Project 3

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```
library(glmnet)
```

```
## Warning: package 'glmnet' was built under R version 3.4.4
```

```
## Loading required package: Matrix
```

```
## Loading required package: foreach
```

```
## Loaded glmnet 2.0-13
```

```
library(gurobi)
```

```
## Loading required package: slam
```

```
library(knitr)
load("data.Rdata")
```

```
df <- data.frame(cbind(X,y))

fit_lasso_cv <- cv.glmnet(X, y, alpha = 1)

#Lasso min lambda
lasso_min <- glmnet(X,y, alpha = 1, lambda = fit_lasso_cv$lambda.min)
min_lambda_coef<- coef(lasso_min)[-1]
nonzero_min <- length(which(min_lambda_coef!=0))

#Lasso onese lambda
lasso_1se<- glmnet(X,y, alpha = 1, lambda = fit_lasso_cv$lambda.1se)
onese_lambda_coef<- coef(lasso_1se)[-1]
nonzero_onese <-length(which(onese_lambda_coef!=0))
```

For lasso, we tried both lambda that minimizes cross-validated error and that which has one standard deviation of the error away from the minimum.

2

$$(Y - X\beta)^T(Y - X\beta) = (Y^T - (X\beta)^T)(Y - X\beta)$$
$$= Y^TY - Y^TX\beta - \beta^TX^TY + \beta^TX^TX\beta$$
$$= Y^TY - 2 * Y^TX\beta + \beta^TX^TX\beta \quad \text{(If A is a 1x1 A } = A^T. Thus,\ \beta^TX^TY = Y^TX\beta)$$

$$= Y^TY + (-2) * (X^TY)^T\beta + \beta^TX^TX\beta$$

This tells us that $constant = \frac{1}{2} * Y^TY,\ C = -X^TY,\ and\ Q = \frac{1}{2} * X^TX$

```
##Function to estimate betas using MIQP
MIQP <- function(X,y,num_beta,k,M){
    model <- list()
    sum_z_constr <- matrix(c(rep(0,num_beta),rep(1,num_beta)),nrow=1)
```

```
    upper_m_constr <- cbind(diag(rep(1,num_beta)),-diag(rep(M,num_beta)))
    lower_m_constr <- cbind(diag(rep(-1,num_beta)),-diag(rep(M,num_beta)))
    model$A <- rbind(sum_z_constr,upper_m_constr,lower_m_constr)

    model$obj <- c((-t(X)%*%y),rep(0,num_beta))
    model$objcon <- (0.5)*(t(y)%*%y)
    Q <- matrix(0,2*num_beta,2*num_beta)
    Q[(1:num_beta),(1:num_beta)] <- t(X)%*%X
    model$Q <- 0.5*Q
    model$sense <- c(rep("<=",nrow(model$A)))
    model$vtype <- c(rep("C",num_beta),rep("B",num_beta))
    model$rhs <- c(k,rep(0,nrow(model$A)-1))
    model$modelsense <- "min"
    ans <- gurobi(model)
    return(ans)
}
```

```
##Solve MIPQ repeatedly
done <- FALSE
num_beta <- 64
k <- 8
M <- 0.01
while(!done){
    ans <- MIQP(X,y,num_beta,k,M)
    if(max(abs(ans$x[1:num_beta])) >= M){
        M <- 2*M
        next
    }
    else{
        break
    }
}
```

```
## Warning for adding variables: zero or small (< 1e-13) coefficients, ignored
## Optimize a model with 129 rows, 128 columns and 320 nonzeros
## Model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
## Coefficient statistics:
##   Matrix range     [1e-02, 1e+00]
##   Objective range  [3e+00, 6e+02]
##   QObjective range [4e-03, 6e+02]
##   Bounds range     [1e+00, 1e+00]
##   RHS range        [8e+00, 8e+00]
## Found heuristic solution: objective 2391.2839442
## Presolve removed 64 rows and 0 columns
## Presolve time: 0.01s
## Presolved: 65 rows, 128 columns, 192 nonzeros
## Presolved model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
##
## Root relaxation: objective 2.350639e+03, 133 iterations, 0.00 seconds
##
##     Nodes    |    Current Node    |     Objective Bounds      |     Work
##  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
```

```
##
## *    0     0               0    2350.6389598 2350.63896  0.00%     -    0s
##
## Explored 0 nodes (133 simplex iterations) in 0.03 seconds
## Thread count was 4 (of 4 available processors)
##
## Solution count 2: 2350.64 2391.28
##
## Optimal solution found (tolerance 1.00e-04)
## Best objective 2.350638959810e+03, best bound 2.350638959810e+03, gap 0.0000%
## Warning for adding variables: zero or small (< 1e-13) coefficients, ignored
## Optimize a model with 129 rows, 128 columns and 320 nonzeros
## Model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
## Coefficient statistics:
##   Matrix range     [2e-02, 1e+00]
##   Objective range  [3e+00, 6e+02]
##   QObjective range [4e-03, 6e+02]
##   Bounds range     [1e+00, 1e+00]
##   RHS range        [8e+00, 8e+00]
## Found heuristic solution: objective 2391.2839442
## Presolve removed 64 rows and 0 columns
## Presolve time: 0.00s
## Presolved: 65 rows, 128 columns, 192 nonzeros
## Presolved model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
##
## Root relaxation: objective 2.310396e+03, 133 iterations, 0.00 seconds
##
##     Nodes    |    Current Node    |     Objective Bounds      |     Work
##  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
##
## *    0     0               0    2310.3957164 2310.39572  0.00%     -    0s
##
## Explored 0 nodes (133 simplex iterations) in 0.00 seconds
## Thread count was 4 (of 4 available processors)
##
## Solution count 2: 2310.4 2391.28
##
## Optimal solution found (tolerance 1.00e-04)
## Best objective 2.310395716422e+03, best bound 2.310395716422e+03, gap 0.0000%
## Warning for adding variables: zero or small (< 1e-13) coefficients, ignored
## Optimize a model with 129 rows, 128 columns and 320 nonzeros
## Model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
## Coefficient statistics:
##   Matrix range     [4e-02, 1e+00]
##   Objective range  [3e+00, 6e+02]
##   QObjective range [4e-03, 6e+02]
##   Bounds range     [1e+00, 1e+00]
##   RHS range        [8e+00, 8e+00]
## Found heuristic solution: objective 2391.2839442
## Presolve removed 64 rows and 0 columns
## Presolve time: 0.00s
```

```
## Presolved: 65 rows, 128 columns, 192 nonzeros
## Presolved model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
##
## Root relaxation: objective 2.231114e+03, 133 iterations, 0.00 seconds
##
##     Nodes    |    Current Node    |     Objective Bounds      |     Work
##  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
##
## *    0     0               0    2231.1144526 2231.11445  0.00%     -    0s
##
## Explored 0 nodes (133 simplex iterations) in 0.00 seconds
## Thread count was 4 (of 4 available processors)
##
## Solution count 2: 2231.11 2391.28
##
## Optimal solution found (tolerance 1.00e-04)
## Best objective 2.231114452564e+03, best bound 2.231114452564e+03, gap 0.0000%
## Warning for adding variables: zero or small (< 1e-13) coefficients, ignored
## Optimize a model with 129 rows, 128 columns and 320 nonzeros
## Model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
## Coefficient statistics:
##   Matrix range     [8e-02, 1e+00]
##   Objective range  [3e+00, 6e+02]
##   QObjective range [4e-03, 6e+02]
##   Bounds range     [1e+00, 1e+00]
##   RHS range        [8e+00, 8e+00]
## Found heuristic solution: objective 2391.2839442
## Presolve removed 64 rows and 0 columns
## Presolve time: 0.00s
## Presolved: 65 rows, 128 columns, 192 nonzeros
## Presolved model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
##
## Root relaxation: objective 2.077373e+03, 133 iterations, 0.00 seconds
##
##     Nodes    |    Current Node    |     Objective Bounds      |     Work
##  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
##
## *    0     0               0    2077.3728165 2077.37282  0.00%     -    0s
##
## Explored 0 nodes (133 simplex iterations) in 0.00 seconds
## Thread count was 4 (of 4 available processors)
##
## Solution count 2: 2077.37 2391.28
##
## Optimal solution found (tolerance 1.00e-04)
## Best objective 2.077372816532e+03, best bound 2.077372816532e+03, gap 0.0000%
## Warning for adding variables: zero or small (< 1e-13) coefficients, ignored
## Optimize a model with 129 rows, 128 columns and 320 nonzeros
## Model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
## Coefficient statistics:
```

```
##   Matrix range     [2e-01, 1e+00]
##   Objective range  [3e+00, 6e+02]
##   QObjective range [4e-03, 6e+02]
##   Bounds range     [1e+00, 1e+00]
##   RHS range        [8e+00, 8e+00]
## Found heuristic solution: objective 2391.2839442
## Presolve removed 64 rows and 0 columns
## Presolve time: 0.00s
## Presolved: 65 rows, 128 columns, 192 nonzeros
## Presolved model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
##
## Root relaxation: objective 1.789173e+03, 133 iterations, 0.00 seconds
##
##     Nodes    |    Current Node    |     Objective Bounds      |     Work
##  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
##
## *    0     0               0    1789.1731112 1789.17311  0.00%     -    0s
##
## Explored 0 nodes (133 simplex iterations) in 0.00 seconds
## Thread count was 4 (of 4 available processors)
##
## Solution count 2: 1789.17 2391.28
##
## Optimal solution found (tolerance 1.00e-04)
## Best objective 1.789173111194e+03, best bound 1.789173111194e+03, gap 0.0000%
## Warning for adding variables: zero or small (< 1e-13) coefficients, ignored
## Optimize a model with 129 rows, 128 columns and 320 nonzeros
## Model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
## Coefficient statistics:
##   Matrix range     [3e-01, 1e+00]
##   Objective range  [3e+00, 6e+02]
##   QObjective range [4e-03, 6e+02]
##   Bounds range     [1e+00, 1e+00]
##   RHS range        [8e+00, 8e+00]
## Found heuristic solution: objective 2391.2839442
## Presolve removed 64 rows and 0 columns
## Presolve time: 0.00s
## Presolved: 65 rows, 128 columns, 192 nonzeros
## Presolved model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
##
## Root relaxation: objective 1.289903e+03, 136 iterations, 0.00 seconds
##
##     Nodes    |    Current Node    |     Objective Bounds      |     Work
##  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
##
##      0     0 1289.90320    0    2 2391.28394 1289.90320  46.1%     -    0s
## H    0     0                     1289.9079674 1289.90320  0.00%     -    0s
##
## Explored 1 nodes (136 simplex iterations) in 0.01 seconds
## Thread count was 4 (of 4 available processors)
##
```

```
## Solution count 2: 1289.91 2391.28
##
## Optimal solution found (tolerance 1.00e-04)
## Best objective 1.289907967424e+03, best bound 1.289903202697e+03, gap 0.0004%
## Warning for adding variables: zero or small (< 1e-13) coefficients, ignored
## Optimize a model with 129 rows, 128 columns and 320 nonzeros
## Model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
## Coefficient statistics:
##   Matrix range     [6e-01, 1e+00]
##   Objective range  [3e+00, 6e+02]
##   QObjective range [4e-03, 6e+02]
##   Bounds range     [1e+00, 1e+00]
##   RHS range        [8e+00, 8e+00]
## Found heuristic solution: objective 2391.2839442
## Presolve removed 64 rows and 0 columns
## Presolve time: 0.00s
## Presolved: 65 rows, 128 columns, 192 nonzeros
## Presolved model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
##
## Root relaxation: objective 5.992193e+02, 139 iterations, 0.00 seconds
##
##     Nodes    |    Current Node    |     Objective Bounds      |     Work
##  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
##
##      0     0  599.21926    0    5 2391.28394  599.21926  74.9%     -    0s
## H    0     0                     599.9147475  599.21926  0.12%     -    0s
##      0     0     cutoff    0       599.91475  599.91475  0.00%     -    0s
##
## Explored 1 nodes (143 simplex iterations) in 0.01 seconds
## Thread count was 4 (of 4 available processors)
##
## Solution count 2: 599.915 2391.28
##
## Optimal solution found (tolerance 1.00e-04)
## Best objective 5.999147475143e+02, best bound 5.999147475143e+02, gap 0.0000%
## Warning for adding variables: zero or small (< 1e-13) coefficients, ignored
## Optimize a model with 129 rows, 128 columns and 320 nonzeros
## Model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
## Coefficient statistics:
##   Matrix range     [1e+00, 1e+00]
##   Objective range  [3e+00, 6e+02]
##   QObjective range [4e-03, 6e+02]
##   Bounds range     [1e+00, 1e+00]
##   RHS range        [8e+00, 8e+00]
## Found heuristic solution: objective 2391.2839442
## Presolve removed 64 rows and 0 columns
## Presolve time: 0.00s
## Presolved: 65 rows, 128 columns, 192 nonzeros
## Presolved model has 2080 quadratic objective terms
## Variable types: 64 continuous, 64 integer (64 binary)
##
```

```
## Root relaxation: objective 2.933035e+02, 88 iterations, 0.00 seconds
##
##     Nodes    |    Current Node    |     Objective Bounds      |     Work
##  Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
##
##      0     0  293.30355    0   29 2391.28394  293.30355  87.7%     -    0s
## H    0     0                     306.4524276  293.30355  4.29%     -    0s
##      0     0     cutoff    0       306.45243  306.45243  0.00%     -    0s
##
## Explored 1 nodes (125 simplex iterations) in 0.01 seconds
## Thread count was 4 (of 4 available processors)
##
## Solution count 2: 306.452 2391.28
##
## Optimal solution found (tolerance 1.00e-04)
## Best objective 3.064524275542e+02, best bound 3.064524275542e+02, gap 0.0000%
```

Coefficient of the best solution from MIQP

```
ans$x[1:num_beta]
```

```
##  [1] 0.8930573 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
##  [8] 0.0000000 1.0892370 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [15] 0.0000000 0.0000000 0.9921031 0.0000000 0.0000000 0.0000000 0.0000000
## [22] 0.0000000 0.0000000 0.0000000 1.1158345 0.0000000 0.0000000 0.0000000
## [29] 0.0000000 0.0000000 0.0000000 0.0000000 0.9797358 0.0000000 0.0000000
## [36] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 1.0029896 0.0000000
## [43] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 1.0201952
## [50] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [57] 1.0389175 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [64] 0.0000000
```

```
sol <- ans$x[1:num_beta]
nonzero_MIQP <- length(sol[sol!= 0])
num_nonzero <- c(nonzero_MIQP,nonzero_min,nonzero_onese)
num_nonzero_df <- data.frame(num_nonzero)
colnames(num_nonzero_df) <- c("Number Non-zero coefficient")
rownames(num_nonzero_df) <- c("MIQP","Min_Lasso","1se_Lasso")
kable(num_nonzero_df)
```

	Number Non-zero coefficient
MIQP	8
Min_Lasso	25
1se_Lasso	9

We can see that the number of nonzero coefficients from MIQP (8) is in between those using min and 1se lambdas.

3

```
##Helper function to get the error
norm_2 <- function(v){
    return(t(v)%*%v)
}

##Function to get the error
pred_error <- function(X,beta_hat,beta_0){
    estimate <- X%*%beta_hat
    truth <- X%*%beta_0
    error <- estimate-truth
    numerator <- norm_2(error)
    denominator <- norm_2(truth)
    return(numerator/denominator)
}
```

```
##Find the errors for each regression
beta_hat_MIQP <- ans$x[1:num_beta]
beta_hat_min_lasso <- min_lambda_coef
beta_hat_1se_lasso <- onese_lambda_coef
beta_0 <- beta_real

MIQP_error <- pred_error(X,beta_hat_MIQP,beta_0)
min_lasso_error <- pred_error(X,beta_hat_min_lasso,beta_0)
onese_lasso_error <- pred_error(X,beta_hat_1se_lasso,beta_0)

error_vec <- c(MIQP_error,min_lasso_error,onese_lasso_error)
names(error_vec) <- c("MIQP","Min_Lasso","1SE_Lasso")
error_df <- data.frame(error_vec)
colnames(error_df) <- "Error"
kable(error_df)
```

	Error
MIQP	0.0044561
Min_Lasso	0.0067197
1SE_Lasso	0.0174882