Experimentalphysik IV
Uebung 06

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[23] anomaler Zeeman-Effekt:

(a) $\Delta E = C \cdot g_j \, m_j$ $\quad g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$

Konst., $\propto B$

3^3S_1: $l = 0$ $j = 1$ $s = 1$ $\quad g_j = 1 + \frac{2 + 2 - 0}{2 \cdot 2} = 2$

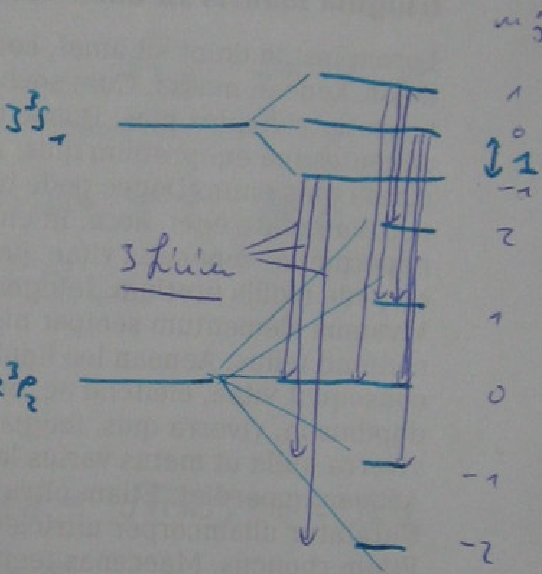
$m_j \in \{-1, 0, 1\}$

$\Delta E \propto -2, 0, 2$

2^3P_2: $l = 1$, $j = 2$, $s = 1$ $\quad g_j = 1 + \frac{6 + 2 - 2}{2 \cdot 6} = 1 + \frac{1}{2} = \frac{3}{2}$

$m_j \in \{-2, -1, 0, 1, 2\}$

$\Delta E \propto -3, -\frac{3}{2}, 0, \frac{3}{2}, 3$



Auswahlregel: $\Delta l = \pm 1$

$\Delta m = \pm 1, 0$

(b) 3 Linien → rechts

(b) äquidistant ⇈

$\boxed{24}$

(a) $E = 0$ falls

- $\mu_0 = 0$
- $\cancel{\mu_1 \parallel \mu_2}$ $\mu_1 \perp \mu_2$, $\mu_1 \perp r$, $\mu_2 \perp r$

$\mu_1 \mu_2 = \mu_1 \mu_2 \cos\varphi$, $\mu_i \, r = \mu_i \, r \cos\vartheta_i$

- $\cos\varphi - 3\cos\vartheta_1 \cos\vartheta_2 = 0$

(b) $\mu_1 \propto \mu_2$ $\quad E \propto \frac{\mu_1\mu_2}{r^3} - 3 \frac{\mu_1 \mu_2 r^2}{r^5}\cos^2\vartheta$

$\propto 1 - 3\cos^2\vartheta$

$E' = 3 \cdot 2 \cdot \cos\vartheta \sin\vartheta = 0$ für $\cos\vartheta = 0$ $\vartheta = \frac{\pi}{2}, \frac{3\pi}{2}$; $\sin\vartheta = 0$ $\vartheta = 0, \pi$

$\Rightarrow \vartheta = k \cdot \frac{\pi}{2}$ $k \in \mathbb{Z}$

$\uparrow\uparrow$ $\quad \cancel{\downarrow\uparrow}$ $\quad \uparrow\rightarrow$ $\quad \uparrow\downarrow$ $\quad \uparrow\leftarrow$

$r \; \mu$

(c) Elektron: s-Quantenzahl: $\pm \frac{1}{2}$ $\Rightarrow \|S\| = \sqrt{s(s+1)}\,\hbar = \frac{\sqrt{3}}{2}\hbar$

$\mu^{e} = 2{,}0023 \frac{\mu_B}{\hbar} \cdot S$ $\qquad \|\mu\| = \mu^e = \underbrace{2{,}0023}_{g} \cdot \frac{\mu_B}{\cancel{\hbar}} \cdot \frac{\sqrt{3}}{2}\cancel{\hbar}$ $\qquad \mu^p = -\mu^{el} \cdot \frac{m_e}{m_p}$

Sei $\vartheta = \frac{\pi}{2}$ $\qquad \|\mu^p\| = -\frac{m_e}{m_p}\mu^e$

$E^{el\,el} = \frac{\mu_0}{4\pi}\left(\frac{\mu^{el}\mu^{el}}{r^3} \underset{\uparrow}{\cdots}\right) = 8{,}\ldots \cdot 10^{-29}\,J$ $\quad$ 3,2

fällt weg da $\mu \perp r$ $\qquad \mu_B = \frac{e}{2m_e}\hbar$

$E^{el} = \frac{\mu_0}{4\pi} \frac{\cancel{g}^2 \cdot e^2 \; 3 \; \hbar^2}{\cancel{4} m_e^2 \; 4 \; r^3}$

$E^{eP_r} = 1{,}76 \cdot 10^{-27}\,J$

$E^{PrPr} = 9{,}57 \cdot 10^{-31}\,J$

$B = \frac{\mu_0}{4\pi}\left(\frac{\mu_2}{r^3} - 3\frac{r(\mu_2 r)}{r^5}\right)$ $\quad$ Multipolentwicklung

26 $\underline{\underline{I}} = \frac{2}{5} m R^2 \mathbb{1}$ $\quad \underline{L} = \underline{\underline{I}}\omega \quad , \quad \underline{M} = \dot{\underline{L}} = \underline{\underline{I}}\dot{\omega}$

(a) $\sqrt{\frac{3}{4}}\hbar = \frac{2}{5} m R^2 \omega \overset{!}{=}$; $v = \dfrac{\sqrt{\frac{3}{4}}\hbar}{\frac{2}{5} m r} = \dfrac{\sqrt{3}\cdot 5 \cdot \hbar}{4 \; m r}$. $\qquad \omega = \frac{d\phi}{dt} = \frac{ds}{dt}\frac{1}{r} = \frac{v}{r}$

$v(r = 1{,}4 \cdot 10^{-15} m) = 1{,}79 \cdot 10^{11} \frac{m}{s}$

$v(r = 10^{-18} m) = 2{,}51 \cdot 10^{4} \frac{m}{s}$

(b) $E^{Rot} = \frac{1}{2} I \omega^2 = \cancel{\frac{1}{2}\frac{2}{5} m r^2 \cdot \frac{5}{5}\frac{3}{4}\frac{\hbar^2}{4}\frac{25}{m^2}} = \frac{2}{5}\frac{75}{32}\frac{\hbar^2}{m r^2}$

$E = \frac{1}{2} I \omega^2 \qquad \omega = \frac{L}{I} \qquad E = \frac{1}{2}\frac{1}{I}L^2 = \frac{1}{2}\frac{5}{2 m r^2}\frac{3}{4}\hbar^2$

$E^{Rot}(r = 1{,}4 \cdot 10^{-15} m) = 5{,}84 \cdot 10^{-3}\, J \approx 71000 \cdot E^{Ruhe}; E_m^{Ruhe} = m_e c^2$

$E^{Rot}(r = 10^{-8} m) = 0{,}0114 \approx 1{,}38 \cdot 10^{11} \cdot E^{Ruhe}$

25 $\underline{J} = \underline{L} + \underline{S} \qquad L, S > 0$

$J \in \{|L-S|, \dots, |L+S|\}$ (Spinaddi. theorem)

o.B.d.A. $L > S$

Anzahl Summanden = Mächtigkeit von $\{|L-S|, \dots, |L+S|\}$

$\sum 2J = \underbrace{\text{Anzahl}}_{n} \cdot \underbrace{\text{Durchschnitt}}_{D}$ (Elias auf?)

$n = |L+S| - |L-S| + 1 = 2S + 1$

$D = \dfrac{2(|L+S| + |L-S|)}{2} = \underbrace{2}_{2J}\dfrac{(|L+S| - 1 + |L-S| + 1)}{2} = \dots = 2\frac{2L}{2} = 2L$

$\left.\begin{array}{l} \sum 2J = 2L \cdot (2S+1) \\ \sum 1 = (2S+1) \end{array}\right\} \quad \sum (2J+1) = (2L+1)(2S+1)$

Der Term gibt genau die Entartung wieder!

$(2L+1)$ ist Entart. in L^2, m_L; $(2S+1)$ ist Entart. in S^2, m_S.

$\rightarrow (2L+1)(2S+1)$ Entartungen insgesamt.