Uebung Theoretische Physik
Blatt 12

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Theo (12)

$\boxed{2}$ $g(x) = (n \ln(x) - x)$

g hat Max. bei x_m; $g' = \frac{n}{x_m} - 1 = 0 \Rightarrow n = x_m$

$$g(x_m + h) = n(\ln(n) - 1) - \frac{h^2}{2n} + \mathcal{O}(h^3)$$ [Ordn. h^1 fällt weg da Max.]

$$g(x) \approx n(\ln(n) - 1) - \frac{(x-n)^2}{2n}$$ [$g'' = -\frac{n}{x^2} \Rightarrow g''|_{x=n} = -\frac{1}{n}$

$$n! \approx \int_0^\infty \exp\left[n(\ln(n)-1) - \frac{(x-n)^2}{2n}\right] dx$$

$e^{\alpha+\beta} = e^\alpha \cdot e^\beta$

$$= \exp(n(\ln(n)-1)) \cdot \int_0^\infty e^{-\frac{(x-n)^2}{2n}} dx$$

$x - n = \mu \quad dx = d\mu$

$$= \exp(n(\ln(n)-1)) \cdot \int_{-n}^\infty e^{-\mu^2/2n} d\mu$$

$\int_{-\infty}^{-n} \dots$ ist sehr klein.

$$\approx \text{''} \quad \int_{-\infty}^\infty e^{-\mu^2/2n} d\mu$$

$$= \text{''} \quad 2 \int_0^\infty e^{-\mu^2/2n} d\mu$$

$$= \text{''} \quad 2 \cdot \sqrt{\frac{\pi n}{2}}$$

$$\Rightarrow \ln(n!) \approx n(\ln(n)-1) + \underbrace{\ln\sqrt{2\pi n}}_{\frac{1}{2}\ln(2\pi n) = \frac{1}{2}(\ln(2\pi) + \ln(n))}$$

$\square$

$\boxed{3}$

$$\ln(W_N(r)) = \ln(N!) - \ln(r!) - \ln[(N-r)!] + r\ln(p) + (N-r)\ln(1-p) = \omega(r)$$

$\frac{d}{dr} \cdot = \frac{d(N-r)}{dr} \cdot \frac{d}{d(N-r)} \cdot$

$$\frac{d\omega}{dr} \overset{!}{\approx} -\ln(r) + \ln(N-r) + \ln(p) - \ln(1-p) = 0$$

$$\Rightarrow \frac{N-\bar{r}}{\bar{r}} = \frac{1-p}{p} = \frac{1-\bar{r}/N}{\bar{r}/N} \Rightarrow p = \bar{r}/N \Leftrightarrow \bar{r} = p \cdot N$$

$$\frac{d^2\omega}{dr^2} = -\frac{1}{r} - \frac{1}{N-r} \qquad \frac{d^2\omega}{dr^2}\Big|_{\bar{r}} = -\left(\frac{1}{Np} + \frac{1}{N(1-p)}\right) = -\frac{1-p+p}{Np(1-p)} = -\frac{1}{Np(1-p)}$$

$$\omega(\bar{r}) = \ln\left[\frac{N!}{(pN)!(N(1-p))!} p^{pN} (1-p)^{N(1-p)}\right]$$

$$\approx (N+\tfrac{1}{2})\ln N - N + \tfrac{1}{2}\ln(2\pi) - (pN+\tfrac{1}{2})\ln(pN) + pN - (N(1-p)+\tfrac{1}{2})\ln(N(1-p)) + N(1-p)$$
$$+ pN \ln p + N(1-p)\ln(1-p)$$

$$= -\frac{1}{2}\ln(2\pi) + \ln N\left[\cancel{N} + \cancel{\tfrac{1}{2}} - pN - \cancel{\tfrac{1}{2}} - N(1-p) - \tfrac{1}{2}\right] + \ln p\left[-\cancel{pN} - \tfrac{1}{2} + \cancel{pN}\right]$$
$$+ \ln(1-p)\left[\cancel{-N(1-p)} - \tfrac{1}{2} + \cancel{N(1-p)}\right] = -\frac{1}{2}\ln(2\pi N p(1-p))$$

$$W_N(r) \approx \exp\left[w(\bar r) + 0 + \frac{d^2 w}{dr^2}\Big|_{\bar r} \frac{(r-\bar r)^2}{2} \right] = W(r) \qquad \square$$

4 $\quad P_m^{(N+1)} = P_{m-1}^{(N)} \cdot p + P_{m+1}^{(N)} (1-p)$ (Teilchen kann nur vom rechten oder linken Nachbarn kommen)

$$P(x,t) \xrightarrow{D} P(m,N) := P(m \cdot a, N \cdot \tau)$$

$$\partial_t P(x,t) \xrightarrow{D} \frac{1}{\tau}\left[P(m,N+1) - P(m,N)\right] \qquad \frac{1}{2\tau}\left[P(m,N+1) - P(m,N-1)\right]$$

$$\partial_x^2 P(x,t) \xrightarrow{D} \frac{1}{a^2}\left[P(m+1,N) - 2P(m,N) + P(m-1,N)\right]$$

$$\partial_x P(x,t) \longrightarrow \frac{1}{2a}\left[P(m+1,N) - P(m-1,N)\right]$$

$$\partial_t P = D\partial_x^2 P + F\partial_x P \xrightarrow{D}$$

$$\frac{1}{\tau}\left[P(m,N+1) - P(m,N)\right] = D\frac{1}{a^2}\left[P(m+1,N) - 2P(m,N) + P(m-1,N)\right] + F\frac{1}{2a}\left[P(m+1,N) - P(m-1,N)\right]$$

$$P(m,N+1) = P(m+1,N)\underbrace{\left[\overset{\delta}{D\frac{\tau}{a^2}} + \overset{\varphi}{F\frac{\tau}{2a}}\right]}_{(1-p)} + P(m-1,N)\underbrace{\left[D\frac{\tau}{a^2} - F\frac{\tau}{2a}\right]}_{p} + P(m,N)\underbrace{\left[1 - 2D\frac{\tau}{a^2}\right]}_{0}$$

$$\delta + \varphi = 1-p \qquad 2\delta = 1$$
$$\delta - \varphi = p \qquad 2\varphi = 1-2p$$

$$D = \frac{1}{2}\frac{a^2}{\tau} \qquad F = (1-2p)\frac{a}{\tau}$$

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15 — 6 ohne Malheur
15 — 9 mit " — 8 Sonnenbrand — 5 Stürze
15 — 9 mit " — 8 Sonnenbrand — 4 stürzen nicht
15 — 9 mit " — 1 kein "

15 Kind. — 6 ohne Malheur
15 Kind. — 1 stürzt, kein SB
15 Kind. — 4 stürzt nicht, SB
15 Kind. — 4 stürzt, SB

B : Sonnenbrand
F : Sturz
U : Unverletzt.

$P(B) = 8/15 \qquad P(F) = 5/15 \qquad P(U) = 6/15$

(b) $P(F \cap B) = 4/15 \neq P(F) \cdot P(B) = \frac{8}{45}$

(a) $P(F \cap B^c) = 1/15$