

Q) Find the global minimum point and value for the function $f(x, y) = x^2 + y^2 + 10$ (manual calculations for 2 itr's)

Sol: step 1: $x=1, y=2, \text{epochs}=2, \eta=0.1$

step 2: $\text{itr}=1$

step 3: $\frac{\partial f}{\partial x} = 2x$

$$\left. \frac{\partial f}{\partial x} \right|_{x=1} = 2(1) = 2$$

$$\frac{\partial f}{\partial y} = 2y$$

$$\left. \frac{\partial f}{\partial y} \right|_{y=2} = 2(2) = 4$$

step 4: $\Delta x = -\eta \frac{\partial f}{\partial x} = -(0.1)(2) = -0.2$

$$\Delta y = -\eta \frac{\partial f}{\partial y} = -(0.1)(4) = -0.4$$

step 5: $x = x + \Delta x = 1 - 0.2 = 0.8$

$$y = y + \Delta y = 2 - 0.4 = 1.6$$

step 6: $\text{itr} = \text{itr} + 1 = 1 + 1 = 2$

step 7: $\text{if}(\text{itr} > \text{epochs}) = (2 > 2) \Rightarrow (\text{false})$

go to next step

else

go to step 3

step 3: $\frac{\partial f}{\partial x} = 2x \Rightarrow \left. \frac{\partial f}{\partial x} \right|_{x=0.8} = 2(0.8) = 1.6$

$$\frac{\partial f}{\partial y} = 2y \Rightarrow \frac{\partial f}{\partial y} \Big|_{y=1.6} = 2(1.6) = 3.2$$

step 4: $\Delta x = -\eta \frac{\partial f}{\partial x} = -(0.1)(1.6) = -0.16$

$$\Delta y = -\eta \frac{\partial f}{\partial y} = -(0.1)(3.2) = -0.32$$

step 5: $x = x + \Delta x = 0.8 - 0.16 = 0.64$

$$y = y + \Delta y = 1.6 - 0.32 = 1.28$$

step 6: $itr = itr + 1 = 2 + 1 = 3$

step 7: if ($itr > epochs$)

$$3 > 2 \Rightarrow \text{True}$$

go to nextstep

else

goto step-3

step 8: print x and y values after 2 iterations

$$x = 0.64$$

$$y = 1.28$$

$$f(x, y) = x^2 + y^2 + 10$$

$$= (0.64)^2 + (1.28)^2 + 10$$

$$= 0.4096 + 1.6384 + 10$$

$$= 12.048$$