Shape Sensitivity Analysis for Coupled Fluid-Solid Interaction Problems

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Abstract

In this paper, a robust continuum sensitivity formulation for the shape sensitivity analysis of weakly coupled aero-structural systems is derived. In this method, the solid boundaries are modelled using the immersed boundary approach. This simplifies the grid generation for the complex and deforming geometries since the computational mesh does not need to conform to the boundaries. The sensitivity analysis consists of differentiating the continuum form of the governing equations where the effect of the solid boundaries are modelled as additional forcing terms in these equations. By differentiating the governing equations, it is possible to reuse the operators utilized for solving the governing equations. Therefore, there is no need to develop new solvers for the solution of sensitivity response. This methodology is applied to different demonstration problems including flow over a cylinder and a simplified aeroelastic model of a wing. The wing structure is modelled as a beam with the lifting surface mounted at the tip where the load is transfered to the structure through the mounting point. The sensitivity results with this approach compares well with the complex step method results. Moreover, it is shown that the methodology is capable of handing complex shapes with high Reynolds numbers.

Chapter 1

Introduction

1.1 Motivation

Fluid-structure interaction (FSI) problems play important role in many scientific and engineering fields, such as automotive, aerospace, and biomedical industry. Despite the wide application, a comprehensive study of FSI systems still remains a challenge due to their strong nonlinearity and multidisciplinary nature. For most FSI problems, analytical solutions to are not available, and physical experiments are limited in scope. Therefore, to get more insight in the physics involved in the complex interaction between fluids and solids, numerical simulations are used. The numerical solutions are conducted based on Computational Fluid Dynamics (CFD) models for the flow and Finite Element Analysis (FEA) for the structural response. Nevertheless, the prohibitive amount of computations has been one of the major issues in the design optimization of such coupled multidisciplinary systems. The other bottle neck is generating an appropriate computational domain that represents the fluid and solid regions. The effort and time required to take a geometry from a CAD package, clean up the model, and generate a mesh is usually a large portion of the overall human time required for the simulation. This cannot be automated for complex and moving domains. The Immersed Boundary (IB) method, reduces the amount of time needed for the fluid flow simulations and provides fast results by directly addressing the challenges associated with this issue.

Due to the large amount of computations involved in the FSI simulation, the gradient based methods are the best candidates for design optimization of such problems. Sensitivity analysis is the integral part of gradient based methods. Although there are analytical techniques for efficient and accurate sensitivity calculation, they have not yet implemented in commercial CFD packages. Therefore, most gradient optimization techniques relay on finite difference method for sensitivity calculation when solving FSI problem that are prone to errors.

The motivation for the research proposed in this document is in two areas.

First, we want to have sensitivity analysis capabilities that can treat the solvers as black-box. This means that we can solve both the governing equations and sensitivity response using the same code. Second, a robust analysis technique for the coupled FSI system based on IB method is formulated. The current approach of IB is not suited for the sensitivity analysis due to the discontinuities in its formulation. This will be explained in more details in the following Chapters.

1.2 Sensitivity Analysis Overview

Sensitivity analysis consists of computing the derivatives of solution of the governing equations, i.e. displacement, velocity, or pressure, with respect to one of several independent design variables, i.e. shape of boundaries or size of elements. There various applications for sensitivity information such as improving the accuracy of surrogate models as in gradient enhanced Kriging [1] or uncertainty quantification [2]. However, our main motivation is the use of this information in gradient-based optimization. The calculation of gradients is often the most expensive step in the optimization cycle therefore, using efficient methods for accurate calculation of sensitivities are vital to the optimization process. As shown in Figure 1.1, methods for sensitivity calculation can be put into three methods: i) numerical, ii) analytical, and iii) automatic differentiation.

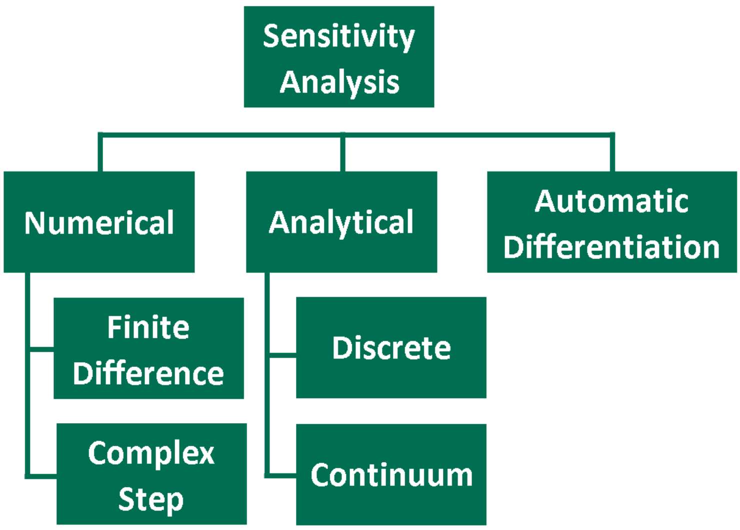

Figure 1.1: Sensitivity calculation techniques.

The Finite Difference (FD) method is probably the easiest method to implement for calculating the sensitivity of a variable. The fact that they can be implemented even when a given computational model is a black box makes most gradient based optimzation algorithms perform finite differences by default when the used does not provide the required gradients. However, the computational cost associated with finite difference for large systems can become very large. For a system with n number of design variables, the analysis needs to be done $n+1$ time to calculate the design sensitivities. Furthermore, to ensure the accuracy, convergence study needs to be done for selecting the appropriate step size for finite difference. The inaccuracy of finite differencing could result in convergence difficulties and inaccurate optimum results. On the other hand, Complex Step (CS) method avoids the loss of precision in finite differences approximation of sensitivities by employing complex arithmetic [3]. The complex step derivative is calculated as shown in Equation (1.1).

$$\mathcal{F}'(u;b) = \frac{\mathrm{Im}\left[\mathcal{F}(u;b+ih)\right]}{h} \tag{1.1}$$

This means that we perturb the design variable by an imaginary value of ih and then look at the imaginary portion of the resulting response. Using the complex step method, we can choose a small step size for h without loosing accuracy. However, many commercial packages such as ANSYS or Nastran cannot handle complex arithmetic which makes the implementation of complex step method infeasible. Moreover, the high cost of finite difference is still associated with the complex method as well.

Automatic differentiation (AD) is based on the systematic application of the differentiation to computer programs [**?**]. In the AD approach, the chain rule of differentiation is applied to every line in the program. This assumes that the computer program consists of a sequence of explicit functions that act successively on some variables. Therefore, by differentiating each of these functions and applying the chain rule, it is possible to calculate the sensitivities.

There has been many research on utilizing AD for optimization. Bischof et al. used AD for calculating the sensitivities using a CFD solver. They used ADIFOR for differentiating the source code of their CFD code (TLNS3D). They used the differentiated code for calculating the sensitivity of a transonic flow to change in the boundary conditions. Hascout et al., used AD for a sonic boom reduction under a supersonic aircraft. However, in all of the research done on AD, it is needed to have access to the source code and modify the solver extensively to able to calculate the sensitivities. This make the use of this method for general perpose codes infeasible since the source code is usually not available.

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