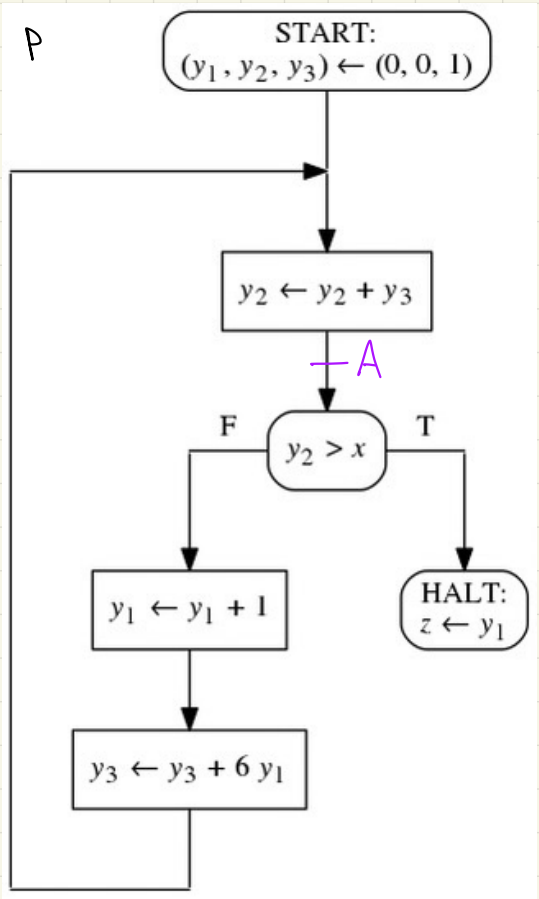
Задание 1.2



$\mathcal{D}_x = \mathcal{D}_{y_1} = \mathcal{D}_{y_2} = \mathcal{D}_{y_3} = \mathcal{D}_z = \mathbb{Z}$

$\varphi(x) \equiv x \geq 0$

$\psi(x,z) \equiv z^3 \leq x < (z+1)^3$

$S_f - A_p \quad x \geq 0 \Rightarrow p_A(x, 0, 1, 1)$

$A_p - A_p: \; x \geq 0 \wedge p_A(x, y_1, y_2, y_3) \wedge \neg\, y_2 > x \Rightarrow$
$\Rightarrow p_A(x, y_1+1, y_2 + y_3 + 6y_1 + 6, y_3 + 6y_1 + 6)$

$A_p - H_h: \; x \geq 0 \wedge p_A(x, y_1, y_2, y_3) \wedge y_2 > x \Rightarrow$
$\Rightarrow y_1^3 \leq x < (y_1+1)^3$

Пусть $p_A(x, y_1, y_2, y_3) = y_1^3 \leq x \wedge y_2 = (y_1+1)^3 \wedge y_3 = y_2 - y_1^3$. Тогда:

$A_p - H_h: \; x \geq 0 \wedge y_1^3 \leq x \wedge y_2 = (y_1+1)^3 \wedge y_3 = y_2 - y_1^3 \wedge y_2 > x \Rightarrow$

$\Rightarrow y_1^3 \leq x < (y_1+1)^3$ ✓

$S_f - A_p: \; x \geq 0 \Rightarrow 0^3 \leq x \wedge 1 = (0+1)^3 \wedge 1 = 1 - 0^3$ ✓

$0 \leq x \wedge 1 = 1 \wedge 1 = 1$

$A_p - A_p: \; x \geq 0 \wedge y_1^3 \leq x \wedge y_2 = (y_1+1)^3 \wedge y_3 = y_2 - y_1^3 \wedge x \geq y_2 \Rightarrow$

$\Rightarrow (y_1+1)^3 \leq x \wedge y_2 + y_3 + 6y_1 + 6 = (y_1+2)^3 \wedge y_3 + 6y_1 + 6 = y_2 + y_3 + 6y_1 + 6 - (y_1+1)^3$ ✓

$y_2 + y_3 + 6y_1 + 6 = y_1^3 + 6y_1^2 + 12y_1 + 8$

$y_2 + y_2 - y_1^3 + 6y_1 + 6 = y_1^3 + 6y_1^2 + 12y_1 + 8$

$2y_2 = 2y_1^3 + 6y_1^2 + 6y_1 + 2$

$y_2 = y_1^3 + 3y_1^2 + 3y_1 + 1$

$y_2 = (y_1+1)^3$

~~$y_3 + 6y_1 + 6 = y_2 + y_3 + 6y_1 + 6 - (y_1+1)^3$~~

$y_2 = (y_1+1)^3$

Все условия верификации выполнены $\Rightarrow \{\varphi\}\, P\, \{\psi\}$, ч.т.д.