

CECS 463 System On Chip II

FALL 2020



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Assignment #04 – The Z-Transform

10/15/2020

```
% Kuldeep Gohil
% CECS 463 Fall20
% Assignment #04 Due: 10/15/2020

clc; close all;
```

4.1a

$$x(n) = (0.8)^n * u(n - 2)$$

```
clc; close all;
A = [1, -0.8];
B = [0, 0, 0.64];
[D, N2] = impseq( 0, 0, 10 );
[U, N2] = stepseq( 2, 0, 10 );
X1 = filter( B, A, D );
X2 = ( (0.8) .^N2 ).* U;
Z = max ( abs ( X1 - X2 ));
fprintf('Problem 4.1a) Error is: %4.4d', Z);
```

Problem 4.1a) Error is: 1.1102e-16

4.1b

$$x(n) = (n + 1)(3)u(n)$$

```
clc; close all;
A = [1, -9, 27, -27];
B = [1, -3];
[D, N2] = impseq( 0, 0, 7 );
X1 = filter( B, A, D );
[U, N2] = stepseq( 0, 0, 7 );
X2 = ( ( N2 + 1 ) .*( 3.^ N2 ) ).* U;
Z = max ( abs ( X1 - X2 ));
fprintf('\nProblem 4.1b) Error is: %1d', Z);
```

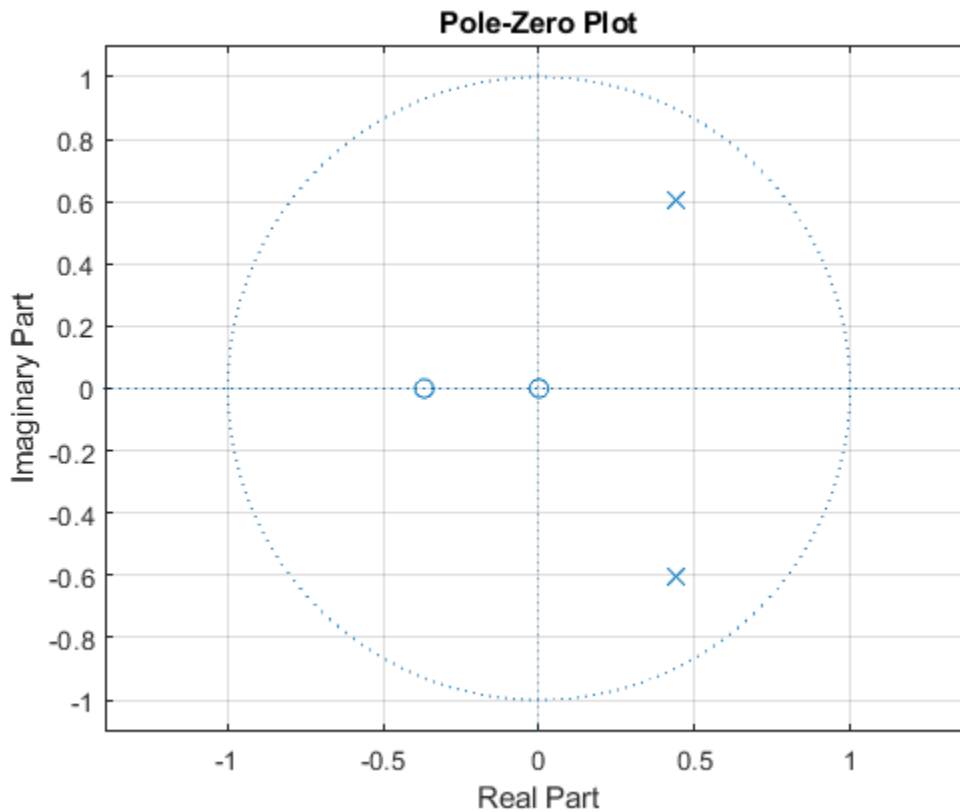
Problem 4.1b) Error is: 0

4.2a

$$x(n) = 3(0.75)^n \cos(0.3\pi n) u(n) + 4(0.75)^n \sin(0.3\pi n) u(n)$$

```
clc; close all;
A = [1, -1.5*cos(0.3*pi), 0.5625];
B = [3, (3 * sin(0.3 * pi) - 2.25 * cos(0.3*pi))];
[D, N2] = impzseq( 0, 0, 7 );
X1 = filter( B, A, D );
[U, N2] = stepzseq( 0, 0, 7 );
X2 = 3 * (((0.75).^N2) .* cos(0.3 * pi * N2)) .* U + 4 * (((0.75).^N2) .* sin(0.3 * pi *
N2)) .* U;
Z = max( abs( X1 - X2 ) );
fprintf('\nProblem 4.2a) Error is: %4.4d', Z);
figure('NumberTitle','off','Name','Problem 4.2a');
hold on; grid on;
title('Pole-Zero Plot');
[~, ~, ~] = zplane( B, A );
```

Problem 4.2a) Error is: 4.4409e-16

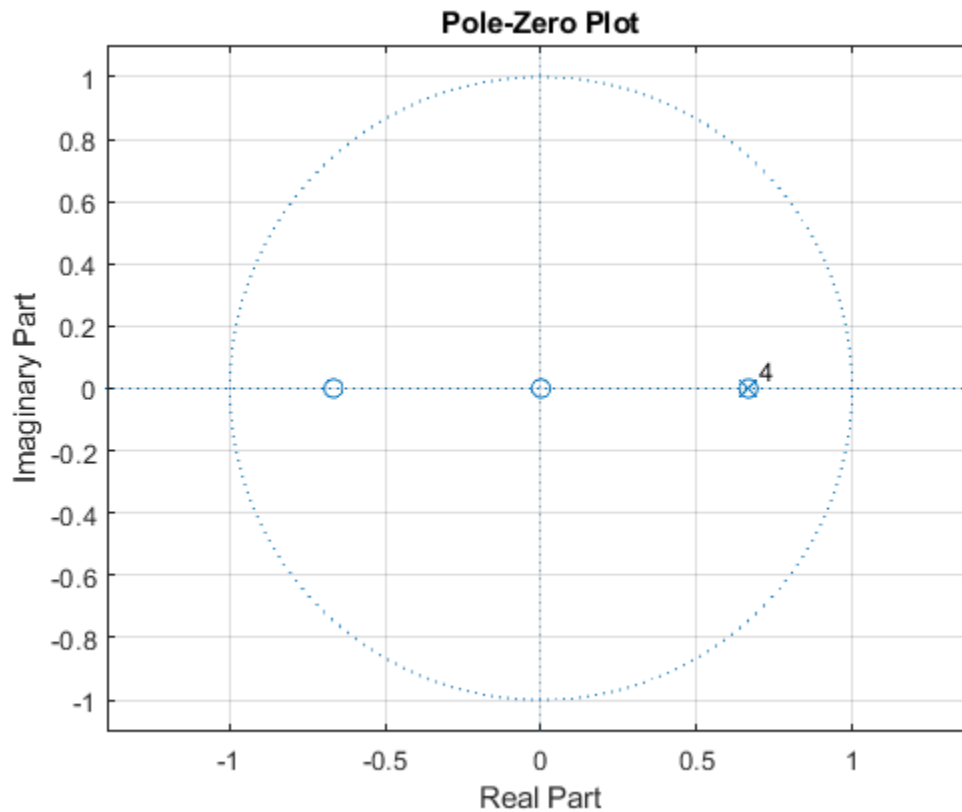


4.2b

$$x(n) = n^2 \cdot (2/3)^{n-2} \cdot u(n-1)$$

```
clc; close all;
A = [1, -8/3, 8/3, -32/27, 16/81];
B = 3/2 * [0, 1, 0, -4/9];
[D,N2] = impseq(0, 0, 8);
X1 = filter(B, A, D);
[U, N2] = stepseq(1, 0, 8);
X2 = ( (N2.^2) .* ((2/3).^( N2 - 2 ))) .* U;
Z = max( abs ( X1 - X2 ));
fprintf('\nProblem 4.2b) Error is: %4.4d\n', Z);
figure('NumberTitle','off','Name','Problem 4.2b');
hold on; grid on;
title('Pole-Zero Plot');
[~, ~, ~] = zplane( B, A );
```

Problem 4.2b) Error is: 9.7700e-15



4.3a

Z Transform of $x(n)$ is $X(z) == 1/(1+0.5z^{-1})$, $z \geq 0.5$ $x(n) = x(3 - n) + x(n - 3)$

```
clc; close all;
fprintf('\nProblem 4.3a');
fprintf('\nX(z) = Z[x(n)]');
fprintf('\n      = Z [ x(3 - n) + x(n - 3) ]');
fprintf('\n      = Z[x {-n(n - 3)}] + Z[ x(n - 3) ]');
fprintf('\n      = (z^-3 * X(1/z)) + (z^-3 * X(z))');
fprintf('\n      = z^-3[( 1/1 + 0.5z ) + 1/( 1+ 0.5z^-1 )], 0.5 < |z| < 2');
fprintf('\n      = ((0.5*z^-3) + (2*z^-4) + (0.5*z^-5)) / (0.5 + (1.25*z^-1) + (0.5z^-2)), 0.5 < |z| < 2\n');
```

Problem 4.3a)

$$\begin{aligned} X(z) &= Z[x(n)] \\ &= Z [x(3 - n) + x(n - 3)] \\ &= Z[x \{-n(n - 3)\}] + Z[x(n - 3)] \\ &= (z^{-3} * X(1/z)) + (z^{-3} * X(z)) \\ &= z^{-3}[(1/1 + 0.5z) + 1/(1+ 0.5z^{-1})], \quad 0.5 < |z| < 2 \\ &= ((0.5*z^{-3}) + (2*z^{-4}) + (0.5*z^{-5})) / (0.5 + (1.25*z^{-1}) + (0.5z^{-2})), \quad 0.5 < |z| < 2 \end{aligned}$$

4.3b

Z Transform of $x(n)$ is $X(z) == 1/(1+0.5z^{-1})$, $z \geq 0.5$ $x(n) = (1/2)^n * x(n - 2)$

```
clc; close all;
fprintf('\nProblem 4.3b');
fprintf('\nX(z) = Z[(1/2)^n * x(n - 2)]');
fprintf('\n      = Z[x(n - 2)] * |(1/2)^-1z|');
fprintf('\n      = Z [ x(n - 2) ] * 2z');
fprintf('\n      = [z^-2 * X(z)] * |2z|');
fprintf('\n      = [(z^-2) / (1 + 0.5z^-1)], |z| > 0.5 |2z|');
fprintf('\n      = ((0.25z^-2) / (1 + 0.25z^-1)), |z| > 0.25\n');
```

Problem 4.3b)

$$\begin{aligned} X(z) &= Z[(1/2)^n * x(n - 2)] \\ &= Z[x(n - 2)] * |(1/2)^{-1}z| \\ &= Z [x(n - 2)] * 2z \\ &= [z^{-2} * X(z)] * |2z| \\ &= [(z^{-2}) / (1 + 0.5z^{-1})], \quad |z| > 0.5 |2z| \\ &= ((0.25z^{-2}) / (1 + 0.25z^{-1})), \quad |z| > 0.25 \end{aligned}$$

4.4a

Z Transform of $X(z)$ is $x(n) = (1/2)^n * u(n)$ $X(z) = ((z - 1) / z) * X(z)$

```
clc; close all;
fprintf('\nProblem 4.4a ');
fprintf('\nx(n) = Z^-1[ X(z) ]');
fprintf('\n      = Z^-1[ (1 - (1/2)) * X(z) ]');
fprintf('\n      = Z^-1[ X(z) - z^-1 * X(z) ]');
fprintf('\n      = x(n) - x(n - 1)');
fprintf('\n      = 0.5^n * u(n) - 0.5^(n - 1) * u(n - 1)');
fprintf('\n      = 1 - 0.5^n * u(n-1)\n');
```

Problem 4.4a)

$$\begin{aligned}x(n) &= Z^{-1}[X(z)] \\&= Z^{-1}[(1 - (1/2)) * X(z)] \\&= Z^{-1}[X(z) - z^{-1} * X(z)] \\&= x(n) - x(n - 1) \\&= 0.5^n * u(n) - 0.5^{(n - 1)} * u(n - 1) \\&= 1 - 0.5^n * u(n-1)\end{aligned}$$

4.4b

Z Transform of $X(z)$ is $x(n) = (1/2)^n * u(n)$ $X(z) = X(z) * X(z^{-1})$

```
clc; close all;
fprintf('\nProblem 4.4b ');
fprintf('\nx(n) = Z^-1 [X(z) ]');
fprintf('\n      = Z^-1 [X(z) * X(z^-1)]');
fprintf('\n      = x(n) * x(-n)');
fprintf('\n      = [0.5^n * u(n)] * [2^n * u(-n)]');
fprintf('\n      = (0.5)^k u(k) * 2^(n - k) * u(-n + k)');
fprintf('\n      = (4/3) * 2^|n|\n');
```

Problem 4.4b)

$$\begin{aligned}x(n) &= Z^{-1}[X(z)] \\&= Z^{-1}[X(z) * X(z^{-1})] \\&= x(n) * x(-n) \\&= [0.5^n * u(n)] * [2^n * u(-n)] \\&= (0.5)^k u(k) * 2^{(n - k)} * u(-n + k) \\&= (4/3) * 2^{|n|}\end{aligned}$$

4.5a

$$X(z) = ((1 - 2z^{-1}) + (3z^{-2}) - (4z^{-3})) * ((4 + 3z^{-1}) - (2z^{-2}) - (z^{-3}))$$

```
clc; close all;
fprintf('\nProblem 4.5a\n');
N1 = [0:3];
N2 = [0:3];
Y1 = [1, -2, 3, -4];
Y2 = [4, 3, -2, 1];
[X, N] = conv_m(Y1, N1, Y2, N2)
disp('X(z) = (4 - (5z^{-1}) + (4z^{-2}) - (2z^{-3}) - (20z^{-4}) + 11 + (z^{-5}) - (4z^{-6}))');
```

Problem 4.5a)

X =

4 -5 4 -2 -20 11 -4

N =

0 1 2 3 4 5 6

$$X(z) = (4 - (5z^{-1}) + (4z^{-2}) - (2z^{-3}) - (20z^{-4}) + 11 + (z^{-5}) - (4z^{-6}))$$

4.5b

$$X(z) = (((z^{-1})(-3z^{-3})) + (2z^{-5}) + ((5z^{-7})(-z^{-9}))(z + 3z^2 + 2z^3 + 4z^4))$$

```
clc; close all;
fprintf('\nProblem 4.5b\n');
N1 = [0:9];
N2 = [-4:0];
Y1 = [0, 1, 0, -3, 0, 2, 0, 5, 0, -1];
Y2 = [4, 2, 3, 1, 0];
[X, N] = conv_m(Y1, N1, Y2, N2)
disp('X(z) = 4z^3 + 2z^2 - 9z^1 - 5 - z^-1 + z^-2 + 26z^-3 + 12z^-4 + 11z^-5 + 3z^-6 - 3z^-7 - z^-8');
```

Problem 4.5b)

X =

Columns 1 through 13

0 4 2 -9 -5 -1 1 26 12 11 3 -3 -1

Column 14

0

N =

Columns 1 through 13

-4 -3 -2 -1 0 1 2 3 4 5 6 7 8

Column 14

9

$$X(z) = 4z^3 + 2z^2 - 9z^1 - 5 - z^{-1} + z^{-2} + 26z^{-3} + 12z^{-4} + 11z^{-5} + 3z^{-6} - 3z^{-7} - z^{-8}$$