

# CECS 463 System On Chip II

## FALL 2020



***Kuldeep Gohil***

***Assignment #05 – More Z-Transforms of Chapter 4***

***10/22/2020***

```
% Kuldeep Gohil
% CECS 463 Fall20
% Assignment #05 Due: 10/22/2020

clc; close all;
```

## Problem 1: Check your function on following operation.

```
% ((z^2 + z + 1 + z^-1 + z^-2 + z^-3) / (z + 2 + z^-1)) = ...
% = (z - 1 + 2z^-1 - 2z^-2) + ((3z^-2 + 3z^-3) / (z + 2 + z^-1))

disp('Lab 05');
disp('Problem 1:');
A = [1, 2, 1];
B = [1, 1, 1, 1, 1, 1];
NA = [-1 : 1];
NB = [-2 : 3];
[P, NP, R, NR] = deconv_m(B, NB, A, NA)
disp(' = (z - 1 + 2z^-1 - 2z^-2) + ( (3z^-2 + 3z^-3)/(z + 2 + z^-1) )');
```

Lab 05

Problem 1:

P = 1      -1      2      -2

NP = -1      0      1      2

R = 0      0      0      0      3      3

NR = -2      -1      0      1      2      3

= (z - 1 + 2z^-1 - 2z^-2) + ( (3z^-2 + 3z^-3)/(z + 2 + z^-1) )

## Problem 2: Determine the following inverse z-transforms using the partial fraction expansion method.

```
% x(z) = ( z / (z^3 + 2z^2 + 1.25z + 0.25)) with ROC |z| > 1
% = ( (z^-2) / (1 + 2z^-1 + 1.25z^-2 + 0.25z^-3) )

disp('Problem 2:');
A = [1, 2, 1.25, 0.25];
B = [0, 0, 1];
[R, P, K] = residuez(B, A)
disp('Inverse z-transform: ');
disp('( 4/(1-(-0.5z^-1)) ) + ( 4z((0.5z^-1)/([1-(-0.5)z^-1]^2)) ...');
disp('... -(4 (1/(1 - (0.5^-1))) ) + (4z ((0.5z^-1)/((1 - (0.5^-1)^2)), |z| < 0.5');
```

Problem 2:

R = 4.0000 + 0.0000i

0.0000 + 0.0000i

-4.0000 + 0.0000i

P = -1.0000 + 0.0000i

-0.5000 + 0.0000i

-0.5000 - 0.0000i

K = []

Inverse z-transform:

( 4/(1-(-0.5z^-1)) ) + ( 4z((0.5z^-1)/([1-(-0.5)z^-1]^2)) ...

... -(4 (1/(1 - (0.5^-1))) ) + (4z ((0.5z^-1)/((1 - (0.5^-1)^2)), |z| < 0.5

## Problem 3: Solved on Paper

```
disp('Problem 3:');
prob3_1 = imread('Prob3.1.jpg');
prob3_2 = imread('Prob3.2.jpg');
prob3_3 = imread('Prob3.3.jpg');
prob3_4 = imread('Prob3.4.jpg');

figure('NumberTitle','off','Name','Problem 3.i');
%Prob3.i
imshow(prob3_1)
figure('NumberTitle','off','Name','Problem 3.ii');
%Prob3.ii
imshow(prob3_2)
figure('NumberTitle','off','Name','Problem 3.iii');
%Prob3.iii
imshow(prob3_3)
figure('NumberTitle','off','Name','Problem 3.iv');
%Prob3.iv
imshow(prob3_4)
```

Problem 3:

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3)  $h(n) = n\left(\frac{1}{3}\right)^n \cdot u(n) + \left(-\frac{1}{4}\right)^n \cdot u(n)$

i)  $H(z) = Z[h(n)] = Z\left[n\left(\frac{1}{3}\right)^n \cdot u(n) + \left(-\frac{1}{4}\right)^n \cdot u(n)\right]$

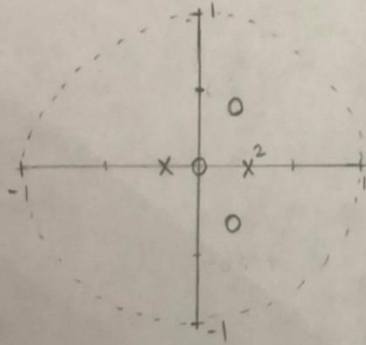
$$= \frac{\left(\frac{1}{3}\right)z^{-1}}{\left[1 - \left(\frac{1}{3}\right)z^{-1}\right]^2} + \frac{1}{1 + \left(\frac{1}{4}\right)z^{-1}}, \quad |z| > \left(\frac{1}{3}\right)$$
$$= \frac{1 - \left(\frac{1}{3}\right)z^{-1} + \left(\frac{7}{36}\right)z^{-2}}{1 - \left(\frac{5}{12}\right)z^{-1} - \left(\frac{1}{18}\right)z^{-2} + \left(\frac{1}{36}\right)z^{-3}}, \quad |z| > \left(\frac{1}{3}\right)$$

ii)  $y(n) = x(n) - \left(\frac{1}{3}\right)x(n-1) + \left(\frac{7}{36}\right)x(n-2) + \left(\frac{5}{12}\right)y(n-1) \dots$

$$\dots + \left(\frac{1}{18}\right)y(n-2) - \left(\frac{1}{36}\right)y(n-3)$$

iii)

### Pole-Zero Plot



- should look something like that

iv)  $Y(z) = H(z) X(z), |z| > 1/3$

$$H(z) = \left( \frac{1 - (1/3)z^{-1} + \frac{7}{36}z^{-2}}{1 - \frac{5}{12}z^{-1} - \frac{1}{18}z^{-2} + \frac{1}{36}z^{-3}} \right)$$

$$X(z) = \left( \frac{1}{1 - 0.25z^{-1}} \right)$$

$$Y(z) = \frac{-16}{1 - (1/3)z^{-1}} + \frac{4}{(1 - (1/3)z^{-1})^2} + \frac{(1/2)}{(1 + 1/4)z^{-1}} + \frac{25/2}{(1 - 1/4)z^{-1}}, |z| > 1/3$$

$$= \frac{-16}{1 - (1/3)z^{-1}} + 12z \left( \frac{(1/3)z^{-1}}{(1 - (1/3)z^{-1})^2} \right) + \left( \frac{(1/2)}{(1 + 1/4)z^{-1}} \right) + \left( \frac{(25/2)}{(1 - (1/4)z^{-1})} \right), |z| > 1/3$$

$$y(n) = -16(1/3)^n u(n) + 12(n+1)(1/3)^{n+1} u(n+1) + \frac{1}{2} (-1/4)^n u(n) + \frac{25}{2} (1/4)^n u(n)$$

## Problem 4: Solved on Paper

```
disp('Problem 4:');
prob4_1 = imread('Prob4.1.jpg');
prob4_2 = imread('Prob4.2.jpg');
prob4_3 = imread('Prob4.3.jpg');
prob4_4 = imread('Prob4.4.jpg');

figure('NumberTitle','off','Name','Problem 4.i');
%Prob4.i
imshow(prob4_1)
figure('NumberTitle','off','Name','Problem 4.ii');
%Prob4.ii
imshow(prob4_2)
figure('NumberTitle','off','Name','Problem 4.iii');
%Prob4.iii
imshow(prob4_3)
figure('NumberTitle','off','Name','Problem 4.iv');
%Prob4.iv
imshow(prob4_4)
```

Problem 4:

Handwritten solution for Problem 4(i):

4)  $H(z) = \frac{(z+1)}{(z-0.5)}$  casual system

i) Inverse z-transform of  $H(z)$ :

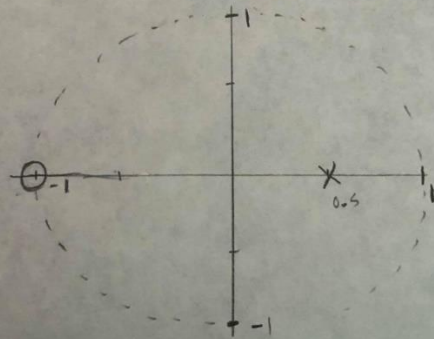
$$h(n) = Z^{-1}[H(z)] = (0.5)^n u(n) + (0.5)^{n-1} u(n-1)$$

Handwritten solution for Problem 4(ii):

ii) Difference equation representation:

$$y(n) = x(n) + x(n-1) + 0.5y(n-1)$$

iii) Pole-zero plot



iv)  $y(n)$  output if input is  $x(n) = 5 \left(\frac{1}{4}\right)^n \cdot u(n)$

$$X(z) = Z \left[ 3 \cos \left( \frac{\pi n}{3} \right) \cdot u(n) \right] = 3 \left( \frac{1 - [\cos(\pi/3)] \cdot z^{-1}}{1 - [2 \cos(\pi/3)] z^{-1} + z^{-2}} \right)$$

$$= 3 \left( \frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}} \right), |z| > 1$$

z-transform of  $y(n)$ :

$$Y(z) = H(z) X(z)$$

$$H(z) = \left( \frac{1 + z^{-1}}{1 - 0.5z^{-1}} \right)$$

$$X(z) = 3 \left( \frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}} \right)$$

$$Y(z) = 3 \left( \frac{1 + z^{-1}}{1 - z^{-1} + z^{-2}} \right) = 3 \left( \frac{1 - 0.5z^{-1} + 1.5z^{-1}}{1 - z^{-1} + z^{-2}} \right)$$

$$= 3 \left( \frac{1 - 0.5z^{-1}}{1 - z^{-1} + z^{-2}} \right) + 3\sqrt{3} \left( \frac{\left(\frac{\sqrt{3}}{2}\right)z^{-1}}{1 - z^{-1} + z^{-2}} \right), |z| > 1$$

$$\boxed{y(n) = 3 \cos \left( \frac{\pi n}{3} \right) \cdot u(n) + 3\sqrt{3} \sin \left( \frac{\pi n}{3} \right) \cdot u(n)}$$

## Problem 5: Solved on Paper

```
disp('Problem 5:');
prob5_yn = imread('Prob5.jpg');
prob5_1 = imread('Prob5.1.jpg');
prob5_2 = imread('Prob5.2.jpg');
prob5_3 = imread('Prob5.3.jpg');
prob5_4 = imread('Prob5.4.jpg');

figure('NumberTitle','off','Name','Problem 5 y(n)');
%Prob5 Solution of y(n)
imshow(prob5_yn)
figure('NumberTitle','off','Name','Problem 5.i');
%Prob5.i
imshow(prob5_1)
figure('NumberTitle','off','Name','Problem 5.ii');
%Prob5.ii
imshow(prob5_2)
figure('NumberTitle','off','Name','Problem 5.iii');
%Prob5.iii
imshow(prob5_3)
figure('NumberTitle','off','Name','Problem 5.iv');
%Prob5.iv
imshow(prob5_4)
```

Problem 5:

5) Solve difference equation for  $y(n)$ ,  $n \geq 0$

$$y(n) - 0.4y(n-1) - 0.45y(n-2) = 0.45x(n) + 0.4x(n-1) - x(n-2)$$

driven by input  $x(n) = [2 + (1/2)^n] \cdot u(n)$   
and subject to  $y(-1) = 0$ ,  $y(-2) = 3$ ;  $x(-1) = x(-2) = 2$

$$y^+(z) - 0.4y(-1) - 0.4z^{-1}y^+(z) - 0.45y(-2) - 0.45y(-1)z^{-1} - 0.45z^{-2}y^+(z) = 0.45x^+(z) + 0.4x(-1) + 0.4z^{-1}x^+(z) - x(-2) - x(-1)z^{-1} - z^{-2}x^+(z)$$

$$y^+(z) = \frac{0.45 + 0.4z^{-1} - z^{-2}}{1 - 0.4z^{-1} - 0.45z^{-2}} x^+(z)$$

$$x^+(z) = \left( \left( \frac{2}{1-z^{-1}} \right) + \left( \frac{1}{1-0.5z^{-1}} \right) \right)$$

$$y^+(z) = \left( \frac{0.45 + 0.4z^{-1} - z^{-2}}{1 - 0.4z^{-1} - 0.45z^{-2}} \right) \left( \frac{2}{1-z^{-1}} + \frac{1}{1-0.5z^{-1}} \right) + \frac{0.15 - 2z^{-1}}{1 - 0.4z^{-1} - 0.45z^{-2}}$$

$$= \frac{1.35 + 0.3z^{-1} - 3.6z^{-2} + 2z^{-3}}{(1-0.9z^{-1})(1+0.5z^{-1})(1-z^{-1})(1-0.5z^{-1})} + \frac{0.15 - 2z^{-1}}{1 - 0.4z^{-1} - 0.45z^{-2}}$$

$$= \frac{-2}{1-z^{-1}} + \frac{2.1116}{1-0.9z^{-1}} + \frac{1.7188}{1-0.5z^{-1}} - \frac{0.3304}{1+0.5z^{-1}}$$

$$y(n) = [-2 + 2.1116(0.9)^n + 1.7188(0.5)^n - 0.3304(-0.5)^n] \cdot u(n)$$



i) Transient Response:

$$y_{tr}(n) = [2.1116(0.9)^n + 1.7188(0.5)^n - 0.3303(-0.5)^n] \cdot u(n)$$

ii) Steady-State Response:

$$y_{ss}(n) = -2$$

iii) Zero-input Response:

$$Y_{zi}^+(z) = \frac{0.15 - 2z^{-1}}{1 - 0.4z^{-1} - 0.45z^{-2}} = \frac{-1.3321}{1 - 0.9z^{-1}} + \frac{1.4821}{1 + 0.5z^{-1}}$$

or

$$y_{zi}(n) = [-1.3321(0.9)^n + 1.4821(-0.5)^n] \cdot u(n)$$

iv) Zero-state Response:

$$Y_{zs}^+(z) = \frac{1.35 + 0.3z^{-1} - 3.8z^{-2} + 2z^{-3}}{(1 - 0.9z^{-1})(1 + 0.5z^{-1})(1 - z^{-1})(1 - 0.5z^{-1})}$$

$$= \frac{-2}{1 - z^{-1}} + \frac{3.4438}{1 - 0.9z^{-1}} + \frac{1.7187}{1 - 0.5z^{-1}} - \frac{1.8125}{1 + 0.5z^{-1}}$$

or

$$Y_{zs}(n) = [-2 + 3.4438(0.9)^n + 1.7187(0.5)^n - 1.8125(-0.5)^n] \cdot u(n)$$