(7) a) $\vec{v}(t) = \begin{pmatrix} v_0 \cos\alpha \\ v_0 \sin\alpha - gt \end{pmatrix}$

$$\vec{r}(t) = \int_0^t \vec{v}(t^*)\,dt^* + \begin{pmatrix} 0 \\ h \end{pmatrix} = \begin{pmatrix} v_0 \cos\alpha\, t \\ v_0 \sin\alpha \cdot t - \frac{1}{2} g t^2 + h \end{pmatrix}$$

b) $r_x(t) = v_0 \cos\alpha\, t \qquad r_y(t) = -\frac{1}{2} g t^2 + v_0 \sin\alpha\, t + h$

$x = v_0 \cos\alpha\, t \Rightarrow t = \frac{x}{v_0 \cos\alpha} \qquad t$ in $r_y(t)$ eingesetzt:

$r_y(x) = -\frac{1}{2} g \left(\frac{1}{v_0 \cos\alpha}\right)^2 x^2 + \tan\alpha\, x + h$

Dies ist die allgemeine Form der Parabelgleichung mit

$y = Ax^2 + Bx + C$

c) $r = 2m, \quad v_0 = 115 \frac{km}{h} \hat{=} 31 \frac{17}{18} \frac{m}{s}$

Im Kreis: $\Delta s(\varphi) = \frac{\varphi}{2\pi} \cdot 2\pi r = \varphi r$

$\dot{\varphi} = \omega = \frac{d\varphi}{dt} = 2\pi f \quad \dot{s} = \frac{ds}{d\varphi} \cdot \frac{d\varphi}{dt} = 2\pi r f = v$

$\frac{v}{r} = \omega$

$\omega = \frac{31 \frac{17}{18} \frac{m}{s}}{2m} \approx 15{,}97 \frac{1}{s}$

$\frac{115m}{3{,}6} \cdot \frac{1}{2\pi 2m} = f$

$\omega = 2\pi f = \frac{115m}{3{,}6} \cdot \frac{1}{2\pi 2m} \cdot 2\pi = 15{,}97 \frac{1}{s}$

$\omega 2\pi r = v \quad \frac{s}{t} \quad \frac{s}{h} \cdot \frac{1}{t}$

d) $\vec{r}(t_0) = \begin{pmatrix} x_0 \\ 0 \end{pmatrix} \qquad -\frac{1}{2} g t_0^2 + v_0 \sin\alpha\, t_0 + h = 0$

$t_{0_x} = \frac{-v_0 \sin\alpha \pm \sqrt{v_0^2 \sin^2\alpha + 2hg}}{-g} \qquad t_0 = \frac{v_0 \sin\alpha}{g} + \frac{\sqrt{v_0^2 \sin^2\alpha + 2hg}}{g}$

$\vec{v}(t_0) = \begin{pmatrix} v_0 \cos\alpha \\ v_0 \sin\alpha \mp \sqrt{v_0^2 \sin^2\alpha + 2hg} - v_0 \sin\alpha \end{pmatrix}$

$\vec{v}(t_0) = \begin{pmatrix} v_0 \cos\alpha \\ -\sqrt{v_0^2 \sin^2\alpha + 2hg} \end{pmatrix}$

(7d) $\tan\beta = \dfrac{-\sqrt{v_0^2 \sin^2\alpha + 2hg}}{v_0 \cos\alpha}$ $\quad |\,()^2$

$$+v_0^2\cos^2\alpha\tan^2\beta = v_0^2\sin^2\alpha + 2hg \qquad \cos^2\alpha = (1 - \sin^2\alpha)$$

$$\cancel{\frac{v_0^2\cos^2\alpha\tan^2\beta - 2hg}{v_0^2} = \sin^2\alpha}$$

$$(1-\sin^2\alpha)(v_0^2\tan^2\beta) = v_0^2\sin^2\alpha + 2hg$$

$$\cancel{v_0^2}\tan^2\beta - \sin^2\alpha\,\cancel{v_0^2}\tan^2\beta = \cancel{v_0^2}\sin^2\alpha + \frac{2hg}{v_0^2}$$

$$\cos^2\alpha\tan^2\beta = \sin^2\alpha + \frac{2hg}{v_0^2}$$

$$\underbrace{\cos^2\alpha}_{1-\sin^2\alpha}\tan^2\beta - \sin^2\alpha = \frac{2hg}{v_0^2}$$

$$\tan^2\beta - \sin^2\alpha\tan^2\beta - \sin^2\alpha = +\frac{2hg}{v_0^2}$$

$$\tan^2\beta - \sin^2\alpha(\tan^2\beta + 1) = +\frac{2hg}{v_0^2}$$

$$+\sin^2\alpha = \frac{-\frac{2hg}{v_0^2} + \tan^2\beta}{\tan^2\beta + 1}$$

$$\sin^2\alpha = \frac{-2hg + v_0^2\tan^2\beta}{v_0^2\tan^2\beta + v_0^2}$$

$$\alpha = \arcsin\sqrt{\frac{-2hg + v_0^2\tan^2\beta}{v_0^2\tan^2\beta + v_0^2}}$$

$v_0 = \frac{115}{3{,}6}\frac{m}{s}$ $\quad \beta = 44°$ $\quad h = 2m$

$\alpha = 42{,}99°$ | 42,4 | 43,38 | 42,86 | 42,86

~~33,02~~ $\quad \bar{\alpha} = 44{,}898°$; $\sigma \approx 4$

(8) c) $F_g + F_L = 0$

$-m \cdot g = -\gamma v^2$

$\frac{mg}{\gamma} = v^2 \qquad \sqrt{\frac{mg}{\gamma}} = v \approx 72,53 \frac{m}{s} \hat{=} 260,40 \frac{km}{h}$

a) $F_g + F_L = F_R \qquad F_R = m \cdot a$

$v = a \cdot t$

$\frac{v}{t} = a \qquad a = \dot{v}$

$m g - \gamma v^2 = m \cdot a = m \cdot \dot{v}$

$-\gamma v^2 = ma - mg$

$v^2 = \frac{m(a-g)}{-\gamma}$

$v = \sqrt{\frac{m(a-g)}{-\gamma}}$

$v = \sqrt{\frac{m(\dot{v}-g)}{-\gamma}}$

b)

$v(t) = \sqrt{\frac{mg}{\gamma}} \tanh\left(\sqrt{\frac{g\gamma}{m}}\, t + \sqrt{\frac{g\gamma v_0^2}{mg}}\right) \qquad \sqrt{\frac{g\gamma}{m}}$

$v(t) = b \tanh(at + c)$

$v(t) = \frac{be^{at+c} - be^{-at-c}}{e^{at+c} + e^{-at-c}}$

$\dot{v}(t) = b\,(1 - \tanh^2(at+c)) \cdot a$

$ab - ab \underbrace{\tanh^2(at+c)}_{v(t)}$

$a \cdot b = \sqrt{\frac{mg^2\gamma}{\gamma m}} = g$

$v^2(t) = b^2 \tanh^2(\underbrace{at+c}_{x})$

Einsetzen:

$mg - \cancel{\gamma} \frac{mg}{\cancel{\gamma}} \tanh^2(x) = m \cdot g - mg \tanh^2(x)$