MMP Blatt 3 . Michael Kopp 2439093

1 a)

1) $\int dx \frac{1}{1+x} = \left[\ln|1+x|\right]$

2) $\int dx \; x \sin(x) = \left[-x\cos x\right] + \int dx \cos x$

$= \left[-x\cos x\right] + \left[\sin x\right]$

3) $\int dx \frac{\sin x}{\cos x} = \int dx \sin x \frac{1}{\cos x} = \left[-\ln(\cos x)\right] = -\left[\ln(\cos x)\right]$

4) $\int dx \frac{f'(x)}{f(x)} = \int dx \frac{\frac{df}{dx}}{f(x)} = \int df \; \frac{1}{f(x)}$

$+ \int dx \; f'(x) \cdot \frac{1}{f(x)} = \left[\ln|f(x)|\right]$

1 b)

1) $\int_0^{2\pi} dx \sin x = \left[-\cos x\right]_0^{2\pi} = 0$

2) $\int_0^{2\pi} dx \, 1 \cdot \cos^2(x) = = \left[x\cos^2 x\right]_0^{2\pi} - \int_0^{2\pi} dx \, -2\cos x \sin x = \left[x\cos^2 x\right]_0^{2\pi} + 2\int_0^{2\pi} dx \cos x \sin x$

$= \left[x\cos^2 x\right]_0^{2\pi} + 2\left(\left[\sin^2 x\right]_0^{2\pi} - \int_0^{2\pi} dx \cos^2 x\right)$

$\leadsto \int_0^{2\pi} dx \cos^2 x = \left[x\cos^2 x\right]_0^{2\pi} + 2\left[\sin^2 x\right] - 2\int_0^{2\pi} dx \cos^2 x \quad | + 2\int_0^{2\pi} dx \cos^2 x \quad | : 3$

$\int_0^{2\pi} dx \cos^2 x = \frac{\left[x\cos^2 x\right]_0^{2\pi} + 2\left[\sin^2 x\right]_0^{2\pi}}{3} = \frac{2\pi}{3}$

3) $\int_0^{\infty} dx \, e^{-ax} = \left[-\frac{1}{a} e^{-ax}\right]_0^{\to\infty} \cdot (\to 0) - (-\frac{1}{a}) = \frac{1}{a}$

4) $\int_0^{\infty} dx \, x \, e^{-ax^2} \qquad x = \sqrt{y} \qquad \frac{dx}{dy} = \frac{1}{2\sqrt{y}}$

$\int_0^{\infty} dy \frac{1}{2\sqrt{y}} \cdot \sqrt{y} \cdot e^{-ay} = \left[\frac{-1}{2a} e^{-ay}\right]_0^{\infty} \overset{\text{Rücksubst.}}{\Rightarrow} \left[-\frac{1}{2a} e^{-ax^2}\right]_0^{\infty}$

$= 0 - \left(-\frac{1}{2a}\right) = \frac{1}{2a}$

① c) $\int dx \frac{x}{1-x^2} = \frac{1}{2}\int dx \frac{2x}{1-x^2} = \frac{1}{2}\int dx \frac{1}{1+x} - \frac{1}{1-x} = \frac{1}{2}\int dx \frac{1}{1+x} - \frac{1}{2}\int dx \frac{1}{1-x} =$

$[\frac{1}{2}\ln(1+x)] - \frac{1}{2}[\ln(1-x)]$

② a) $\Gamma(v) = \int_0^\infty dx\, x^{v-1} e^{-x} = \overbrace{[-x^{v-1} \cdot e^{-x}]_0^\infty}^{0} - \int_0^\infty dx\, (v-1) x^{v-2} \cdot e^{-x} = (v-1)\int_0^\infty dx\, x^{v-2} e^{-x}$

$\Gamma(v+1) = ((v+1)-1) \cdot \int_0^\infty dx\, v^{(v+1)-2} \cdot e^{-x} = v \cdot \underbrace{\int_0^\infty dx\, v^{v-1} \cdot e^{-x}}$

$F(v+1) = v \cdot \Gamma(v)$

b) $\Gamma(1) = \int_0^\infty dx\, x^0 \cdot e^{-x} = \int_0^\infty dx\, e^{-x} = [-e^{-x}]_0^\infty = (\to 0) - (-1) = 1$

$\to$ Teil (a)

(IA): $n = 1$ $\quad \Gamma(2) = \int_0^\infty dx\, x \cdot e^{-x} = 1 \cdot \Gamma(1) = 1 = 1!$ (wahr)

(IV): Sei $\Gamma(n+1) = n!$ wahr für n

(IS): ZZ.: $\Gamma(n+2) = (n+1)!$

$\downarrow$ Teil (a)

$(n+1)\,\Gamma(n+1) \stackrel{?}{=} n! \cdot (n+1)$ $\qquad$ 1: $\Gamma(n+1)$ bzw $n!$ (nach IV identisch)

$(n+1) = (n+1)$ wahr . $\square$

identisch (wegen $x = y$)

c) $\int_{-\infty}^\infty dx\, e^{-x^2} = \Gamma(\frac{1}{2}) = \int_0^\infty dx \frac{1}{\sqrt{x}} \cdot e^{-x}$

$\int_{-\infty}^0 dx\, e^{-x^2} + \int_0^\infty dx\, e^{-x^2} = \int_0^\infty dx\, e^{-x^2} - \int_0^{-\infty} dx\, e^{-x^2}$

Substitution: $\qquad x = \sqrt{y} \qquad x = -\sqrt{y}$

$\int_0^\infty dy \frac{1}{2\sqrt{y}} \cdot e^{-y} + \int_0^\infty dy \frac{1}{2\sqrt{y}} \cdot e^{-y} = 2 \cdot \int_0^\infty dy \frac{1}{2\sqrt{y}} \cdot e^{-y} = \int_0^\infty dy \frac{1}{\sqrt{y}} \cdot e^{-y}$

MMP Übung Blatt 3 Michael Wopp 2439093

(2) d) 1) $\int_{-\infty}^{\infty} dx\, e^{-\lambda x^2} = \int_0^{\infty} dx\, e^{-\lambda x^2} + \int_{-\infty}^{0} dx\, e^{-\lambda x^2} =$

$\int_0^{\infty} dx\, e^{-\lambda x^2} - \int_0^{-\infty} dx\, e^{-\lambda x^2}$

$x = \frac{1}{\sqrt{\lambda}} \sqrt{y} \qquad x = \frac{-1}{\sqrt{\lambda}} \cdot \sqrt{y}$

$\int_0^{\infty} dy\, \frac{1}{2\sqrt{\lambda}\sqrt{y}} \cdot e^{-y} - \int_0^{\infty} dy\, \frac{-1}{2\sqrt{y}\sqrt{\lambda}} \cdot e^{-y} =$

$\quad -\text{''}- \quad + \int_0^{\infty} dy\, \frac{1}{2\sqrt{y}\sqrt{\lambda}} \cdot e^{-y} =$

$2 \int_0^{\infty} dy\, \frac{1}{2\sqrt{\lambda} \cdot \sqrt{y}} \cdot e^{-y} = \frac{1}{\sqrt{\lambda}} \underbrace{\int_0^{\infty} dy\, \frac{1}{2\sqrt{y}} \cdot e^{-y}}_{\Gamma(\frac{1}{2}) \rightsquigarrow c)}$

$= \frac{1}{\sqrt{\lambda}} \cdot \Gamma(\tfrac{1}{2}) = \frac{1}{\sqrt{\lambda}} \cdot \sqrt{\pi} = \sqrt{\frac{\pi}{\lambda}}$