# Integer Programming Marketing Optimization Report

Maximizing ROI under Tiered Investment and Strategic Constraints

Archie Don, Jake Embleton, Gabriel Kinshuk, Vinay Sangamalli

RM 294 - Optimization I
Project 2

November 2, 2025

# Contents


**1 Summary** **3**

**2 Summary** **3**

**3 Introduction** **3**

3.1 ROI Optimization . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3

3.2 Tiered Investment & Strategic Constraints . . . . . . . . . . . . . . . . . . . . 4

3.3 Project Objectives . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4

**4 Methodology** **5**

4.1 Data Loading & Preprocessing . . . . . . . . . . . . . . . . . . . . . . . . 5

4.2 Linear Program Formulation . . . . . . . . . . . . . . . . . . . . . . . . . 5

4.3 Mixed Integer Program Formulation . . . . . . . . . . . . . . . . . . . . . 6

4.4 Model Allocation Cross-Evaluation . . . . . . . . . . . . . . . . . . . . . . 7

4.5 Platform Cap Constraint Analysis . . . . . . . . . . . . . . . . . . . . . . 8

4.6 Mixed Integer Program with Minimum Spend Per Platform . . . . . . . . . . 9

4.7 Mixed Integer Program with Monthly Reinvesting . . . . . . . . . . . . . . . 10

4.8 Allocation Stability . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 10

**5 Results & Discussion** **11**

5.1 Optimal Linear Program Budget Allocation . . . . . . . . . . . . . . . . . . 11

5.2 Mixed-Integer Program Budget Allocation . . . . . . . . . . . . . . . . . . 11

5.3 Model Cross-Evaluation . . . . . . . . . . . . . . . . . . . . . . . . . . . 12

5.3.1 Model Cross-Evaluation Summary . . . . . . . . . . . . . . . . . . . . 12

5.4 Platform Cap Constraint Analysis . . . . . . . . . . . . . . . . . . . . . . 13

5.5 MIP with Minimum Spend . . . . . . . . . . . . . . . . . . . . . . . . . . 13

5.6 MIP with Monthly Reinvesting . . . . . . . . . . . . . . . . . . . . . . . . 14

5.7 Allocation Stability Analysis . . . . . . . . . . . . . . . . . . . . . . . . . 15

5.7.1 Modeling Stability with Hard Constraints . . . . . . . . . . . . . . . . 16

**6 Conclusion** **18**

**7 Appendix** **19**

# 1 Summary

# 2 Summary

This report evaluates strategies for allocating a \$10M marketing budget across multiple platforms using linear programming (LP) and mixed-integer programming (MIP) approaches. The MIP consistently produced more robust allocations, with a smaller out-of-sample ROI drop (39.3%) compared to the LP (48.9%), and better maintained strategic diversification through platform caps and minimum spend constraints. Relaxing platform caps provided only modest ROI improvements (2.5% for LP, 3.1% for MIP), confirming that caps primarily support diversification.

The rolling monthly reinvestment model yielded the highest theoretical returns (70.8% annual growth), but allocations were highly unstable, with 32 month-to-month violations exceeding \$1M. This volatility highlights potential operational challenges and switching costs in practical deployment. We also have yet to quantify the potential positive effect of running campaigns over a longer period, as opposed to multiple short-term campaigns.

Overall, the findings suggest that the MIP with platform caps and minimum spend thresholds is the most practical approach for real-world budget allocation. Future work should investigate methods to smooth monthly allocations and assess operational costs associated with frequent budget adjustments, in order to access the high returns of the reinvestment model while maintaining feasible implementation.

# 3 Introduction

## 3.1 ROI Optimization

In marketing analytics, firms are increasingly focused on optimizing return on investment (ROI) across a range of advertising platforms. Similar to how portfolio managers balance risk and return, marketing analysts must determine how to allocate limited budgets to maximize total profit while satisfying strategic and operational constraints. A naïve approach might be to direct all funds toward the medium with the highest expected ROI. However, this strategy often fails in practice because the marginal effectiveness of investment typically diminishes as spending increases. In other words, additional investment in a given platform yields progressively smaller returns beyond certain thresholds. Additionally, investing the entire budget in a single platform may be overly susceptible to poor ROI forecasts due to the lack of diversification.

To address this, data-driven optimization techniques can be employed to systematically determine how to allocate marketing resources across multiple channels. By formulating the problem mathematically, it becomes possible to compare potential strategies objectively, quantify trade-offs, and identify allocations that maximize total expected return under realistic business constraints.

## 3.2 Tiered Investment & Strategic Constraints

Each marketing platform's ROI is modeled as a piecewise constant function, where the total return increases in segments or "tiers" as the investment grows. The first tier typically provides the highest marginal return, with subsequent tiers offering progressively lower rates. This piecewise structure creates a concave relationship between investment and total return for each individual platform when ROI decreases monotonically, allowing the optimization to be formulated as a linear program.

However, in many practical cases, ROI data may not follow a strictly decreasing pattern, making the total return function non-concave. In such cases, a linear programming formulation is no longer sufficient, and the problem must be addressed through integer programming techniques that explicitly enforce the logical relationships between tiers. This is usually done using binary variables that "track" if the previous tiers for a platform have been filled.

In addition to these nonlinear investment dynamics, marketing budget allocations are often subject to a range of strategic and operational constraints. These may include limits on how much can be spent across certain categories of media, proportional spending requirements between traditional and digital channels, and upper bounds on individual platform investments. In many cases, such constraints also capture implicit dependencies among marketing activities. For example, meaningful investment in search engine optimization (SEO) may depend on maintaining adequate spending in social media or display advertising, since SEO efforts are most effective when there is sufficient promotional content for users to engage with once they arrive. These interdependencies reflect the interconnected nature of modern marketing strategies, where the performance of one channel is often reinforced by activity in others. Incorporating such considerations into the optimization framework ensures that the resulting allocation remains both realistic and strategically coherent.

## 3.3 Project Objectives

In this report, we use ROI data from two consulting firms to determine optimal marketing budget allocations across multiple advertising platforms for a fixed $10 million budget. We first construct a linear programming model assuming concave ROI data, then extend the formulation to a mixed-integer program to handle non-monotonic ROI patterns and enforce logical tier dependencies. We explore several scenarios, including the impact of platform-specific investment limits, proportional allocation requirements, minimum investment thresholds, and multi-period reinvestment strategies. Additionally, we assess allocation stability over time and investigate the sensitivity of the results to different ROI estimates. The goal of these analyses is to evaluate how different optimization approaches perform under realistic business constraints and to provide actionable recommendations for allocating marketing resources efficiently.

# 4 Methodology

## 4.1 Data Loading & Preprocessing

We begin by loading ROI forecasts provided by a consulting firm and converting each column to the appropriate data type. These forecasts define both the expected ROI and the investment bounds for each tier of every platform. To facilitate tiered investment in the optimization models, we transform the tier bounds so that, for each tier, the lower bound is set to zero and the upper bound is the difference between the original upper and lower limits. For terminal tiers with no explicit upper bound, we replace infinity with a suitably large number. In this case, we use the overall budget cap of $10M.

## 4.2 Linear Program Formulation

To determine the optimal allocation of a fixed marketing budget across multiple platforms under concave ROI assumptions, we formulate a linear program that maximizes total expected return. Let $x_i$ denote the investment in tier $i$ of a platform. Each tier has a lower bound $L_i = 0$ and an upper bound $U_i$ equal to the width of the tier. Let $\mathrm{ROI}_i$ denote the return per unit investment in tier $i$. The LP can be written as:

Maximize:

$$\sum_{i=1}^{n} \mathrm{ROI}_i \, x_i$$

This is not the usual description, the de

Subject to:

$$\begin{aligned}
\sum_{i=1}^{n} x_i &\leq B && \text{(Budget)} \\
\sum_{i \in \text{Print, TV}} x_i &\leq \sum_{i \in \text{Facebook, Email}} && \text{(Trad} \leq \text{digital)} \\
\sum_{i \in \text{Social Media}} x_i &\geq 2 \sum_{i \in \text{SEO, AdWords}} && \text{(Social} \geq 2\times \text{ search)} \\
\sum_{i \in \text{Platform } p} x_i &\leq C_p && \forall p \text{ (Platform cap)} \\
0 \leq x_i &\leq U_i && \forall i \text{ (Tier bounds)}
\end{aligned}$$

Here, $B$ is the total budget (e.g., $10M), and $C_p$ is the optional maximum allocation per platform (e.g., $3M). This constraint is treated as optional, as we later evaluate the model both with and without it.

Each variable $x_i$ represents the amount invested in a given tier. The objective sums the products of tier investments and their corresponding ROIs to compute total expected return. The first constraint ensures that total investment does not exceed the budget. The next two constraints enforce strategic allocation rules across platforms. The optional platform cap prevents over-investment in any single platform. Finally, the bounds on $x_i$ ensure that each tier receives a feasible amount of investment. Solving this LP with Gurobi yields the allocation $x_i^*$ for each tier that maximizes expected return while satisfying all constraints.

We define a function to optimize the budget allocation by:

```
def solve_lp_model(roi_df, budget, use_platform_cap=True,
    platform_cap=3.0, verbose=True):
...
    # Objective
    mod.setObjective(gp.quicksum(ROI[i] * x[i] for i in range(n)),
        sense=gp.GRB.MAXIMIZE)
...
```

With an illustrative constraint:

```
...
    # Print + TV <= Facebook + Email
    mod.addConstr(
        gp.quicksum(x[i] for i in range(n) if platforms[i] in ['
            Print', 'TV']) <=
        gp.quicksum(x[i] for i in range(n) if platforms[i] in ['
            Facebook', 'Email'])
    )
...
```

## 4.3 Mixed Integer Program Formulation

To incorporate the sequential activation of investment tiers, we extend the linear model into a mixed integer program (MIP). This formulation introduces binary variables to enforce logical dependencies between tiers within each platform while preserving all other allocation and strategic constraints from the LP.

Let $x_i$ again denote the investment in tier $i$, and let $y_i$ be a binary activation variable such that $y_i = 1$ if tier $i$ is active (i.e., receives any investment), and 0 otherwise. The objective remains to maximize total expected return:

$$\max \sum_{i=1}^{n} \mathrm{ROI}_i \, x_i$$

Subject to all previous LP constraints, plus the following:

this is not a piecewise linearization math

$$
\begin{aligned}
x_i &\leq U_i\, y_i && \forall i \text{ (Tier activation link)} \\
y_{i+1}^{(p)} &\leq y_i^{(p)} && \forall p \text{ (Sequential activation)} \\
x_i^{(p)} &\geq U_i^{(p)}\, y_{i+1}^{(p)} && \forall p \text{ (Cumulative investment)} \\
y_i \in \{0,1\}, \quad & x_i \geq 0 && \forall i \text{ (Binary, nonnegativity)}
\end{aligned}
$$

The first new constraint ensures that a tier can only receive investment if it is activated. The sequential activation constraint enforces that higher tiers cannot be selected unless all lower tiers in the same platform are active. The continuity constraint guarantees that investing in a higher tier implies fully utilizing the previous tier's capacity.

All other LP constraints—such as the total budget limit, relative platform ratios, and optional per-platform caps—remain in place.

We define a function to optimize the MIP with additional constraints:

```
def solve_mip_model(roi_df, budget, use_platform_cap=True,
    platform_cap=3.0, verbose=True):
    ...
    # Big-M linking (only upper bound, since per-tier LB=0)
    mip_mod.addConstrs((mip_x[i] <= UB[i] * y[i] for i in range(n)))

    # Sequential tier ordering
    for p in np.unique(platforms):
        idxs = np.where(platforms == p)[0]
        idxs = idxs[np.argsort(tiers[idxs])]
        for i1, i2 in zip(idxs[:-1], idxs[1:]):
            mip_mod.addConstr(y[i2] <= y[i1])
            mip_mod.addConstr(mip_x[i1] >= UB[i1] * y[i2])
    ...
```

## 4.4 Model Allocation Cross-Evaluation

To evaluate the robustness and generalization of the optimized allocations, we perform a cross-evaluation procedure between the Linear Program (LP) and Mixed Integer Program (MIP) models. Each model is trained on one company's ROI data and tested on another, assessing how well each allocation strategy transfers across market environments.

We first extract the total investment per platform from the optimized tier-level allocations:

```
def extract_platform_investments(solution_x, platforms):
    """Extract total investment per platform from solution vector"""
    platform_investments = {}
    for p in np.unique(platforms):
        total = sum(solution_x[i] for i in range(len(platforms)) if
            platforms[i] == p)
        if total > 1e-6:
            platform_investments[p] = total
    return platform_investments
```

Each platform's total investment is then re-evaluated on the alternative company's ROI structure to measure out-of-sample performance. This is done by reconstructing the implied ROI contribution from each tier:

```
def calculate_roi_with_details(platform_investments, roi_df):
    """Recompute total ROI under a different ROI structure"""
    total_roi = 0.0
    for platform, investment in platform_investments.items():
        platform_data = roi_df[roi_df['Platform'] == platform].
            sort_values('Tier')
        remaining = investment
        for _, row in platform_data.iterrows():
            tier_cap = row['UpperBound'] - row['LowerBound']
            invest = min(remaining, tier_cap)
            total_roi += invest * row['ROI']
            remaining -= invest
    return total_roi
```

This process allows direct comparison of how each model's optimal allocation, when trained on one dataset, would perform under the ROI conditions of another company.

## 4.5 Platform Cap Constraint Analysis

To assess the effect of the optional per-platform investment cap, we compare the optimized allocations with and without the cap applied. For both the Linear Program (LP) and Mixed Integer Program (MIP), we re-run the optimization models while disabling the platform cap constraint.

```
# LP Model without platform cap
lp_no_cap = solve_lp_model(roi_company1, budget=10, use_platform_cap
    =False)

# MIP Model without platform cap
mip_no_cap = solve_mip_model(roi_company2, budget=10,
    use_platform_cap=False)
```

After obtaining these alternative allocations, we extract the total ROI under both scenarios. We then compute the difference in ROI between the capped and uncapped solutions, both in absolute terms and as a percentage gain. This quantifies the impact of the platform cap on total expected return and demonstrates how relaxing the cap may change the allocation of investments across platforms.

## 4.6 Mixed Integer Program with Minimum Spend Per Platform

To model scenarios where each platform has a minimum required investment, we extend the MIP by introducing binary platform selection variables $z_p$. A platform $p$ is either selected ($z_p = 1$) and receives at least its minimum investment $M_p$, or it is not selected ($z_p = 0$) and receives no investment. Tier-level investments $x_i$ are linked to both the tier activation $y_i$ and the platform selection $z_p$. All standard MIP constraints (sequential tier ordering, total budget, strategic allocation rules, and optional platform caps) still apply.

The additional constraints can be expressed as:

incorrect expression

$$
\begin{aligned}
y_i &\leq z_p && \forall i \in \text{tiers of platform } p \text{ (tier} \leq \text{platform)} \\
\sum_{i \in \text{tiers of } p} x_i &\geq M_p \, z_p && \forall p \text{ (minimum platform spend)}
\end{aligned}
$$

With the code implementation of the extra constraint by:

```
...
    # All-or-nothing by platform with minimum spend
    for p in uniq_platforms:
        p_idxs = np.where(platforms == p)[0]
        spend_p = gp.quicksum(x[i] for i in p_idxs)

        # Link tiers to platform selection
        min_spend_mod.addConstrs((y[i] <= z[p] for i in p_idxs),
           name=f"tier_leq_platform_{p}")

        # If selected, spend at least MinInvestment; else 0
        m_p = float(min_req.get(p, 0.0))
        if m_p > 0:
            min_spend_mod.addConstr(spend_p >= m_p * z[p], name=f"
               min_spend_{p}")

        # Cap if selected (and 0 if not)
        if use_platform_cap:
            min_spend_mod.addConstr(spend_p <= platform_cap * z[p],
               name=f"cap_if_selected_{p}")
...
```

## 4.7 Mixed Integer Program with Monthly Reinvesting

This model extends the baseline MIP to operate on a rolling monthly horizon, where each month is solved as a separate optimization problem using the budget carried forward from the previous period. The reinvestment rule increases the next month's available budget by 50% of the returns earned in the current month:

$$B_{t+1} = B_t + 0.5\, R_t$$

where $B_t$ is the available budget at the start of month $t$ and $R_t$ is the total monetary return from the optimal allocation in that month.

For each monthly iteration, the ROI dataset is filtered to include only entries corresponding to the current month $t$. The resulting subset defines the ROI parameters and investment bounds for that period's optimization. The MIP is then solved as a standard single-period allocation problem, after which the next month's budget is updated according to the reinvestment rule.

Each month's optimization problem is thus solved sequentially with:

$$\sum_{i \in I_t} x_{i,t} \leq B_t,$$

where $B_t$ is updated recursively using the equation above.

## 4.8 Allocation Stability

To assess whether the month-to-month investment decisions produced by the rolling MIP model are stable, we perform a stability analysis on the resulting allocations. The purpose is to verify that spending on each platform does not fluctuate excessively between consecutive months.

Formally, for each platform $i$ and consecutive months $t$ and $t+1$, the change in allocation is computed as:

$$\Delta x_{i,t} = x_{i,t+1} - x_{i,t}.$$

An allocation is considered *stable* if

$$|\Delta x_{i,t}| \leq T,$$

this is not a mathematical formula to your MIL

where $T$ is a predefined stability threshold ($1M here).

We define a function to identify stability violations by:

```
# Run stability analysis with a $1M threshold
stability_analysis = analyze_allocation_stability(
    rolling_result, stability_threshold=1.0)
```

The function iterates through each consecutive pair of months in the rolling optimization results, computes the absolute change in allocation per platform, and flags violations where the change exceeds the threshold. Summary statistics such as the total number of violations, the largest deviation, and the platforms with the most frequent violations are then reported.

# 5 Results & Discussion

## 5.1 Optimal Linear Program Budget Allocation

| Platform | Tier | Allocation ($M) |
|---|---|---|
| TV | 1 | 3.000 |
| AdWords | 1 | 1.000 |
| Instagram | | 3.000 |
| Tier 1 | 1 | 2.900 |
| Tier 2 | 2 | 0.100 |
| Email | 1 | 3.000 |
| **Total ROI** | | 0.5436 |

Table 1: Optimal Allocation by Platform and Tier

## 5.2 Mixed-Integer Program Budget Allocation

| Platform | Tier | Allocation ($M) |
|---|---|---|
| Print | | 3.000 |
| Tier 1 | 1 | 2.600 |
| Tier 2 | 2 | 0.400 |
| AdWords | | 2.333 |
| Tier 1 | 1 | 1.200 |
| Tier 2 | 2 | 1.133 |
| Facebook | 1 | 3.000 |
| LinkedIn | 1 | 1.667 |
| **Total ROI** | | 0.4528 |

Table 2: Optimal Allocation by Platform and Tier

These values are not correctly match the s

### 5.3 Model Cross-Evaluation

| Platform | Tier | Allocation ($M) |
|---|---|---|
| AdWords | 1 | 1.000 |
| Email | | 3.000 |
| Tier 1 | 1 | 1.900 |
| Tier 2 | 2 | 1.000 |
| Tier 3 | 3 | 0.100 |
| Instagram | | 3.000 |
| Tier 1 | 1 | 1.600 |
| Tier 2 | 2 | 1.400 |
| TV | | 3.000 |
| Tier 1 | 1 | 1.800 |
| Tier 2 | 2 | 1.200 |
| **Total ROI** | | 0.2777 |

Table 3: LP Allocation Tested on Company 2 ROI

| Platform | Tier | Allocation ($M) |
|---|---|---|
| AdWords | | 2.333 |
| Tier 1 | 1 | 2.000 |
| Tier 2 | 2 | 0.300 |
| Tier 3 | 3 | 0.033 |
| Facebook | | 3.000 |
| Tier 1 | 1 | 1.000 |
| Tier 2 | 2 | 0.800 |
| Tier 3 | 3 | 1.200 |
| LinkedIn | | 1.667 |
| Tier 1 | 1 | 0.700 |
| Tier 2 | 2 | 0.967 |
| Print | | 3.000 |
| Tier 1 | 1 | 0.700 |
| Tier 2 | 2 | 2.000 |
| Tier 3 | 3 | 0.300 |
| **Total ROI** | | 0.2749 |

Table 4: MIP Allocation Tested on Company 1 ROI

#### 5.3.1 Model Cross-Evaluation Summary

| Model | Train ROI | Test ROI | Absolute Drop | Relative Drop (%) |
|---|---|---|---|---|
| LP (Company1) | 0.5436 | 0.2777 | 0.2659 | 48.91 |
| MIP (Company2) | 0.4528 | 0.2749 | 0.1779 | 39.29 |
| *Better generalizing model: MIP* | | | | |

Table 5: In-sample vs. out-of-sample ROI comparison for LP and MIP models. Absolute and relative drops indicate generalization loss.

Here we see that the MIP model generalizes better, showing a smaller out-of-sample ROI drop (39.3%) compared to the LP model (48.9%). While the LP model optimized on Company 1 ROI yields a higher in-sample return, its larger generalization loss suggests greater sensitivity to forecast errors. If the true ROI forecast is uncertain, the MIP allocation would be the more robust choice, despite its slightly lower in-sample return.

## 5.4 Platform Cap Constraint Analysis

**LP Model – Without Cap**

| Platform | Tier | Allocation ($M) |
|---|---|---|
| TV | 1 | 3.550 |
| Instagram | 1 | 2.900 |
| Email | 1 | 3.550 |
| Total ROI | | 0.5573 |
| With cap ROI: 0.5436 | | |
| Without cap ROI: 0.5573 | | |
| ROI improvement: 0.0137 | | |
| Percentage gain: 2.52% | | |

**MIP Model – Without Cap**

| Platform | Tier | Allocation ($M) |
|---|---|---|
| Facebook | 1 | 4.600 |
| Facebook | 2 | 5.400 |
| Total ROI | | 0.4670 |
| With cap ROI: 0.4528 | | |
| Without cap ROI: 0.4670 | | |
| ROI improvement: 0.0142 | | |
| Percentage gain: 3.13% | | |

Table 6: Comparison of LP and MIP allocations with and without the 3M platform cap. ROI metrics show the impact of removing the cap on total return.

Removing the platform cap increases total ROI modestly (2.5–3%) but leads to more concentrated investments in fewer platforms. To maintain diversification and reduce exposure to inaccurate ROI forecasts, keeping the platform cap is recommended.

## 5.5 MIP with Minimum Spend

| Platform | Tier | Allocation ($M) |
|---|---|---|
| Print | | 3.000 |
| Tier 1 | 1 | 2.600 |
| Tier 2 | 2 | 0.400 |
| AdWords | | 2.333 |
| Tier 1 | 1 | 1.200 |
| Tier 2 | 2 | 1.133 |
| Facebook | 1 | 3.000 |
| LinkedIn | 1 | 1.667 |
| **Total ROI** | | 0.4528 |

Table 7: Optimal allocation by platform and tier under minimum spend constraints.

Notably, the minimum spend constraint did not change the budget allocation for the MIP.

yeah it should not have a change, but your

## 5.6 MIP with Monthly Reinvesting

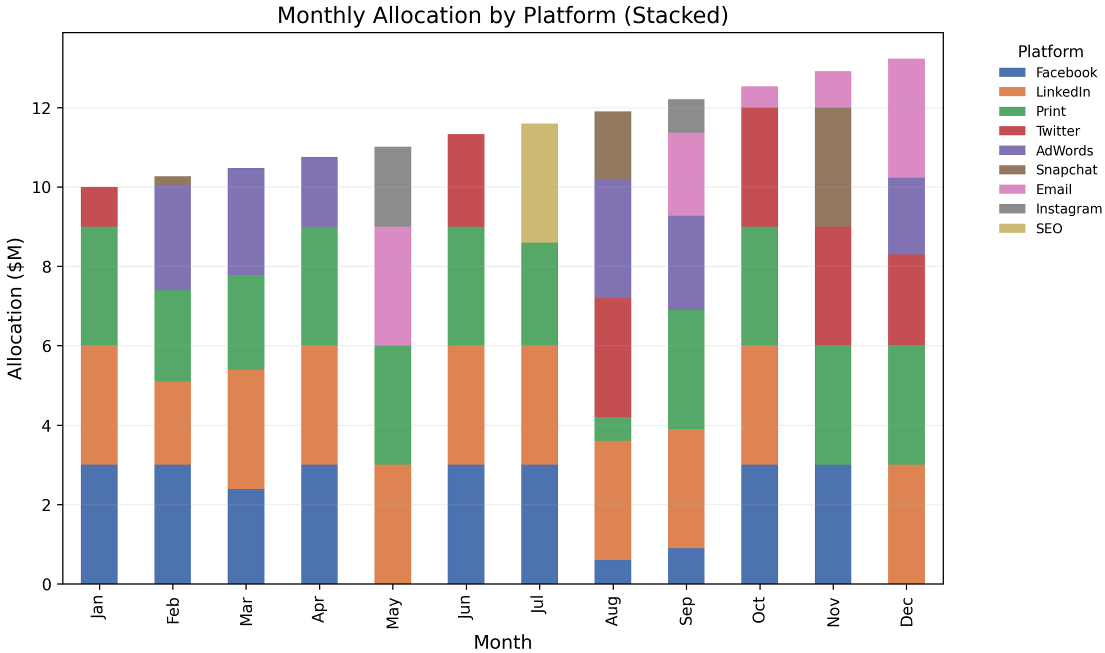


Figure 1: Monthly allocation of the marketing budget by platform using the rolling MIP model with reinvested returns. Each bar shows the total spend at the end of the month, stacked by platform.

As seen in the figure, monthly allocations fluctuate considerably. The model frequently invests heavily in *Print*, *LinkedIn*, and *Facebook*, but in some months completely withdraws from these platforms before reinvesting fully in the following month. In practice, there may be costs associated with initiating investments versus maintaining ongoing spending, which are not accounted for here and could materially reduce the projected returns. Additionally, the model does not consider the effects of sustained presence on marketing effectiveness over time. The detailed month-by-month allocations are provided in the appendix.

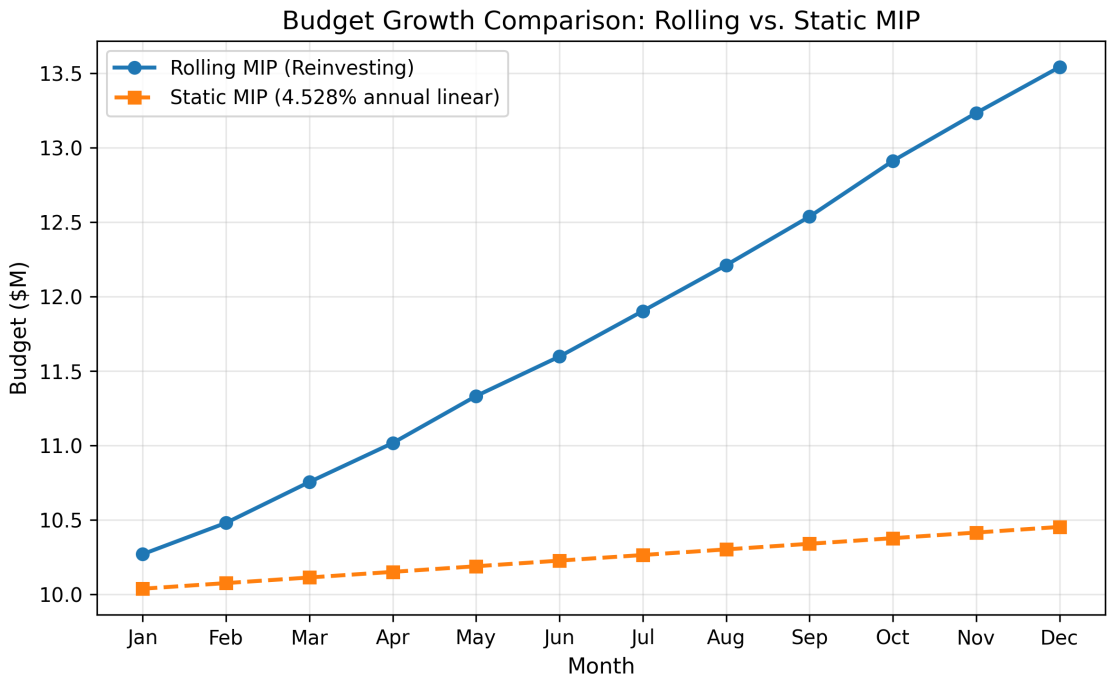


Figure 2: End-of-month budget growth comparison: Rolling MIP model with 50% reinvestment of monthly returns (final return \$7.084M, 70.8% growth) versus static MIP model with linear annual growth of 4.528%.

The figure above illustrates the difference in projected returns between the rolling monthly reinvestment model and the static MIP model. The rolling model produces substantially higher returns, achieving 70.8% growth compared to 4.53% for the static allocation.

## 5.7 Allocation Stability Analysis

To evaluate temporal consistency in spending, we assessed whether monthly allocations changed by more than \$1M between consecutive months. Results indicate that the solution is not stable because 32 violations of the \$1M threshold were identified.

Table 8: Summary of Stability Analysis Results

| **Criterion** | **Result** |
|---|---|
| Stability threshold | \$1.0M |
| Overall status | Not Stable |
| Total violations | 32 |
| Most frequent violators | AdWords, Facebook, Email, Twitter (5 each) |
| Other affected platforms | Snapchat (4), Instagram (2), SEO (2), Print (2), LinkedIn (2) |

Table 9: Top Allocation Instability Events (sorted by magnitude)

| **Platform** | **Month Transition** | **Change ($M)** | **Violation ($M)** |
|---|---|---|---|
| Facebook | Apr → May | −3.00 | 2.00 |
| Email | Apr → May | +3.00 | 2.00 |
| Facebook | May → Jun | +3.00 | 2.00 |
| Email | May → Jun | −3.00 | 2.00 |
| SEO | Jun → Jul | +3.00 | 2.00 |
| Twitter | Jul → Aug | +3.00 | 2.00 |
| AdWords | Jul → Aug | +3.00 | 2.00 |
| SEO | Jul → Aug | −3.00 | 2.00 |
| Twitter | Aug → Sep | −3.00 | 2.00 |
| Twitter | Sep → Oct | +3.00 | 2.00 |

Several platforms undergo extreme reallocations, often shifting from the maximum cap ($3M) to zero and back again in successive months.

Such volatility implies that the rolling monthly optimization reacts aggressively to changing ROI parameters, prioritizing short-term returns at the expense of allocation stability. These fluctuations would be operationally challenging in real-world media management, increasing switching costs and campaign discontinuity risks.

Table 10: Month-to-Month Platform Allocation Changes ($M)

| Platform | Jan→Feb | Feb→Mar | Mar→Apr | Apr→May | May→Jun | Jun→Jul | Jul→Aug | Aug→Sep | Sep→Oct | Oct→Nov | Nov→Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|
| AdWords | +2.66** | +0.04 | -0.94 | -1.75** | +0.00 | +0.00 | +3.00** | -0.63 | -2.37** | +0.00 | +1.93** |
| Email | +0.00 | +0.00 | +0.00 | +3.00** | -3.00** | +0.00 | +0.00 | +2.10** | -1.56** | +0.37 | +2.09** |
| Facebook | +0.00 | -0.61 | +0.61 | -3.00** | +3.00** | +0.00 | -2.40** | +0.30 | +2.10** | +0.00 | -3.00** |
| Instagram | +0.00 | +0.00 | +0.00 | +2.02** | -2.02** | +0.00 | +0.00 | +0.84 | -0.84 | +0.00 | +0.00 |
| LinkedIn | -0.90 | +0.90 | +0.00 | +0.00 | +0.00 | +0.00 | +0.00 | +0.00 | +0.00 | -3.00** | +3.00** |
| Print | -0.70 | +0.09 | +0.61 | +0.00 | +0.00 | -0.40 | -2.00** | +2.40** | +0.00 | -0.00 | +0.00 |
| SEO | +0.00 | +0.00 | +0.00 | +0.00 | +0.00 | +3.00** | -3.00** | +0.00 | +0.00 | +0.00 | +0.00 |
| Snapchat | +0.21 | -0.21 | +0.00 | +0.00 | +0.00 | +0.00 | +1.70** | -1.70** | +0.00 | +3.00** | -3.00** |
| Twitter | -1.00 | +0.00 | +0.00 | +0.00 | +2.33** | -2.33** | +3.00** | -3.00** | +3.00** | +0.00 | -0.70 |

** indicates violation of the stability constraint ($|\Delta| > \$1M$).

### 5.7.1 Modeling Stability with Hard Constraints

To enforce allocation stability, we introduce constraints that limit month-to-month changes in platform spending to at most $1M. Implementing these constraints requires optimizing all 12 months simultaneously in a single model, rather than solving each month sequentially.

#### Decision Variables

- $x_{i,t}$: investment in tier $i$ during month $t$, for all tiers $i$ and months $t = 1, \ldots, 12$
- $y_{i,t} \in \{0, 1\}$: binary variable indicating whether tier $i$ is active in month $t$

**Platform Spend** Let $\mathcal{T}_p$ denote the set of tiers belonging to platform $p$. Total spend on platform $p$ in month $t$ is:

$$S_{p,t} = \sum_{i \in \mathcal{T}_p} x_{i,t}$$

**Stability Constraints** For each platform $p$ and each month $t = 2, \ldots, 12$, month-to-month changes are constrained by:

$$-1 \leq S_{p,t} - S_{p,t-1} \leq 1$$

or equivalently as two linear inequalities:

$$S_{p,t} - S_{p,t-1} \leq 1, \quad S_{p,t} - S_{p,t-1} \geq -1$$

**Budget Dynamics** The rolling budget is modeled internally as:

$$B_1 = 10 \text{ (initial budget in \$M)}, \quad B_{t+1} = B_t + 0.5 \cdot R_t, \quad t = 1, \ldots, 11$$

where $R_t = \sum_i \text{ROI}_{i,t} \cdot x_{i,t}$ is the total return in month $t$.

**Monthly Budget Constraint**

$$\sum_i x_{i,t} \leq B_t \quad \forall t$$

**Objective**

$$\max \sum_{t=1}^{12} R_t = \max \sum_{t=1}^{12} \sum_i \text{ROI}_{i,t} \cdot x_{i,t}$$

**Expected Trade-offs** Enforcing strict stability is likely to:

- **Reduce total 12-month ROI** compared to the unconstrained solution, since optimal reallocations are restricted
- **Smooth allocation patterns**, limiting dramatic entries and exits from platforms
- **Increase operational feasibility**, producing more predictable spending and lower switching costs

# 6 Conclusion

The analysis shows that the mixed-integer program (MIP) produces more robust allocations than the linear program (LP), with an out-of-sample ROI drop of 39.3% compared to 48.9% for the LP, making the MIP better suited for real-world application. Relaxing platform caps provides only minor ROI gains (2.5% for LP, 3.1% for MIP), indicating that the cap primarily helps maintain diversification. Incorporating minimum spend requirements preserves baseline investment in key platforms without significantly reducing total ROI (MIP ROI remains 0.4528).

The rolling monthly reinvestment model achieves 70.8% budget growth over the year, but allocations are highly volatile, with 32 month-to-month violations exceeding $1M. This level of instability could increase operational complexity and switching costs.

For practical implementation, we recommend using the MIP with platform caps and minimum spend constraints, while investigating ways to smooth monthly allocations. Given the exceptionally high returns of the rolling monthly reinvestment model (70.8% growth), future work should quantify the operational costs of rapid budget reallocations to extend the model toward a more stable middle ground between growth and allocation stability.

# 7 Appendix

**Monthly Allocation Summaries (Jan–Apr)**

| Platform | Tier | Allocation ($M) |
|---|---|---|
| Facebook | 1 | 3.000 |
| LinkedIn | | 3.000 |
| Tier 1 | 1 | 0.400 |
| Tier 2 | 2 | 2.200 |
| Tier 3 | 3 | 0.400 |
| Print | 1 | 3.000 |
| Twitter | 1 | 1.000 |
| | **Return** | 0.5394M (5.39%) |
| | **Next Budget** | 10.2697M |

Jan

| Platform | Tier | Allocation ($M) |
|---|---|---|
| AdWords | | 2.657 |
| Tier 1 | 1 | 2.100 |
| Tier 2 | 2 | 0.557 |
| Facebook | | 3.000 |
| Tier 1 | 1 | 0.800 |
| Tier 2 | 2 | 1.000 |
| Tier 3 | 3 | 1.200 |
| LinkedIn | | 2.100 |
| Tier 1 | 1 | 0.500 |
| Tier 2 | 2 | 1.600 |
| Print | 1 | 2.300 |
| Snapchat | 1 | 0.213 |
| | **Return** | 0.4209M (4.10%) |
| | **Next Budget** | 10.4802M |

Feb

| Platform | Tier | Allocation ($M) |
|---|---|---|
| AdWords | | 2.696 |
| Tier 1 | 1 | 2.200 |
| Tier 2 | 2 | 0.496 |
| Facebook | | 2.392 |
| Tier 1 | 1 | 2.392 |
| LinkedIn | | 3.000 |
| Tier 1 | 1 | 0.900 |
| Tier 2 | 2 | 0.800 |
| Tier 3 | 3 | 1.300 |
| Print | | 2.392 |
| Tier 1 | 1 | 1.100 |
| Tier 2 | 2 | 0.700 |
| Tier 3 | 3 | 0.592 |
| | **Return** | 0.5488M (5.24%) |
| | **Next Budget** | 10.7545M |

Mar

| Platform | Tier | Allocation ($M) |
|---|---|---|
| AdWords | 1 | 1.755 |
| Facebook | | 3.000 |
| Tier 1 | 1 | 2.400 |
| Tier 2 | 2 | 0.600 |
| LinkedIn | | 3.000 |
| Tier 1 | 1 | 0.800 |
| Tier 2 | 2 | 1.800 |
| Tier 3 | 3 | 0.400 |
| Print | | 3.000 |
| Tier 1 | 1 | 1.900 |
| Tier 2 | 2 | 1.100 |
| | **Return** | 0.5237M (4.87%) |
| | **Next Budget** | 11.0164M |

Apr

**Monthly Allocation Summaries (May–Aug)**

| Platform | Tier | Allocation ($M) |
|---|---|---|
| Email | | 3.000 |
| Tier 1 | 1 | 2.800 |
| Tier 2 | 2 | 0.200 |
| Instagram | 1 | 2.016 |
| LinkedIn | 1 | 3.000 |
| Print | | 3.000 |
| Tier 1 | 1 | 2.000 |
| Tier 2 | 2 | 0.500 |
| Tier 3 | 3 | 0.500 |
| **Return** | | 0.6315M (5.73%) |
| **Next Budget** | | 11.3322M |

May

| Platform | Tier | Allocation ($M) |
|---|---|---|
| Facebook | | 3.000 |
| Tier 1 | 1 | 1.900 |
| Tier 2 | 2 | 1.100 |
| LinkedIn | 1 | 3.000 |
| Print | | 3.000 |
| Tier 1 | 1 | 2.200 |
| Tier 2 | 2 | 0.800 |
| Twitter | 1 | 2.332 |
| **Return** | | 0.5305M (4.68%) |
| **Next Budget** | | 11.5974M |

Jun

| Platform | Tier | Allocation ($M) |
|---|---|---|
| Facebook | | 3.000 |
| Tier 1 | 1 | 0.700 |
| Tier 2 | 2 | 1.200 |
| Tier 3 | 3 | 0.300 |
| Tier 4 | 4 | 0.800 |
| LinkedIn | | 3.000 |
| Tier 1 | 1 | 2.400 |
| Tier 2 | 2 | 0.600 |
| Print | | 2.597 |
| Tier 1 | 1 | 0.500 |
| Tier 2 | 2 | 2.000 |
| Tier 3 | 3 | 0.097 |
| SEO | | 3.000 |
| Tier 1 | 1 | 2.500 |
| Tier 2 | 2 | 0.500 |
| **Return** | | 0.6114M (5.27%) |
| **Next Budget** | | 11.9031M |

Jul

| Platform | Tier | Allocation ($M) |
|---|---|---|
| AdWords | | 3.000 |
| Tier 1 | 1 | 1.700 |
| Tier 2 | 2 | 1.300 |
| Facebook | 1 | 0.600 |
| LinkedIn | | 3.000 |
| Tier 1 | 1 | 1.100 |
| Tier 2 | 2 | 1.900 |
| Print | 1 | 0.600 |
| Snapchat | 1 | 1.703 |
| Twitter | | 3.000 |
| Tier 1 | 1 | 2.600 |
| Tier 2 | 2 | 0.400 |
| **Return** | | 0.6167M (5.18%) |
| **Next Budget** | | 12.2115M |

Aug

**Monthly Allocation Summaries (Sep–Dec)**

| Platform | Tier | Allocation ($M) |
|---|---|---|
| AdWords | | 2.370 |
| Tier 1 | 1 | 0.500 |
| Tier 2 | 2 | 1.300 |
| Tier 3 | 3 | 0.570 |
| Email | 1 | 2.100 |
| Facebook | 1 | 0.900 |
| Instagram | 1 | 0.841 |
| LinkedIn | | 3.000 |
| Tier 1 | 1 | 1.200 |
| Tier 2 | 2 | 0.700 |
| Tier 3 | 3 | 0.900 |
| Tier 4 | 4 | 0.200 |
| Print | | 3.000 |
| Tier 1 | 1 | 1.500 |
| Tier 2 | 2 | 0.700 |
| Tier 3 | 3 | 0.800 |
| **Return** | | 0.6519M (5.34%) |
| **Next Budget** | | 12.5374M |

Sep

| Platform | Tier | Allocation ($M) |
|---|---|---|
| Email | | 0.537 |
| Facebook | | 3.000 |
| LinkedIn | | 3.000 |
| Tier 1 | 1 | 1.600 |
| Tier 2 | 2 | 1.100 |
| Tier 3 | 3 | 0.300 |
| Print | | 3.000 |
| Tier 1 | 1 | 1.800 |
| Tier 2 | 2 | 1.200 |
| Twitter | | 3.000 |
| Tier 1 | 1 | 1.500 |
| Tier 2 | 2 | 1.100 |
| Tier 3 | 3 | 0.400 |
| **Return** | | 0.7483M (5.97%) |
| **Next Budget** | | 12.9116M |

Oct

| Platform | Tier | Allocation ($M) |
|---|---|---|
| Email | | 0.912 |
| Facebook | | 3.000 |
| Tier 1 | 1 | 1.800 |
| Tier 2 | 2 | 1.200 |
| Print | | 3.000 |
| Tier 1 | 1 | 0.700 |
| Tier 2 | 2 | 1.900 |
| Tier 3 | 3 | 0.400 |
| Snapchat | 1 | 3.000 |
| Twitter | 1 | 3.000 |
| **Return** | | 0.6440M (4.99%) |
| **Next Budget** | | 13.2336M |

Nov

| Platform | Tier | Allocation ($M) |
|---|---|---|
| AdWords | | 1.934 |
| Email | | 3.000 |
| Tier 1 | 1 | 1.100 |
| Tier 2 | 2 | 1.900 |
| LinkedIn | | 3.000 |
| Tier 1 | 1 | 1.000 |
| Tier 2 | 2 | 1.600 |
| Tier 3 | 3 | 0.400 |
| Print | | 3.000 |
| Tier 1 | 1 | 2.300 |
| Tier 2 | 2 | 0.700 |
| Twitter | 1 | 2.300 |
| **Return** | | 0.6165M (4.66%) |
| **Next Budget** | | 13.5418M |

Dec