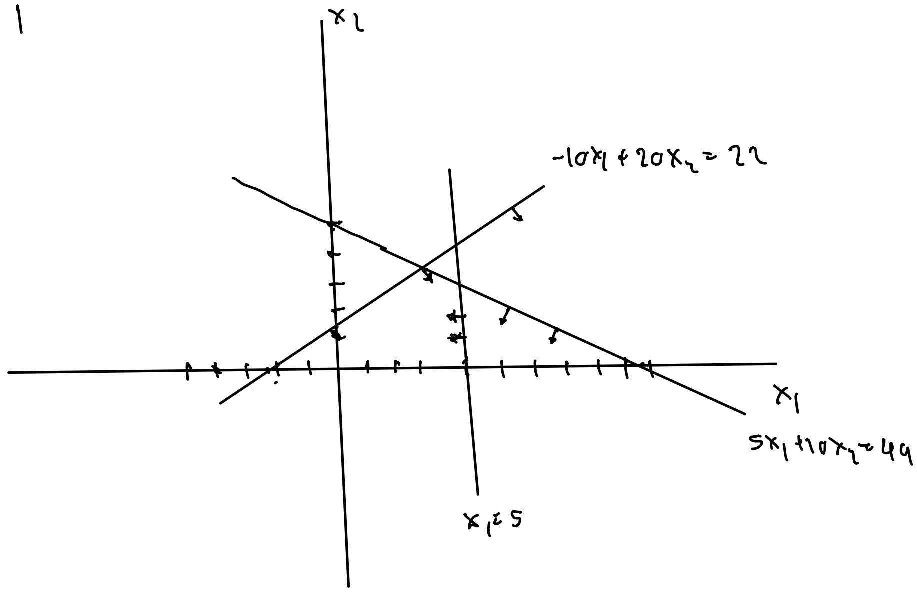
Problem 1



$(x_1=3.8,\ x_2=3)$ (8.2)

$x_1 \le 3$
$(x_1=3,\ x_2=2.6)$
(7.4)

$x_1 \ge 4$
$(x_1=4,\ x_2=2.9)$
$(7.6$

$x_1 \le 3$
$x_2 \le 2$
$(x_1=1.8,\ x_2=2)$
6.2

$x_1 \le 3$
$x_2 \ge 3$
not feasible

$x_1 \ge 4$
$x_2 \le 2$
$(x_1=4,\ x_2=2)$
(4)

$x_1 \ge 4$
$x_2 \ge 3$
not feasible

$x_1 \le 3$
$x_2 \le 2$
$x_1 \le 1$
$(x_1=1,\ x_2=1.6)$
(5.4)
no need to
go further

$x_1 \le 3$
$x_2 \le 2$
$x_1 \ge 2$
$(x_1=2,\ x_2=2)$
(6)

$x_1 \le 3$
$x_2 \le 2$
$x_1 \le 1$
$x_2 \le 1$

$x_1 \le 3$
$x_2 \le 2$
$x_1 \le 1$
$x_2 \ge 2$

$(x_1=1, x_2=1)$ $(x_1=1, x=2)$

(3) not feasible

The optimal solution is $x_1=2$, $x_2=2$

and the objective value is 6

2) R gave same answer. See code below

3) If we stop early difference = $8-2=6$

~ do not stop early " $= 10-3=7$

Problem 2.

Let x_i = # of factory in city i $\quad i=1,2$

y_i = # of warehouse in city i $\quad i=1,2$

Maximize: $f = 9x_1 + 5x_2 + 6y_1 + 4y_2$

subject to:

$$6x_1 + 3x_2 + 5y_1 + 2y_2 \leq 11$$
$$x_1 \leq 1$$
$$x_2 \leq 1$$
$$y_1 \leq 1$$
$$y_2 \leq 1$$
$$y_1 + y_2 \leq 1$$
$$-x_1 - x_2 \leq 1$$
$$x_i, y_i \text{ - integers}$$

They should open a factory in Austin and Dallas and a warehouse in Dallas. With this strategy, they will achieve a profit of \$18 M

Problem 3

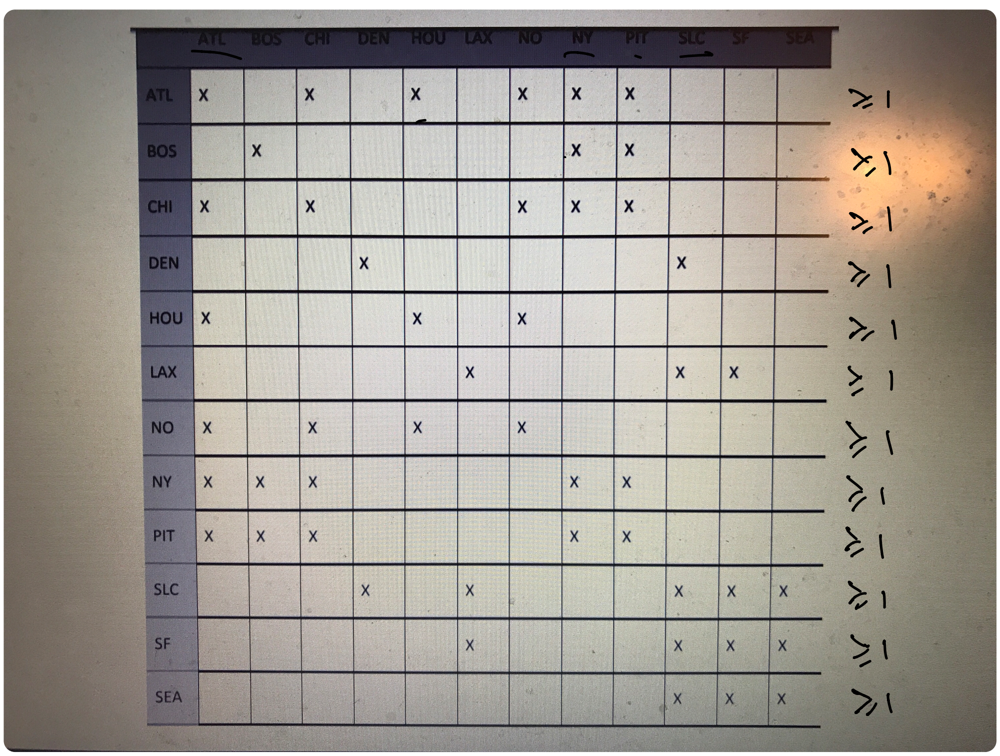
Y_i = whether to open a hub at location i

C_{ij} = whether hub i cover city j

$$\text{minimize} \quad \sum_i Y_{ij}$$

$$\text{subject to:} \quad \sum_i C_{ij} \geq 1 \quad \forall j$$

$$Y_i \in \{0,1\}$$



	ATL	BOS	CHI	DEN	HOU	LAX	NO	NY	PIT	SLC	SF	SEA	
ATL	X		X		X		X	X	X				≥ 1
BOS		X						X	X				≥ 1
CHI	X		X				X	X	X				≥ 1
DEN				X						X			≥ 1
HOU	X				X		X						≥ 1
LAX						X				X	X		≥ 1
NO	X		X		X		X						≥ 1
NY	X	X	X					X	X				≥ 1
PIT	X	X	X					X	X				≥ 1
SLC				X		X				X	X	X	≥ 1
SF						X				X	X	X	≥ 1
SEA										X	X	X	≥ 1

Answer we should build 3 hubs at x_1, x_8, x_{10}.
ATL NY SLC

Problem 4

Let $\quad p_i$ = allowable pattern i

$$p_i = \begin{bmatrix} a_{i1} \\ a_{i2} \\ \vdots \\ a_{ij} \end{bmatrix} \quad \text{where } a_{ij} = \#\text{ of finals } j \text{ in pattern } i$$

c_i = waste produced by pattern $i \quad i = 1, 2, \ldots, 17$

x_i = number of pattern i used $\quad i = 1, 2, \ldots, 17$

d_j = demand for finals $j \quad j = 1, 2, 3$

$$\text{minimize} \quad \sum_{i=1}^{17} c_i x_i$$

$$\text{subject to:} \quad \sum_{i=1}^{n} x_i a_{ij} = d_j \quad \forall j$$

The optimal strategy is to produce

7 of pattern $\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

3 of pattern $\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$

92 of pattern $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

47 of pattern $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

Problem 5

x_i = # of workers on day i

C_i : Cost of hiring a worker on day i

minimize $\sum_i C_i x_i$

subject to: $\sum_{j \in A} x_j \geq n_i \quad \forall i$ where

$A = \{ j : j = \{0, 1, \ldots, 6\}$ and $j \neq i+1 \pmod 7$ and $j \neq i+2 \pmod 7 \}$

The optimal schedule is to hire

1 worker Sun-Thur

8 workers Mon-Fri

2 workers Tue-Sat

0 worker Wed-Sun

3 workers Thur-Mon

1 worker Fri-Tue

This strategy will cost them 4,830.

3) most popular is Mon-Fri.