1) a) $\frac{d}{dx}\psi(x) - a\frac{d^3}{dx^3}\psi(x) = x\psi(x) \quad | \mathcal{F}$

$$\int_{-\infty}^{\infty}\frac{d}{dx}\psi(x)\, e^{-i\xi x} - \int_{-\infty}^{\infty} a\frac{d^3}{dx^3}\psi(x)\, e^{-i\xi x} = \int_{-\infty}^{\infty} x\cdot\psi(x)\, e^{-i\xi x}\,dx$$

$$+i\xi\,\hat{\psi}(\xi) - (i\xi)^3 a\,\hat{\psi}(\xi) = i\frac{d}{dk}\hat{\psi}(\xi)$$

$$\frac{d}{dk}\hat{\psi}(\xi) = \xi\hat{\psi}(\xi) - a\xi^3\hat{\psi}(\xi)$$

$\Rightarrow$ homog. Diff'gl. 1 Ordn.

b) $$\frac{d^2}{dx^2}\psi(x) - \underbrace{\int_{-\infty}^{\infty}dz\; h(x-z)\,\psi(z)}_{\text{Faltung}} = \delta(x)$$

$$(i\xi)^2\hat{\psi}(\xi) - \hat{h}(\xi)\cdot\hat{\psi}(\xi) = 1$$

$$\hat{h}(\xi) = \int_{-\infty}^{\infty}dx\; e^{-i\xi x}\cdot e^{-x} = \int_0^{\infty}dx\; e^{-x(i\xi+1)}$$

$$= \left[\frac{-1}{i\xi+1}e^{-x}\right]_0^{\infty} = 0 - \frac{-1}{i\xi+1} = \frac{1}{i\xi+1}$$

$$-\xi^2\hat{\psi}(\xi) - \frac{1}{i\xi+1}\hat{\psi}(\xi) = 1$$

$$\hat{\psi}(\xi)\left(-\xi^2 - \frac{1}{i\xi+1}\right) = 1$$

$$\hat{\psi}(\xi) = \frac{-1}{\xi^2 + \frac{1}{i\xi+1}} = -\frac{i\xi+1}{i\xi^3+\xi^2+1}$$

$$\psi(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}d\xi\; e^{+i\xi x}\cdot\frac{i\xi+1}{i\xi^3+\xi^2+1}$$

u' $\quad v$

$v' = \Big(-\xi^3 i\xi^2 + i - (-3\xi^3 + 3i\xi^2 + 2i\xi^2 + 2\xi)\Big) \div \Big((i\xi^3+\xi^2+1)^2\Big)$

$$= \frac{-2\xi^3 - 4i\xi^2 - 2\xi + i}{(i\xi^3+\xi^2+1)^2}$$

② $\vec{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} t \\ t^2 \end{pmatrix} \quad t \in [0,1] \qquad \frac{d\vec{r}}{dt} = \begin{pmatrix} 1 \\ 2t \end{pmatrix} \quad \left|\frac{d\vec{r}}{dt}\right| = \sqrt{1+4t^2}$

(i) $\int_C d\vec{r} \cdot \vec{r} = \int_C \frac{d\vec{r}}{dt}\, dt\, \vec{r} = \int_0^1 \begin{pmatrix} 1 \\ 2t \end{pmatrix} \begin{pmatrix} t \\ t^2 \end{pmatrix} dt =$

$\int_0^1 t + 2t^3 \, dt = \left[\frac{1}{2}t^2 + \frac{1}{2}t^4 \right]_0^1 = 1$

(ii) $\int_C d\vec{r}\, x(t)\, y(t) = \int_C \frac{d\vec{r}}{dt}\, dt \; t^3 = \int_0^1 \begin{pmatrix} t^3 \\ 2t^4 \end{pmatrix} dt$

$= \left[\begin{pmatrix} \frac{1}{4}t^4 \\ \frac{2}{5}t^5 \end{pmatrix} \right]_0^1 = \begin{pmatrix} \frac{1}{4} \\ \frac{2}{5} \end{pmatrix}$

(iii) $\int_C ds \frac{1}{\sqrt{1+4x^2}} = \int_C \left|\frac{d\vec{r}}{dt}\right| dt \; \frac{1}{\sqrt{1+4t^2}}$

$= \int_0^1 dt \cdot 1 = 1$

(iv) $\int_C ds = \int_0^1 \sqrt{1+4t^2}\, dt \quad \overset{2t = y}{\underset{\frac{dy}{dt} = 2}{=}} \quad \int_0^2 \frac{1}{2} \sqrt{1+y^2}\, dy$

$= \left[\frac{1}{4} y \sqrt{1+y^2} + \frac{1}{2} \operatorname{arsinh} y \right]_0^2$

$= 0{,}837$

(3)

$$\vec{e}_\phi = \frac{\partial r}{\partial \phi} = \begin{pmatrix} -R \sin\phi \sin\theta \\ R\cos\phi \sin\theta \\ 0 \end{pmatrix}$$

$$\vec{e}_\theta = \frac{\partial r}{\partial \theta} = \begin{pmatrix} R\cos\phi\cos\theta \\ R\sin\phi\cos\theta \\ -R\sin\theta \end{pmatrix}$$

$$\vec{e}_\phi \times \vec{e}_\theta = \begin{pmatrix} -R^2\cos\phi\sin^2\theta \\ -R^2\sin\phi\sin^2\theta \\ (-R^2\sin^2\phi\sin\theta\cos\theta - R^2\cos^2\phi\sin\theta\cos\theta) \\ (-R^2\sin\theta\cos\theta\,\underbrace{(\sin^2\phi+\cos^2\phi)}_{1}) \end{pmatrix}$$

$$= R^2 \begin{pmatrix} -\cos\phi\sin^2\theta \\ -\sin\phi\sin^2\theta \\ -\sin\theta\cos\theta \end{pmatrix}$$

$$|\vec{e}_\phi \times \vec{e}_\theta| = R^2\sqrt{\cos^2\phi\sin^4\theta + 2\cos\phi\sin^2\theta\sin\phi\cos^2\theta + \sin^2\phi\cos^4\theta + \cos^2\phi\cos^4\theta - 2\cos\phi\cos^2\theta\sin\phi\sin^2\theta + \sin^2\phi\sin^4\theta + \sin^2\theta\cos^2\theta}$$

$$= R^2\sqrt{\cos^4\theta + \sin^4\theta + \sin^2\theta\cos^2\theta}$$

$$= R^2\sqrt{\cos^4\theta + \sin^2\theta} = R^2\sqrt{\cos^4\theta + 1 - \cos^2\theta}$$

$$|\vec{e}_\phi \times \vec{e}_\theta| = \sqrt{\cos^2\phi\sin^4\theta + \sin^2\phi\sin^4\theta + \sin^2\theta\cos^2\theta}$$

$$= \sqrt{\sin^4\theta + \sin^2\theta\cos^2\theta}$$

$$= \sqrt{\sin^2\theta}$$

$$= \sin\theta$$

$$\int = \int_0^\pi d\theta \int_0^{2\pi} d\phi\, R^2 \sin\theta = \int_0^\pi d\theta\, 2\pi R^2 \sin\theta = 4\pi R^2$$