Exercise 1

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```
##Read in the data
df <- read.csv("mathtest.csv")
df <- as.matrix(df)
n_vec <- as.numeric(table(df[,1]))
K <- length(unique(df[,1]))
y <- df[,2]
```

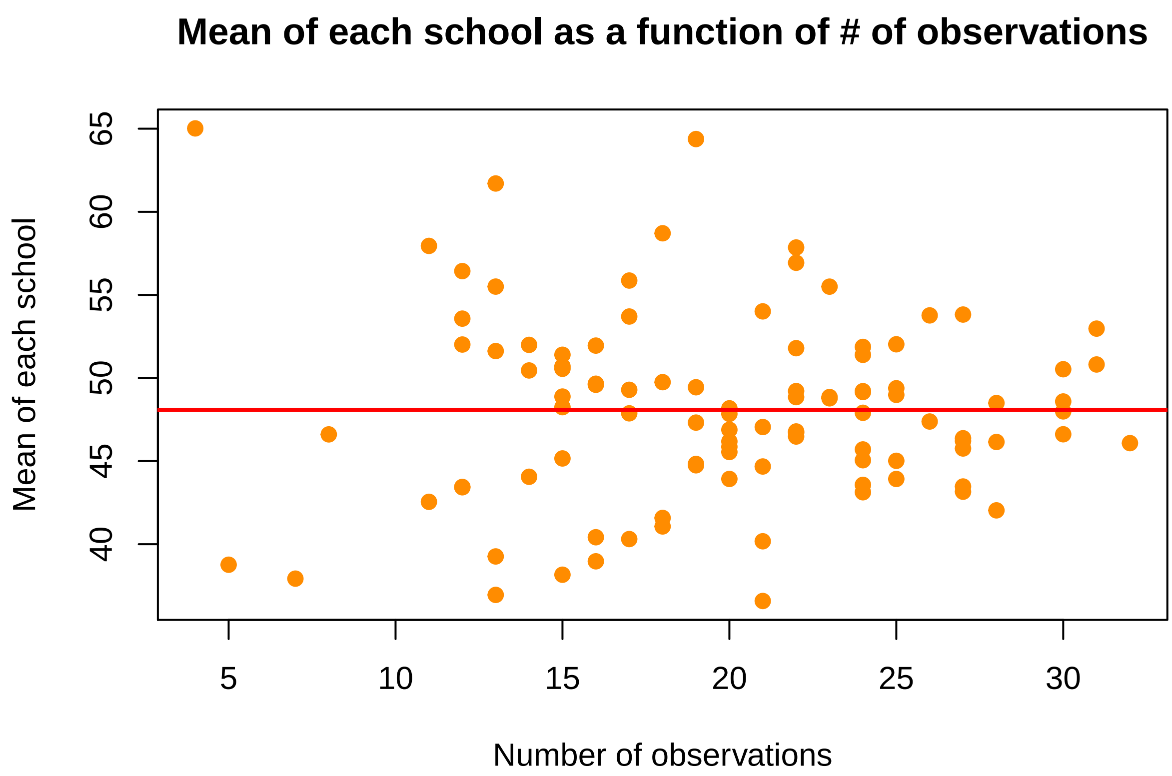
```
##Prior hyperparameters
a <- 1
b <- 1
c <- 1
d <- 1
v <- 0
phi <- 1/10^(9)

B <- 10000
```

```
##Initialize current parameters
cur_mu <- 0
cur_lambda <- 0.01
cur_psi <- 0.01
theta_vec <- rep(0, K)

##Data structure to store samples
par_mat <- matrix(NA, nrow = B, ncol = 3)
theta_mat <- matrix(NA, nrow = B, ncol = length(theta_vec))
```

```
for(iter in 1:B){
    ##Update mu
    mu_var <- 1/(K*cur_lambda*cur_psi + phi)
    cur_mu <- rnorm(1, mean = mu_var*(cur_lambda*cur_psi*sum(theta_vec) + phi*v), sd = sqrt(mu_var))

    ##Update lambda
    ss <- 0
    for(i in 1:K){
        cur_school <- df[df[, 1] == i, 2]
        for(j in 1:n_vec[i]){
            ss <- ss + (cur_school[j] - theta_vec[i])^2
        }
    }
    cur_lambda <- rgamma(1, sum(n_vec)/2 + K/2 + a, ss/2 + cur_psi*sum((theta_vec-cur_mu)^2)/2 + b)

    ##Update psi
    cur_psi <- rgamma(1, K/2 + c, cur_lambda*sum((theta_vec-cur_mu)^2)/2 + d)

    ##Update theta_i
    for(i in 1:K){
        cur_school <- cur_school <- df[df[, 1] == i, 2]
```

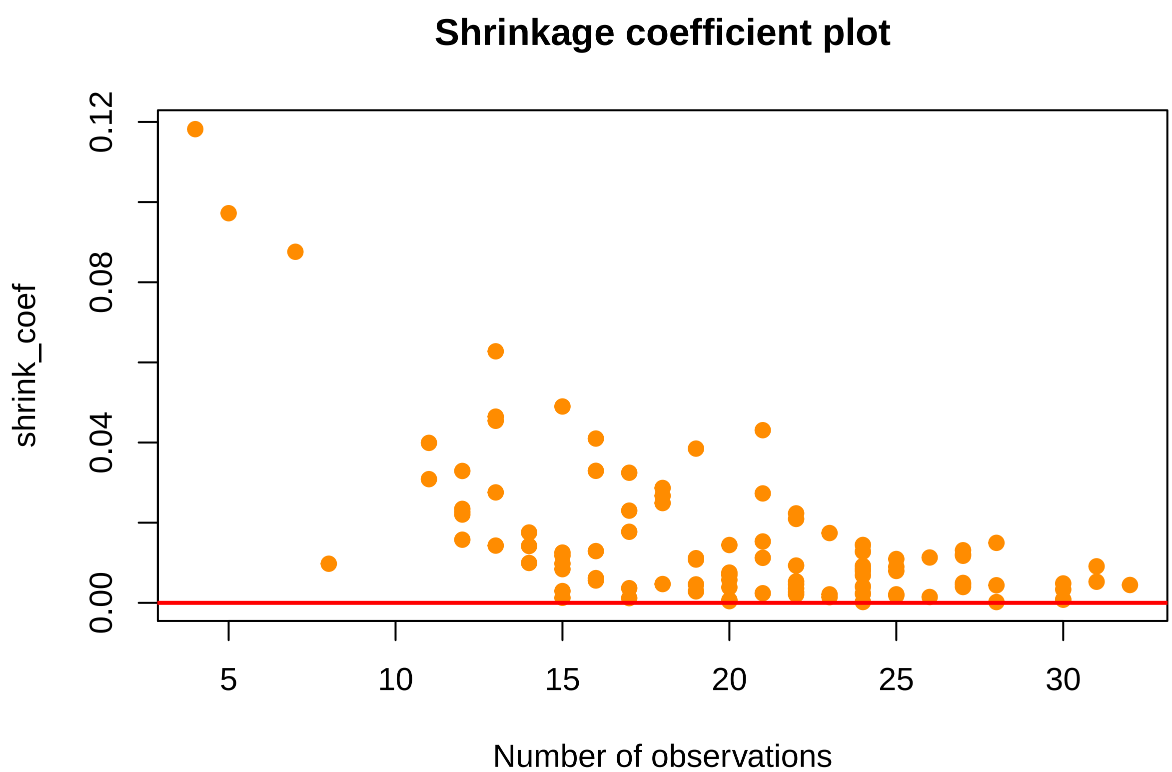
```r
        theta_var <- 1/(n_vec[i]*cur_lambda + cur_lambda*cur_psi)
        theta_vec[i] <- rnorm(1, mean = theta_var*(cur_lambda*sum(cur_school) + cur_lambda*cur_psi*cur_
    }

    par_mat[iter, ] <- c(cur_mu, cur_lambda, cur_psi)
    theta_mat[iter, ] <- theta_vec
}
```

```r
##
burn_idx <- 0.1*B
final_idx <- burn_idx:B
pos_mean <- apply(par_mat[final_idx, ], 2, mean)
pos_theta <- apply(theta_mat[final_idx, ], 2, mean)
shrink_coef <- rep(NA, K)
y_bar_vec <- rep(NA, K)
for(i in 1:K){
    cur_school <- df[df[, 1] == i, 2]
    y_bar_i <- mean(cur_school)
    y_bar_vec[i] <- y_bar_i
    shrink_coef[i] <- abs((y_bar_i - pos_theta[i])/y_bar_i)
}
plot(n_vec, y_bar_vec, col = "darkorange", pch = 19, main = "Mean of each school as a function of # of 
abline(h = mean(y), col = "red", lwd = 2)
```

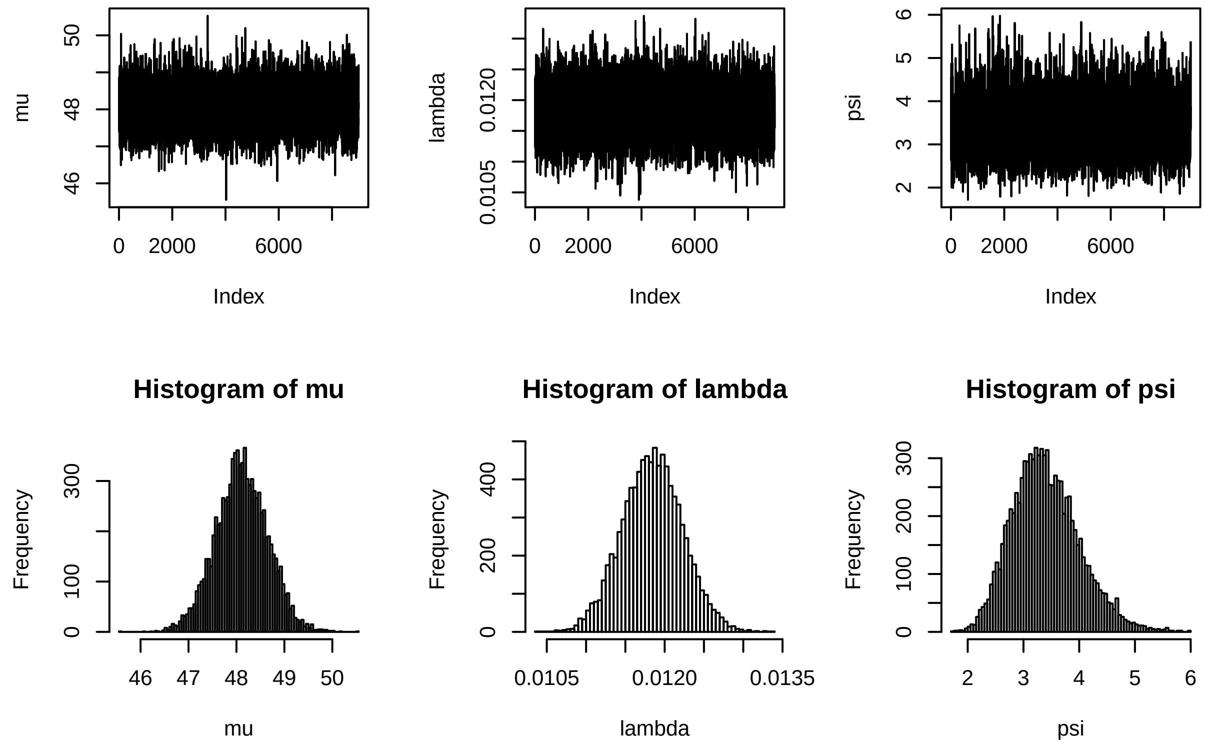



```r
##shrinkage plot
plot(n_vec, shrink_coef, col = "darkorange", pch = 19, main = "Shrinkage coefficient plot", xlab = "Num
```

```r
abline(h=0, col="red", lwd = 2)
```




```r
##Diagnostic plot
par(mfrow = c(2,3))
plot(par_mat[final_idx, 1], ylab = "mu", type= "l")
plot(par_mat[final_idx, 2], ylab = "lambda", type= "l")
plot(par_mat[final_idx, 3], ylab = "psi", type= "l")

hist(par_mat[final_idx, 1], xlab = "mu", breaks = 100, main = "Histogram of mu")
hist(par_mat[final_idx, 2], xlab = "lambda", breaks = 100, main = "Histogram of lambda")
hist(par_mat[final_idx, 3], xlab = "psi", breaks = 100, main = "Histogram of psi")
```



mu
46
48
50
0
2000
6000
Index
lambda
0.0105
0.0120
0
2000
6000
Index
psi
2
3
4
5
6
0
2000
6000
Index
Histogram of mu
Frequency
0
100
300
46
47
48
49
50
mu
Histogram of lambda
Frequency
0
200
400
0.0105
0.0120
0.0135
lambda
Histogram of psi
Frequency
0
100
200
300
2
3
4
5
6
psi