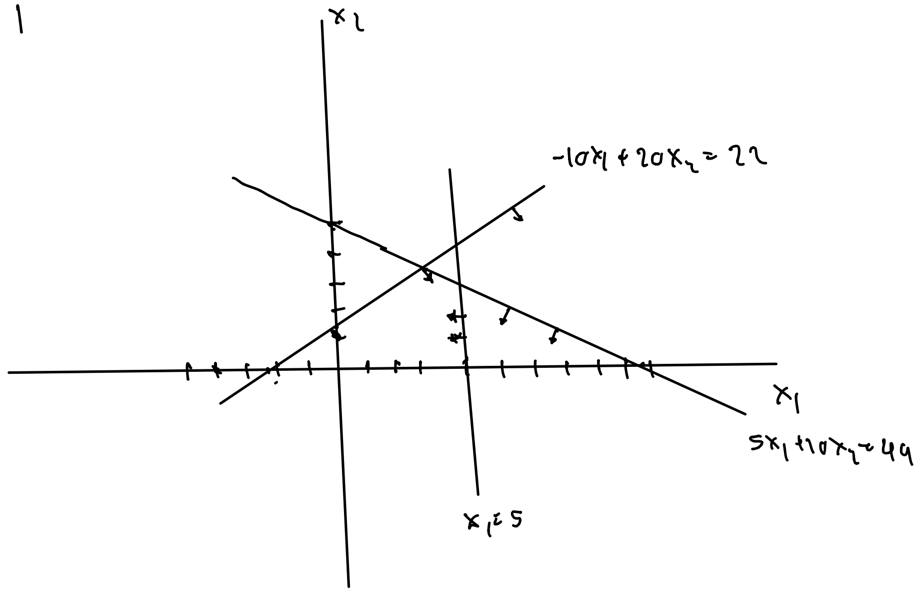
Problem 1



$(x_1=3.8,\ x_2=3)$ (8.2)

$x_1 \leq 3$

$(x_1=3,\ x_2=2.6)$

(7.4)

$x_1 \geq 4$

$(x_1=4,\ x_2=2.9)$

(7.6

$x_1 \leq 3$

$x_2 \leq 2$

$(x_1=1.8,\ x_2=2)$

6.2

$x_1 \leq 3$

$x_2 \geq 3$

not feasible

$x_1 \geq 4$

$x_2 \leq 2$

$(x_1=4,\ x_2=2)$

(4)

$x_1 \geq 4$

$x_2 \geq 3$

not feasible

$x_1 \leq 3$

$x_2 \leq 2$

$x_1 \leq 1$

$(x_1=1,\ x_2=1.6)$

(5.4)

no need to go further

$x_1 \leq 3$

$x_2 \leq 2$

$x_1 \geq 2$

$(x_1=2,\ x_2=2)$

(6)

$x_1 \leq 3$

$x_2 \leq 2$

$x_1 \leq 1$

$x_2 \leq 1$

$x_1 \leq 3$

$x_2 \leq 2$

$x_1 \leq 1$

$x_2 \geq 2$

$(x_1=1, x_2=1)$ $(x_1=1, x_2=2)$

(3) not feasible

The optimal solution is $x_1=2$, $x_2=2$

and the objective value is 6

2) R gave same answer. See code below

3) If we stop early difference $= 8-2 = 6$

— do not stop early " $= 10-3 = 7$

Problem 2.

Let x_i = # of factory in city i $\quad i=1,2$

y_i = # of warehouse in city i $\quad i=1,2$

Maximize: $f = 9x_1 + 5x_2 + 6y_1 + 4y_2$

subject to:

$$6x_1 + 3x_2 + 5y_1 + 2y_2 \leq 11$$
$$x_1 \leq 1$$
$$x_2 \leq 1$$
$$y_1 \leq 1$$
$$y_2 \leq 1$$
$$y_1 + y_2 \leq 1$$
$$-x_1 - x_2 \leq 1$$
$$x_i, y_i \text{ - integers}$$

They should open a factory in Austin and Dallas and a warehouse in Dallas. With this strategy, they will achieve a profit of \$18 M

Problem 3

Y_i = whether to open a hub at location i

C_{ij} = whether hub i cover city j

minimize $\sum_i Y_{ij}$

subject to: $\sum_i C_{ij} \geq 1 \quad \forall j$

$Y_i \in \{0,1\}$

	ATL	BOS	CHI	DEN	HOU	LAX	NO	NY	PIT	SLC	SF	SEA	
ATL	X		X		X		X	X	X				≥ 1
BOS		X						X	X				≥ 1
CHI	X		X				X	X	X				≥ 1
DEN				X						X			≥ 1
HOU	X				X		X						≥ 1
LAX						X				X	X		≥ 1
NO	X		X		X		X						≥ 1
NY	X	X	X					X	X				≥ 1
PIT	X	X	X					X	X				≥ 1
SLC				X		X				X	X	X	≥ 1
SF						X				X	X	X	≥ 1
SEA										X	X	X	≥ 1

Answer we should build 3 hubs at x_1, x_8, x_{10}.
ATL NY SLC

Problem 4

Let p_i = allowable pattern i

$$p_i = \begin{bmatrix} a_{i1} \\ a_{i2} \\ \vdots \\ a_{ij} \end{bmatrix}$$ where a_{ij} = # of finals j in pattern i

c_i = waste produced by pattern i $i = 1, 2, \ldots, 17$

x_i = number of pattern i used $i = 1, 2, \ldots, 17$

d_j = demand for finals j $j = 1, 2, 3$

minimize $\sum_{i=1}^{17} c_i x_i$

subject to: $\sum_{i=1}^{n} x_i a_{ij} = d_j \quad \forall j$

The optimal strategy is to produce

7 of pattern $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

3 of pattern $\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$

92 of pattern $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

47 of pattern $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

Problem 5

x_i = # of workers on day i

c_i = Cost of hiring a worker on day i

minimize $\sum_i c_i x_i$

subject to: $\sum_{j \in A} x_j \geq n_i \quad \forall i$ where

$A = \{ j : j = \{0, 1, \ldots, 6\}$ and $j \neq i+1 \pmod 7$ and $j \neq i+2 \pmod 7 \}$

The optimal schedule is to hire

1 worker Sun-Thur

8 workers Mon-Fri

2 workers Tue-Sat

0 worker Wed-Sun

3 workers Thur-Mon

1 worker Fri-Tue

This strategy will cost them 4,830.

3) most popular is Mon-Fri.

Code for HW3

Korawat Tanwisuth

February 20, 2018

1

```
library(lpSolve)
#1
A = matrix(c(-10,5,1,20,10,0), ncol =2)
b = matrix(c(22,49,5),ncol =1)
signs = rep("<=", 3)
z = c(-1,4)
ans =lp("max",z , A,signs,b,int.vec = 1:2)
ans$solution
```

```
## [1] 2 2
```

```
ans
```

```
## Success: the objective function is 6
```

2

```
#2
b <- c(11,rep(1,6))
A <- matrix(0,ncol=4,nrow=7)
A[1,] <- c(6,3,5,2)
A[2:5,] <- diag(4)
A[6,] <- c(0,0,1,1)
A[7,] <- c(-1,-1,0,0)
z = c(9,5,6,4)
signs <- rep("<=",7)
ans = lp("max",z,A,signs,b,int.vec = 1:4)
ans$solution
```

```
## [1] 1 1 0 1
```

```
ans
```

```
## Success: the objective function is 18
```

3

```
b = rep(1,12)
A = matrix(0,ncol=12,nrow=12)
A[1,] <- c(1,0,1,0,1,0,rep(1,3),rep(0,3))
A[2,] <- c(0,1,rep(0,5),1,1,rep(0,3))
A[3,] <- c(1,0,1,rep(0,3),rep(1,3),rep(0,3))
A[4,] <- c(rep(0,3),1,rep(0,5),1,rep(0,2))
A[5,] <- c(1,rep(0,3),1,0,1,rep(0,5))
```

```r
A[6,] <- c(rep(0,5),1,rep(0,3),1,1,0)
A[7,] <- c(rep(c(1,0),4),rep(0,4))
A[8,] <- c(rep(1,3),rep(0,4),1,1,rep(0,3))
A[9,] <- c(rep(1,3),rep(0,4),1,1,rep(0,3))
A[10,] <- c(rep(0,3),1,0,1,rep(0,3),rep(1,3))
A[11,] <- c(rep(0,5),1,rep(0,3),rep(1,3))
A[12,] <- c(rep(0,9),rep(1,3))
z = rep(1,12)
signs <- rep(">=",12)
ans = lp("min",z,A,signs,b,binary.vec = 1:12)
ans$solution
```

```
##  [1] 1 0 0 0 0 0 0 1 0 1 0 0
```

```r
ans
```

```
## Success: the objective function is 3
```

4

```r
b <- matrix(c(233,148,106),ncol=1)
finals <- c(25,37,54)
mincut <- floor(120/finals)
res <- matrix(0,ncol = 3)
for(i in 0:mincut[1]){
    for(j in 0:mincut[2]){
        for(k in 0:mincut[3]){
            pattern <- c(i,j,k)
            if(sum(pattern*finals)<=120 & sum(pattern)!= 0){
                res <- rbind(res,pattern)
            }
        }
    }
}
A <- matrix(t(res[2:nrow(res),]),ncol=nrow(res)-1)
z <- matrix(120-t(t(A)%*%finals))
signs <- rep("=", nrow(A))
ans = lp("min",z,A,signs,b,int.vec=1:17)
ans$solution
```

```
##  [1]  0  7  0  0  0  3  0  0  0 92  0  0  0  0  0 47  0
```

```r
ans
```

```
## Success: the objective function is 855
```

5

```r
A <- matrix(0,ncol=7,nrow = 7)
for(i in 0:6){
    temp <- rep(1,7)
    except <- c(((i+1)%%7)+1,((i+2)%%7)+1)
    temp[except] <- rep(0,length(except))
    A[i+1,] <- temp
```

```r
}


b <- c(5,13,12,10,14,8,6)
signs <- rep(">=",7)
z <- c(330,300,330,360,360,360,360)
ans = lp("min",z,A,signs,b,int.vec = 1:7)
ans$solution
```

```
## [1] 1 8 2 0 3 0 1
```

```r
ans
```

```
## Success: the objective function is 4830
```