Übung Integrale – $\boxed{\text{Lösung}}$

1)

a) $F(x) = x^3 - 2x^2 + 2x + c$

b) $F(x) = -x^{-2} + x^3 + c$

c) $F(x) = -\cos(x) + c$

d) $F(x) = \frac{2}{3}x^3 - \sin(x) + c$

e) $F(x) = \frac{1}{5\cdot 3}(3x)^5 - \frac{2}{3}x^3 + \sin(a)\cdot x + c$

2)

a) $F(x) = -\cos(x)$ $\quad I_1 = [\pi; 2\pi[\quad A = \left|\left[-\cos(x)\right]_{\pi}^{2\pi}\right| = 2$

$I_2 = [2\pi; 3\pi] \quad A = \left|\left[-\cos(x)\right]_{2\pi}^{3\pi}\right| = 2$

$A_{ges} = 2$

b) $F(x) = \frac{1}{4}x^4 \quad A = \left[\frac{1}{4}x^4\right]_1^{10} = \frac{1}{4}10^4 - \frac{1}{4}10 =$

$\frac{1}{4}\cdot(10^4 - 10) = 2499{,}75$

c) $F(x) = \frac{1}{4}x^4 - 4x \quad A_1 = \left|\left[\frac{1}{4}x^4 - 4x\right]_{-2}^{\sqrt[3]{4}}\right| \approx 16{,}76$

$f(x_i) = 0 \quad x = \sqrt[3]{4} \approx 1{,}587 \quad A_2 = \left|\left[\frac{1}{4}x^4 - 4x\right]_{\sqrt[3]{4}}^{2}\right| \approx 0{,}76$

$A_{ges} \approx 17{,}52$

$\int_{-2}^{2} f(x)\,dx = \left[\frac{1}{4}x^4 - 4x\right]_{-2}^{2} = -16$

$\Rightarrow$ Das reine Integral ist negativ, der Flächeninhalt positiv

Das reine Integral ist um 1,52 (9,5%) kleiner

3) $A = \int_0^{2\pi} |\sin(x) - 3\cos(x)|\,dx \approx 12{,}65$

Schnittpunkte: $x_1 \approx 1{,}25 \quad x_2 \approx 4{,}39$

$0 \le x \le 1{,}25$: $A_1 = \int_0^{1{,}25} (g(x) - f(x))\,dx \approx 2{,}16$

$1{,}25 < x < 4{,}39$: $A_2 = \int_{1{,}25}^{4{,}39} (f(x) - g(x))\,dx \approx 6{,}32$

$4{,}39 \le x \le 2\pi$: $A_3 = \int_{4{,}35} (g(x) - f(x))\,dx \approx 4{,}16$

4) $F(x) = \frac{3}{3}x^3 - 2x^2 + x \quad \int_4^6 f(x)\,dx = \left[\frac{3}{3}x^3 - 2x^2 + x\right]_4^6 = 63\frac{1}{3} \quad m = \frac{63\frac{1}{3}}{6-4} = 31\frac{2}{3}$

5) $f(x) = 2\cdot x^{-2} \quad (f(x))^2 = 4x^{-4} = g(x) \quad G(x) = -\frac{4}{3}x^{-3} \quad V = \pi\int_2^4 (f(x))^2\,dx = \pi\cdot\left[-\frac{4}{3}x^{-3}\right]_2^4 \approx 0{,}146\pi$

Zusatz: $V = \pi\cdot\left[-\frac{4}{3}x^{-3}\right]_{x=2}^{x\to\infty} = \pi \lim_{x\to\infty}\left(-\frac{4}{3}x^{-3}\right) - \left(-\frac{4}{3}2^{-3}\right) = \pi\cdot\left(0 + \frac{1}{6}\right) = \frac{1}{6}\pi$