Elektrodynamik
Uebung 08
NOTIZEN – enth"alt noch viele Fehler...

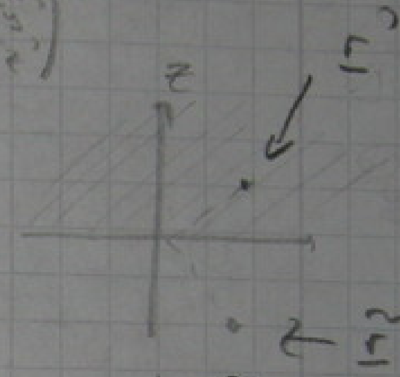
Vgl f"ur Aufgabe 1:
http://itp.tugraz.at/ schnizer/AnalyticalMethods/AnMe11.pdf

Michael Kopp

June 20, 2010

$\boxed{2}$

(a) $G(\underline{r},\underline{r}') = \left(\frac{q/q}{\|\underline{r}-\underline{r}'\|} + \frac{-q/q}{\|\underline{r}-\tilde{\underline{r}}'\|}\right)$ $\quad \tilde{\underline{r}}' = \begin{pmatrix} r'_x \\ r'_y \\ -r'_z \end{pmatrix} = \underline{r}' - 2(\underline{r}'\underline{e}_z)\underline{e}_z$

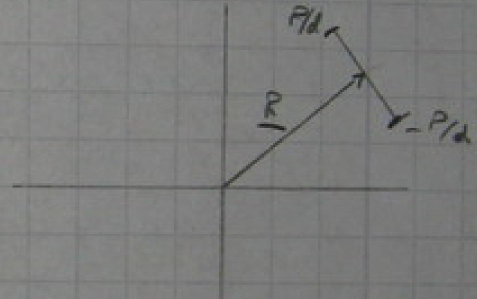


Für $\underline{r},\underline{r}' \in V$ gilt G $\Delta G = -4\pi\delta(\underline{r}-\underline{r}')$

und für $\underline{r} \in \{\underline{r} \in \mathbb{R}^3 \mid \underline{r}\underline{e}_z = 0\} = \partial V$

$G(\underline{r},\underline{r}') = 0$.

(b) $\overset{(\underline{r})}{\phi} = \overset{(\underline{r},\underline{r}')}{G} * \overset{\rho(\underline{r}')}{\rho}$



$\rho(\underline{r}') = \frac{p}{d}\,\delta\left(\underline{r}'-(\underline{R}+\tfrac{1}{2}\underline{d})\right) - \frac{p}{d}\,\delta\left(\underline{r}'-(\underline{R}-\tfrac{1}{2}\underline{d})\right)$

$$\phi = \int_V \left(\frac{1}{\|\underline{r}-\underline{r}'\|} - \frac{1}{\|\underline{r}-\underline{r}'+2(\underline{r}'\underline{e}_z)\underline{e}_z\|}\right)\cdot\frac{p}{d}\left(\delta\left(\underline{r}'-(\underline{R}+\tfrac{1}{2}\underline{d})\right) - \delta\left(\underline{r}'-(\underline{R}-\tfrac{1}{2}\underline{d})\right)\right) dV'$$

$$= \frac{p/d}{\|\underline{r}-\underline{R}-\frac{\underline{d}}{2}\|} - \frac{p/d}{\|\underline{r}-\underline{R}-\frac{\underline{d}}{2}+2(\underline{R}\underline{e}_z+\frac{1}{2}\underline{d}\underline{e}_z)\underline{e}_z\|} - \frac{p/d}{\|\underline{r}-\underline{R}+\frac{\underline{d}}{2}\|} + \frac{p/d}{\|\underline{r}-\underline{R}+\frac{\underline{d}}{2}+2\underline{e}_z\cdot(\underline{R}\underline{e}_z-\frac{1}{2}\underline{d}\underline{e}_z)\|}$$

Verw. $\underline{r}-\underline{R}=\underline{\alpha}$: $\|\underline{r}-\underline{R}-\frac{\underline{d}}{2}\| = \sqrt{\alpha^2+\frac{1}{4}d^2-\alpha d\cos\theta}$

Taylor $\frac{1/d}{\sqrt{\cdot}}$ in $d=0$, 0-te Ordnung (*)

$\cancel{\frac{p}{\alpha d}} + \frac{p\cos\theta}{2\alpha^2}$ $\qquad -\left(\cancel{\frac{p}{\alpha d}} - \frac{p\cos\theta}{2\alpha^2}\right)$

(*) Taylor kann man elegant machen, wenn man in

$\ldots = \underline{\alpha}: \quad \| \underline{r} - \underline{R} - \frac{\underline{d}}{2} \| = \sqrt{\alpha^2 + \frac{1}{4} d^2 - \alpha d \cos\vartheta}$ $\qquad (\underline{R}\, \underline{e}_z - \frac{1}{2} \underline{d}\, \underline{e}_z)$

Taylor $\sqrt{\ldots}^{\,-1}$ in $d = 0$, 0-te Ordnung (*)

$$\cancel{\frac{p}{\alpha d}} + \frac{p\cos\vartheta}{2\alpha^2} \qquad\qquad - \left(\cancel{\frac{p}{\alpha d}} - \frac{p\cos\vartheta}{2\alpha^2} \right)$$

(*) Taylor kann man elegant machen, wenn man in Legendre-Polynome entw. und bei $l = 1$ abbricht.

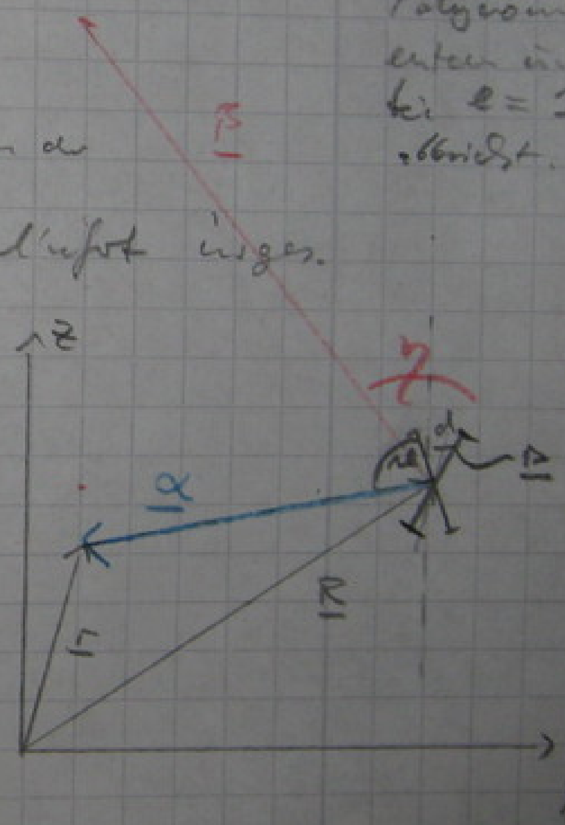
Die andren Klammern kann man umschreiben; daß dazu

$$\underline{\beta} := \begin{pmatrix} r_x - R_x \\ r_y - R_y \\ r_z + R_z \end{pmatrix}, \quad \underline{D} := \begin{pmatrix} d_x \\ d_y \\ -d_z \end{pmatrix},$$

die andren Klammern sind dann nicht von der Form $\sqrt{\beta^2 + \frac{1}{4} D^2 \pm \beta D \cos\eta}$, Entw. läuft insges.

$$\lim_{d \to 0} \phi(\underline{r}) = \frac{p\cos\vartheta}{\|\underline{r} - \underline{R}\|^2} - \frac{p\cos\eta}{\beta^2}$$

$$\beta^2 = \left\| \begin{pmatrix} r_x - R_x \\ r_y - R_y \\ r_z + R_z \end{pmatrix} \right\|^2$$



mit $\underline{\beta} = \underline{\alpha} + 2R_z \underline{e}_z$, $\underline{D} = \underline{d} - 2d_z \underline{e}_z$:

$\langle \underline{\beta}, \underline{D} \rangle = \ldots = \alpha d \cos\vartheta - 2d_z r_z = \beta \cdot D \cos\eta$

$\beta^2 = \alpha^2 + 4R_z r_z$, $D = d$

$$\cos\eta = \frac{\alpha d \cos\vartheta - 2d_z r_z}{d \cdot \sqrt{\alpha^2 + 4R_z r_z}}$$

$\underline{r}' = x'\underline{e}_x$ n' zeigt nach Innen

(c) $\phi(\underline{r}) = \int_V \rho(\underline{r}')\, G(\underline{r},\underline{r}')\, dV' + \frac{1}{4\pi} \int_{\partial V} \phi(\underline{r}')\, \underbrace{(\underline{n}'\nabla')}_{\partial_{z'}}\, G(r,r')\, df'$

$\rho = q \cdot \delta(\underline{r}'-\underline{R})$, $\phi(\underline{r}')\big|_{\partial V} = \begin{cases} 0 & x<0 \\ \Phi & x>0 \end{cases}$ G wie in (a)

$$\phi(\underline{r}) = \frac{q}{\|\underline{r}-\underline{R}\|} - \frac{q}{\|\underline{r}-\underline{R}+2R_z\underline{e}_z\|} + \underbrace{\int_0^\infty dx'\, \Phi \cdot \partial_{z'} G(\underline{r},\underline{r}')}_{N}$$

Vw. $\partial_{z'} \frac{1}{\|\underline{r}-\underline{r}'\|}\Big|_{\partial V} = \frac{(z-z')}{\|\underline{r}-\underline{r}'\|^3}\Big|_{\partial V} = \frac{-z}{\|\underline{r}-x'\underline{e}_x\|^3}$

$\partial_{z'} \frac{-1}{\|\underline{r}-\underline{r}'+2z'\underline{e}_z\|}\Big|_{\partial V} = \frac{z+z'}{\|\cdot\|^3}\Big|_{\partial V} = \frac{z}{\|\underline{r}-x'\underline{e}_x\|^3}$

$N = 0$