Problem Set 8

Question 1.

a. What is the Master Theorem? Describe its context and provide the statement.

b. Give three different recurrence relations that fall into the three different outcomes of the Master Theorem.

Question 2. *Quicksort*

a. Given the list $[18, 5, 9, 1, 0, 3, 12, 6]$ draw the procedure of the Quicksort algorithm (Levitin Ch. 5.2) on this list, using the rightmost element as a pivot.

b. From a complexity perspective, what is the most computationally expensive part of the Quicksort algorithm?

c. Given that we are using the strategy of choosing the rightmost element of a list, provide an (unsorted) list of 8 elements that yields the worst-case outcome for Quicksort.

d. For a generic list, what are the bad pivot elements and what are the best pivot elements?

e. Sketch two state-space trees (as you did for Mergesort), for one tree give the scenario where we always choose the worst pivot element and in the other the scenario where we always pick the best pivot. In the state-space tree, let the value of the node be the size of the subproblem. For each level of this tree, provide the total time for the most time consuming sub-algorithm over the states.

f. Express these two scenarios as two recurrence relations.

g. * Given what you know about the Master Theorem, how much time can be dedicated to finding a good pivot element?

Question 3. (Levitin 5.5.1)

a. For the one-dimensional version of the closest-pair problem, i.e., for the problem of finding two closest numbers among a given list of n real numbers for $n > 1$, design a pseudocode algorithm that is directly based on the divide-and-conquer technique.

b. Provide a recurrence relation for the time-complexity of your algorithm.

c. What is the time-complexity of your algorithm?

d. Is it a good algorithm for this problem?