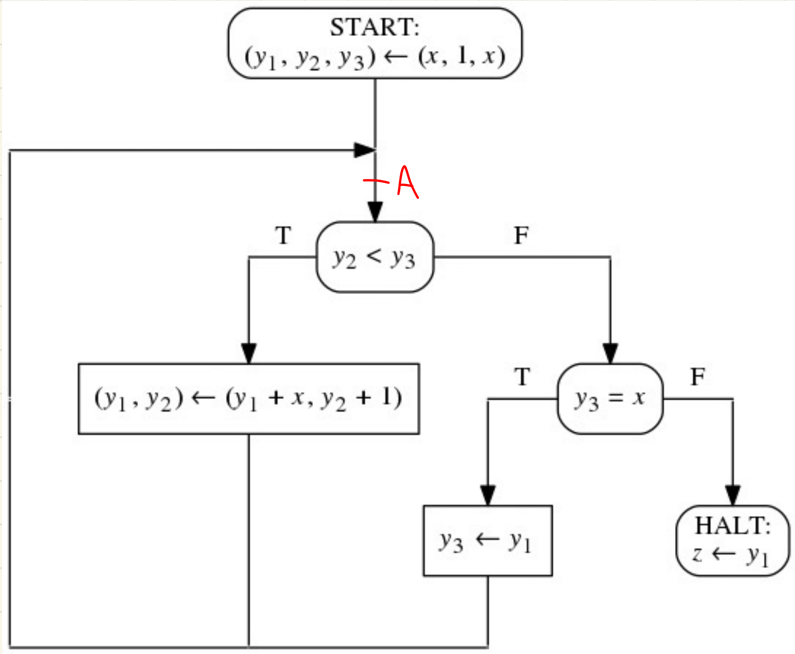
Задание 1.3



$D_x = D_{y_1} = D_{y_2} = D_{y_3} = D_z = \mathbb{Z}$

$\varphi(\bar{x}) = x > 1$

$u(\bar{x}, \bar{y}) = x^3 + x^2 - y_1 - y_3$

$q(\bar{x}, \bar{y}) = y_1 = y_2 \cdot x \wedge (y_2 \le x \wedge y_3 = x \vee x \le y_2 \le x^2 \wedge y_3 = x^2)$

$W = \mathbb{N} \cup \{0\}$

$<$ – арифметич. сравнение

Условия верификации:

(S – A): $x > 1 \Rightarrow x = 1 \cdot x \wedge (1 \le x \wedge x = x \vee x \le 1 \le x^2 \wedge x = x^2)$ ✓

(A $\overset{T}{-}$ A): $x > 1 \wedge y_2 < y_3 \wedge y_1 = y_2 \cdot x \wedge (\overbrace{y_2 \le x \wedge y_3 = x}^{1} \vee \overbrace{x \le y_2 \le x^2 \wedge y_3 = x^2}^{2}) \Rightarrow$

$\Rightarrow y_1 + x = (y_2 + 1) \cdot x \wedge (y_2 + 1 \le x \wedge y_3 = x \vee x \le y_2 + 1 \le x^2 \wedge y_3 = x^2)$ ✓

$y_1 + x = x \cdot y_2 + x$

$y_2 \cdot x + x = x \cdot y_2 + x$

1). $x > 1 \wedge y_2 < y_3 \wedge y_1 = y_2 \cdot x \wedge y_2 \le x \wedge y_3 = x \Rightarrow y_2 + 1 \le x \wedge y_3 = x$ ✓

$y_2 + 1 < y_3 + 1 = x + 1$

2). $x > 1 \wedge y_2 < y_3 \wedge y_1 = y_2 \cdot x \wedge x \le y_2 \le x^2 \wedge y_3 = x^2 \Rightarrow x \le y_2 + 1 \le x^2 \wedge y_3 = x^2$ ✓

$x \le y_2 + 1 < y_3 + 1 = x^2 + 1$

(A $\overset{FT}{-}$ A): $x > 1 \wedge y_2 \ge y_3 \wedge y_3 = x \wedge y_1 = y_2 \cdot x \wedge (\overbrace{y_2 \le x \wedge y_3 = x}^{1} \vee \overbrace{x \le y_2 \le x^2 \wedge y_3 = x^2}^{2}) \Rightarrow$

$\Rightarrow y_1 = y_2 \cdot x \wedge (y_2 \le x \wedge y_1 = x \vee x \le y_2 \le x^2 \wedge y_1 = x^2)$ ✓

1). $x > 1 \wedge y_2 \ge y_3 \wedge y_3 = x \wedge y_1 = y_2 \cdot x \wedge y_2 \le x \wedge y_3 = x \Rightarrow x \le y_2 \le x^2 \wedge y_1 = x^2$ ✓

$y_2 = x$

$x \le x \le x^2$

$y_2 \cdot x = x^2$

$x \cdot x = x^2$

2). $\underbrace{x > 1 \wedge y_2 \ge y_3 \wedge y_3 = x \wedge y_1 = y_2 \cdot x \wedge x \le y_2 \le x^2 \wedge y_3 = x^2}_{F} \Rightarrow \dots$ ✓

Условие корректности выбора оценочной функции:

$x > 1 \wedge y_1 = y_2 \cdot x \wedge (\overbrace{y_2 \le x \wedge y_3 = x}^{1} \vee \overbrace{x \le y_2 \le x^2 \wedge y_3 = x^2}^{2}) \Rightarrow x^3 + x^2 - y_1 - y_3 \ge 0$ ✓

1). $x > 1 \wedge y_1 = y_2 \cdot x \wedge y_2 \le x \wedge y_3 = x \Rightarrow x^3 + x^2 - y_1 - y_3 \ge 0$

$x^3 + x^2 - y_1 - x \ge x^3 + x^2 - x^2 - x = x^3 - x > 0$ ✓

2). $x > 1 \wedge y_1 = y_2 \cdot x \wedge x \le y_2 \le x^2 \wedge y_3 = x^2 \Rightarrow x^3 + x^2 - y_1 - y_3 \ge 0$

$x^3 + x^2 - y_1 - x^2 = x^3 - y_1 \ge x^3 - x^2 > 0$ ✓

Условие завершаемости:

$A \overset{T}{-} A$: $x > 1 \wedge y_2 < y_3 \wedge y_1 = y_2 \cdot x \wedge (y_2 \leq x \wedge y_3 = x \vee x \leq y_2 \leq x^2 \wedge y_3 = x^2) \Rightarrow$ ✓

$\Rightarrow \cancel{x^3} + \cancel{x^2} - (y_1 + x) - \cancel{y_3} < \cancel{x^3} + \cancel{x^2} - y_1 - \cancel{y_3}$ ✓

$-y_1 - x < -y_1$

$-x < 0$

$A \overset{FT}{-} A$: $x > 1 \wedge y_2 \geq y_3 \wedge y_3 = x \wedge y_1 = y_2 \cdot x \wedge (\overbrace{y_2 \leq x \wedge y_3 = x}^{1} \vee \overbrace{x \leq y_2 \leq x^2 \wedge y_3 = x^2}^{2}) \Rightarrow$ ✓

$\Rightarrow \cancel{x^3} - \cancel{x^2} - \cancel{y_1} - y_1 < \cancel{x^3} + \cancel{x^2} - \cancel{y_1} - y_3$

$-y_1 < -y_3$

$y_3 < y_1$

1). $x > 1 \wedge y_2 \geq y_3 \wedge y_3 = x \wedge y_1 = y_2 \cdot x \wedge y_2 \leq x \wedge y_3 = x \Rightarrow y_3 < y_1$ ✓

$$\left.\begin{array}{l} y_3 = x \\ y_1 = y_2 \cdot x \Rightarrow \{ y_2 \geq y_3 = x \wedge y_2 \leq x \} \Rightarrow y_1 = x^2 \end{array}\right\} \Rightarrow y_3 < x$$

2). $\underbrace{x > 1 \wedge y_2 \geq y_3 \wedge y_3 = x \wedge y_1 = y_2 \cdot x \wedge x \leq y_2 \leq x^2 \wedge y_3 = x^2}_{F} \Rightarrow y_3 < y_1$ ✓

Все условия метода фундированных множеств выполнены $\Rightarrow$ P завершается на φ.