Thermodynamik
Uebung 05

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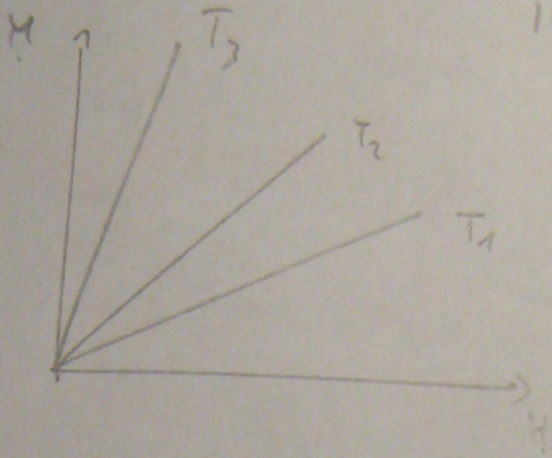
Theo 5

$\boxed{1}$



$T_1 > T_2 > T_3$

(a) $M = \frac{na}{T} H$

(b) $dA = +\underline{H}\cdot d\underline{M}$

$dU = dA + dQ \Rightarrow$

$dQ = dU - dA$

$\frac{dQ}{dT} = nb4T^3 - \underline{H}\cdot d\underline{M}$ da $d\underline{H} = 0$: $0 = d\left(\frac{1}{na} T \underline{M}\right)$

$= \frac{1}{na}\left(\underline{M}\, dT + T\, d\underline{M}\right)$

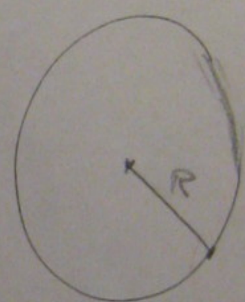
$= nb4T^3 + \frac{na}{T^2} H^2 = c_H$

$\Rightarrow d\underline{M} = -\frac{dT}{T}\underline{M} = -\frac{dT}{T^2} na\, \underline{H}$

(c) $d\underline{M} = 0 \Rightarrow dA = 0 \Rightarrow dU = dQ$

$\frac{dQ}{dT} = \frac{dU}{dT} = nb4T^3$

$\boxed{2}$ $\rho = \frac{3mN}{4\pi R^3}$ $\quad |\underline{r} - \underline{r}'| = (r^2 - 2rr'\cos\vartheta + r'^2)^{1/2}$



(a) $U^{gr} = -\frac{G}{2}\int d^3r\; 2\pi \int_0^R dr' \int_{-1}^{1} dx\; \frac{\rho^2 r'^2}{(r^2 - 2rr'x + r'^2)^{1/2}}$

$= -\frac{G}{2}\rho^2 2\pi \int d^3r \int_0^R dr' \left[r'^2 (r^2 - 2rr'x + r'^2)^{1/2} \right]_{-1}^{1} \frac{\cancel{2}}{-\cancel{2}rr'}$

$= -\frac{G}{2}\rho^2 2\pi \int d^3r \int_0^R dr' \frac{r'}{r}\left[|r + r'| - |r - r'| \right]$ $\quad [\cdot] = \begin{cases} 2r' & r' < r \\ 2r & r' > r \end{cases}$

$= -\pi G \rho^2 \int d^3r \left[\int_0^r dr' \frac{2r'^2}{r} + \int_r^R dr'\, 2r' \right]$

$= -\pi G \rho^2 \int d^3r \left[\frac{1}{r}\frac{2}{3} r^3 + R^2 - r^2 \right]$ $\quad [\cdot] = -\frac{1}{3} r^2 + R^2$

$= -\pi G \rho^2\, 4\pi \int_0^R dr \left(-\frac{1}{3} r^2 + R^2 \right) r^2 = -4\pi^2 G \rho^2 \left(-\frac{1}{15} R^5 + \overset{\frac{5}{15}}{\frac{1}{3}} R^5 \right)$

$= -\frac{16}{15} \pi^2 G \rho^2 R^5$

(b) $U = U^{id} + U^{gr}$

$$\frac{U}{N} = \frac{3}{2}kT - \frac{\cancel{16}^{\,4}}{\cancel{15}_{\,5}}\pi^2 G \frac{m^2 N}{V} \frac{3}{4\pi R^3} R^{\cancel{5}\,2}$$

$$= \frac{3}{2}kT - \frac{4}{5}\pi G m^2 \frac{N}{V} R^2 = \frac{3}{2}kT - \frac{4}{5}\pi G m^2 \frac{N}{V} \underbrace{\left(\frac{4}{3}\pi R^3\right)^{2/3}}_{V^{2/3}} \left(\frac{3}{4\pi}\right)^{2/3}$$

divergiert da $R \to \infty$.

$$= \frac{3}{2}kT - \frac{(36\pi)^{1/3}}{5} G m^2 \frac{N}{V^{1/3}}$$

Es muss $\frac{N}{R} = \text{const.}$ bleiben im Lim. da $\frac{N}{V}R^2 \propto \frac{N}{R}$.

(c) $dU = dA = -p\,dV$

$$dU = \frac{3}{2}k\,dT + \underbrace{\frac{(36\pi)^{1/3}}{5} G m^2 \frac{1}{3} \frac{N^2}{V^{4/3}}}_{=:\,p_g} dV = -p_T\,dV$$

(d) $p_g \overset{!}{=} p_T$ (Flussgleichgew.) $\qquad p_T = \frac{NkT}{V}$

$$V^{1/3} = \frac{1}{kT}\frac{(36\pi)^{1/3}}{15} G m^2 N = \left(\frac{4}{3}\pi\right)^{1/3} R$$

$$R = \frac{1}{5}\frac{G m^2}{k}\frac{N}{T}$$