Simulating Neutrino Oscillation Experiments with GLoBES

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– Theory of Neutrino Oscillations
– State of the Art in Neutrino Physics

Part 2: Joachim Kopp (`jkopp@ph.tum.de`)
– The GLoBES Software Package
– Simulation of Future Experiments

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18.07. - 22.07.2005, Ringberg Castle

Outline

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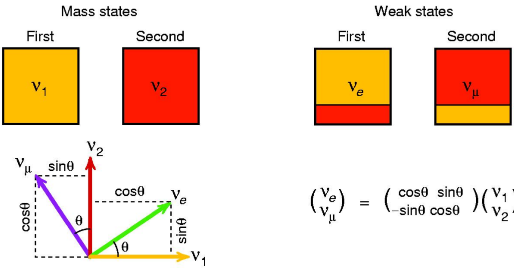
Mass and flavour eigenstates

- For neutrinos, the eigenstates of the mass operator are not equal to the ones of the weak interaction:



$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

- Massless neutrinos: the mass eigenstates can be arbitrarily chosen $\Rightarrow$ can be defined as identical to the flavour eigenstates $\Rightarrow$ no oscillations! $\Rightarrow$ oscillations are the evidence for non-zero neutrino mass!

The formalism

- Let $(|\nu_e\rangle, |\nu_\mu\rangle)$ be the flavour and $(|\nu_1\rangle, |\nu_2\rangle)$ the mass eigenstates $\Rightarrow$ every flavour-state can be written as a linear combination of mass eigenstates (since it is a rotation one can use a rotation matrix):

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

- Time evolution:

$$|\nu_e(t)\rangle \; = \; \cos\theta \, e^{-iE_1 t} |\nu_1(t=0)\rangle \; + \; \sin\theta \, e^{-iE_2 t} |\nu_2(t=0)\rangle$$

- Energy momentum relation:

$$E_i^2 = p_i^2 + m_i^2 \Rightarrow E_i = \sqrt{p_i^2 + m_i^2} \approx p_i + \frac{m_i^2}{2p_i}$$

Diasappearance Probability

- Assumption: both of the mass eigenstates have the same momentum p (in fact, this does not have to be true, but the longer calculation gives the same result) $\Rightarrow$ with $p = E_\nu$: $E_2 = E_1 - \frac{\Delta m^2}{2E_\nu}$ where $\Delta m^2 = m_1^2 - m_2^2$
- E. g. for the muon-neutrino we get:

$$|\nu_\mu\rangle = e^{-i(p+\frac{m_1^2}{2p})t} \cdot \left[-\sin\theta \, |\nu_1(0)\rangle + \cos\theta \, e^{\frac{i\Delta m^2}{2E_\nu}} |\nu_2(0)\rangle \right]$$

- With a simple calculation (trigonometric identities, etc.) one can construct the probability for detecting a ν_μ as a ν_e at a time t:

$$P_{\nu_\mu \to \nu_e}(t) = |\langle \nu_e(0) | \nu_\mu(t) \rangle|^2 = |(\cos\theta \, \langle \nu_1(0)| + \sin\theta \, \langle \nu_2(0)|) \cdot |\nu_\mu(t)\rangle|^2$$

$= \sin^2(2\theta) \cdot \sin^2(\frac{t\Delta m^2}{4E_\nu}) \Rightarrow$ oscillates with time!

Survival Probability

- Probability to see the ν_μ again as a ν_μ:

$$P_{\nu_\mu \to \nu_\mu}(t) = 1 - P_{\nu_\mu \to \nu_e}(t) = 1 - \sin^2(2\theta) \cdot \sin^2(\frac{t \Delta m^2}{4E_\nu})$$

- Neutrinos travel (nearly) with the speed of light $\Rightarrow$ one can set t=x, where x is the distance to the place where the neutrino was born
- Oscillation length: distance after which the ν_μ is again a 100 % ν_μ

$$\sin^2(\frac{x \Delta m^2}{4E_\nu}) \equiv \sin^2(\frac{\pi x}{L_0}) \Rightarrow L_0 = \frac{4\pi E_\nu}{\Delta m^2} \propto \frac{E_\nu}{\Delta m^2}$$

$\rightarrow$ important quantity for experiments

- Notice, that only the squared mass difference Δm^2 can be determined by oscillation experiments!

Matter Effects

- The Hamiltonian in the basis of the mass eigenstates is diagonal $\Rightarrow$ Schrödinger's equation (one can skip the first p from $E = p + \frac{m^2}{2p}$ since it just gives a global phase):

$$i\frac{d}{dt}\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = H_M \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} = \frac{1}{2p} \cdot \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

- To go into the flavour basis, one has to rotate the states as well as the Hamiltonian by a unitary matrix U, which leeds to the Hamiltonian H_F in the flavour basis:

$$H_F = \frac{1}{2p} \cdot \begin{pmatrix} m_{ee}^2 & m_{e\mu}^2 \\ m_{e\mu}^2 & m_{\mu\mu}^2 \end{pmatrix} = U H_M U^\dagger =$$

$$= \frac{1}{2p} \cdot \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \cdot \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} =$$

Matter Effects

$$= \frac{1}{4p}\left[\Sigma\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + D\begin{pmatrix}-\cos(2\theta) & \sin(2\theta)\\ \sin(2\theta) & \cos(2\theta)\end{pmatrix}\right]$$

where $\Sigma = m_1^2 + m_2^2$ and $D = m_2^2 - m_1^2$

- Since it is diagonal, the first term just gives a common phase $\Rightarrow$ has no physical meaning
- Coherent forward scattering in matter: ν_e and $\overline{\nu}_e$ can - different from all other flavours - interact with the electrons via CC-reactions $\Rightarrow$ change of the oscillation probabilities!
- The Hamiltonian in the flavour basis has to be changed:

$$H_F \rightarrow H_{F,mat} = H_F + \frac{1}{2p}\begin{pmatrix}A & 0\\ 0 & 0\end{pmatrix}$$

where $A = 2\sqrt{2}G_F N_e p$ (G_F: Fermi's constant, N_e: electron density)

Matter Effects

- Back to the mass eigenstates $\Rightarrow$ after a lengthy calculation we get:

$$H_{M,mat} = \frac{1}{2p} \cdot \begin{pmatrix} m_1^2 + A\cos^2(\theta) & A\sin(\theta)\cos(\theta) \\ A\sin(\theta)\cos(\theta) & m_2^2 + A\sin^2(\theta) \end{pmatrix}$$

  $\Rightarrow$ this Hamiltonian is not diagonal anymore $\Rightarrow$ the mass eigenstates of the vacuum are not eigenstates in matter
- So one has to diagonalize again to get the new mass eigenstates $|\nu_{1m}\rangle$ and $|\nu_{2m}\rangle$ in matter, which gives

$$m_{1,2m}^2 = \frac{1}{2} \cdot \left[(\Sigma + A) \pm \sqrt{(A - D\cos(2\theta))^2 + D^2\sin^2(2\theta)} \right]$$

  and so the squared mass difference in matter is (remember that $D = m_2^2 - m_1^2$)

$$D_m = m_{2m}^2 - m_{1m}^2 = D \cdot \sqrt{(\frac{A}{D} - \cos(2\theta))^2 + \sin^2(2\theta)}$$

Matter Effects

- So we also have in matter a new mixing angle θ_{mat} and so a new transition probability

$$P_{mat,\nu_e \to \nu_\mu}(x) = \sin^2(2\theta_{mat})\sin^2\left(\frac{xD_m}{4p}\right)$$

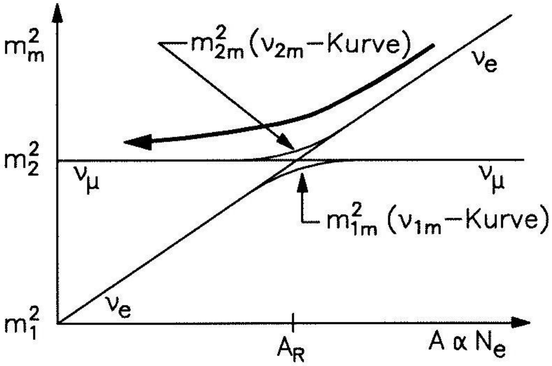
- The oscillation amplitude is

$$\sin^2(2\theta_{mat}) = \frac{\sin^2(2\theta)}{(\frac{A}{D} - \cos(2\theta))^2 + \sin^2(2\theta)} = f(A/D) = f(E_\nu N_e)$$

$\Rightarrow$ the ratio $\frac{A}{D}$ determines the oscillation probabilities in matter $\Rightarrow$ for $\frac{A}{D} \approx \cos(2\theta)$ we have a resonance effect $\Rightarrow$ **a muon-neutrino can at certain distances be a 100% electron-neutrino!!!**

Flavour-Flipping/The MSW-Effect

(Mikheyev-Smirnov-Wolfenstein)



e.g. the first mass eigenstate (ν_{1m}-curve) is most probably a ν_e below and most probably a ν_μ above the resonant matter density A_R

3-Flavour Oscillations

- For 3 neutrino-flavours the mixing looks like:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \cdot \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}$$

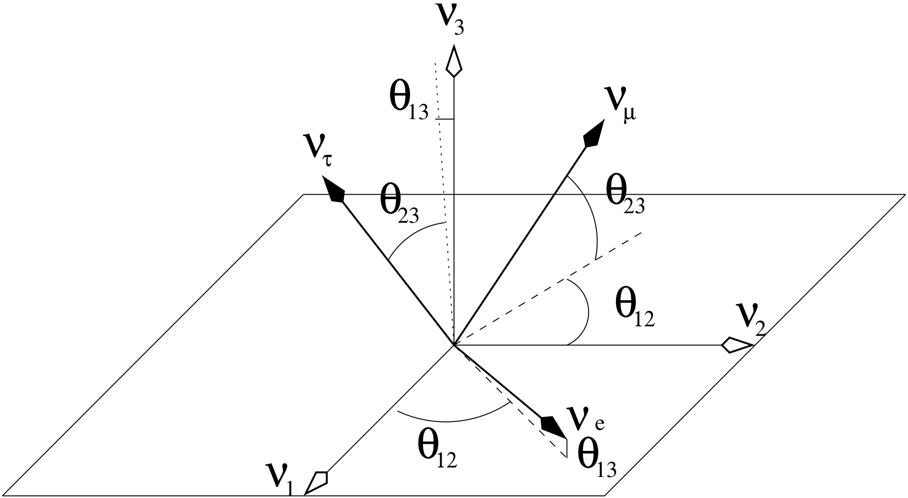
  with $U^\dagger = U^{-1}$ and

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

  where $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$

- Parameters: 3 mixing angles (θ_{12}, θ_{23} and θ_{13}), 1 CP-violating phase (δ) and 2 squared mass differences ($(\Delta m^2)_{sol} = (\Delta m^2)_{21}$ and $(\Delta m^2)_{atm} = (\Delta m^2)_{31}$)

3-Neutrino-Mixing



Outline

The Oscillation Parameters

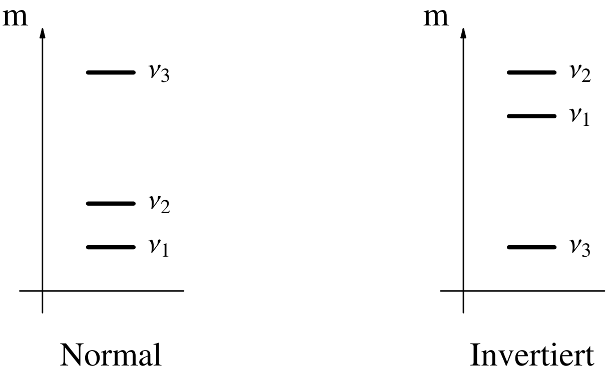
Parameter	Best fit value and $\pm 3\sigma$ range	Experiments
$\theta_{12} \approx 32.6^\circ$	$\sin^2(\theta_{12}) = 0.29^{+0.08}_{-0.06}$	solar/reactor neutrinos
$\theta_{23} \approx 45.0^\circ$	$\sin^2(\theta_{23}) = 0.50 \pm 0.16$	atmospheric/accelerator
$\theta_{13} \lesssim 9.6^\circ$	$\sin^2(\theta_{13}) = 0.000 + 0.047$	CHOOZ-experiment
δ	?????	several proposals
$(\Delta m^2)_{21}$	$(8.1^{+1.0}_{-0.8}) \cdot 10^{-5}\ \mathrm{eV}^2$	solar/reactor
$(\Delta m^2)_{31}$	$(2.2^{+1.1}_{-0.8}) \cdot 10^{-3}\ \mathrm{eV}^2$	atmospheric/accelerator

taken from Maltoni, Schwetz et al., New J. Phys. 6 (2004)
see also: Bahcall et al., JHEP 08 (2004) or Bandyopadhyay et al., Phys. Lett. B608 (2005)

- Two large mixing angles and one small mixing angle ($\neq$CKM!).
- No information about CP violation.
- Precise measurement of neutrino mixing parameters is essential for a deeper theoretical understanding of flavour physics and physics beyond the Standard Model.

Mass Hierarchies

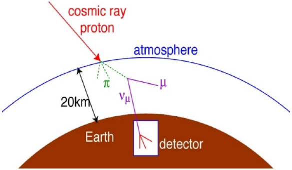
Problem: since oscillation only measures squared mass differences, and we do not know the sign of $(\Delta m^2)_{atm}$ yet, there could be different hierarchies



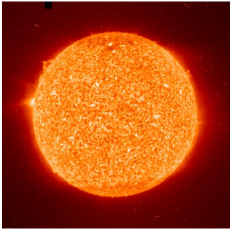
$\Rightarrow$ from the oscillations we get only the information, that neutrinos do have mass, but nothing about the absolute values

Where do we know all that from?

- Super-Kamiokande (atmospheric ν's), K2K,... $\Rightarrow \theta_{23}$, $(\Delta m^2)_{31}$



- Solar neutrinos, KamLAND, ... $\Rightarrow \theta_{12}$, $(\Delta m^2)_{21}$



- CHOOZ $\Rightarrow \theta_{13}$

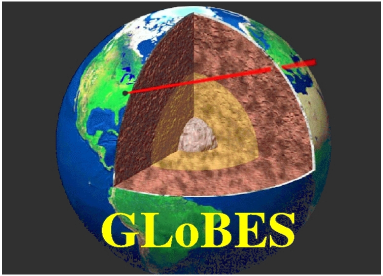
The next steps

- Achieving a better precision for the known mixing parameters
- Measurement of the small mixing angle θ_{13}
- Measurement of the CP-phase δ

AND (what we do): Making Simulations with GLoBES

Outline

The GLoBES software package



- C library for simulating neutrino oscillation experiments
- Provides sophisticated functions for high level and low level access to the simulation
- AEDL — Abstract Experiment Definition Language for defining experimental setups
- Developed at TUM by P. Huber, M. Lindner and W. Winter

Comput. Phys. Commun. 167:195, 2005, hep-ph/0407333

Components of a GLoBES simulation

Neutrino Sources

- Energy and flavour spectrum
- Signal and Background

Oscillations

- Fundamental parameters: θ_{12}, θ_{13}, θ_{23}, δ, Δm^2_{21}, Δm^2_{31}.
- Baseline and matter density profile

Detector

- Interaction cross sections
- Energy resolution
- Systematical errors

Analysis

- Number and width of energy bins
- Fitting parameters to the simulated data
- χ^2 analysis

Calculation of event rates

$$\frac{dn_f^{\mathrm{IT}}}{dE'} \propto \sum_{f_i} \int \int dE\, d\hat{E} \quad \underbrace{\Phi_{f_i}(E)}_{\text{Production}} \times$$

$$\underbrace{\frac{1}{L^2} P_{(f_i \to f)}(E, L, \rho; \theta_{23}, \theta_{12}, \theta_{13}, \Delta m_{31}^2, \Delta m_{21}^2, \delta)}_{\text{Propagation}} \times$$

$$\underbrace{\sigma_f^{\mathrm{IT}}(E) k_f^{\mathrm{IT}}(E - \hat{E})}_{\text{Interaction}} \times \underbrace{T_f(\hat{E}) V_f(\hat{E} - E')}_{\text{Detection}}$$

P. Huber, M. Lindner, W. Winter, Nucl. Phys. B645:3-48, 2002, hep-ph/0204352

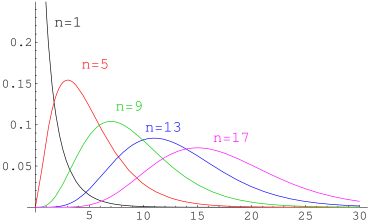
E	= Neutrino energy	$P_{(f_i \to f)}(\ldots)$	= Oscillation probability
$\hat{E}$	= Charged Lepton energy	$\sigma_f^{\mathrm{IT}}(E)$	= Cross sections
E'	= Reconstructed energy	$k_f^{\mathrm{IT}}(E - \hat{E})$	= Charged lepton distribution
$\Phi_{f_i}(E)$	= Source flux	$T_f(\hat{E})$	= Threshold function
L	= Baseline	$V_f(\hat{E} - E')$	= Energy resolution function

Sensitivity of a reactor experiment to $\sin^2(2\theta_{13})$

1. Define what you want to measure: What 90 % C.L. limit can the experiment set on $\sin^2(2\theta_{13})$, assuming the true value is 0?
2. Define the setup using AEDL (Abstract Experiment Definition Language)
3. Calculate the “true” event rates.
4. Now calculate the expected event rates for nonzero values of $\sin^2(2\theta_{13})$.
5. Perform a χ^2 analysis to find out which values produce event rates that are still compatible with $\sin^2(2\theta_{13}) = 0$ at the chosen C.L.

χ^2 Analysis in GLoBES

- If X_i are normally distributed with $\langle X_i \rangle = 0$ and $\sigma = 1$, then $\chi^2 = \sum_{i=1}^{n} X_i^2$ is distributed according to the χ^2 distribution with n degrees of freedom.



- For n energy bins with event rates following the Poisson distribution

$$\chi^2 = \sum_{i=1}^{n} 2(N_i(\vec{\Lambda}) - N_i(\vec{\Lambda}_0)) + 2N_i(\vec{\Lambda}) \log \frac{N_i(\vec{\Lambda})}{N_i(\vec{\Lambda}_0)}$$

follows a χ^2 distribution with $\dim \vec{\Lambda}$ degrees of freedom. Here:

$$\begin{aligned} N_i(\vec{\Lambda}_0) &= \text{Event rates for true parameter values } \vec{\Lambda}_0. \\ N_i(\vec{\Lambda}) &= \text{Event rates for hypothesised parameters } \vec{\Lambda}. \end{aligned}$$

χ^2 Analysis in GLoBES

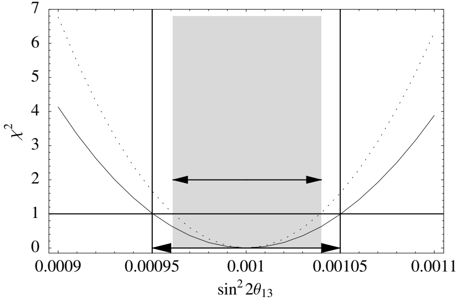
Inclusion of systematics and external input

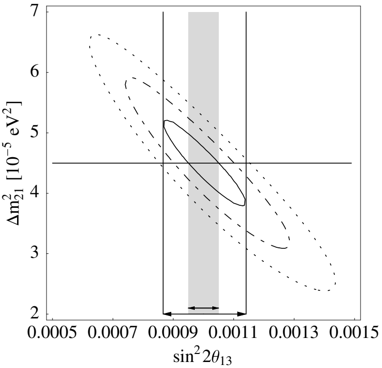
$$\chi^2 = \underbrace{\sum_{i=1}^{n} 2(N_i(\vec{\Lambda},\vec{x}) - N_i(\vec{\Lambda}_0,\vec{x}_0)) + 2N_i(\vec{\Lambda},\vec{x}) \log \frac{N_i(\vec{\Lambda},\vec{x})}{N_i(\vec{\Lambda}_0,\vec{x}_0)}}_{\text{Statistical part}}$$

$$+ \underbrace{\sum_{j=1}^{\dim \vec{\Lambda}} \frac{(\vec{\Lambda}_j - \vec{\Lambda}_{0j})^2}{\sigma^2_{\vec{\Lambda}j}}}_{\text{External input}} + \underbrace{\sum_{k=1}^{\dim \vec{x}} \frac{(\vec{x}_k - \vec{x}_{0k})^2}{\sigma^2_{\vec{x}k}}}_{\text{Systematics}}$$

$\vec{\Lambda}, \vec{\Lambda}_0$	=	True and hypothesised oscillation parameters
$\vec{x}, \vec{x}_0$	=	Systematics parameters
$N_i(\vec{\Lambda},\vec{x})$	=	Expected event rates
$\sigma_{\vec{\Lambda}j}$	=	Standard errors of external parameter input
$\sigma_{\vec{x}k}$	=	Standard errors of systematics parameters

χ^2 Analysis in GLoBES

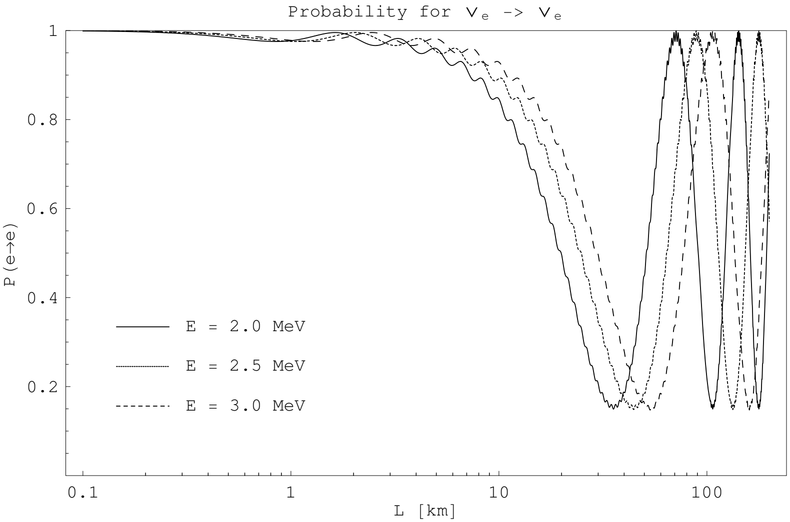




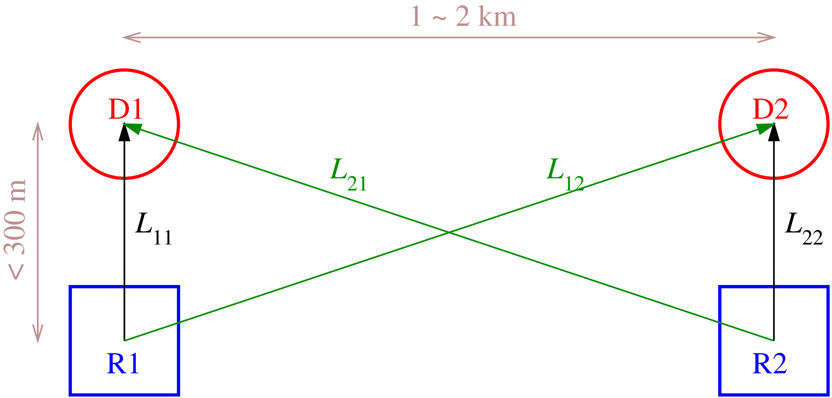
P. Huber, M. Lindner, W. Winter, Nucl. Phys. B645:3-48, 2002, hep-ph/0204352

Outline

The $\bar{\nu}_e$ disappearance channel



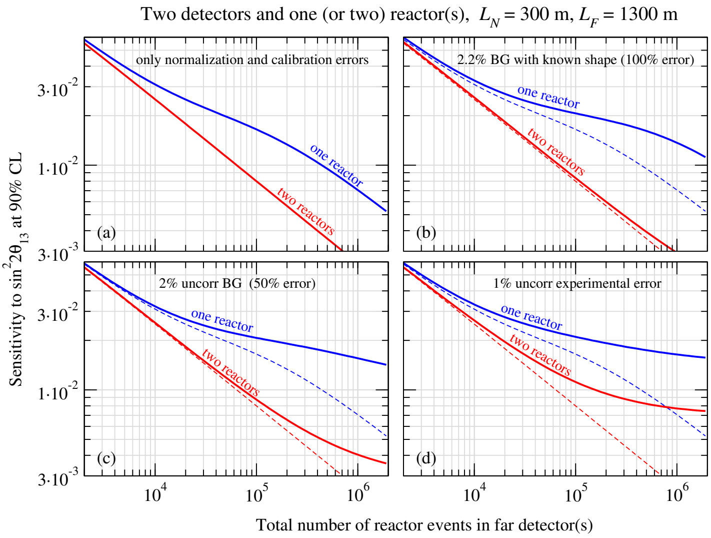
The R2D2 Setup



P. Huber, M. Lindner, T. Schwetz, JHEP 0502:029, 2005, hep-ph/0411166

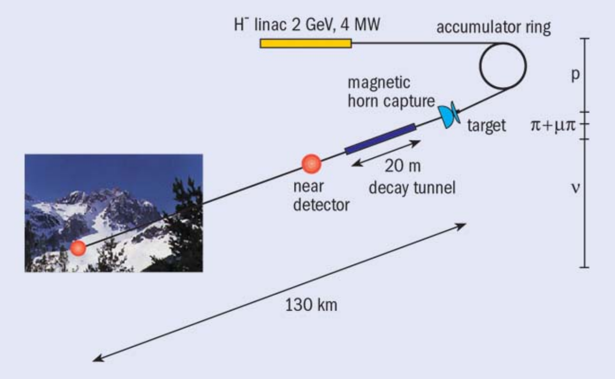
- Near and far detectors $\longrightarrow$ Elimination of flux normalization error
- Two reactors $\longrightarrow$ Elimination of detector systematics

The R2D2 Setup



P. Huber, M. Lindner, T. Schwetz, JHEP 0502:029, 2005, hep-ph/0411166

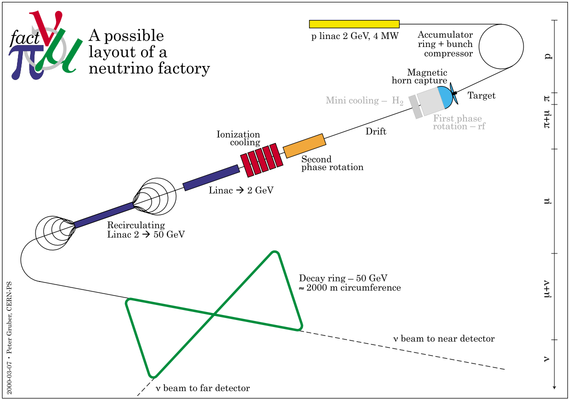
Superbeams



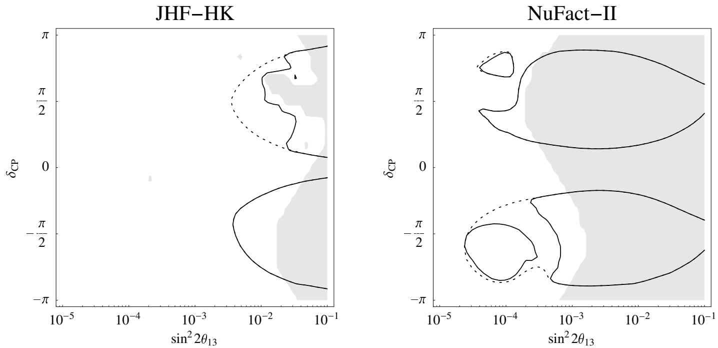
J. Bouchez, CERN Courier 42, No. 5

- High neutrino energy $\longrightarrow$ Easy to detect
- ν_μ beam with ν_e, $\bar{\nu}_e$ and $\bar{\nu}_\mu$ contamination $\longrightarrow$ limited physics potential

Neutrino Factories



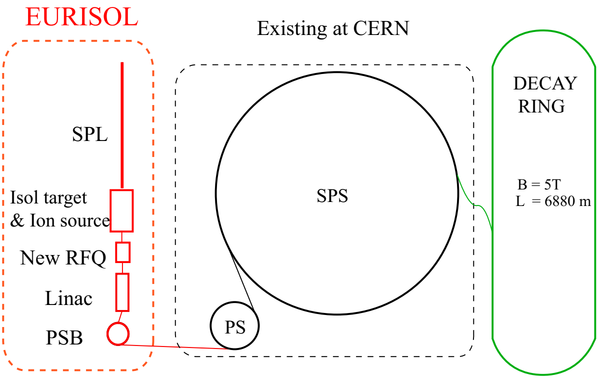
- High neutrino energy $\longrightarrow$ Easy to detect
- Long Baselines $\longrightarrow$ Exploiting matter effects
- Beam contains only ν_μ and $\bar{\nu}_e$ $\longrightarrow$ "Wrong sign muon" signature: Detection of μ^+ indicates oscillation



P. Huber, M. Lindner, W. Winter, Nucl. Phys. B645:3-48, 2002, hep-ph/0204352

- JHF-HK: E_p = 50 GeV, L = 295 km, Target Power 4 MW, 1 Mt Water Čerenkov detector
- Nu-Fact II: E_μ = 50 GeV, L = 3000 km, Target Power: 4 MW, $5 \cdot 10^{20} \frac{\mu}{\text{year}}$, 50 kt magnetized iron calorimeter

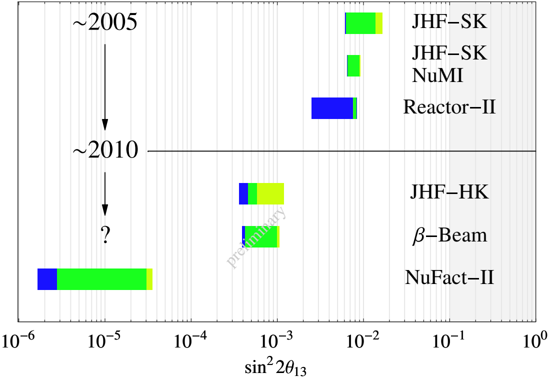
Beta Beams



J. Bouchez, M. Lindroos, M. Mezzetto, AIP Conf. Proc. 721:37-47, 2004, hep-ex/0310059

- High neutrino energy $\longrightarrow$ Easy to detect
- Long Baselines $\longrightarrow$ Exploiting matter effects
- Pure ν_e or $\bar{\nu}_e$ beam

Limitations of Neutrino Oscillation Experiments



C. Albright et.al., physics/0411123

- Correlations: Sensitivity only to combination of parameters
- Degeneracies: Distinct regions in parameter space can explain the experiment

Conclusions

- Neutrino oscillations are theoretically well understood and experimentally clearly verified.
- A precise determination of the oscillation parameters is desirable to achieve a deeper understanding of particle physics (flavour structure, . . .).
- GLoBES is a tool for simulating future neutrino oscillation experiments and determine their physics potential.
- Nuclear reactors, Superbeams, Neutrino Factories and Beta Beams are excellent tools for solving many open questions.