
```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##       [,1]  [,2]  [,3]
## [1,] -0.20  0.58 -0.89
## [2,] -0.61 -0.76  0.45
## [3,] -0.76  0.29  0.00
```

```
Rank(E)
```

```
## [1] 3
```

Problem 4

```
r1 <- matrix(c(0.8,-0.6,0), nrow =1)
r2 <- matrix(c(0.6,0.8,0), nrow =1)
r3 <- matrix(c(3,3,1), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]  0.8 -0.6    0
## [2,]  0.6  0.8    0
## [3,]  3.0  3.0    1
```

```
Rank(A)
```

```
## [1] 3
```

```
L <- diag(eigen(A)$values)
print(round(L,2))
```

```
##      [,1]     [,2]     [,3]
## [1,] 1+0i 0.0+0.0i 0.0+0.0i
## [2,] 0+0i 0.8+0.6i 0.0+0.0i
## [3,] 0+0i 0.0+0.0i 0.8-0.6i
```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##      [,1]        [,2]        [,3]
## [1,] 0+0i -0.13+0.07i -0.13-0.07i
## [2,] 0+0i  0.07+0.13i  0.07-0.13i
## [3,] 1+0i  0.98+0.00i  0.98+0.00i
```

Section 5.2

```
r1 <- matrix(c(-2,4,2), nrow =1)
r2 <- matrix(c(-4.8,8.4,1.6), nrow =1)
r3 <- matrix(c(-4.8,2.4,5.6), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,] -2.0  4.0  2.0
## [2,] -4.8  8.4  1.6
## [3,] -4.8  2.4  5.6
```

```
Rank(A)
```

```
## [1] 3
```

```
Q<- A - diag(eigen(A)$values[1],3)

print(Q)
```

```
##      [,1] [,2] [,3]
## [1,] -8.0  4.0  2.0
## [2,] -4.8  2.4  1.6
## [3,] -4.8  2.4 -0.4
```

```
Rank(Q)
```

```
## [1] 2
```

Section 5.3

```
r1 <- matrix(c(1,4,2), nrow =1)
r2 <- matrix(c(5,5,8), nrow =1)
r3 <- matrix(c(-3,69,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    4    2
## [2,]    5    5    8
## [3,]   -3   69    9
```

```
L <- diag(eigen(A)$values)
print(round(L,2))
```

```
##       [,1]   [,2] [,3]
## [1,] 31.19   0.00 0.00
## [2,]  0.00 -16.31 0.00
## [3,]  0.00   0.00 0.12
```

```r
E <- eigen(A)$vectors
print(round(E,2))
```

```
##      [,1]  [,2]  [,3]
## [1,] 0.10 -0.03 -0.81
## [2,] 0.31 -0.35 -0.11
## [3,] 0.95  0.94  0.57
```

```r
A <- matrix(c(0,0,2.4,1.2), nrow=2)
print(A)
```

```
##      [,1] [,2]
## [1,]    0  2.4
## [2,]    0  1.2
```

```r
#check eigendecomposition
eigen(A)
```

```
## eigen() decomposition
## $values
## [1] 1.2 0.0
##
## $vectors
##           [,1] [,2]
## [1,] 0.8944272    1
## [2,] 0.4472136    0
```

```r
#create L
L <- diag(eigen(A)$values)
L
```

```
##      [,1] [,2]
## [1,]  1.2    0
## [2,]  0.0    0
```

```r
#check if L is singular
inv(L)
```

```
## Warning in inv(L): Matrix appears to be singular.
```

```
##      [,1] [,2]
## [1,]  Inf  Inf
## [2,]  Inf  Inf
```

Section 5.4

Problem 1

```
r1 <- matrix(c(1,2,2), nrow =1)
r2 <- matrix(c(2,4,8), nrow =1)
r3 <- matrix(c(3,6,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    2
## [2,]    2    4    8
## [3,]    3    6    9
```

```
E <- eigen(A)$vectors
print(round(E,2))
```

```
##       [,1]  [,2]  [,3]
## [1,] -0.20  0.58 -0.89
## [2,] -0.61 -0.76  0.45
## [3,] -0.76  0.29  0.00
```

```
x<- matrix(c(2,4,8),nrow=3)
print(x)
```

```
##      [,1]
## [1,]    2
## [2,]    4
## [3,]    8
```

```
h<- inv(E)%*%x

h[1]%*%E[,1] + h[2]%*%E[,2] + h[3]%*%E[,3]
```

```
##      [,1] [,2] [,3]
## [1,]    2    4    8
```

Problem 2

```
L <- diag(c(1.2,0.7))
e1 <- c(2,1)
e2 <- c(1,0)
E <- cbind(e1,e2)
A <- E%*%L%*%inv(E)
print(A)
```

```
##      [,1] [,2]
## [1,]  0.7  1.0
## [2,]  0.0  1.2
```

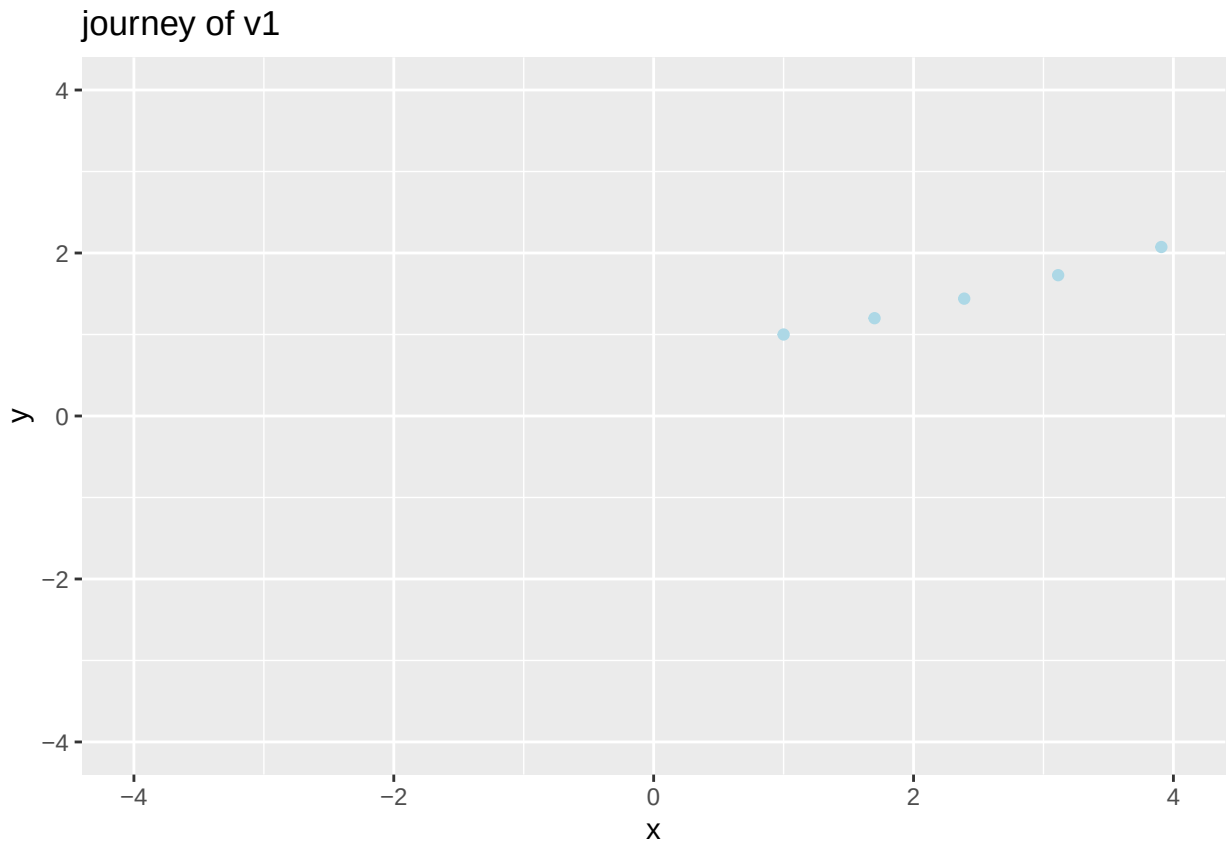
reproducing graph

```
#plot of v1
v1 <- c(1,1)

p0 <- v1
p1 <- A%*%v1
p2 <- A%*%A%*%v1
p3 <- A%*%A%*%A%*%v1
p4 <- A%*%A%*%A%*%A%*%v1
p <- as.data.frame(t(cbind(p0, p1,p2,p3,p4)))
colnames(p) <- c("x","y")


f1 <- p %>% ggplot(aes(x=x,y=y)) +
            geom_point(color = "lightblue")+
            xlim(c(-4,4)) + ylim(c(-4,4)) +
            ggtitle("journey of v1")

f1
```



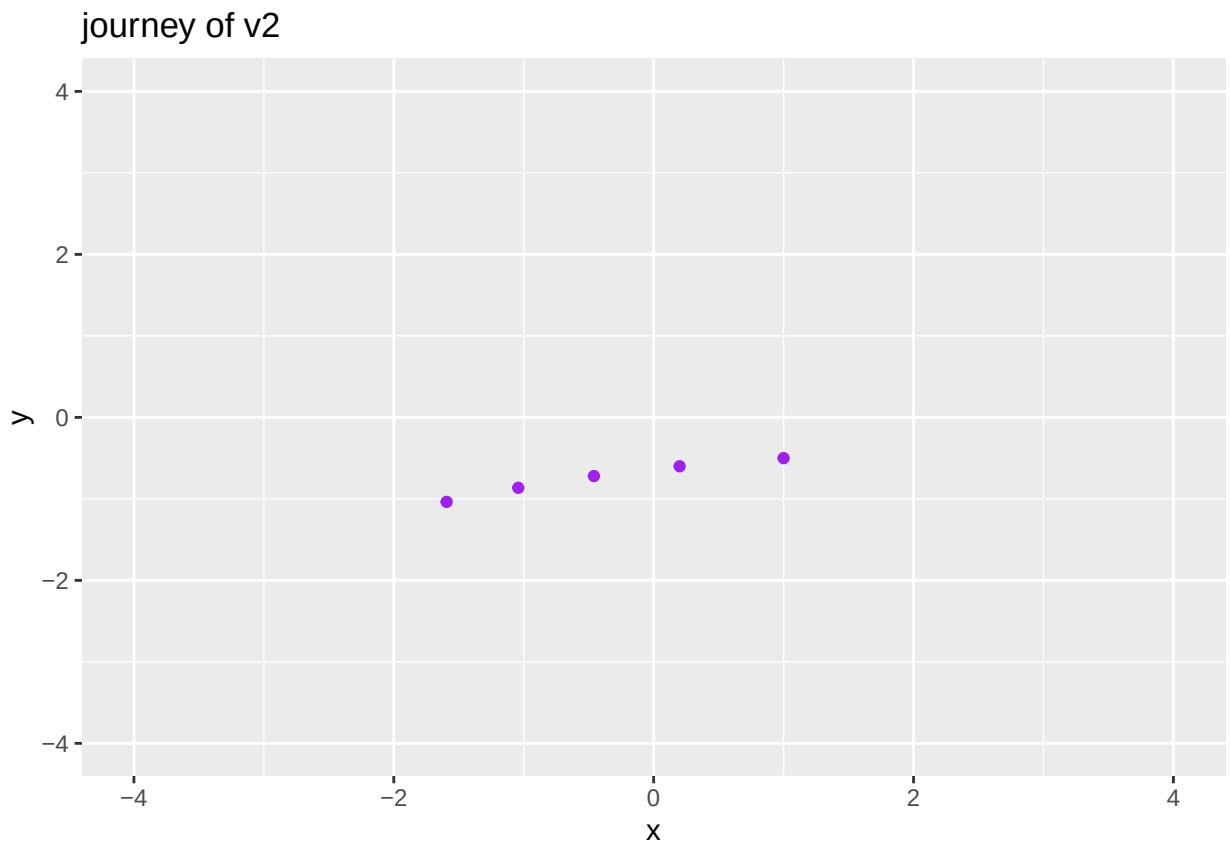

```
#plot of v2
v2 <- c(1,-0.5)

p0 <- v2
```

```
p1 <- A%*%v2
p2 <- A%*%A%*%v2
p3 <- A%*%A%*%A%*%v2
p4 <- A%*%A%*%A%*%A%*%v2
p <- as.data.frame(t(cbind(p0,p1,p2,p3,p4)))
colnames(p) <- c("x","y")


f2 <- p %>% ggplot(aes(x=x,y=y)) +
            geom_point(color = "purple") +
            xlim(c(-4,4)) + ylim(c(-4,4))+
            ggtitle("journey of v2")

f2
```

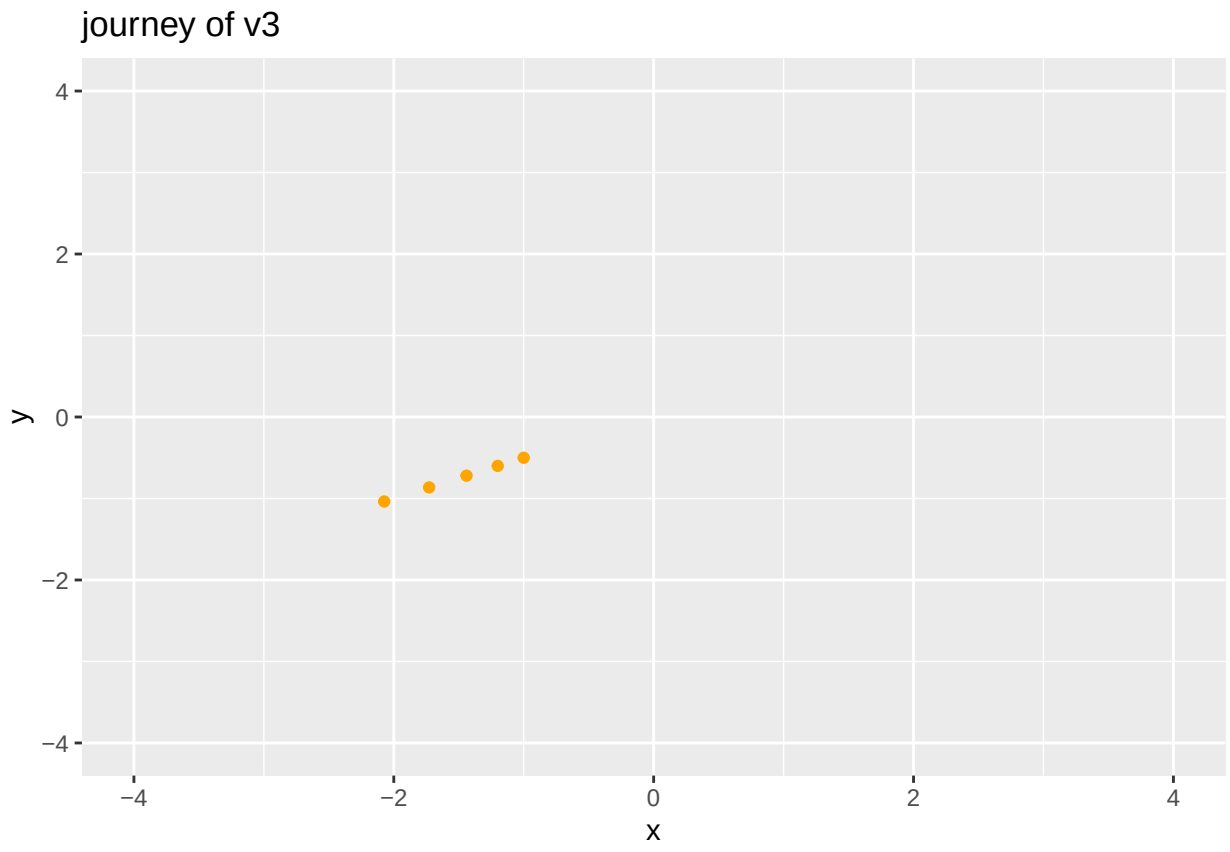



```
#plot of v3
v3 <- c(-1,-0.5)

p0 <- v3
p1 <- A%*%v3
p2 <- A%*%A%*%v3
p3 <- A%*%A%*%A%*%v3
p4 <- A%*%A%*%A%*%A%*%v3
p <- as.data.frame(t(cbind(p0,p1,p2,p3,p4)))
colnames(p) <- c("x","y")
```

```
f3 <- p %>% ggplot(aes(x=x,y=y)) +
            geom_point(color = "orange") +
            xlim(c(-4,4)) + ylim(c(-4,4))+
            ggtitle("journey of v3")

f3
```



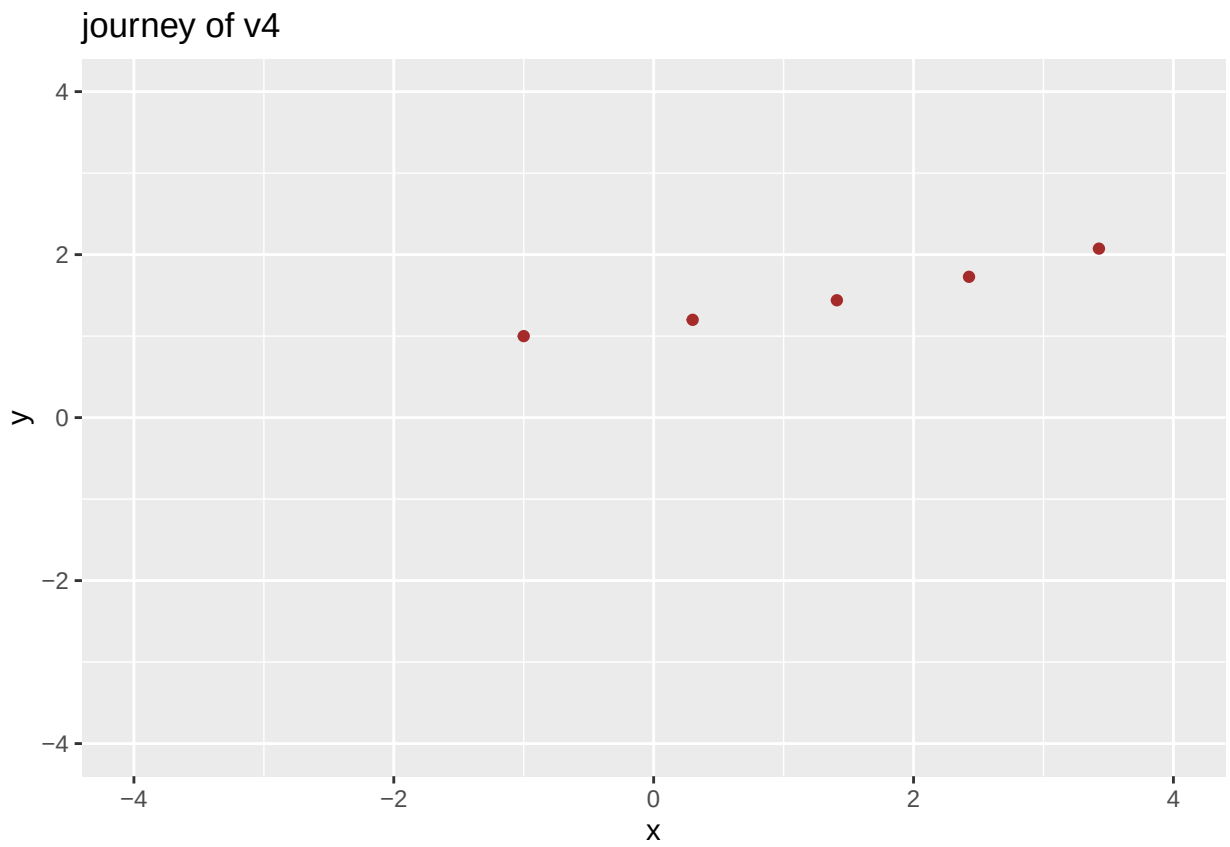

```
#plot of v4

v4 <- c(-1,1)

p0 <- v4
p1 <- A%*%v4
p2 <- A%*%A%*%v4
p3 <- A%*%A%*%A%*%v4
p4 <- A%*%A%*%A%*%A%*%v4
p <- as.data.frame(t(cbind(p0,p1,p2,p3,p4)))
colnames(p) <- c("x","y")


f4 <- p %>% ggplot(aes(x=x,y=y)) +
            geom_point(color = "brown") +
            xlim(c(-4,4)) + ylim(c(-4,4))+
            ggtitle("journey of v4")
```

```
f4
```




```
#plotting all together
(f1|f2)/(f3|f4)
```

Section 5.5

Problem 1

```
r1 <- matrix(c(1,2,2), nrow =1)
r2 <- matrix(c(2,4,8), nrow =1)
r3 <- matrix(c(3,6,9), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2    2
## [2,]    2    4    8
## [3,]    3    6    9
```

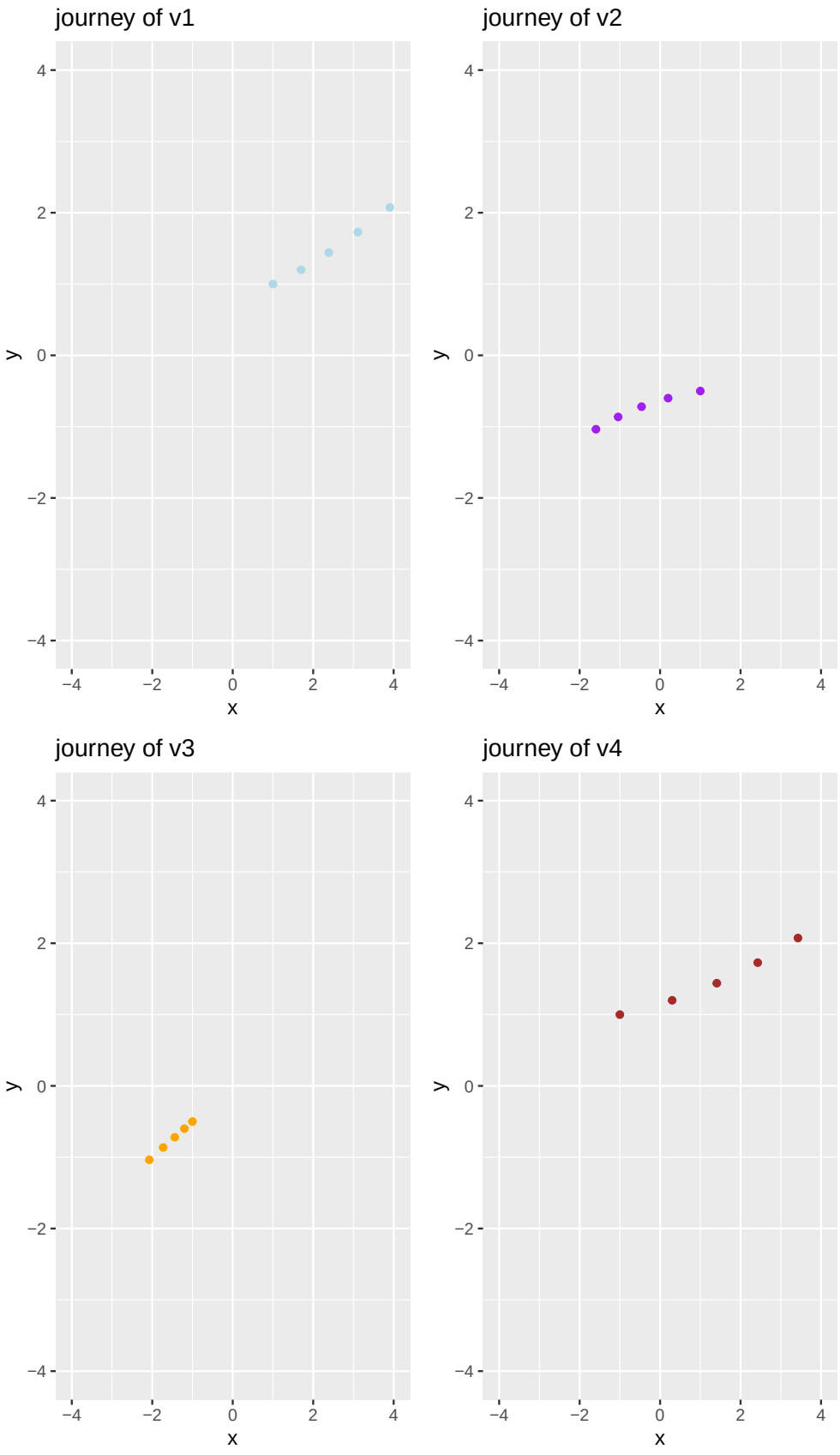


Figure 1: journey of vectors

```
Rank(A)
```

```
## [1] 2
```

```
GA <- t(A)%*%A
print(GA)
```

```
##      [,1] [,2] [,3]
## [1,]   14   28   45
## [2,]   28   56   90
## [3,]   45   90  149
```

```
L <- as.data.frame(round(diag(eigen(GA)$values,3),1))
print(L)
```

```
##      V1  V2 V3
## 1 217.6 0.0  0
## 2   0.0 1.4  0
## 3   0.0 0.0  0
```

```
E <- round(eigen(GA)$vectors,3)
E
```

```
##        [,1]   [,2]   [,3]
## [1,] -0.252 -0.370  0.894
## [2,] -0.504 -0.739 -0.447
## [3,] -0.826  0.563  0.000
```

```
t(E)
```

```
##        [,1]   [,2]   [,3]
## [1,] -0.252 -0.504 -0.826
## [2,] -0.370 -0.739  0.563
## [3,]  0.894 -0.447  0.000
```

```
round(inv(E),3)
```

```
##        [,1]   [,2]   [,3]
## [1,] -0.252 -0.504 -0.827
## [2,] -0.369 -0.739  0.564
## [3,]  0.895 -0.448  0.000
```

Problem 2

```
r1 <- matrix(c(1,2,-1), nrow =1)
r2 <- matrix(c(2,2,3), nrow =1)
r3 <- matrix(c(0,3,2), nrow =1)
A <- rbind(r1,r2,r3)
print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -1
## [2,]    2    2    3
## [3,]    0    3    2
```

```
Rank(A)
```

```
## [1] 3
```

```
GA <- t(A)%*%A
print(GA)
```

```
##      [,1] [,2] [,3]
## [1,]    5    6    5
## [2,]    6   17   10
## [3,]    5   10   14
```

```
L <- round(diag(eigen(GA)$values,2),1)
print(L)
```

```
##      [,1] [,2]
## [1,] 28.2  0.0
## [2,]  0.0  5.4
```

```
E <- round(eigen(GA)$vectors,3)
E
```

```
##        [,1]   [,2]   [,3]
## [1,] -0.319 -0.037  0.947
## [2,] -0.719 -0.641 -0.267
## [3,] -0.617  0.766 -0.178
```

```
round(t(E),2)
```

```
##       [,1]  [,2]  [,3]
## [1,] -0.32 -0.72 -0.62
## [2,] -0.04 -0.64  0.77
## [3,]  0.95 -0.27 -0.18
```

```
round(inv(E),2)
```

```
##       [,1]  [,2]  [,3]
## [1,] -0.32 -0.72 -0.62
## [2,] -0.04 -0.64  0.77
## [3,]  0.95 -0.27 -0.18
```

Section 5.6

Problem 1

```
M <- matrix(c(0.7,0.25,0.3,0.75),
            nrow=2, byrow = T)

print(M)
```

```
##      [,1] [,2]
## [1,]  0.7 0.25
## [2,]  0.3 0.75
```

```
E <- eigen(M)$vectors
print(round(eigen(M)$vectors,2))
```

```
##       [,1]  [,2]
## [1,] -0.64 -0.71
## [2,] -0.77  0.71
```

```
print(eigen(M)$values)
```

```
## [1] 1.00 0.45
```

```
x0 <- matrix(c(0.6,0.4),nrow=2)
print(x0)
```

```
##      [,1]
## [1,]  0.6
## [2,]  0.4
```

```
p1 <- E[,1]/sum(E[,1])
print(round(p1,2))
```

```
## [1] 0.45 0.55
```

Section 5.7

```
Q <- matrix(c(0,0.5,0,0.5,0,0.5,0,0.5,0),
            nrow = 3, byrow =T)
print(Q)
```

```
##      [,1] [,2] [,3]
## [1,]  0.0  0.5  0.0
## [2,]  0.5  0.0  0.5
## [3,]  0.0  0.5  0.0
```

```
N <- inv(diag(3)-Q)
print(N)
```

```
##      [,1] [,2] [,3]
## [1,]  1.5    1  0.5
## [2,]  1.0    2  1.0
## [3,]  0.5    1  1.5
```

```
R <- matrix(c(0.5,0,0,0,0,0.5), nrow=3, byrow = T)
print(R)
```

```
##      [,1] [,2]
## [1,]  0.5  0.0
## [2,]  0.0  0.0
## [3,]  0.0  0.5
```

Problem 1

```
N <- inv(diag(3)-Q)
print(N)
```

```
##      [,1] [,2] [,3]
## [1,]  1.5    1  0.5
## [2,]  1.0    2  1.0
## [3,]  0.5    1  1.5
```

Problem 2

```
c1 <- matrix(c(1,1,1), nrow=3)
print(N%*%c1)
```

```
##      [,1]
## [1,]    3
## [2,]    4
## [3,]    3
```

Problem 3

```
print(N%*%R)
```

```
##      [,1] [,2]
## [1,] 0.75 0.25
## [2,] 0.50 0.50
## [3,] 0.25 0.75
```

Chapter 6

Section 6.1

```
A <- matrix(c(1,2,-2,-1,-1,-3),
            nrow=2, byrow = TRUE)

print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -2
## [2,]   -1   -1   -3
```

```
Rank(A)
```

```
## [1] 2
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##      [,1] [,2]
## [1,] 3.63 0.00
## [2,] 0.00 2.61
```

```
U <- svd(A)$u
print(round(U,2))
```

```
##       [,1]  [,2]
## [1,] -0.58 -0.81
## [2,] -0.81  0.58
```

```
print(round((svd(A)$u)%*%S),2)
```

```
##      [,1] [,2]
## [1,]   -2   -2
## [2,]   -3    2
```

```
V <- svd(A)$v
print(round(V,2))
```

```
##       [,1]  [,2]
## [1,]  0.06 -0.53
## [2,] -0.10 -0.84
## [3,]  0.99 -0.05
```

```
print(round(A%*%(svd(A)$v)),2)
```

```
##      [,1] [,2]
## [1,]   -2   -2
## [2,]   -3    2
```

```
U%*%S%*%t(V)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -2
## [2,]   -1   -1   -3
```

Problem 1

```
A <- matrix(c(1,1,2,1,-1,3),
            nrow=2, byrow = TRUE)

print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    2
## [2,]    1   -1    3
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##      [,1] [,2]
## [1,] 3.87 0.00
## [2,] 0.00 1.41
```

```
U <- svd(A)$u
print(round(U,2))
```

```
##       [,1]  [,2]
## [1,] -0.55 -0.83
## [2,] -0.83  0.55
```

```
V <- svd(A)$v
print(round(V,2))
```

```
##       [,1]  [,2]
## [1,] -0.36 -0.20
## [2,]  0.07 -0.98
## [3,] -0.93  0.00
```

```
print(round(A%*%(svd(A)$v),0))
```

```
##      [,1] [,2]
## [1,]   -2   -1
## [2,]   -3    1
```

```
print(round(svd(A)$u%*%S,0))
```

```
##      [,1] [,2]
## [1,]   -2   -1
## [2,]   -3    1
```

```
print(U%*%S%*%t(V))
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    2
## [2,]    1   -1    3
```

Problem 2

```
A <- matrix(c(1,4,7,1,1,9),
            nrow=3, byrow = TRUE)

print(A)
```

```
##      [,1] [,2]
## [1,]    1    4
## [2,]    7    1
## [3,]    1    9
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##       [,1] [,2]
## [1,] 10.26 0.00
## [2,]  0.00 6.61
```

```
U <- svd(A)$u
print(round(U,2))
```

```
##      [,1]  [,2]
## [1,] 0.40 -0.07
## [2,] 0.33  0.94
## [3,] 0.86 -0.33
```

```
V <- svd(A)$v
print(round(V,2))
```

```
##      [,1]  [,2]
## [1,] 0.35  0.94
## [2,] 0.94 -0.35
```

```
print(U%*%S%*%t(V))
```

```
##      [,1] [,2]
## [1,]    1    4
## [2,]    7    1
## [3,]    1    9
```

```
print(U%*%S%*%inv(V))
```

```
##      [,1] [,2]
## [1,]    1    4
## [2,]    7    1
## [3,]    1    9
```

Section 6.2

Problem 1

```
A <- matrix(c(1,2,-2,-1,-1,-3),
            nrow=2, byrow = TRUE)

print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    2   -2
## [2,]   -1   -1   -3
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##      [,1] [,2]
## [1,] 3.63 0.00
## [2,] 0.00 2.61
```

```
U <- svd(A)$u
print(round(U,2))
```

```
##       [,1]  [,2]
## [1,] -0.58 -0.81
## [2,] -0.81  0.58
```

```
V <- svd(A)$v
print(round(V,2))
```

```
##       [,1]  [,2]
## [1,]  0.06 -0.53
## [2,] -0.10 -0.84
## [3,]  0.99 -0.05
```

```
A_inv <- V%*%inv(S)%*%inv(U)
print(round(A_inv,2))
```

```
##       [,1]  [,2]
## [1,]  0.16 -0.13
## [2,]  0.28 -0.17
## [3,] -0.14 -0.23
```

```
result <- A%*%A_inv
print(round(result,2))
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

Problem 2

```
A <- matrix(c(1,1,2,1,-1,3),
            nrow=2, byrow = TRUE)

print(A)
```

```
##      [,1] [,2] [,3]
## [1,]    1    1    2
## [2,]    1   -1    3
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##      [,1] [,2]
## [1,] 3.87 0.00
## [2,] 0.00 1.41
```

```
U <- svd(A)$u
print(round(U,2))
```

```
##       [,1]  [,2]
## [1,] -0.55 -0.83
## [2,] -0.83  0.55
```

```
V <- svd(A)$v
print(round(V,2))
```

```
##       [,1]  [,2]
## [1,] -0.36 -0.20
## [2,]  0.07 -0.98
## [3,] -0.93  0.00
```

```
A_inv <- V%*%inv(S)%*%inv(U)
print(round(A_inv,2))
```

```
##      [,1] [,2]
## [1,] 0.17  0.0
## [2,] 0.57 -0.4
## [3,] 0.13  0.2
```

```
result <- A%*%A_inv
print(round(result,2))
```

```
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

Section 6.3

```
v1 <- matrix(c(5,2),nrow=2)
v2 <- matrix(c(10,0),nrow=2)
v3 <- matrix(c(0,1),nrow=2)

#project onto v2
print(t(v1)%*%v2/(t(v2)%*%v2))
```

```
##      [,1]
## [1,]  0.5
```

```
#project onto v2
print(t(v1)%*%v3/(t(v3)%*%v3))
```

```
##      [,1]
## [1,]    2
```

- How to normalize a vector

```
#how to normalize v2
nv2 <- v2 / sqrt(sum(v2^2))
print(nv2)
```

```
##      [,1]
## [1,]    1
## [2,]    0
```

```
print(Norm(nv2))
```

```
## [1] 1
```

```r
#how to normalize v2
nv3 <- v3 / sqrt(sum(v3^2))
print(nv3)
```

```
##      [,1]
## [1,]    0
## [2,]    1
```

```r
print(Norm(nv3))
```

```
## [1] 1
```

- Projecting onto normalized vector

```r
#project onto v2
print(t(v1)%*%nv2/(t(nv2)%*%nv2))
```

```
##      [,1]
## [1,]    5
```

```r
#project onto v2
print(t(v1)%*%nv3/(t(nv3)%*%nv3))
```

```
##      [,1]
## [1,]    2
```

- orthogonal matrix and a vector multiplication

```r
A <- matrix(c(1,2,3,-1,3,2), nrow = 3)
A
```

```
##      [,1] [,2]
## [1,]    1   -1
## [2,]    2    3
## [3,]    3    2
```

```r
Rank(A)
```

```
## [1] 2
```

```r
OA <- orthonormalization(A)
OA
```

```
##           [,1]       [,2]       [,3]
## [1,] 0.2672612 -0.7715167  0.5773503
## [2,] 0.5345225  0.6172134  0.5773503
## [3,] 0.8017837 -0.1543033 -0.5773503
```

```
x <- matrix(c(2,1,5),nrow = 3)
x
```

```
##      [,1]
## [1,]    2
## [2,]    1
## [3,]    5
```

```
b <- OA%*%x
b
```

```
##           [,1]
## [1,]  2.649757
## [2,]  4.573010
## [3,] -1.437487
```

```
t(OA)%*%b
```

```
##      [,1]
## [1,]    2
## [2,]    1
## [3,]    5
```

```
t(b)%*%OA
```

```
##      [,1] [,2] [,3]
## [1,]    2    1    5
```

- principal component analysis

```
A <- matrix(c(1,2,3,4,5,
              0.2,0.8,0.7,1.1,1.7),
              nrow=2, byrow = TRUE)

print(A)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  1.0  2.0  3.0  4.0  5.0
## [2,]  0.2  0.8  0.7  1.1  1.7
```

```
S <- diag(svd(A)$d)
print(round(S,2))
```

```
##      [,1] [,2]
## [1,] 7.76 0.00
## [2,] 0.00 0.36
```

```r
U <- svd(A)$u
print(round(U,2))
```

```
##       [,1]  [,2]
## [1,] -0.96 -0.29
## [2,] -0.29  0.96
```

```r
V <- svd(A)$v
print(round(V,2))
```

```
##       [,1]  [,2]
## [1,] -0.13 -0.29
## [2,] -0.28  0.50
## [3,] -0.40 -0.59
## [4,] -0.53 -0.34
## [5,] -0.68  0.45
```

```r
P <- t(A)%*%U
print(round(P,2))
```

```
##       [,1]  [,2]
## [1,] -1.01 -0.10
## [2,] -2.15  0.18
## [3,] -3.07 -0.21
## [4,] -4.15 -0.12
## [5,] -5.28  0.16
```