Uebung 06, Thermodynamik

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December 2, 2010

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(1.1) S nur von Systemgrößen abh.

(1.2) S ist additiv bzgl. Teilsyst.

(1.3) ~~Gl~~ Im Gl. ~~Gew~~ ist S maximal

(1.4) $\Delta S = 0$ bei quasistat. rev. Prozesse

(2.1) S stetig part. diffbar

(2.2) S strikt mon. wachsend in U, V

(3.) $\partial U / \partial S = 0 \Rightarrow S = 0$

(a) Verletzt (1.2) da: Teile Syst. in zwei Teile mit α, $(1-\alpha)$ auf:

$S(\alpha) + S(1-\alpha) = (\alpha^{6/5} + (1-\alpha)^{6/5})\, S(1) \neq S(1) \Rightarrow$ nicht verträgl.

(b) $U = c_2 (VN)^{1/2} \frac{N}{S}$ $\quad \frac{\partial U}{\partial S} = -c_2 (VN)^{1/2} \frac{N}{S^2} = 0 \Rightarrow S \neq 0$

und nicht mit U, V wachsend. $\Rightarrow$ nicht verträgl.

(c) Verl. (1.2) nicht!

$S(\alpha) + S(1-\alpha) = \alpha S(1) + (1-\alpha) S(1) = S(1)$

$U = \left[\frac{1}{c_3} \frac{S N^{3/2}}{V^2} \right]^2$ $\quad \frac{\partial U}{\partial S} = 2 \frac{1}{c_3} \frac{S N^{3/2}}{V^2} \cdot \frac{1}{c_3} \frac{N^{3/2}}{V^2} \stackrel{!}{=} 0 \Rightarrow S = 0$

$\Rightarrow$ verträglich

(d) $U = \left(\frac{1}{c_4} S V^{-1/4} \right)^{4/3}$ $\quad \frac{\partial U}{\partial S} = \frac{4}{3} \frac{1}{c_4} S V^{-1/4} \frac{1}{c_4} V^{-1/4} \stackrel{!}{=} 0 \Rightarrow S = 0$

$\Rightarrow$ verträglich

[2] Theo (5)

$$U^i = U^i(T) = C^{(i)}(T-T_0) + U_0^i \qquad i \in \{1,2\}$$

Anfangs T_a^i, dann therm. Kontakt

(a) $U = U^1 + U^2 = \text{const} \Rightarrow$ $\quad T_e^1 = T_e^2 = T_e$

$$C^1(T_a^1 - T_0) + U_0^1 + C^2(T_a^2 - T_0) + U_0^2 = C^1(T_e - T_0) + C^2(T_e - T_0) + \cancel{U_0^1} + \cancel{U_0^2}$$

$$= (C^1 + C^2)(T_e - T_0)$$

$$\Rightarrow \frac{(C^1+C^2)(-T_0) + C^1T_a^1 + C^2T_a^2}{C^1+C^2} + T_0 = T_e = \frac{C^1T_a^1 + C^2T_a^1}{C^1+C^2}$$

(b) $dU = TdS - pdV + \mu dN \qquad dV = dN = 0$

$dU^i = C^i dT^i$

Int. über $[T_a^i, T_e]$

$C^i dT^i = T^i dS^i \Rightarrow C^i \frac{dT^i}{T^i} = dS^i \Rightarrow$

$C^i \ln \frac{T_e^i}{T_a^i} = S^i = S^i(T)$

was hier mit S bez. ist, ist eig. ΔS

$$\Delta S \geq S_e^1 + S_e^2 - S_a^1 - S_a^2$$

$$= C \ln \frac{C^1T_a^1 + C^2T_a^2}{(C^1+C^2)T_0} \cdot (C^1 + C^2) - C^1 \ln \frac{T_a^1}{T_0} - C^2 \ln \frac{T_a^2}{T_0}$$

$$= C^1 \ln \frac{C^1T_a^1 + C^2T_a^2}{(C^1+C^2)T_a^1} + C^2 \ln \frac{C^1T_a^1 + C^2T_a^2}{(C^1+C^2)T_a^2} \quad \geq$$

sei oBdA $C^1 < C^2$

$$T_e = \frac{T_a^1}{1 + \underbrace{C^2/C^1}_{>1}} + \frac{T_a^2}{1 + \underbrace{C^1/C^2}_{<1}} \Rightarrow T_a^1 > T_e > T_a^2$$

$$\geq C^1 - C^1 \frac{(C^1+C^2)T_a^1}{C^1T_a^1 + C^2T_a^2} + C^2 - C^2 \frac{(C^1+C^2)T_a^2}{C^1T_a^1 + C^2T_a^2} =$$

$$(C^1+C^2) - \frac{C^1T_a^1 + C^2T_a^2}{C^1T_a^1 + C^2T_a^2}(C^1 + C^2) = 0$$

$$\ln x \geq \frac{x-1}{x} = 1 - \frac{1}{x}$$

□

(c) $C^1 = C^2 = C$:

$T_a^2 = T_a^1 - \Delta T$

$T_a^1 = T_a^2 + \Delta T$

$$S^1 = C \cdot \ln \frac{T_e}{T_a^1} = C \ln \frac{T_a^1 + T_a^2}{2T_a^1} = C \ln \frac{2T_a^1 - \Delta T}{2T_a^1}$$

$$S^2 = C \cdot \ln\left(1 + \frac{\Delta T}{2T_a^2}\right) = C \frac{\Delta T}{2T_a^2} - C\frac{1}{2}\left(\frac{\Delta T}{2T_a^2}\right)^2 + O[(\Delta T)^3]$$

$$S^1 = C \ln \frac{2T_a^2 + \Delta T \pm \Delta T}{2(T_a^2 + \Delta T)} \dots = C \ln\left(1 - \frac{\Delta T}{2(T_a^2 + \Delta T)}\right) = C \ln\left(1 - \frac{1}{2}\frac{\Delta T / T_a^2}{1 + \Delta T / T_a^2}\right)$$

kompliziert!

$$= C \ln \frac{1}{2}\left(1 + \frac{1}{1 + (\Delta T / T_a^2)}\right)$$

$$S^1 = C \cdot \ln\left(1 - \frac{1}{2}\frac{\Delta T}{T_a^1}\right) = -C\left[\frac{\Delta T}{2T_a^1} + \frac{1}{2}\left(\frac{\Delta T}{2T_a^1}\right)^2 + O[(\Delta T)^3]\right]$$

$$= -C\left[\frac{\Delta T}{2T_a^2 + 2\Delta T} + \dots\right]$$

$$= -C\left[1 - \frac{1}{1+(\Delta T/2T_a^2)} + \frac{1}{2} - \frac{2/2}{1+(\Delta T/2T_a^2)} + \frac{1/2}{(1+(\Delta T/ \dots)^2}\right]$$

$T_a^1 = T_a^2 + \Delta T$

$$\frac{\Delta T}{2T_a^2 + 2\Delta T} = \frac{\Delta T/2T_a^2}{1+\Delta T/2T_a^2} = \frac{\xi \pm 1}{1+\xi} = 1 - \frac{1}{1+\xi}$$

$$(\ )^2 = 1 + \frac{1}{(1+\xi)^2} - \frac{2}{1+\xi}$$

$\boxed{T_1 \text{ ist wärmer}}$

$$\xi = \frac{\Delta T}{2T_a^2} \quad \ominus \; -C\left[\frac{3}{2} - \frac{2}{1+\xi} + \frac{1}{2(1+\xi)^2}\right]$$

$$S^2 = C\left[\xi - \xi^2/2\right]$$

$$\Delta S = C\left(\ln\frac{T_e}{T_a^1} + \ln\frac{T_e}{T_a^2}\right)$$

$$= C\left(\ln\frac{T_a^1 + T_a^2}{2T_a^1} + \ln\frac{T_a^1 + T_a^2}{2T_a^2}\right)$$

$\boxed{3}$ $dU = dQ$; $dW = 0$ da $dV = 0$, außerdem $dN = 0$.

$$\frac{\partial U}{\partial S} = T \qquad\qquad dQ = \Sigma_i \, C_V^i(T)\, dT$$

$$dU = T\,dS = dQ$$

$$T\,dS = \Sigma_i \, C_V^i(T)\, dT \quad |:T \,|\int$$

$$\Delta S^i = \int_{T_a^i}^{T_e} \frac{C_V^i(T)\,dT}{T}$$

$$\Delta S = \Sigma_i \int_{T_a^i}^{T_e} \frac{C_V^i(T)}{T}\,dT$$

Mittelwertsatz d. Integralrechnung:

$$\Delta S = \Sigma_i \, C_V^i(\tilde{T}_i) \int_{T_a^i}^{T_e} \frac{1}{T}\,dT =: \Sigma_i \, \gamma^i \ln\frac{T_e}{T_a^i} \geq$$

$$\min_i \gamma^i \; \Sigma_i \ln\frac{T_e}{T_a^i} =: \Gamma \ln\frac{(T_e)^\mu}{\Pi_i T_a^i} \;\textcircled{$\geq$}$$

Da $C_V^i(T) > 0 \Rightarrow \gamma^i > 0 \Rightarrow \Gamma > 0$

Im $\{T_a^i\}$-Raum hat der Hyperwürfel $T_e \times \dots \times T_e$ das maximale Volumen, also $(T_e)^\mu / \Pi_i T_a^i \geq 1$: $\textcircled{$\geq$}\; \Gamma \ln 1 = 0$