

本节主题



乘法的运算过程

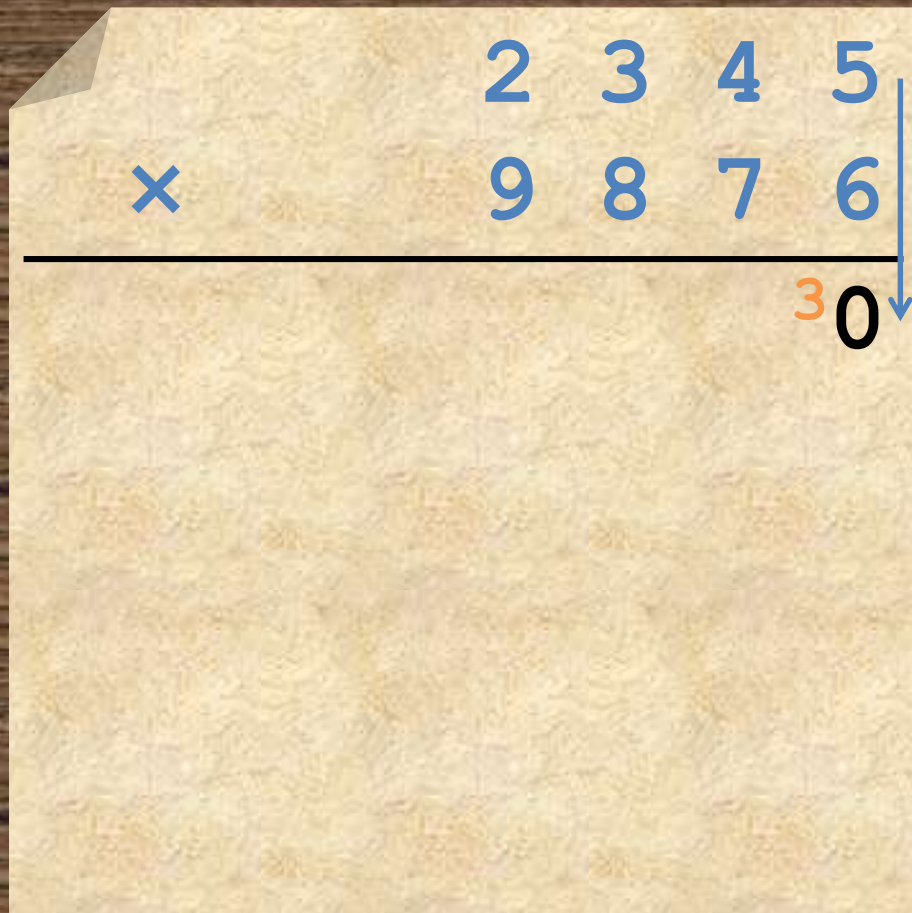
北京大学·慕课
计算机组成
制作人：陆俊林



手工进行乘法运算

$$\begin{array}{r} \times \quad \quad \quad 2 \quad 3 \quad 4 \quad 5 \\ \quad \quad \quad 9 \quad 8 \quad 7 \quad 6 \\ \hline \end{array}$$

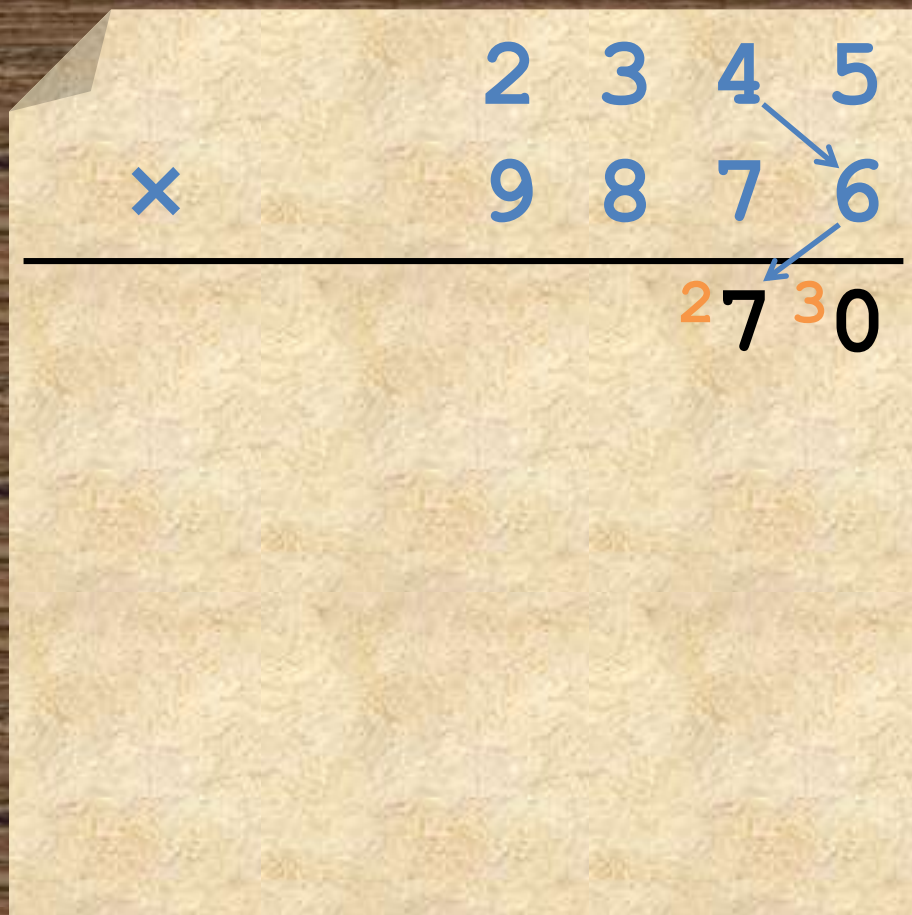
手工进行乘法运算



A piece of yellow paper with a folded top-left corner, showing a handwritten multiplication problem. The numbers are written in blue ink. The multiplier 2345 is on the top line, and the multiplicand 9876 is on the bottom line. A horizontal line is drawn below the multiplicand. To the left of the numbers is a blue 'x' symbol. A blue arrow points downwards from the rightmost digit (5) of the multiplier to the '0' in the product '30'. The '3' is written in orange ink.

$$\begin{array}{r} \times \quad 2345 \\ 9876 \\ \hline \quad \quad \quad 30 \end{array}$$

手工进行乘法运算



A piece of yellow paper with a folded top-left corner, showing a handwritten multiplication problem. The numbers are written in blue ink, except for the multiplier '6' which is black. A horizontal line separates the multiplicand from the multiplier. Below the line, the product '2730' is written, with the '2' and '3' in orange and the '7' and '0' in black. Two blue arrows point from the '4' in the multiplicand to the '6' in the multiplier, and from the '6' in the multiplier to the '7' in the product, indicating the multiplication step.

$$\begin{array}{r} \times \quad 2345 \\ 9876 \\ \hline 2730 \end{array}$$

手工进行乘法运算

A piece of yellow paper with a folded top-left corner, showing a handwritten multiplication problem. The numbers 2345 and 9876 are written in blue ink. A horizontal line separates the numbers from the product 20730, which is written in orange ink. Blue arrows indicate the multiplication steps: one from the '3' in 2345 to the '6' in 9876, and another from the '4' in 2345 to the '7' in 9876.

$$\begin{array}{r} \times \quad 2345 \\ 9876 \\ \hline 20730 \end{array}$$

手工进行乘法运算

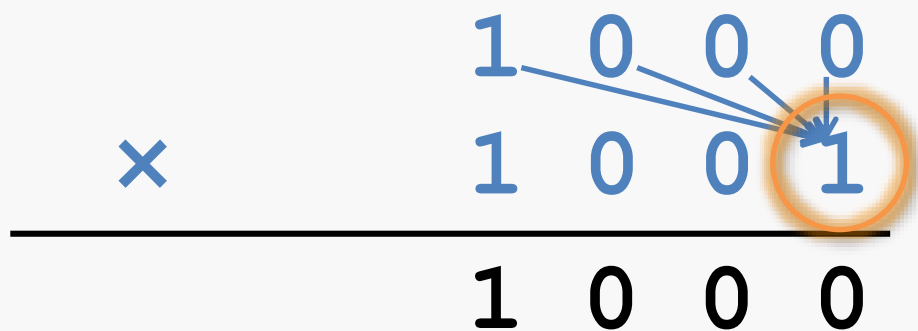
A handwritten multiplication problem on a piece of paper. The problem is 2345×9876 . The numbers are written in blue ink. A horizontal line is drawn below the multiplier. The product is written in black ink below the line. Blue arrows indicate the calculation steps: one arrow points from the '2' in the multiplicand to the '6' in the multiplier, and another arrow points from the '5' in the multiplicand to the '6' in the multiplier. The product is 1420730 .

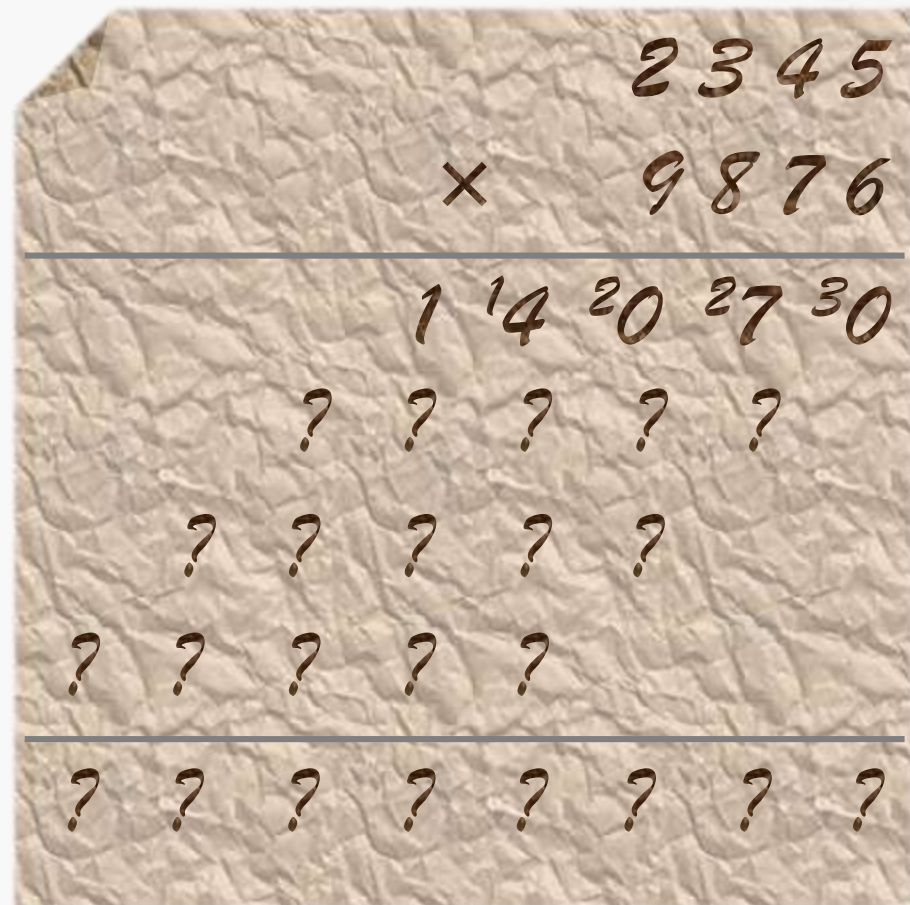
$$\begin{array}{r} 2345 \\ \times 9876 \\ \hline 1420730 \end{array}$$

手工进行乘法运算

				2	3	4	5
×				9	8	7	6
				1	4	0	7
				1	4	0	7
		?	?	?	?	?	?
		?	?	?	?	?	?
?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?

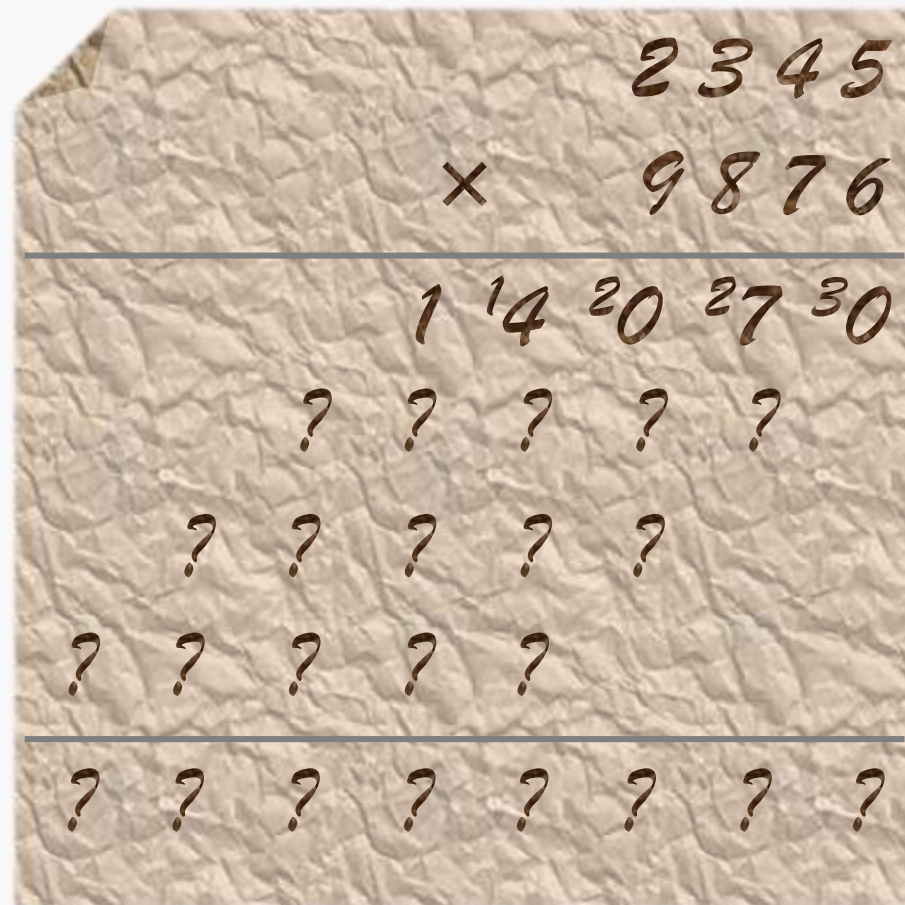
较为简单的数字

$$\begin{array}{r} \times \quad 1000 \\ 1000 \\ \hline 1000 \end{array}$$



$$\begin{array}{r} 2345 \\ \times 9876 \\ \hline 14202730 \\ ?\ ?\ ?\ ?\ ? \\ ?\ ?\ ?\ ?\ ? \\ ?\ ?\ ?\ ?\ ? \\ \hline ?\ ?\ ?\ ?\ ?\ ?\ ?\ ? \end{array}$$

较为简单的数字

$$\begin{array}{r} \times \quad \quad \quad 1 \ 0 \ 0 \ 0 \\ 1 \ 0 \ 0 \ 1 \\ \hline 1 \ 0 \ 0 \ 0 \\ 0 \ 0 \ 0 \ 0 \end{array}$$

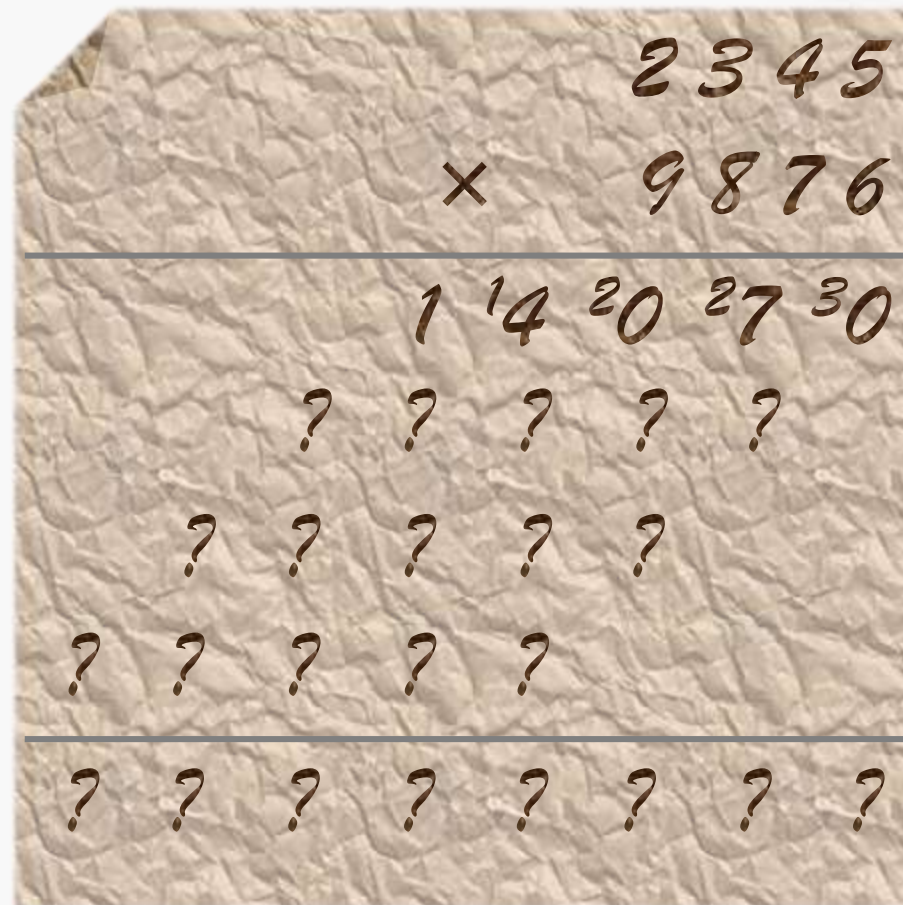


A photograph of a piece of crumpled, light-brown paper with handwritten mathematical content in dark ink. The paper is folded at the top-left corner. The content includes a multiplication problem and a grid of numbers and question marks.

$$\begin{array}{r} \times \quad \quad \quad 2 \ 3 \ 4 \ 5 \\ 9 \ 8 \ 7 \ 6 \\ \hline 1 \ 14 \ 20 \ 27 \ 30 \\ ? \ ? \ ? \ ? \ ? \\ ? \ ? \ ? \ ? \ ? \\ ? \ ? \ ? \ ? \ ? \\ ? \ ? \ ? \ ? \ ? \end{array}$$

较为简单的数字

$$\begin{array}{r} \times \\ 1000 \\ 1001 \\ \hline 1000 \\ 0000 \\ 0000 \end{array}$$

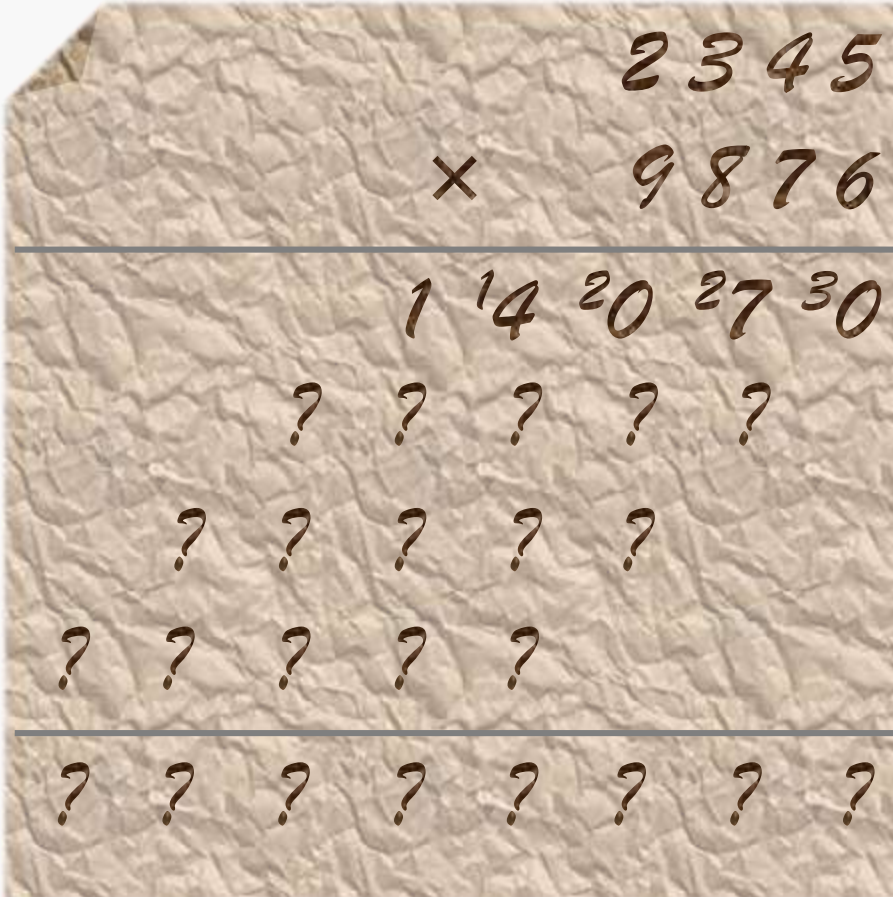


A photograph of a piece of crumpled, aged paper with handwritten numbers in brown ink. The paper is folded at the top-left corner. The handwriting is in a cursive style. The numbers are arranged in a grid-like pattern, with a horizontal line separating the top row from the rest. The top row contains the numbers 2, 3, 4, 5. Below this, there is a multiplication sign followed by the numbers 9, 8, 7, 6. Below that, there is a row of numbers: 1, 14, 20, 27, 30. Below this row, there are four rows of question marks, each with five question marks. Below the last row of question marks, there is a horizontal line, and then another row of eight question marks.

$$\begin{array}{r} 2345 \\ \times 9876 \\ \hline 114202730 \\ ? ? ? ? ? \\ ? ? ? ? ? \\ ? ? ? ? ? \\ ? ? ? ? ? \\ \hline ? ? ? ? ? ? ? ? \end{array}$$

较为简单的数字

$$\begin{array}{r} \times 1000 \\ 1000 \\ \hline 1000 \\ 0000 \\ 0000 \\ 1000 \end{array}$$

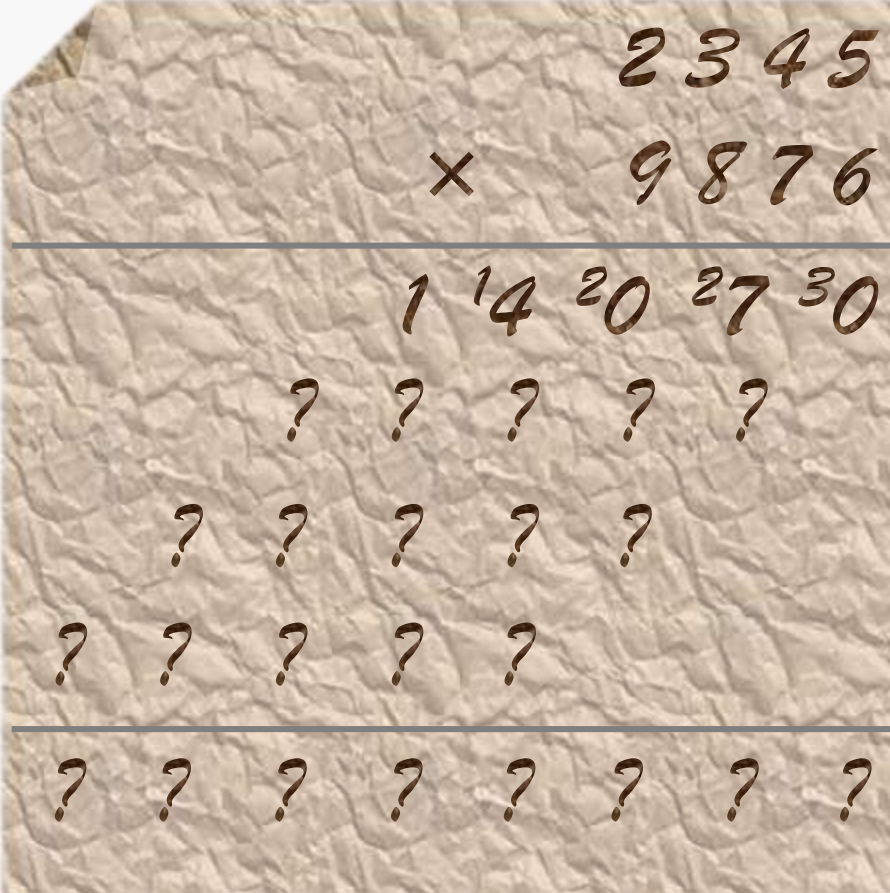


A photograph of a piece of crumpled, light-brown paper with handwritten numbers in dark ink. The paper is folded at the top-left corner. The handwriting is in a cursive style. The multiplication problem is written as follows:

$$\begin{array}{r} 2345 \\ \times 9876 \\ \hline 114202730 \\ ?\ ?\ ?\ ?\ ? \\ ?\ ?\ ?\ ?\ ? \\ ?\ ?\ ?\ ?\ ? \\ ?\ ?\ ?\ ?\ ? \end{array}$$

较为简单的数字

$$\begin{array}{r} 1000 \\ 1001 \\ \times \\ \hline 1000 \\ 0000 \\ 0000 \\ 1000 \\ \hline 1001000 \end{array}$$


$$\begin{array}{r} 2345 \\ 9876 \\ \times \\ \hline 14202730 \\ ??? \\ ??? \\ ???? \\ ???? \\ \hline ???? \end{array}$$



简化后的运算过程

				1	0	0	0	被乘数 Multiplicand
×				1	0	0	1	乘数 Multiplier
<hr/>								
				1	0	0	0	
			0	0	0	0		
		0	0	0	0			
	1	0	0	0				
<hr/>								
	1	0	0	1	0	0	0	乘积 Product



简化后的运算过程

				1	0	0	0	被乘数 Multiplicand
×				1	0	0	1	乘数 Multiplier
<hr/>								
				1	0	0	0	
			0	0	0	0	0	
		0	0	0	0			
	1	0	0	0				
<hr/>								
	1	0	0	1	0	0	0	乘积 Product

如果当前参与运算的乘数位为1，
则直接将乘数放置在对位位置上



简化后的运算过程

				1	0	0	0
				1	0	0	1
×				1	0	0	0
<hr/>							
				1	0	0	0
			0	0	0	0	0
		0	0	0	0		
	0	0	0	0			
1	0	0	0				
<hr/>							
1	0	0	1	0	0	0	

被乘数 Multiplicand

乘数 Multiplier

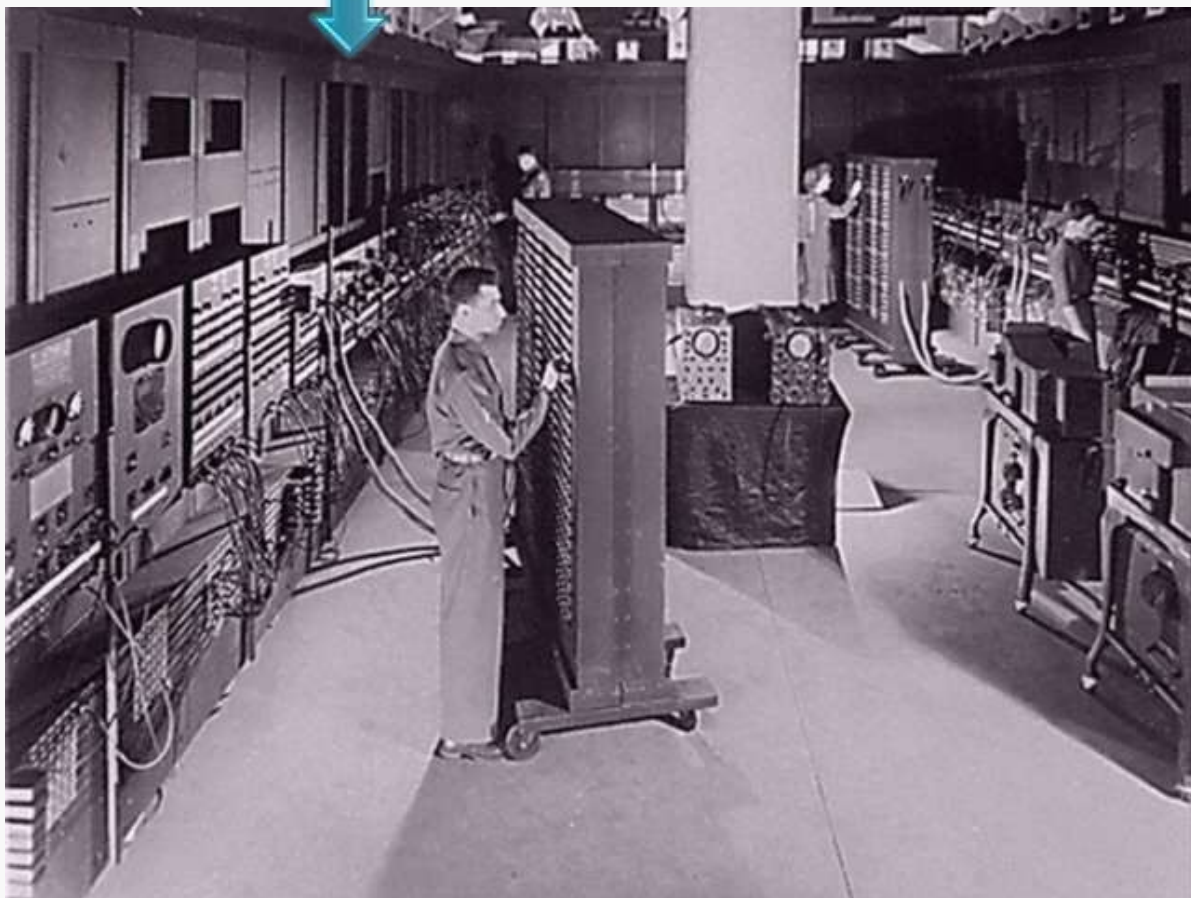
如果当前参与运算的乘数位为1，
则直接将被乘数放置在对应位置上

如果当前参与运算的乘数位为0，
则直接将“0”放置在对应位置上

乘积 Product

十进制和二进制运算的选择

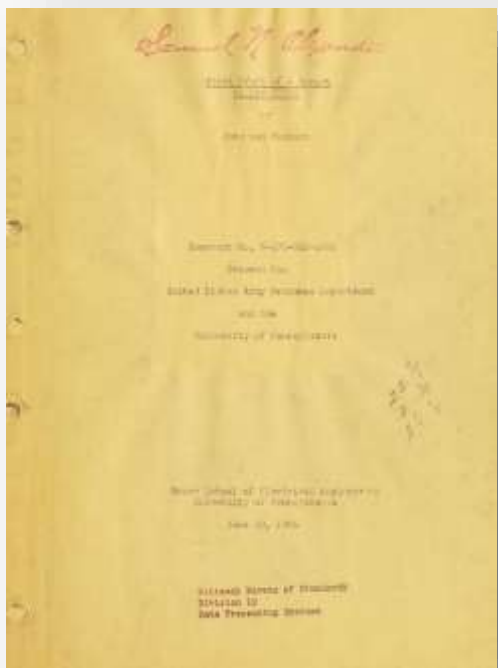
采用十进制的ENIAC



采用二进制的EDVAC



十进制和二进制运算的选择



关于EDVAC的
报告草案
1945

- ❏ 电子管是一种“全或无”设备（all-or-none），适合表示只有两个数值的系统，即二进制。
- ❏ 二进制可以大幅度地简化乘法和除法的运算过程。尤其是对于乘法，不再需要十进制乘法表，也不再需要两轮的加法。
- ❏ **必须要记住**，十进制才是适合人使用的。因此，输入输出设备需要承担二进制和十进制之间的转换工作。



约翰·冯·诺依曼
John Von Neumann
1903~1957



二进制乘法的运算过程

		1	0	0	0	被乘数 Multiplicand
×		1	0	0	1	乘数 Multiplier

				1	0	0	0
			0	0	0	0	0
		0	0	0	0	0	0
	1	0	0	0			

如何面向硬件调整运算过程？

1	0	0	1	0	0	0	乘积 Product
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运算过程的进一步调整



		1 0 0 0			
×		1 0 0 1			
		1 0 0 0			
		0 0 0 0			
		0 0 0 0			
1	0 0 0				
1	0 0 1 0 0 0				

被乘数 Multiplicand

乘数 Multiplier

乘积 Product

运算过程的进一步调整



		1	0	0	0	
×		1	0	0	1	
		1	0	0	0	
	0	0	0	0		
	0	0	0	0		
1	0	0	0			
1	0	0	1	0	0	0

被乘数 Multiplicand

乘数 Multiplier

运算开始时，乘积记为“0”

乘积 Product

运算过程的进一步调整



		1	0	0	0
×		1	0	0	1
		1	0	0	0
	0	0	0	0	
	0	0	0	0	
1	0	0	0		
0	0	0	0	0	0

被乘数 Multiplicand

乘数 Multiplier

运算开始时，乘积记为“0”

乘积 Product

运算过程的进一步调整



				1	0	0	0	
×					1	0	0	1
				<u>1</u>	0	0	0	
				0	0	0		
				0	0	0		
				1	0	0		
0	0	0	0	0	0	0	0	

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后
直接与当前的乘积累加

乘积 Product

运算过程的进一步调整



				1	0	0	0	
×					1	0	0	1
				1	0	0	0	
				0	0	0	0	
				0	0	0	0	
1	0	0	0					
0	0	0	1	0	0	0		

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后
直接与当前的乘积累加

每产生一个中间结果
被乘数向左移动一位

乘积 Product

运算过程的进一步调整

			1	0	0	0	
×			1	0	0	1	
			1	0	0	0	
		<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>		
	0	0	0	0			
1	0	0	0				
	0	0	0	1	0	0	0

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后
直接与当前的乘积累加

每产生一个中间结果
被乘数向左移动一位

乘积 Product

运算过程的进一步调整

$$\begin{array}{r} 1000 \\ \times 1001 \\ \hline 1000 \\ 0000 \\ 0000 \\ \underline{ 0000} \\ 1000 \\ \hline 0001000 \end{array}$$

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后
直接与当前的乘积累加

每产生一个中间结果
被乘数向左移动一位

乘积 Product

运算过程的进一步调整

$$\begin{array}{r} 1000 \\ \times \quad 1001 \\ \hline 1000 \\ 0000 \\ 0000 \\ 1000 \\ \hline 0001000 \end{array}$$

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后
直接与当前的乘积累加

每产生一个中间结果
被乘数向左移动一位

乘积 Product

运算过程的进一步调整

$$\begin{array}{r} 1000 \\ \times \quad 1001 \\ \hline 1000 \\ 0000 \\ 0000 \\ 1000 \\ \hline 1001000 \end{array}$$

被乘数 Multiplicand

乘数 Multiplier

每个中间结果产生后
直接与当前的乘积累加

每产生一个中间结果
被乘数向左移动一位

乘积 Product

运算过程的进一步调整



$$\begin{array}{r} 1000 \\ \times \quad 1001 \\ \hline \end{array}$$

1001000

被乘数 Multiplicand

乘数 Multiplier

适合硬件实现的运算过程！

乘积 Product

本节小结



乘法的运算过程

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制作人：陆俊林

