#8.(a) Ridge regression optimization problem

$\Rightarrow$ Minimizing $\sum_{i=1}^{n} (y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j)^2 + \lambda \sum_{j=1}^{p} \beta_j^2$
β

In this setting. $n=2$, $p=2$. , $\hat{\beta}_0 = 0$

$\Rightarrow$ minimizing $\sum_{i=1}^{2} (y_i - \sum_{j=1}^{2} x_{ij}\beta_j)^2 + \lambda \sum_{j=1}^{2} \beta_j^2$
β

$\Leftrightarrow$ minimizing $(y_1 - x_{11}\beta_1 - x_{12}\beta_2)^2 + (y_2 - x_{21}\beta_1 - x_{22}\beta_2)^2 + \lambda(\beta_1^2 + \beta_2^2)$ $\Big|\, Q(\beta)$
β

(b) Under $x_{11} = x_{12}$, $x_{21} = x_{22}$,
$(= x_1)$ $(= x_2)$

$Q(\beta) = (y_1 - x_1(\beta_1+\beta_2))^2 + (y_2 - x_2(\beta_1+\beta_2))^2 + \lambda(\beta_1^2+\beta_2^2)$

1) $\frac{\partial}{\partial \beta_1} Q(\beta) = 2(y_1 - x_1(\beta_1+\beta_2))\cdot(-x_1) + 2(y_2 - x_2(\beta_1+\beta_2))\cdot(-x_2) + 2\lambda\beta_1$

$\therefore \frac{\partial}{\partial \beta_1} Q(\beta)\Big|_{\beta = \hat{\beta}} = 0 \Leftrightarrow -x_1y_1 + x_1^2(\hat{\beta}_1+\hat{\beta}_2) - x_2y_2 + x_2^2(\hat{\beta}_1+\hat{\beta}_2) + \lambda\hat{\beta}_1 = 0$

$\Leftrightarrow \hat{\beta}_1 = \frac{x_1y_1 + x_2y_2 - \hat{\beta}_2(x_1^2+x_2^2)}{\lambda + x_1^2 + x_2^2}$

2) 같은 방법으로, $\hat{\beta}_2 = \frac{x_1y_1 + x_2y_2 - \hat{\beta}_1(x_1^2+x_2^2)}{\lambda + x_1^2 + x_2^2}$ // $\Rightarrow \hat{\beta}_1 = \hat{\beta}_2$

(c) Lasso optimization problem

$$\underset{\beta}{\text{Minimizing}} \quad \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j\right)^2 + \lambda \sum_{j=1}^{p} |\beta_j|$$

In this setting.

$$\underset{\beta}{\text{Minimizing}} \quad \underbrace{(y_1 - x_{11}\beta_1 - x_{12}\beta_2)^2 + (y_2 - x_{21}\beta_1 - x_{22}\beta_2)^2}_{=\,Q(\beta)} + \lambda(|\beta_1| + |\beta_2|)$$

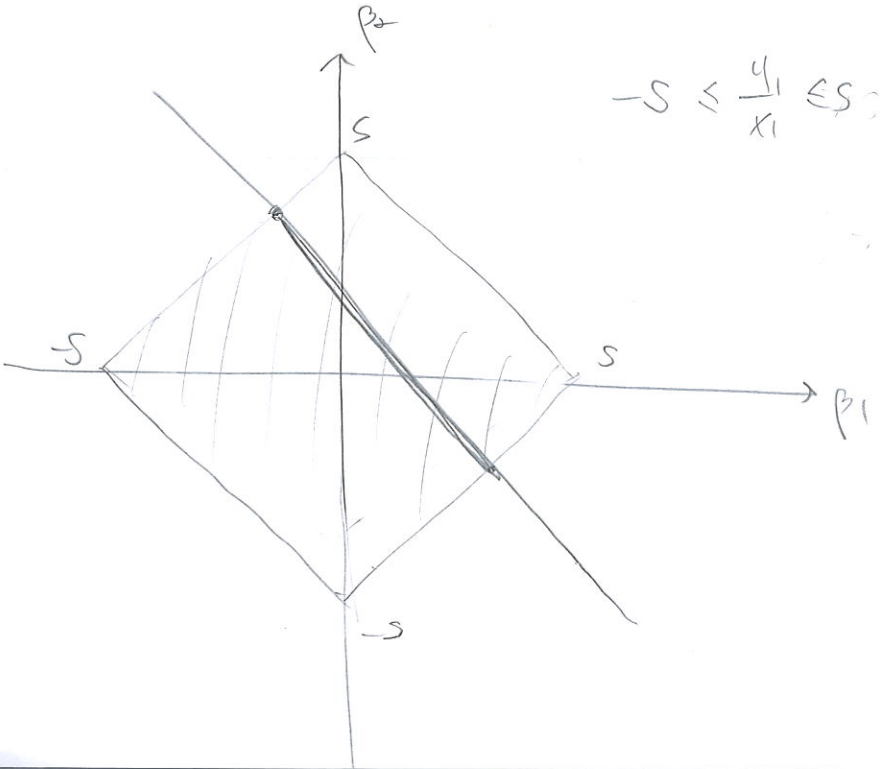
(d) Under $x_{11} = x_{12} = x_1$, $x_{21} = x_{22} = x_2$, $x_1 + x_2 = 0$, $y_1 + y_2 = 0$

$$\rightarrow Q(\beta) = (y_1 - x_1(\beta_1+\beta_2))^2 + (y_2 - x_2(\beta_1+\beta_2))^2$$

$$= (y_1 - x_1(\beta_1+\beta_2))^2 + (-y_1 + x_1(\beta_1+\beta_2))^2$$

$$= \underline{2(y_1 - x_1(\beta_1+\beta_2))^2}$$

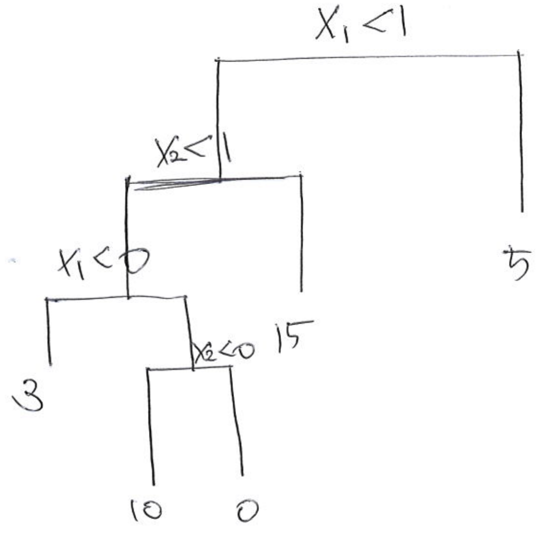
$\therefore$ $Q(\beta)$를 최소화하는 $\beta \Rightarrow \beta_1 + \beta_2 = \dfrac{y_1}{x_1}$ 이게 $|\beta_1| + |\beta_2| \le S$ 인지?



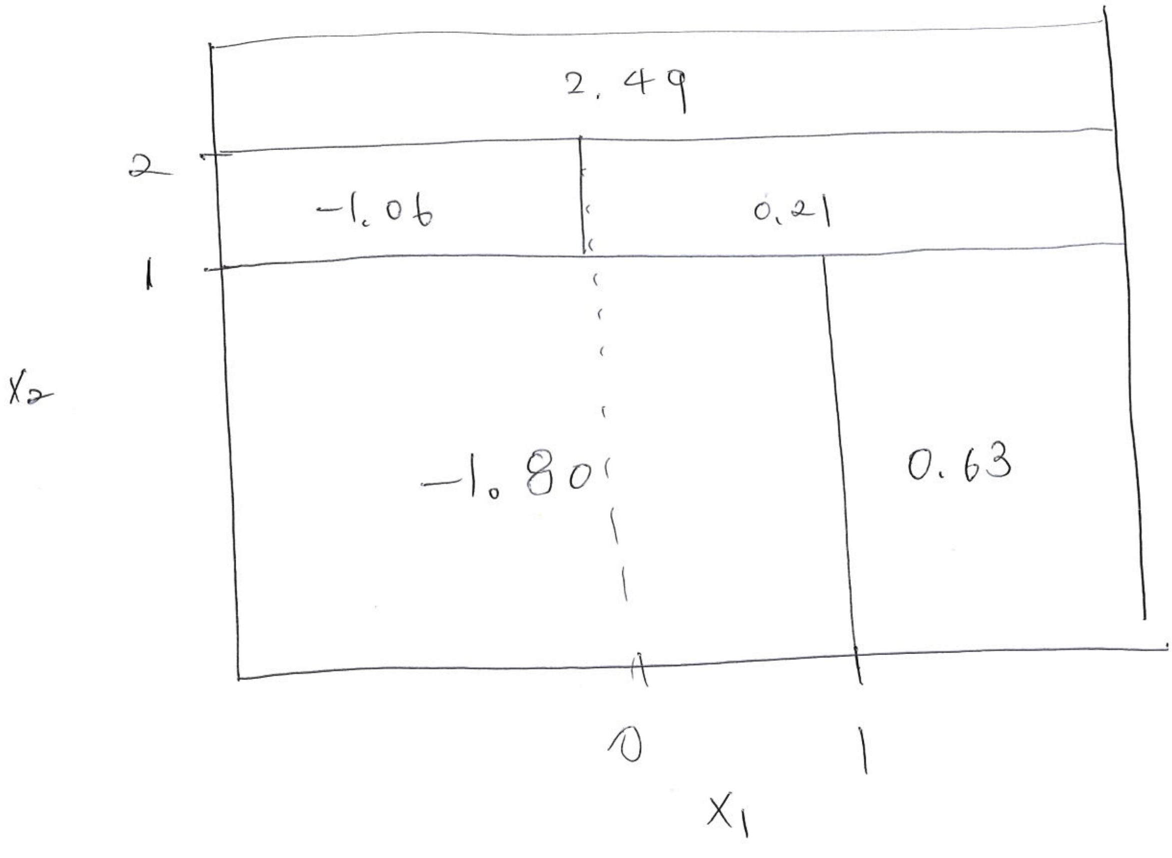
$-S \le \dfrac{y_1}{x_1} \le S \Rightarrow$ There are many sol.

No unique sol.

#9 (a)

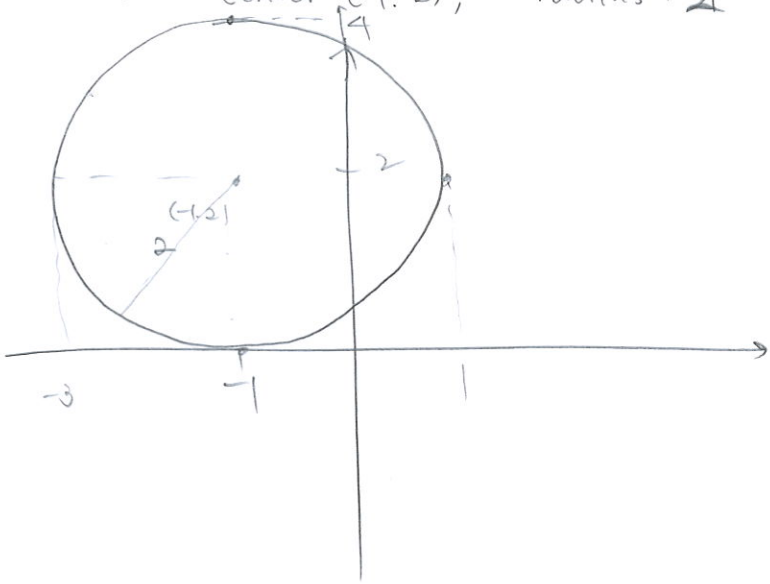


(b)



#11. (a) $(1+X_1)^2 + (2-X_2)^2 = 4$

⇒ center $(-1, 2)$, radius $= 2$



(b) $(1+X_1)^2 + (2-X_2)^2 < 4$ ⇒ 원의 내부

> 4 ⇒ 〃 외부

(c) (0,0) (2,2) (3,8) 는 원의 외부에 있다) 둘은 다른 class로.

(-1,1)은 내부에 있다.

(d) (Typo). X1, X12, X2 and X2

⇒ X_1, X_1^2, X_2 and X_2^2

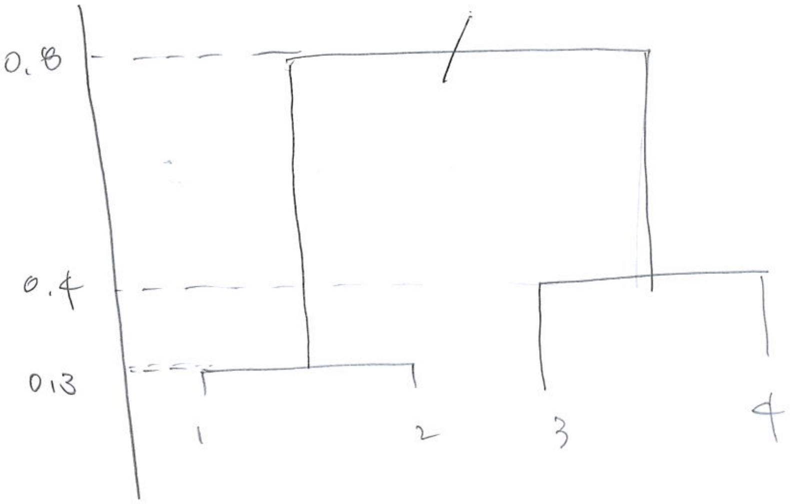
Decision boundary : $(1+X_1)^2+(2-X_2)^2=4$

$\Leftrightarrow 1+2X_1+X_1^2+4-4X_2+X_2^2=4$

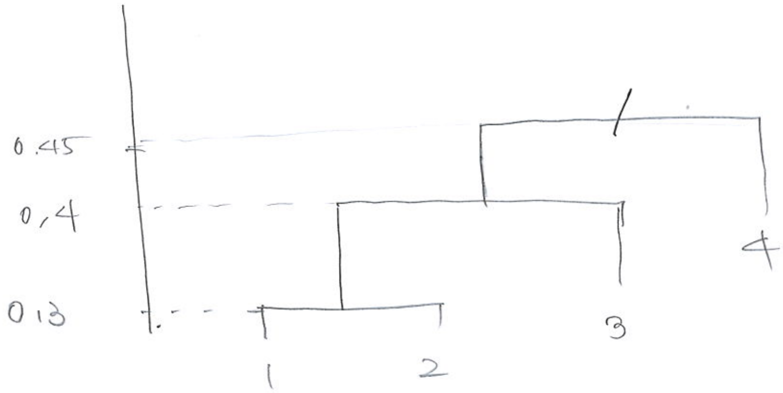
$\Leftrightarrow 2X_1+X_1^2-4X_2+X_2^2+1=0$: linear in terms of

X_1, X_1^2, X_2, X_2^2

#13 (a) complete linkage.



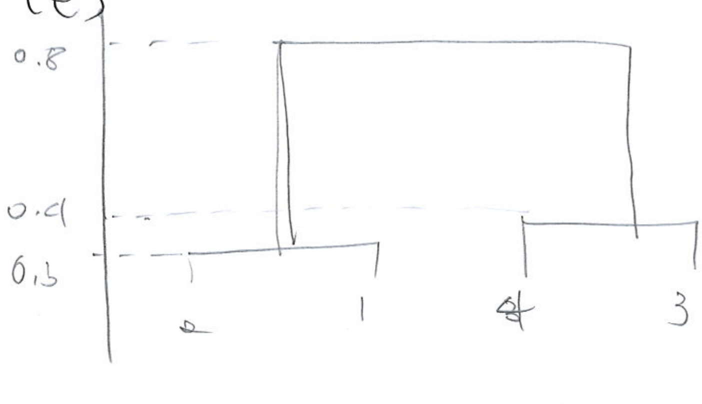
(b) Single linkage



(c) (1,2), (3.4)

(d) (1.2.3), (4)

(e)



Same dendogram in (a)