$$(1 - At + \tfrac{1}{2}A^2t^2 - \tfrac{1}{6}A^3t^3)\,B\,(1 + At + \tfrac{1}{2}A^2t^2 + \tfrac{1}{6}A^3t^3) =$$

$$B + BAt + \tfrac{1}{2}BA^2t^2 + \tfrac{1}{6}BA^3t^3$$

$$- ABt - ABAt^2 - \tfrac{1}{2}ABA^2t^3 - \tfrac{1}{6}ABA^3t^4$$

$$+ \tfrac{1}{2}A^2Bt^2 + \tfrac{1}{2}A^2BAt^3 + \tfrac{1}{4}A^2BA^2t^4 + \tfrac{1}{12}A^2BA^3t^5$$

$$- \tfrac{1}{6}A^3Bt^3 - \tfrac{1}{6}A^3BAt^4 - \tfrac{1}{12}A^3BA^2t^5 - \tfrac{1}{36}A^3BA^3t^6 =$$

$$B + \underbrace{(BA - AB)}_{-[AB]}t + \tfrac{1}{2}\underbrace{(BA^2 - 2ABA + A^2B)}_{0}t^2 + \tfrac{1}{6}(BA^3 - 3ABA^2 + 3A^2BA - A^3B)t^3 + \dots$$

$$BA^2\cdot A - 2ABA\cdot A - A\cdot BA^2 + 2A\cdot ABA + A^2B\cdot A - A\cdot A^2B$$

$$(BA^2 - 2ABA + A^2B)A - A(BA^2 - 2ABA + A^2B)$$

$$[A[AB]]\cdot A - A\cdot[A[AB]]$$

$$= [[A[AB]]A]$$

$$g(x) = \sum_j \frac{1}{j!} (\partial_x^j g) \cdot x^j \qquad , p = -i\hbar\partial_x$$

$$[p, g(x)] =$$

$$= -i\hbar\partial_x \left(\sum_j \frac{1}{j!} (\partial_x^j g)\, x^j \bullet \right) - \sum_j \frac{1}{j!} (\partial_x^j g) \cdot x^j \left(-i\hbar\partial_x \bullet \right) =$$

$$= -i\hbar \left\{ \sum_j \frac{1}{j!} (\partial_x^{j+1} g)\, x^j \bullet + \sum_j \frac{1}{j!} (\partial_x^j g)\, j\, x^{j-1} \bullet + \cancel{\sum_j \frac{1}{j!} (\partial_x^j g)\, x^j \partial_x \bullet} - \cancel{\sum_j \frac{1}{j!} (\partial_x^j g)\, x^j \partial_x \bullet} \right\}$$

$$= -i\hbar \left\{ \sum_j \frac{1}{j!} (\partial_x^{j+1} g)\, x^j \bullet + \sum_j \frac{1}{(j-1)!} (\partial_x^j g)\, x^{j-1} \bullet \right\} \qquad j+1 =: k$$

$$j = 1, \dots, \infty$$

$$k = 2, \dots, \infty$$

$$= -i\hbar \left\{ \sum_k \frac{1}{(k-1)!} (\partial_x^k g)\, x^{k-1} + \sum_j \frac{1}{(j-1)!} (\partial_x^j g)\, x^{j-1} \right\} \bullet$$

$$= -i\hbar \left\{ \underbrace{2 \sum_l \frac{1}{(l-1)!} (\partial_x^l g)\, x^{l-1}}_{\text{soll verschwindet sein...}} - \underbrace{\frac{1}{0!} (\partial_x^1 g)\, x^0}_{\partial_x g} \right\} \bullet \qquad l = 1, \dots, \infty$$

soll „+" sein

$$W = e^{-At} e^{(A+B)t} \qquad \dot W = -A e^{-At} e^{(A+B)t} + e^{-At}(A+B)e^{(A+B)t}$$

$$= -AW + e^{-At}(A+B)\, e^{(A+B)t}$$

$$e^{At} e^{-At} = \mathbb{1}$$

$$= -AW + \underbrace{e^{-At}(A+B)e^{At}}_{\Downarrow\ 2(a)} W$$

$$(A+B) - t[A,(A+B)]$$

$$[A,(A+B)] = A(A+B) - (A+B)A$$

$$= A^2 + AB - A^2 - BA = (AA - AA) + (AB - BA)$$

$$= [A,A] + [A,B]$$

$$\Rightarrow \dot W = \cancel{-AW} + (\cancel{A} + B - t\underbrace{[A,A]}_{0} - t[A,B])W$$

$$\dot w = (B - t[A,B])W$$

$$\Rightarrow \text{Ansatz:} \quad \tilde W = e^{Bt} e^{-\frac{1}{2}t^2[A,B]}$$

Da $[A,B]$ ein Skalar ist, ist die Abl.:

$$\dot{\tilde W} = B \cdot e^{Bt} e^{-\frac12 t^2[A,B]} + (-t[A,B])\, e^{Bt} e^{-\frac12 t^2[A,B]}$$

da $[A,B] \in \mathbb{R}$ darf die mal so rum gezogen werden...

Diese löst die DGL.

Für $t = 1$ erhält man:

$$e^{-A} e^{(A+B)} = e^{B - \frac12[A,B]} \qquad \Big|\ e^{A} \cdot \text{ von links anmult.}$$

$$e^{A+B} = e^{A} \cdot e^{B} \cdot e^{-\frac12[A,B]}$$