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(a) 1) $\hat{f}_2(\xi) = \int_{-\infty}^{\infty} e^{-i\xi x} f(x+a)\,dx$

$x+a := y \qquad \frac{dx}{dy} = 1 \Rightarrow dx = dy$

Grenzen werden beibehalten da $\infty + a = \infty$

$$\hat{f}_2(\xi) = \int_{-\infty}^{\infty} e^{-i\xi(y-a)} f(y)\,dy$$

$$= \int_{-\infty}^{\infty} e^{-i\xi y} \cdot e^{i\xi a} f(y)\,dy$$

$$= e^{i\xi a} \int_{-\infty}^{\infty} e^{-i\xi y} f(y)\,dy$$

$$= e^{i\xi a}\,\hat{f}(\xi)$$

2) $$\hat{f}_3(\xi) = \int_{-\infty}^{\infty} e^{-i\xi x} \underbrace{f(x)\,e^{iqx}}_{f_3(x)}\,dx$$

$$= \int_{-\infty}^{\infty} e^{-i(\xi-q)x} f(x) \qquad \tilde{\xi} = \xi - q$$

$$= \int_{-\infty}^{\infty} e^{-i\tilde{\xi}x} f(x) = \hat{f}(\tilde{\xi})$$

$$\hat{f}_3(\xi) = \hat{f}(\tilde{\xi}) = \hat{f}(\xi - q)$$

„Hilfsrechnung!"

(b) $\hat{f}_6(\xi)$ von $f(x) = e^{-x^2}$ (Gauß mit $\tilde{a} = \sqrt{\tfrac{1}{2}}$):

$$\hat{f}_6(\xi) = \int_{-\infty}^{\infty} dx\, e^{-i\xi x} e^{-x^2} = \sqrt{\tfrac{1}{2}}\sqrt{2\pi}\; e^{-\left(\sqrt{\frac{1}{2}}\right)^2 \xi^2/2}$$

$$= \sqrt{\pi}\, e^{-\xi^2/4}$$

$\Rightarrow$ Bestimmung von $\hat{f}(\xi)$ von $f(x) = e^{-x^2}$

$\hat{f}_f(\xi)$ von $\sin(ax)\, e^{-x^2}$:

$$\hat{f}_f(\xi) = \int_{-\infty}^{\infty} dx\, e^{-i\xi x} \sin(ax)\, e^{-x^2}$$

$$= \int_{-\infty}^{\infty} dx\, e^{-i\xi x} \frac{1}{2i}\left(e^{ixa} - e^{-ixa}\right) e^{-x^2}$$

$$= \int_{-\infty}^{\infty} dx\, e^{-i\xi x} \frac{1}{2i} e^{ixa} e^{-x^2} - \int_{-\infty}^{\infty} dx\, e^{-i\xi x} \frac{1}{2i} e^{-ixa} e^{-x^2}$$

$$= \int_{-\infty}^{\infty} dx\, e^{-i(\xi-a)x} \frac{1}{2i} e^{-x^2} - \int_{-\infty}^{\infty} dx\, e^{-i(\xi+a)x} \frac{1}{2i} e^{-x^2}$$

$$= \frac{1}{2i}\left[\hat{f}_e(\xi-a) - \hat{f}_e(\xi+a)\right]$$

$$= \sqrt{\pi}\, \frac{1}{2i}\left[e^{-(\xi-a)^2/4} - e^{-(\xi+a)^2/4}\right]$$

⇒ Eigentliche Koeffizienten[illegible]

⇒ [illegible] von Er[illegible] (a)