Problem 1

Let,

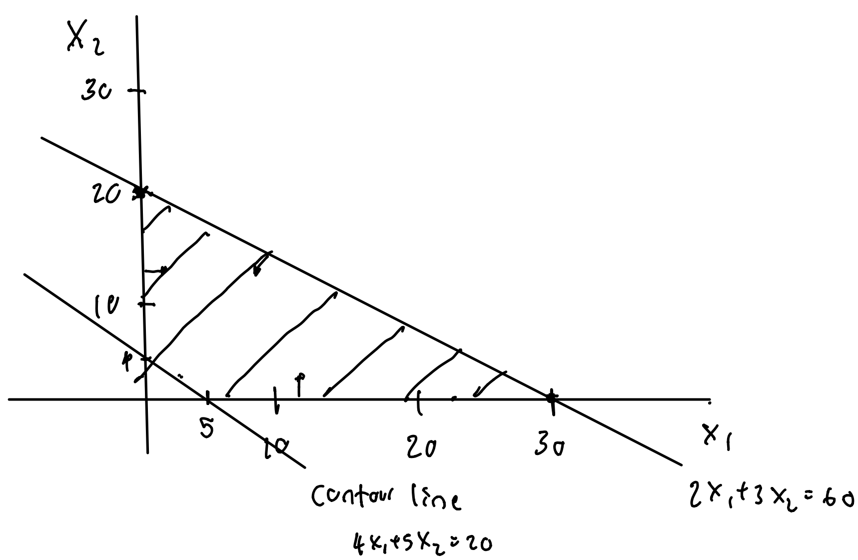
x_1 = # of torte consumed

x_2 = # of apple pie consumed

We want to maximize $f(x_1, x_2) = 4x_1 + 5x_2$

subject to: $2x_1 + 3x_2 \leq 60$

$x_1, x_2 \geq 0$

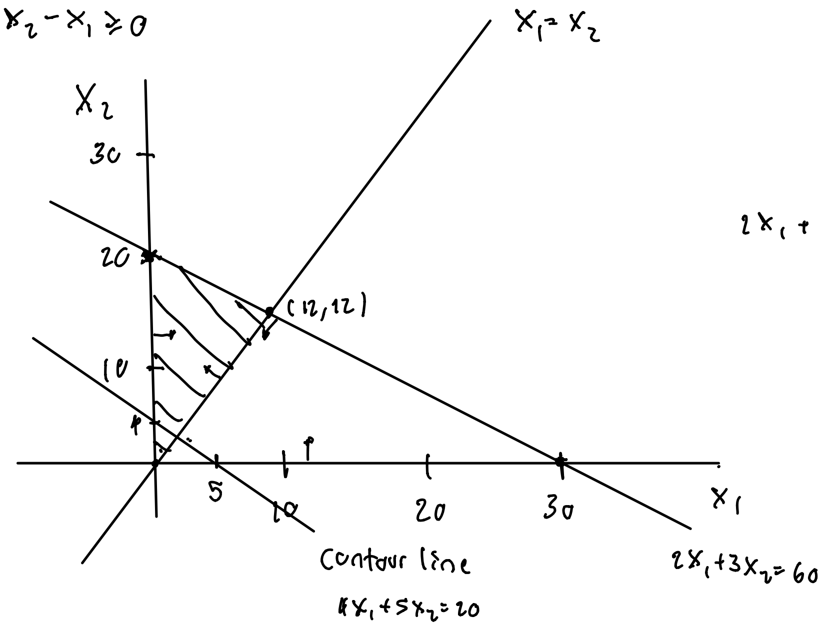


If move contour line parallely in the direction of the gradient, we can see that the last point touching is $x_1 = 30, x_2 = 0$.

Thus, Max should eat 30 tortes and 0 apple pies and he will get 120 points, which is optimal.

Adding new constraint

$x_2 - x_1 \geq 0$



$2x_1 + 3x_1 = 60$

$5x_1 = 60$

$x_1 = 12$

$x_2 = 12$

Now, we can see that our feasible region changes and the new set of solution is $x_1 = 12$, $x_2 = 12$.

Thus, Max should eat 12 tartes and 12 apple pies and he will get 108 points, which is 12 points lower than the optimal point previously calculated.

Problem 2

a) Let

x_1 = # of wheat planted

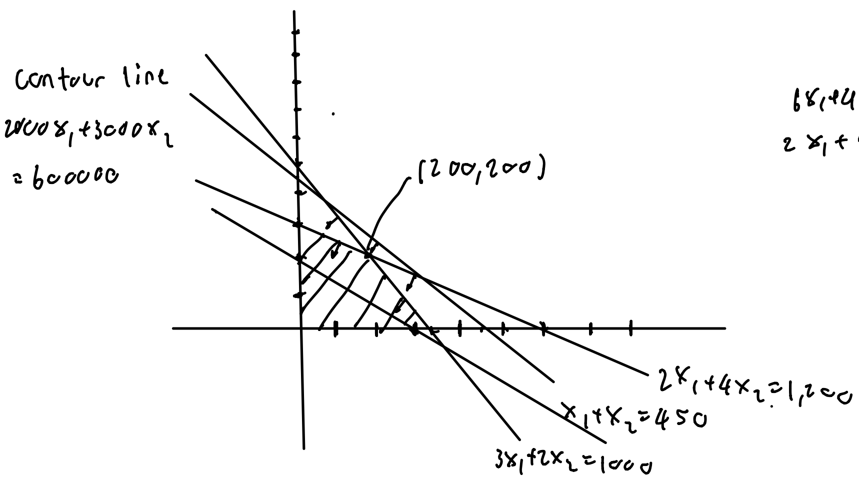
x_2 = # of corn planted

Our objective is to:

maximize $f(x_1, x_2) = 2000x_1 + 3000x_2$

subject to:

$$3x_1 + 2x_2 \leqslant 1{,}000$$
$$2x_1 + 4x_2 \leqslant 1{,}200$$
$$x_1 + x_2 \leqslant 450$$
$$x_1, x_2 \geqslant 0$$



$$6x_1 + 4x_2 = 2000$$
$$2x_1 + 4x_2 = 1200$$
$$4x_1 = 800$$
$$x_1 = 200$$
$$x_2 = 200$$

If move contour line parallely in the direction of the gradient, we can see that the last point touching is $x_1 = 200$, $x_2 = 200$

Thus, the farmer should produce 200 wheat and 200 corn. and he will make 1,000,000 in profit.

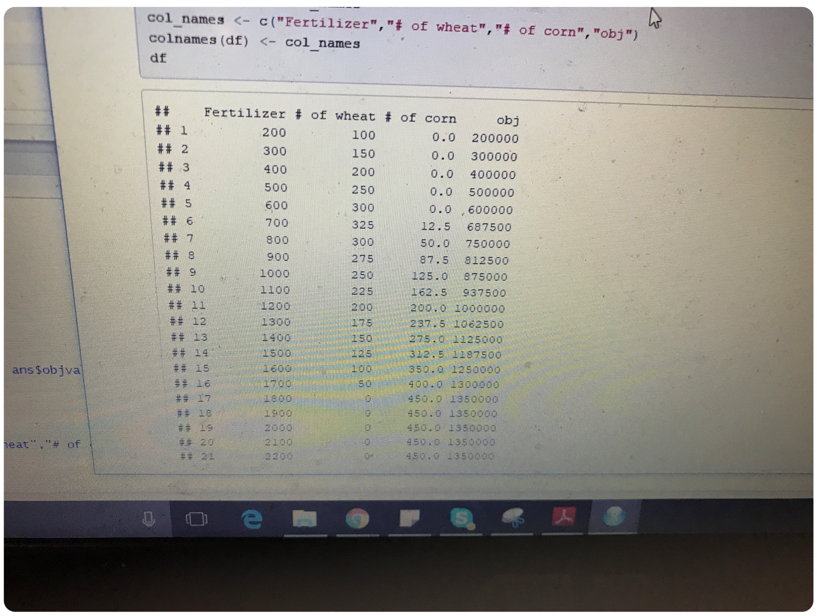
b) We solved the problem in R by inputting

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 4 \\ 1 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad b = \begin{bmatrix} 1000 \\ 1200 \\ 450 \\ 0 \\ 0 \end{bmatrix} \qquad z = \begin{bmatrix} 2000 \\ 3000 \end{bmatrix}$$

constraint coefficient — RHS — coefficient of objective function

R gave the same answer (200 wheat, 200 corn) and the same optimal value 1000000.

c)



```
col_names <- c("Fertilizer","# of wheat","# of corn","obj")
colnames(df) <- col_names
df
```

##	Fertilizer	# of wheat	# of corn	obj
## 1	200	100	0.0	200000
## 2	300	150	0.0	300000
## 3	400	200	0.0	400000
## 4	500	250	0.0	500000
## 5	600	300	0.0	600000
## 6	700	325	12.5	687500
## 7	800	300	50.0	750000
## 8	900	275	87.5	812500
## 9	1000	250	125.0	875000
## 10	1100	225	162.5	937500
## 11	1200	200	200.0	1000000
## 12	1300	175	237.5	1062500
## 13	1400	150	275.0	1125000
## 14	1500	125	312.5	1187500
## 15	1600	100	350.0	1250000
## 16	1700	50	400.0	1300000
## 17	1800	0	450.0	1350000
## 18	1900	0	450.0	1350000
## 19	2000	0	450.0	1350000
## 20	2100	0	450.0	1350000
## 21	2200	0	450.0	1350000

As we can see from the output, the farmer will not produce corn when fertilizer is between 200 and 600. When fertilizer is between 1800 and 2200, the farmer stops producing tree.

We also notice that profit starts increasing once we relax fertilizer constraint and stays the same after fertilizer=1800, meaning that the constraint is no longer binding.

Problem 3

Let $\quad x_i$ = the proportion invested in investment i for $i = 1,2,3,4,5$

$$11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \leq 40$$
$$3x_1 + 6x_2 + 5x_3 + 1x_4 + 34x_5 \leq 20$$

$$x_i \leq 1 \quad \forall\, i = 1,2,\ldots,5$$

$$-x_i \leq 0 \quad \forall\, i = 1,2,\ldots,5$$

We would buy 100% of investment 1, 3, and 4, 20% of investment 2, and 29% in investment 5. We would make 57.44 millions in the end.

Problem 4

x_1 : # of servings for corn

x_2 = # of servings for milk

x_3 = # of serving for wheat bread

minimize $\quad f = 0.18x_1 + 0.23x_2 + 0.05x_3$

subject to:

$$-107x_1 - 500x_2 - 0x_3 \leq -5000$$

$$107x_1 + 500x_2 + 0x_3 \leq 50000$$

$$-72x_1 - 121x_2 - 65x_3 \leq -2000$$

$$72x_1 + 121x_2 + 65x_3 \leq 2250$$

$$x_i \leq 10 \quad \forall i \quad i \in \{1,2,3\}$$

$$-x_i \leq 0 \quad \forall i \quad i \in \{1,2,3\}$$

we would buy 1.94 servings of corn, 10 servings of milk, and 10 servings of wheat bread. This will cost us \$ 3.15.

Problem 5 (formulation 1) *** I solved this problem two ways (please scroll down to see both)

Let x_{ij} = # of tons of wood in unit i in year j

$i = 1,2 \quad j = 1,2,3$

a_{ij} = # of tons of wood available of unit i in year j

b_j = minimum number of wood harvested in year j

c_j = maximum number of wood harvested in year j

maximize $\quad f = \sum_j \sum_i x_{ij}$

subject to:

$$x_{ij} \leq a_{ij} \quad \forall i = 1,2 \ \ \forall j = 1,2,3$$

$$-\sum_i x_{ij} \leq -b_j \quad \forall j = 1,2,3$$

$$\sum_i x_{ij} \leq c_j \quad \forall j = 1,2,3$$

$$-x_{ij} \leq 0 \quad \forall i = 1,2 \ \ \forall j = 1,2,3$$

x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	
1						$\leq 1 \times 2 = 2$
	1					$\leq 1.3 \times 2 = 2.6$
		1				$\leq 1.4 \times 2 = 2.8$
			1			$\leq 1 \times 3 = 3$
				1		$\leq 1.2 \times 3 = 3.6$
					1	$\leq 1.6 \times 3 = 4.8$
-1			-1			≤ -1.2
	-1			-1		≤ -1.5
		-1			-1	≤ -2
1			1			≤ 2
	1			1		≤ 2
		1			1	≤ 3
-1						

-1						≤ 0
	-1					≤ 0
		-1				≤ 0
			-1			≤ 0
						≤ 0
					-1	≤ 0

In unit 1, we would harvest 2, 2, 2.8 in year 1, 2, 3 respectively.
In unit 2, we would harvest 0, 0, 0, 2 in year 1, 2, 3 respectively.
We would achieve 7 tons of wood in the end.

Problem 5 (formulation 2)

Let x_{ij} = # of acre cut of unit i in year j

$i=1,2 \quad j=1,2,3$

$$f: x_{11}+1.3x_{12}+1.4x_{13}+x_{21}+1.2x_{22}+1.6x_{23}$$

$$x_{11}+x_{12}+x_{13} \leq 2$$

$$x_{21}+x_{22}+x_{23} \leq 3$$

$$-x_{11}-x_{21} \leq -1.2$$

$$1.3x_{12}+1.2x_{22} \leq -1.5$$

$$1.4x_{13}+1.6x_{23} \leq -2$$

$$x_{11}+x_{21} \leq 2$$

$$1.3x_{12}+1.2x_{22} \leq 2$$

$$1.4x_{13}+1.6x_{23} \leq 3$$

$$x_{ij} \geq 0 \quad \forall i=1,2 \quad \forall j=1,2,3$$

In unit 1, we would cut 0.46, 1.54, and 0 acre in year 1,2,3 respectively.

In unit 2, we would cut 1.125, 0, and 1.875 acre in year 1,2,3 respectively.

With this strategy, we will get 6.59 tons of wood.