

Neural Network Basic Assignment 1

이름: 구자현

1. Sigmoid Function을 z 에 대해 미분하세요.

$$\sigma'(z) = \frac{d}{dz} \left(\frac{1}{1+e^{-z}} \right)$$

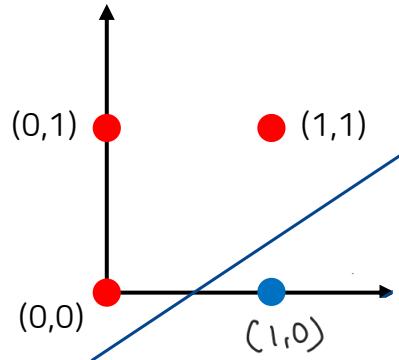
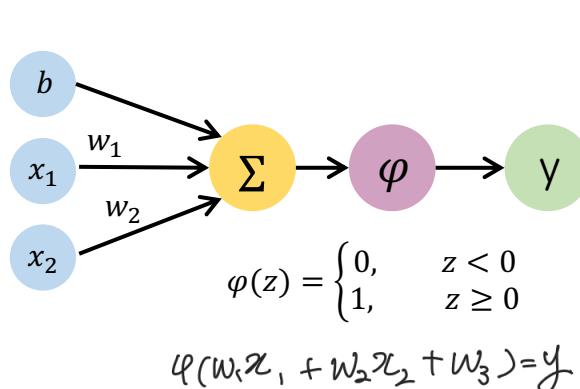
$$= \frac{+e^{-z}}{(1+e^{-z})^2}$$

$$\Rightarrow \frac{1}{(1+e^{-z})} \cdot \frac{e^{-z}}{(1+e^{-z})} = \frac{(1+e^{-z}) - 1}{(1+e^{-z})} = 1 - \sigma(z)$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\therefore \sigma'(z) = \sigma(z)(1-\sigma(z))$$

2. 다음과 같은 구조의 Perceptron과 $\bullet (=1)$, $\circ (=0)$ 을 평면좌표상에 나타낸 그림이 있습니다.



2-1. \bullet , \circ 를 분류하는 임의의 b, w 를 선정하고 분류해보세요.

$$b = -1.0 \quad w_1 = 1.5 \quad w_2 = 1.5$$

x_1	x_2	Σ	$\varphi(\Sigma)$	y
0	0	$0 \cdot (1.5) + 0 \cdot (1.5) + (-1.0) \rightarrow -1.0$	-1.0	0
0	1	$0 \cdot (1.5) + 1 \cdot (1.5) + (-1.0) \rightarrow 0.5$	0.5	1
1	0	$1 \cdot (1.5) + 0 \cdot (1.5) + (-1.0) \rightarrow 0.5$	0.5	1
1	1	$1 \cdot (1.5) + 1 \cdot (1.5) + (-1.0) \rightarrow 2.0$	2.0	1

2-2. Perceptron 학습 규칙에 따라 임의의 학습률을 정하고 b, w 를 1회 업데이트 해주세요.

$$w_i \leftarrow w_i + \eta(y - 0)x_i \quad [n=0.05] \quad (\text{학습률})$$

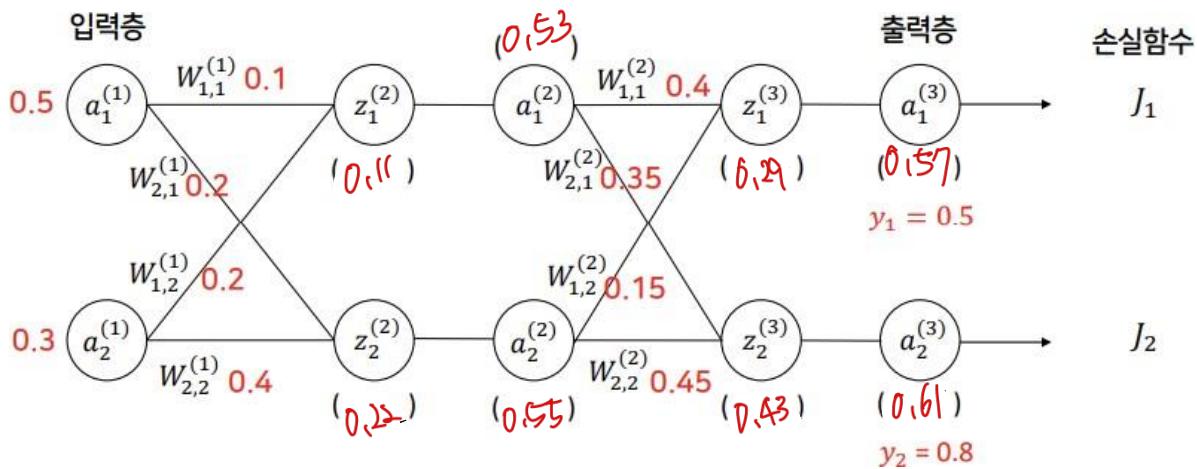
$$b \leftarrow b + 0.05(-0)x_i \Rightarrow -1.0 + 0.05 = -0.95$$

$$w_1 \leftarrow w_1 + 0.05(-0)x_1 \Rightarrow 1.5 + 0.05 = 1.55$$

$$w_2 \leftarrow w_2 + 0.05(-0)x_2 \Rightarrow 1.5 + 0.05 = 1.55$$

1회
업데이트
진행

3. 다음과 같이 입력과 가중치가 주어진 퍼셉트론이 있을 때, 아래의 물음에 답해주세요. 모든 문제는 풀이과정을 자세하게 적어주세요! (3-3까지 있습니다.)



- 3-1. FeedForward가 일어날 때, 각 노드가 갖는 값을 빈칸에 써주세요. 단, 활성화함수는 sigmoid 함수입니다. (모든 계산의 결과는 소수점 셋째자리에서 반올림하여 둘째자리까지만 써주세요.)

$$z_1^{(1)} = W_{1,1}^{(1)} \cdot a_1^{(1)} + W_{1,2}^{(1)} \cdot a_2^{(1)} \quad a_1^{(1)} = \phi(z_1^{(1)}) \Rightarrow \frac{1}{1+e^{-(0.11)}}$$

$$= (0.1) \cdot (0.5) + (0.2) \cdot (0.3) \Rightarrow \frac{1}{1+e^{-(0.11)}} \approx 0.53$$

$$= 0.05 + 0.06 = 0.11$$

$$z_2^{(1)} = W_{2,1}^{(1)} \cdot a_1^{(1)} + W_{2,2}^{(1)} \cdot a_2^{(1)} \quad a_2^{(1)} = \phi(z_2^{(1)}) \Rightarrow \frac{1}{1+e^{-(0.22)}}$$

$$= (0.2) \cdot (0.5) + (0.4) \cdot (0.3) \Rightarrow \frac{1}{1+e^{-(0.22)}} \approx 0.55$$

$$= 0.1 + 0.12 = 0.22$$

$$z_1^{(2)} = W_{1,1}^{(2)} \cdot a_1^{(1)} + W_{1,2}^{(2)} \cdot a_2^{(1)} \quad a_1^{(2)} = \phi(z_1^{(2)}) \Rightarrow \frac{1}{1+e^{-(0.29)}}$$

$$= (0.4) \cdot (0.53) + (0.15) \cdot (0.55) \Rightarrow \frac{1}{1+e^{-(0.29)}} \approx 0.57$$

$$\approx 0.29$$

$$z_2^{(2)} = W_{2,1}^{(2)} \cdot a_1^{(1)} + W_{2,2}^{(2)} \cdot a_2^{(1)} \quad a_2^{(2)} = \phi(z_2^{(2)}) \Rightarrow \frac{1}{1+e^{-(0.43)}}$$

$$= (0.35) \cdot (0.53) + (0.45) \cdot (0.55) \Rightarrow \frac{1}{1+e^{-(0.43)}} \approx 0.43$$

$$\approx 0.43$$

- 3-2. 3-1에서 구한 값을 이용하여 손실함수 J_1 과 J_2 의 값을 구해주세요. (J_1 과 J_2 는 반올림하지 말고 써주세요.)

$$MSE = \frac{1}{2N} \sum_{i=1}^N (y_i - \hat{y}_i)^2 \quad J_1 = \frac{1}{2} (a_1^{(3)} - y_1)^2 \quad a_1^{(3)} = 0.57$$

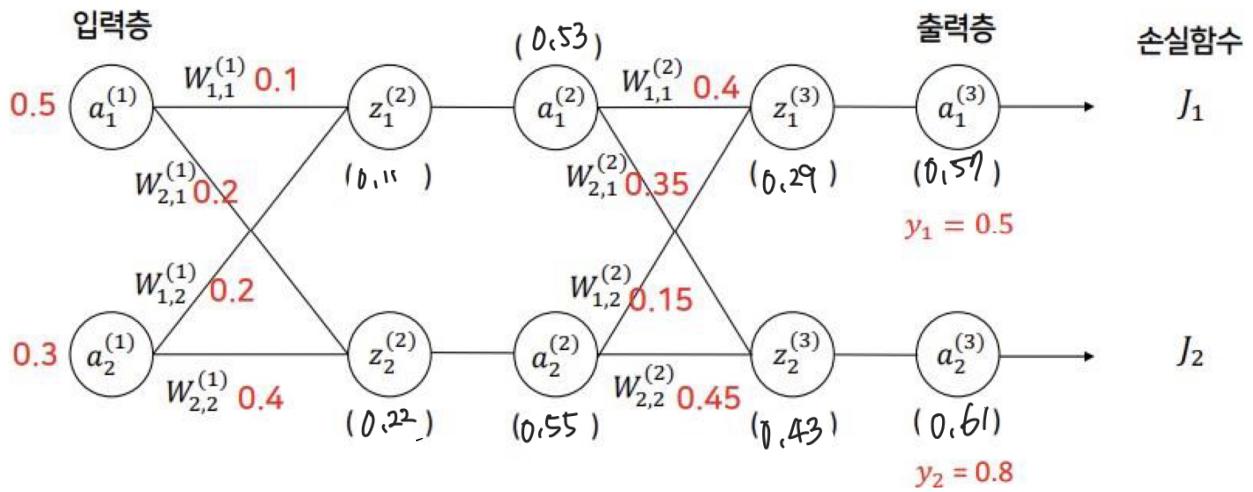
$\cancel{(\text{대입})}$

$$J_1 = \frac{1}{2} (a_1^{(3)} - y_1)^2$$

$$a_2^{(3)} = 0.61 \quad y_2 = 0.8$$

$$J_1 = \frac{1}{2} (0.57 - 0.5)^2 = 0.00245$$

$$J_2 = \frac{1}{2} (0.61 - 0.8)^2 = 0.001805$$



3-3. 위에서 구한 값을 토대로, BackPropagation이 일어날 때 $W_{2,1}^{(1)}$ 과 $W_{2,1}^{(2)}$ 의 조정된 값을 구해주세요.

단, learning rate는 0.1입니다. (계산 과정에서 소수점 넷째자리에서 반올림하여 셋째자리까지만 써주시고, 마지막 결과인 $W_{2,1}^{(1)}$ 과 $W_{2,1}^{(2)}$ 의 값만 반올림하지 말고 써주세요.)

(1) $W_{2,1}^{(1)}$

$$\begin{aligned} \frac{\partial J_{\text{total}}}{\partial W_{2,1}^{(1)}} &= \frac{\partial J_1}{\partial W_{2,1}^{(1)}} + \frac{\partial J_2}{\partial W_{2,1}^{(1)}} \\ &= \left(\frac{\partial J_1}{\partial a_1^{(3)}} \cdot \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \cdot \frac{\partial z_1^{(3)}}{\partial a_2^{(3)}} \cdot \frac{\partial a_2^{(3)}}{\partial z_2^{(3)}} \cdot \frac{\partial z_2^{(3)}}{\partial W_{2,1}^{(1)}} \right) \\ &\quad + \left(\frac{\partial J_2}{\partial a_2^{(3)}} \cdot \frac{\partial a_2^{(3)}}{\partial z_2^{(3)}} \cdot \frac{\partial z_2^{(3)}}{\partial a_1^{(3)}} \cdot \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \cdot \frac{\partial z_1^{(3)}}{\partial W_{2,1}^{(1)}} \right) \end{aligned}$$

$\therefore \frac{\partial J_{\text{total}}}{\partial W_{2,1}^{(1)}} = 0.05 \cdot 0.245 \cdot 0.55 \cdot 0.249 \cdot 0.5 + (-0.19) \cdot 0.238 \cdot 0.45 \cdot 0.241 \cdot 0.5 = -0.002$

$\therefore W_{2,1}^{(1)} = W_{2,1}^{(1)} - 0.1 \left(\frac{\partial J_{\text{total}}}{\partial W_{2,1}^{(1)}} \right) = 0.2 - 0.1(-0.002) = 0.200$

$$\begin{aligned} ① J_1 &= \frac{1}{2} (a_1^3 - y_1)^2 \\ \rightarrow \frac{\partial J_1}{\partial a_1^3} &= \frac{1}{2} \cdot \frac{\partial}{\partial a_1^3} (a_1^3 - y_1)^2 = \frac{1}{2} \cdot 2 (a_1^3 - y_1) = 0.57 - 0.5 = 0.05 \\ ② \frac{\partial a_1^3}{\partial z_1^{(3)}} &= \sigma(z_1^{(3)}) \cdot (1 - \sigma(z_1^{(3)})) = \sigma^3 \cdot (1 - \sigma^3) = 0.57 \cdot (1 - 0.57) \approx 0.245 \\ ③ \frac{\partial z_1^{(3)}}{\partial a_2^{(3)}} &\Rightarrow W_{1,2}^{(2)} = 0.15 \end{aligned}$$

$$\begin{aligned} ④ \frac{\partial a_2^3}{\partial z_2^{(3)}} &= \sigma(z_2^{(3)}) \cdot (1 - \sigma(z_2^{(3)})) = a_2^3 \cdot (1 - a_2^3) = 0.55 \cdot (1 - 0.55) \approx 0.245 \\ ⑤ \frac{\partial z_2^{(3)}}{\partial W_{2,1}^{(1)}} &= a_1^1 = 0.5 \end{aligned}$$

$$\begin{aligned} ① J_2 &= \frac{1}{2} (a_2^3 - y_2)^2 \\ \rightarrow \frac{1}{2} \frac{\partial}{\partial a_2^3} (a_2^3 - y_2)^2 &= (a_2^3 - y_2) = 0.61 - 0.8 = -0.19 \\ ② \frac{\partial a_2^3}{\partial z_2^{(3)}} &= a_2^3 \cdot (1 - a_2^3) = 0.61 \cdot (1 - 0.61) \approx 0.238 \\ ③ \frac{\partial z_2^{(3)}}{\partial a_1^{(3)}} &= W_{1,2}^{(2)} = 0.15 \\ ④ \frac{\partial a_1^3}{\partial z_1^{(3)}} &= a_1^2 \cdot (1 - a_1^2) = 0.55 \cdot (1 - 0.55) \approx 0.245 \\ ⑤ \frac{\partial z_1^{(3)}}{\partial W_{2,1}^{(1)}} &= a_1^1 = 0.5 \end{aligned}$$

$$(2) \quad W_{2,2}^{(2)}$$

$$\frac{\partial J_{\text{total}}}{\partial W_{2,2}^{(2)}} = \underbrace{\frac{\partial J_2}{\partial \alpha_2^3}}_{\textcircled{1}} \cdot \underbrace{\frac{\partial \alpha_2^3}{\partial z_2^3}}_{\textcircled{2}} \cdot \underbrace{\frac{\partial z_2^3}{\partial W_{2,2}^{(2)}}}_{\textcircled{3}}$$

$$\textcircled{1} \quad \frac{\partial J_2}{\partial \alpha_2^3} = (\alpha_2^{(3)} - y_2) = 0,6(-0,8) = -0,48$$

$$\textcircled{2} \quad \frac{\partial \alpha_2^3}{\partial z_2^3} = 0,6 / (1-0,6) \approx 0,238 \quad (\because \alpha_2^3 \cdot (1-\alpha_2^3))$$

$$\textcircled{3} \quad \frac{\partial z_2^3}{\partial W_{2,2}^{(2)}} = \alpha_1^2 = 0,55$$

$$\therefore \frac{\partial J_{\text{total}}}{\partial W_{2,2}^{(2)}} = (-0,48) \cdot (0,238) \cdot (0,55)$$

$$\approx -0,025$$

$$\therefore W_{2,2}^{(2)} = W_{2,2}^{(2)} - 0,1 \left(\frac{\partial J_{\text{total}}}{\partial W_{2,2}^{(2)}} \right)$$

$$= 0,45 - 0,1 (-0,025)$$

$$= 0,4525$$