$\boxed{2}$ (a) Für Gauß-Bündel gilt:

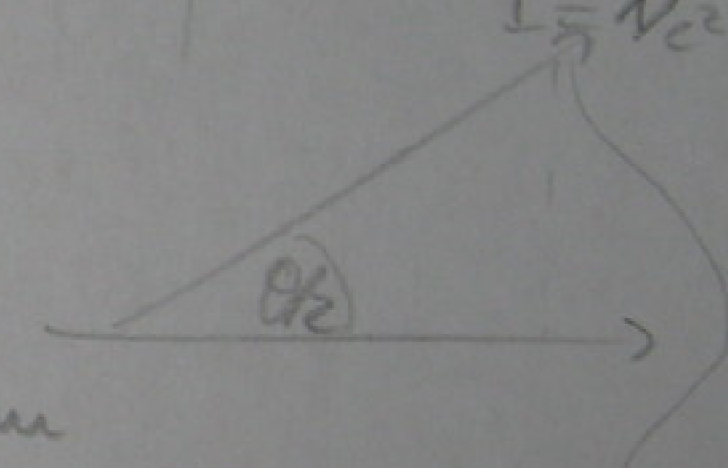
• Nahfeld $w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}$

↳ $z_0 = \frac{\pi w_0^2}{\lambda}$

$1 + \left(\frac{10\,m}{\frac{\pi \cdot 10^{-6}\,m^2}{633 \cdot 10^{-9}\,m}}\right)^2$

$w(10\,m) = 2{,}25$ mm

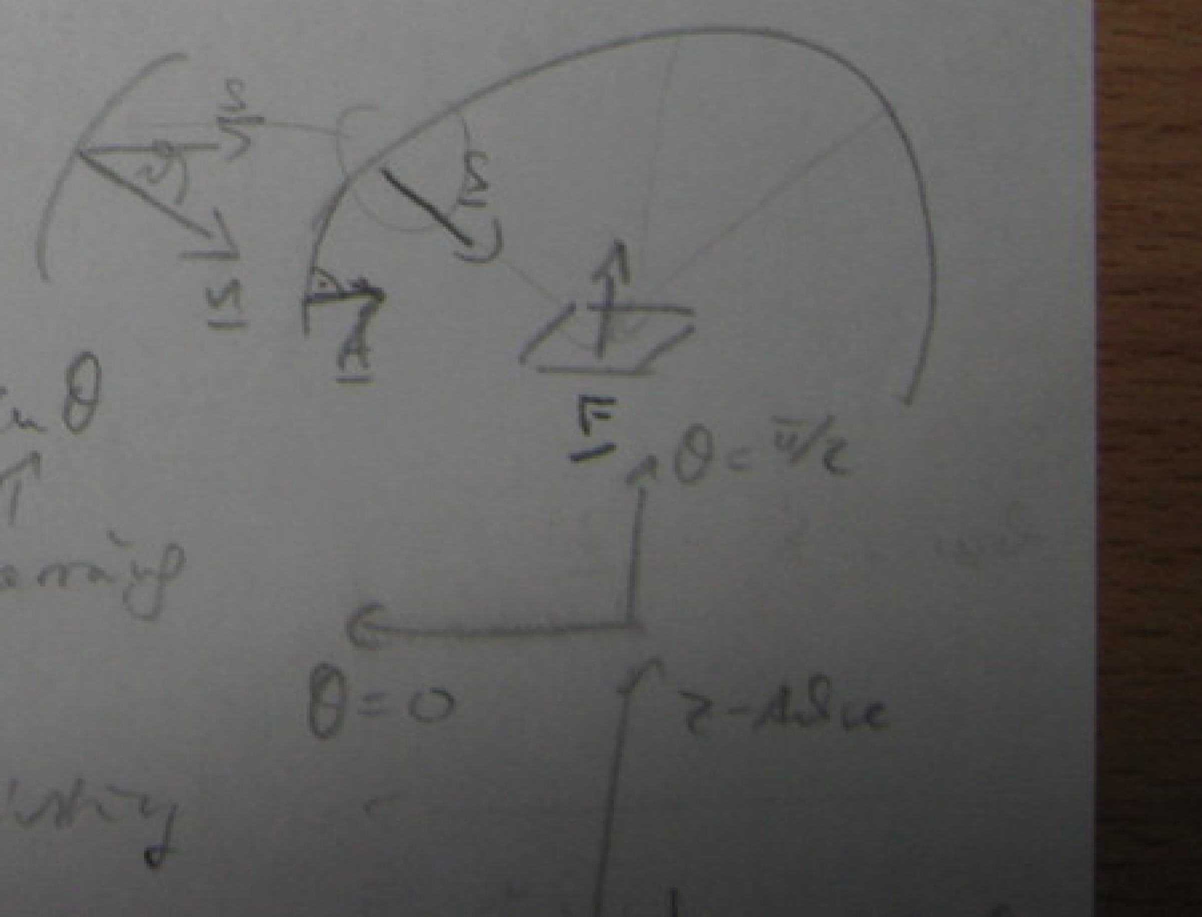
• Fernfeld $\theta_{div} = \frac{\theta}{2} = \arctan\left(\frac{\lambda}{\pi w_0}\right)$



$w_{Mond} = \tan\left(\frac{\theta}{2}\right) \cdot d_M = \frac{\lambda}{\pi w_0} \cdot d_M \approx 76{,}6$ km

(b) $\frac{\lambda d_M}{\pi w_0} = 1\,km \rightarrow w_0 = \frac{\lambda d_M}{\pi \cdot 1\,km} \approx 76{,}6$ mm

$\boxed{3}$ (a) $\underbrace{\left(\int_0^\infty u \, d\nu\right) \cdot c}_{\sigma T^4} \cdot dA \cdot \frac{d\Omega}{4\pi} \cdot \frac{\cos\vartheta}{1}$



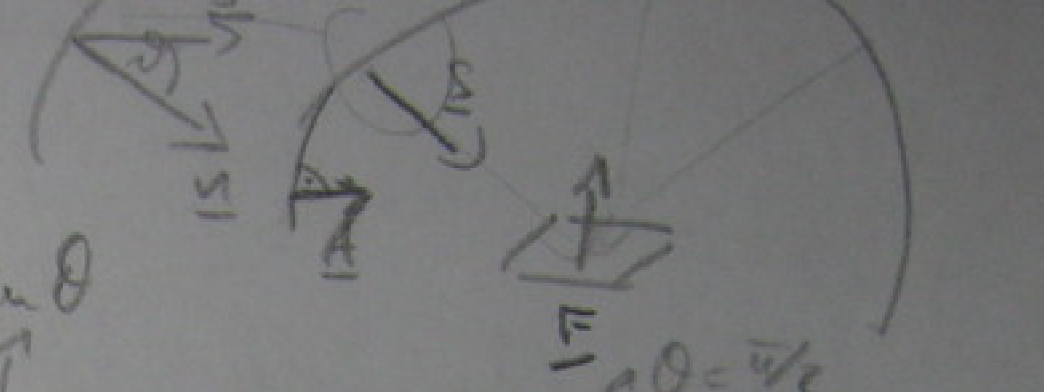
$d\Omega = \frac{1\,mm^2}{R^2} \cdot \sin\theta$

↑ Vorzeichen?

Jacobian: $R^2 \cos\theta$

Flächenelement dA strahlt seitlich

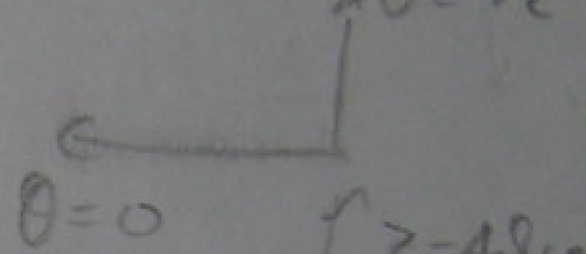
[3] (a) $\underbrace{\left(\int_0^\infty u\,d\nu\right)\cdot c}_{\sigma T^4} \cdot dA \cdot \frac{d\Omega}{4\pi} \cdot \underbrace{\cos\vartheta}_{1}$



$d\Omega = \frac{1\,mm^2}{R^2}\cdot \sin\theta$

Vorsicht

Jacobian: $R^2\cos\theta$



Flächenelement dA strahlt Leistung $\sigma T^4\, dA$ ab. Von dA aus gesehen hat F Raumwinkel $\frac{1\,mm^2}{R^2}\sin\theta$.

$\int\cos\theta\sin\theta\, d\theta = \int \sin\theta\, d(\sin\theta) = \frac{1}{2}\sin^2\theta$

$$P^{ges} = \underbrace{\int_0^{2\pi} d\varphi}\int_0^{\pi/2} d\theta\ \sigma T^4 \cdot \underbrace{R^2\cos\theta}_{dA} \cdot \underbrace{\frac{1\,mm^2}{4\pi R^2}\sin\theta}_{\frac{d\Omega}{4\pi}}$$

$$= \cancel{2\pi}\ \sigma T^4 \cdot \frac{1\,mm^2}{4\cancel{\pi}} \cdot \frac{1}{2}$$

F von Int. mit θ

$$= \frac{\sigma T^4 \cdot 1\,mm^2}{4}$$

Für Raumtemp.: $T = 300\,K$: $114{,}8\,\mu W$.

⇝ von Radius unabhängig: Fläche wird zwar größer, Raumwinkel gleichermaßen kleiner.

(b) $T = \sqrt[4]{\frac{4P}{\sigma\, 1\,mm^2}} \approx 9{,}16\,K \mathrel{\hat=} -264\,°C$