Neural Network Basic Assignment 1

이름: 구자협

1. Sigmoid Function을 z에 대해 미분하세요.

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

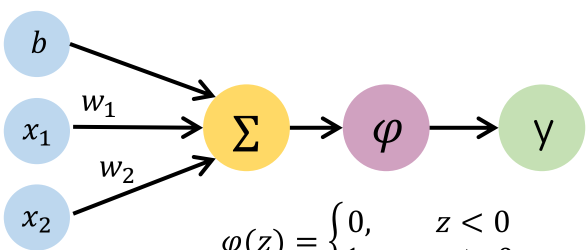
$$\sigma'(z) = \frac{d}{dz}\left(\frac{1}{1+e^{-z}}\right)$$

$$= \frac{+e^{-z}}{(1+e^{-z})^2}$$

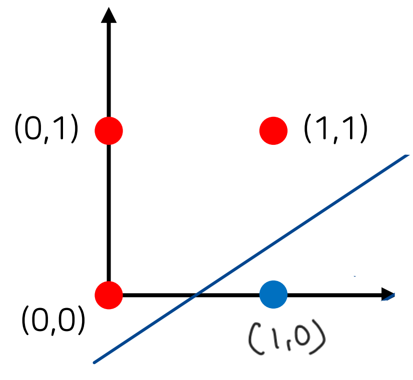
$$\Rightarrow \underbrace{\frac{1}{(1+e^{-z})}}_{\sigma(z)} \cdot \frac{e^{-z}}{(1+e^{-z})} = \frac{(1+e^{-z})-1}{(1+e^{-z})} = 1-\sigma(z)$$

$$\therefore \sigma'(z) = \sigma(z)(1-\sigma(z))$$

2. 다음과 같은 구조의 Perceptron과 ●(=1), ●(=0)을 평면좌표상에 나타낸 그림이 있습니다.



$$\varphi(z) = \begin{cases} 0, & z < 0 \\ 1, & z \geq 0 \end{cases}$$



$\varphi(w_1x_1 + w_2x_2 + w_3) = y$

2-1. ●, ● 를 분류하는 임의의 b, w를 선정하고 분류해보세요.

$b = -1.0 \quad w_1 = 1.5 \quad w_2 = 1.5$

x_1	x_2	Σ	$\varphi(z)$ →	y
0	0	$0\cdot(1.5) + 0\cdot(1.5) + (-1.0)$ →	-1.0	0
0	1	$0\cdot(1.5) + 1\cdot(1.5) + (-1.0)$ →	0.5	1
1	0	$1\cdot(1.5) + 0(1.5) + (-1.0)$ →	0.5	1
1	1	$1\cdot(1.5) + 1\cdot(1.5) + (-1.0)$ →	2.0	1

2-2. Perceptron 학습 규칙에 따라 임의의 학습률을 정하고 b, w를 1회 업데이트 해주세요.

$w_i \leftarrow w_i + \eta(y-0)x_i$ $\quad \boxed{\eta = 0.05}$ (학습률)

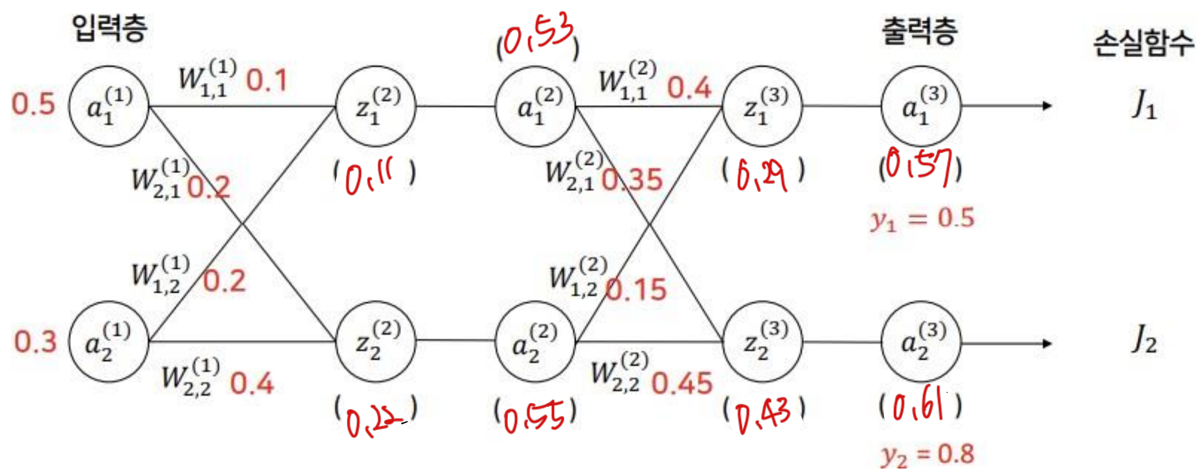
$b \leftarrow b + 0.05((1-0) \times 1 \Rightarrow -1.0 + 0.05 = -0.95$

$w_1 \leftarrow w_1 + 0.05((1-0) \times 1 \Rightarrow 1.5 + 0.05 = 1.55$

$w_2 \leftarrow w_2 + 0.05((1-0) \times 1 \Rightarrow 1.5 + 0.05 = 1.55$

1회 업데이트 진행

3. 다음과 같이 입력과 가중치가 주어진 퍼셉트론이 있을 때, 아래의 물음에 답해주세요. 모든 문제는 풀이과정을 자세하게 적어주세요! (3-3까지 있습니다.)



3-1. FeedForward가 일어날 때, 각 노드가 갖는 값을 빈칸에 써주세요. 단, 활성화함수는 sigmoid 함수입니다. (모든 계산의 결과는 소수점 셋째자리에서 반올림하여 둘째자리까지만 써주세요.)

$$\phi(z) = \frac{1}{1+e^{-z}}$$

$$z_1^{(2)} = W_{1,1}^{(1)} \cdot a_1^{(1)} + W_{1,2}^{(1)} \cdot a_2^{(1)} = (0.1)\cdot(0.5) + (0.2)\cdot(0.3) = 0.05 + 0.06 = 0.11$$

$$z_2^{(2)} = W_{2,1}^{(1)} \cdot a_1^{(1)} + W_{2,2}^{(1)} \cdot a_2^{(1)} = (0.2)\cdot(0.5) + (0.4)\cdot(0.3) = 0.1 + 0.12 = 0.22$$

$$a_1^{(2)} = \phi(z_1^{(2)}) \Rightarrow \frac{1}{1+e^{-(0.11)}} \Rightarrow \frac{1}{1+e^{-(0.11)}} \approx 0.53$$

$$a_2^{(2)} = \phi(z_2^{(2)}) \Rightarrow \frac{1}{1+e^{-(0.22)}} \Rightarrow \frac{1}{1+e^{-(0.22)}} \approx 0.55$$

$$z_1^{(3)} = W_{1,1}^{(2)} \cdot a_1^{(2)} + W_{1,2}^{(2)} \cdot a_2^{(2)} = (0.4)\cdot(0.53) + (0.15)\cdot(0.55) \approx 0.29$$

$$z_2^{(3)} = W_{2,1}^{(2)} \cdot a_1^{(2)} + W_{2,2}^{(2)} \cdot a_2^{(2)} = (0.35)\cdot(0.53) + (0.45)\cdot(0.55) \approx 0.43$$

$$a_1^{(3)} = \phi(z_1^{(3)}) \Rightarrow \frac{1}{1+e^{-(0.29)}} \Rightarrow \frac{1}{1+e^{-(0.29)}} \approx 0.57$$

$$a_2^{(3)} = \phi(z_2^{(3)}) \Rightarrow \frac{1}{1+e^{-(0.43)}} \Rightarrow \frac{1}{1+e^{-(0.43)}} \approx 0.61$$

3-2. 3-1에서 구한 값을 이용하여 손실함수 J_1과 J_2의 값을 구해주세요. (J_1과 J_2는 반올림하지 말고 써주세요.)

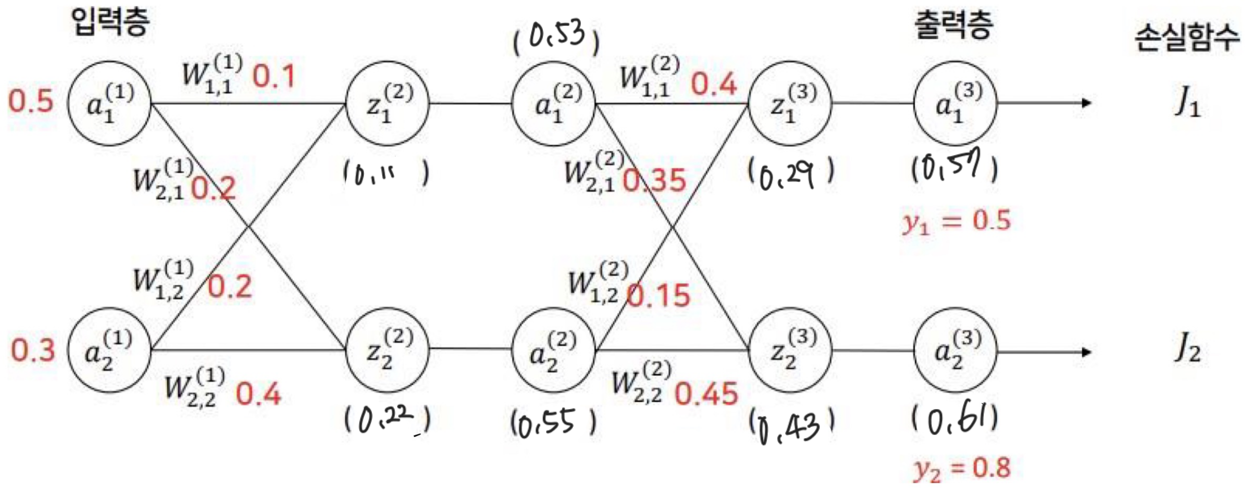
$$MSE = \frac{1}{2N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

$$J_1 = \frac{1}{2}(a_1^{(3)} - y_1)^2 \qquad J_2 = \frac{1}{2}(a_2^{(3)} - y_2)^2$$

$a_1^{(3)} = 0.57$
$y_1 = 0.5$
$a_2^{(3)} = 0.61$
$y_2 = 0.8$

(대입)

$$\begin{cases} J_1 = \frac{1}{2}(0.57 - 0.5)^2 = 0.00245 \\ J_2 = \frac{1}{2}(0.61 - 0.8)^2 = 0.01805 \end{cases}$$



3-3. 위에서 구한 값을 토대로, BackPropagation이 일어날 때 $W^{(2)}_{2,2}$과 $W^{(1)}_{2,1}$의 조정된 값을 구해주세요. 단, learning rate는 0.1입니다. (계산 과정에서 소수점 넷째자리에서 반올림하여 셋째자리까지만 써주시고, 마지막 결과인 $W^{(1)}_{2,1}$과 $W^{(2)}_{2,2}$의 값만 반올림하지 말고 써주세요.)

(1) $W^{(1)}_{2,1}$

$$\frac{\partial J_{total}}{\partial W^{(1)}_{2,1}} = \frac{\partial J_1}{\partial W^{(1)}_{2,1}} + \frac{\partial J_2}{\partial W^{(1)}_{2,1}}$$

$$= \left(\underbrace{\frac{\partial J_1}{\partial a^{(3)}_1}}_{①} \cdot \underbrace{\frac{\partial a^{(3)}_1}{\partial z^{(3)}_1}}_{②} \cdot \underbrace{\frac{\partial z^{(3)}_1}{\partial a^{(2)}_2}}_{③} \cdot \underbrace{\frac{\partial a^{(2)}_2}{\partial z^{(2)}_2}}_{④} \cdot \underbrace{\frac{\partial z^{(2)}_2}{\partial W^{(1)}_{2,1}}}_{⑤} \right)$$

$$+ \left(\underbrace{\frac{\partial J_2}{\partial a^{(3)}_2}}_{①} \cdot \underbrace{\frac{\partial a^{(3)}_2}{\partial z^{(3)}_2}}_{②} \cdot \underbrace{\frac{\partial z^{(3)}_2}{\partial a^{(2)}_2}}_{③} \cdot \underbrace{\frac{\partial a^{(2)}_2}{\partial z^{(2)}_2}}_{④} \cdot \underbrace{\frac{\partial z^{(2)}_2}{\partial W^{(1)}_{2,1}}}_{⑤} \right)$$

$$\therefore \frac{\partial J_{total}}{\partial W^{1}_{2,1}}$$

$$= 0.05 \cdot 0.245 \cdot 0.15 \cdot 0.247 \cdot 0.5$$

$$+ (-0.19) \cdot 0.238 \cdot 0.45 \cdot 0.249 \cdot 0.5$$

$$= -0.002$$

$$\boxed{\begin{aligned} \therefore W^{(1)}_{2,1} &= W^{(1)}_{2,1} - 0.1\left(\frac{\partial J_{total}}{\partial W^{(1)}_{2,1}}\right) \\ &= 0.2 - 0.1(-0.002) = 0.2002 \end{aligned}}$$

① $J_1 = \frac{1}{2}(a^3_1 - y_1)^2$

$\rightarrow \frac{\partial J_1}{\partial a^3_1} = \frac{1}{2} \cdot \frac{\partial}{\partial a^3_1}(a^3_1 - y_1)^2 = \frac{1}{2} \cdot 2(a^3_1 - y_1) = 0.57 - 0.5 = 0.05$

② $\frac{\partial a^{(3)}_1}{\partial z^{(3)}_1}$

$\rightarrow \sigma(z^3_1) \cdot (1 - \sigma(z^3_1)) = a^3_1 \cdot (1 - a^3_1) = 0.57 \cdot (1 - 0.57) \approx 0.245$

③ $\frac{\partial z^3_1}{\partial a^2_2} \Rightarrow W^2_{1,2} = 0.15$

④ $\frac{\partial a^2_2}{\partial z^2_2} = \sigma(z^2_2) \cdot (1 - \sigma(z^2_2)) = a^2_2 \cdot (1 - a^2_2) = 0.55 \cdot (1 - 0.55) \approx 0.247$

⑤ $\frac{\partial z^2_2}{\partial W^1_{2,1}} = a^1_1 = 0.5$

① $J_2 = \frac{1}{2}(a^{(3)}_2 - y_2)^2$

$\rightarrow \frac{1}{2} \frac{\partial}{\partial a^{(3)}_2}(a^{(3)}_2 - y_2)^2 = (a^{(3)}_2 - y_2) = 0.61 - 0.8 = -0.19$

② $\frac{\partial a^3_2}{\partial z^3_2} = a^3_2 \cdot (1 - a^3_2) = 0.61(1 - 0.61) \approx 0.238$

③ $\frac{\partial z^{(3)}_2}{\partial a^{(2)}_2} = W^{(2)}_{2,2} = 0.45$

④ $\frac{\partial a^{(2)}_2}{\partial z^{(2)}_2} = a^{(2)}_2 \cdot (1 - a^{(2)}_2) = 0.55(1 - 0.55)$

≈ 0.249

⑤ $\frac{\partial z^{(2)}_2}{\partial W^{(1)}_{2,1}} = a^1_1 = 0.5$

(2) $W_{2,2}^{(2)}$

$$\frac{\partial J_{total}}{\partial W_{2,2}^{(2)}} = \underbrace{\frac{\partial J_2}{\partial a_2^3}}_{①} \cdot \underbrace{\frac{\partial a_2^3}{\partial z_2^3}}_{②} \cdot \underbrace{\frac{\partial z_2^3}{\partial W_{2,2}^2}}_{③}$$

① $\dfrac{\partial J_2}{\partial a_2^3} = (a_2^{(3)} - y_2) = 0.61 - 0.8 = -0.19$

② $\dfrac{\partial a_2^3}{\partial z_2^3} = 0.61(1-0.61) \simeq 0.238 \quad (\because a_2^3 \cdot (1-a_2^3))$

③ $\dfrac{\partial z_2^3}{\partial W_{2,2}^2} = a_2^2 = 0.55$

$$\therefore \frac{\partial J_{total}}{\partial W_{2,2}^{(2)}} = (-0.19)\cdot(0.238)\cdot(0.55)$$

$$\approx -0.025$$

$$\therefore W_{2,2}^{\prime(2)} = W_{2,2}^{(2)} - 0.1\left(\frac{\partial J_{total}}{\partial W_{2,2}^{(2)}}\right)$$

$$= 0.45 - 0.1(-0.025)$$

$$= 0.4525$$