Probearbeit Kl. 12/13

① $f(x) = \frac{10(x-2)}{x^2}$ $\quad D_f = \mathbb{R} \setminus \{0\}$

$f(x) = \frac{10}{x} - \frac{20}{x^2} = 10x^{-1} - 20x^{-2}$

$f'(x) = -10x^{-2} + 40x^{-3} = \frac{-10x+40}{x^3}$

$f''(x) = 20x^{-3} - 120x^{-4} = \frac{20x-120}{x^4}$

$f'''(x) = -60x^{-4} + 480x^{-5} = \frac{-60x+480}{x^5}$

Asymptoten: $x_a = 0$ $\quad x \to 0\ x<0\ f \to -\infty,\ x \to 0\ x>0\ f \to -\infty$

$x \to \infty\ f \to 0,\ x \to -\infty\ f \to 0 \Rightarrow y_a = 0$

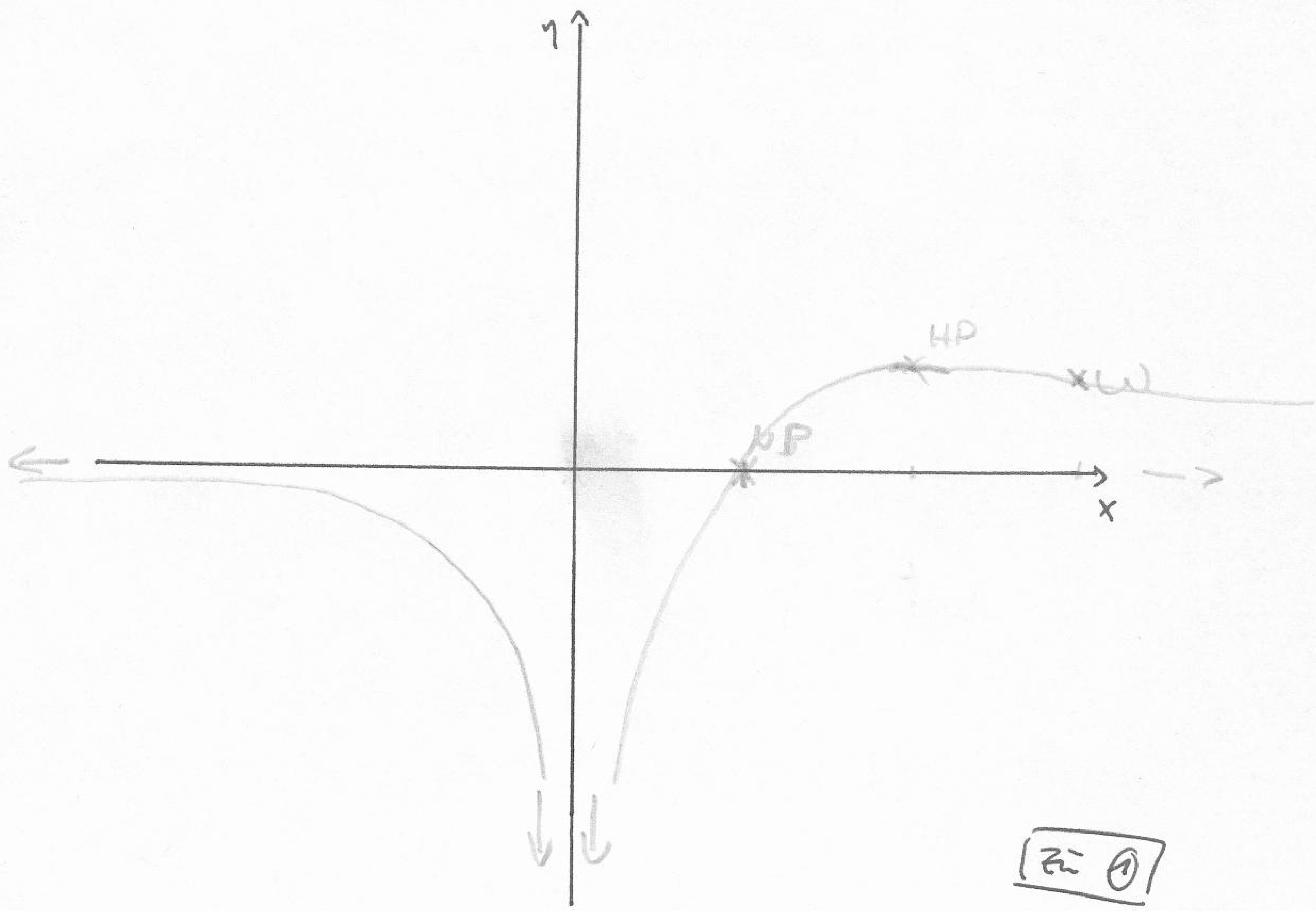
Symmetrie: Keine gerade/ungerade Funktion

$\rightarrow$ nicht sym. zur y-Achse oder zum Ursprung

Nullstellen: $10(x-2) = 0$ $\quad x_0 = 2$

Extremstellen: $f'(x_i) = 0$ $\quad$ ~~f~~ $\quad x = 4$ $\quad f''(4) = \frac{-40}{4^4} < 0 \Rightarrow HP(4|1{,}25)$

Wendestellen: $f''(x_i) = 0$ $\quad x = 6$ $\quad f'''(6) = \frac{120}{6^5} > 0 \Rightarrow W(6|\frac{10}{9})$



Zu ①

② 1) $f(x) = \dfrac{\cancel{x^2}(1-\frac{1}{x^2})}{\cancel{x^2}(2+\frac{2}{x^2})}$ $\quad x \to \infty \quad f \to \frac{1}{2} \quad y_a = \frac{1}{2}$

2) $f(x) = a + \dfrac{b}{x}$ $\quad x \to \infty \quad f \to a \quad y_a = a$

3) $f(x)$ hat <u>keine</u> waagerechte Asymptote – höchstens eine schiefe Asymptote vom Typ $y_a = m \cdot x + c$

③ 1) $(x^2 - 2x + 3) : (2x) = \frac{1}{2}x - 1 + \frac{3}{2}x^{-1}$ $\quad y_a = \frac{1}{2}x - 1$

$$\begin{array}{l} x^2 \\ -2x \\ -2x \quad +3 \end{array}$$

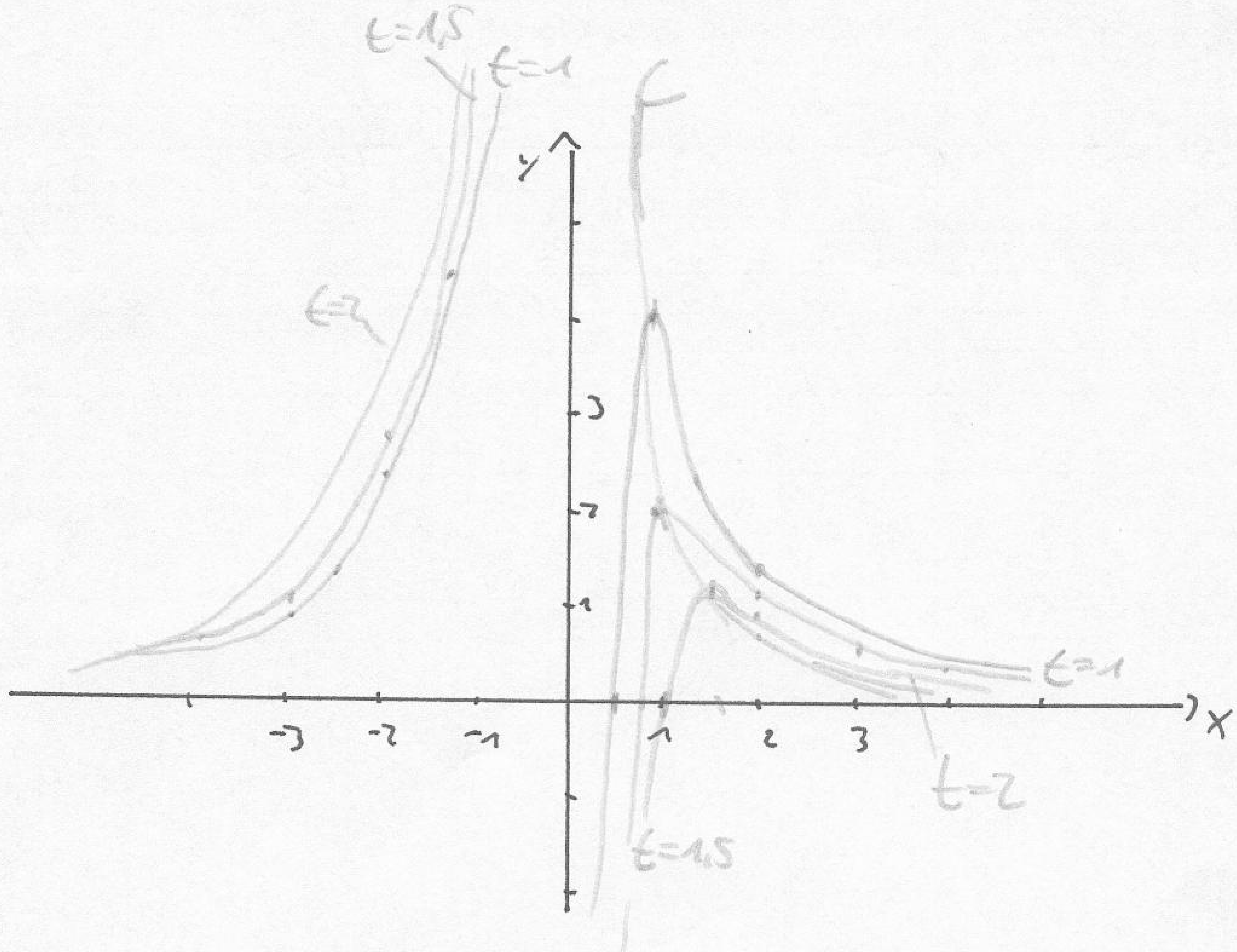
2) $(-2x^2 + 1) : (x+1) = -2x + 2 - \dfrac{1}{x+1}$ $\quad y_a = -2x + 2$

$$\begin{array}{l} -2x^2 \\ -(-2x^2 - 2x) \\ \hline \quad 2x + 1 \\ \quad -(2x + 2) \\ \hline \qquad -1 \end{array}$$

3) $(x^3 + 2x^2) : (x+4) = x^2 - 2x + 8 - \dfrac{32}{x+4}$ $\quad y_a = x^2 - 2x + 8$

$$\begin{array}{l} -(x^3 + 4x^2) \\ \hline \quad -2x^2 \\ \quad -(-2x^2 - 8x) \\ \hline \qquad 8x \\ \qquad -(8x + 32) \\ \hline \qquad\quad -32 \end{array}$$

④ 1)



2) $f_t(x) = \frac{8x-4t}{x^3}$ $D_f = \mathbb{R}\setminus\{0\}$ $t>0$

Symmetrie: Das Schaubild weist keine Symmetrie auf (keine geraden/ ungeraden Hochzahlen)

Asymptoten: $x_a = 0$ $x \to 0$ $x<0$ $f\to\infty$, $x\to 0$ $x>0$ $f \to -\infty$

Nullstellen: $x = \frac{1}{2}t$

Ableitungen: $f_t(x) = 8x^{-2} - 4t\,x^{-3}$

$f'_t(x) = -16x^{-3} + 12t\,x^{-4} = \frac{-16x+12t}{x^4}$

$f''_t(x) = 48x^{-4} - 48t\,x^{-5} = \frac{48x-48t}{x^5}$

$f'''_t(x) = -192x^{-5} + 240t\,x^{-6} = \frac{-192x+240t}{x^6}$

Extremstellen: $f'_t(x_i) = 0$ $-16x+12t=0$ $x = \frac{3}{4}t$

$f''_t(\frac{3}{4}t) = \frac{-12t}{(\frac{3}{4}t)^5} < 0$ $HP(\frac{3}{4}t \mid \frac{128}{27t^2})$

Wendestellen: $f''_t(x_i) = 0$ $x = t$ $f'''_t(t) = \frac{48t}{t^6} > 0$ $W(t \mid -\frac{4}{t^2})$

(4) 3) $H_t\left(\frac{3}{4}t \,\middle|\, \frac{128}{27t^2}\right)$

(I) $x_H = \frac{3}{4}t$ $\quad y_H = \frac{128}{27t^2}$ (II)

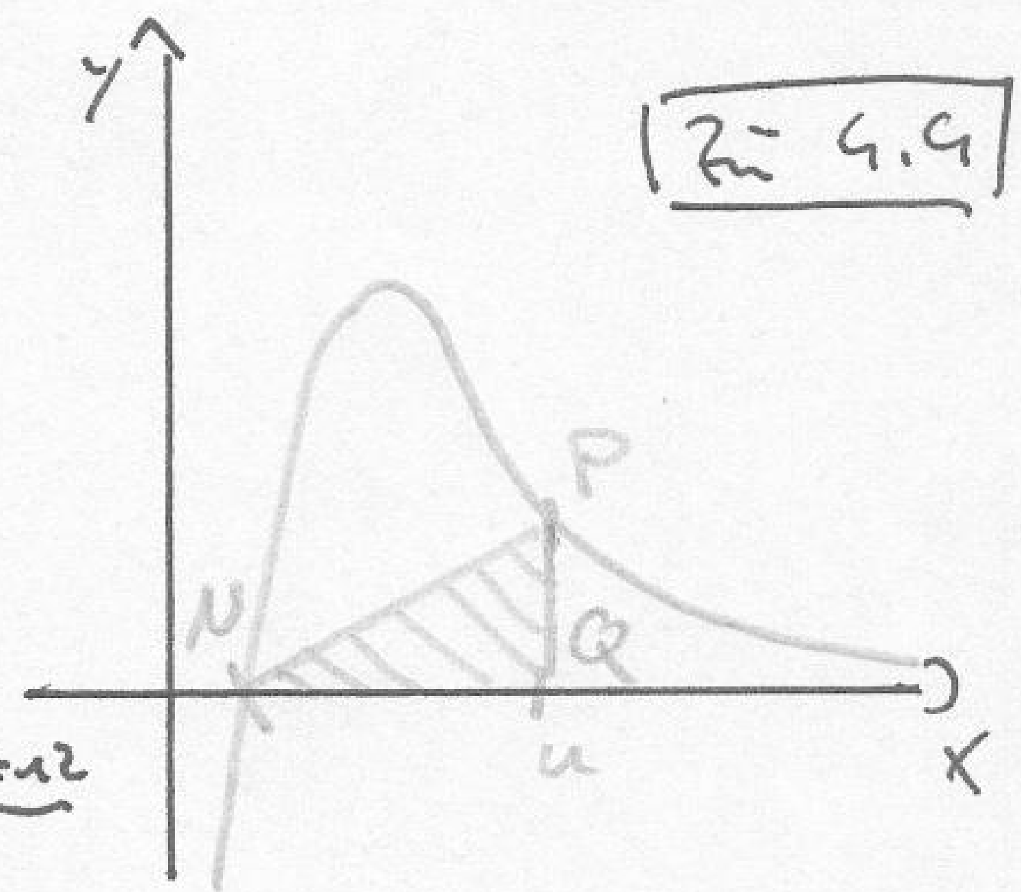
(I') $t = \frac{4}{3}x_H$ $\quad$ (I' in II) $\quad y_H = \frac{128}{27\left(\frac{4}{3}\right)^2 x^2} = \frac{8}{3}\cdot x^{-2}$

C: $x \mapsto \frac{8}{3}x^{-2}$

4) N: $f_2(x) = \frac{8x-8}{x^3}$ $\quad f_2(x_i) = 0$ $\quad x = 1$ $\quad N(1|0)$

Dreieck: $A = \frac{1}{2}\cdot g\cdot h$ $\quad h = f(u)$

$g = (u-1)$



Zu 4.4

$A(u) = \frac{1}{2}\cdot(u-1)\cdot\frac{8u-8}{u^3}$

$A(u) = 4u^{-1} - 8u^{-2} - 4u^{-3}$

$A'(u) = -4u^{-2} + 16u^{-3} + 12u^{-4} = \frac{-4u^2+16u+12}{u^4}$

$A''(u) = 8u^{-3} - 48u^{-4} - 48u^{-5}$

$A'(u_i) = 0$ $\quad u_{1,2} = \frac{-16 \pm \sqrt{256+192}}{-8} = \frac{-2\pm\sqrt{7}}{-1}$ $\quad [u_1 \approx -0{,}646]$

$u_2 \approx 4{,}646$

$A''(u_2) < 0 \Rightarrow HP(u_2 \mid f(u_2))$

$\Rightarrow$ für $u_2 \approx 4{,}646$ wird der Flächeninhalt des Dreiecks extremal!