# Wing Flutter Analysis With An Uncoupled Method

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*M. Sc. Thesis Presentation*

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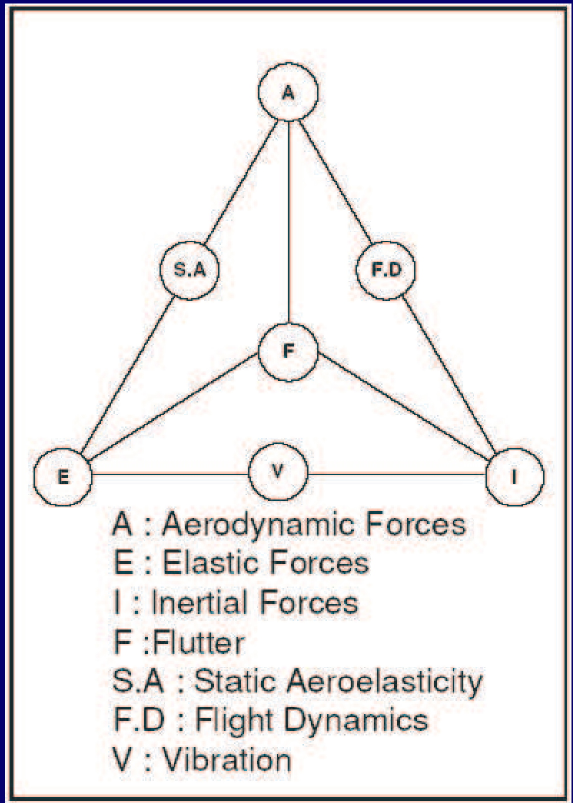
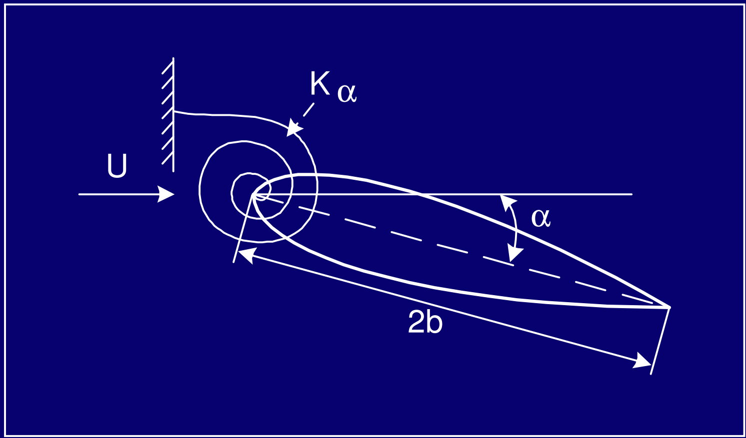
# Introduction




*Flutter*

*Dynamic instability of an elastic body in an airstream caused by the unsteady aerodynamic forces generated from elastic deformations of the structure.*

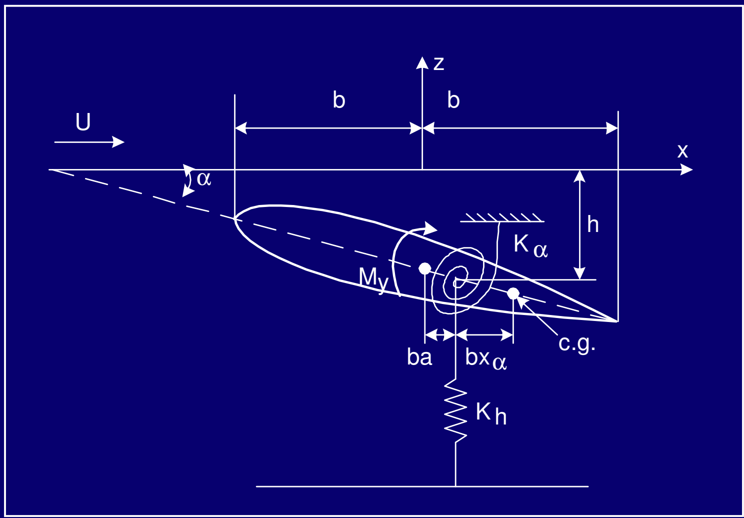
# Flutter Modeling




1 – DOF System

- Rigid Airfoil
- Unit Span
- Oscillation around the Leading Edge

$$I_\alpha \ddot{\alpha} + K_\alpha \alpha = M_y \qquad \alpha = \bar{\alpha}_0 e^{iwt}$$




2 – DOF System

- Rigid Airfoil
- Unit Span
- Pitching and Plunging Motion

$$m\ddot{h} + S_\alpha \ddot{\alpha} + m w_h^2 h = Q_h$$

$$S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + I_\alpha w_\alpha^2 \alpha = Q_\alpha$$

# Flutter Modeling

## P-K Method

- Forced Response Analysis
- Eigenvalue Solution

$$[M]\{\ddot{q}\}+[K]\{q\}=\{f(q,\dot{q},\ddot{q},t)\} \longrightarrow$$ EOM for the system

- Harmonic displacement
- Transform to Modal Coordinates

* $$\left[[M_h]s^2+[K_h]-\frac{1}{2}\rho V^2[Q_h]\right]\{\eta_h\}=0 \longrightarrow$$ Basic EV Flutter Eqn.

$$s=\frac{Vk}{b}(\gamma+i) \qquad p=k(\gamma+i)$$

* Valid when $\gamma = 0$

# Flutter Modeling

## Coupled Methods

- Solution of Aeroelastic Equations every time step
- Grid Transformation
- Coupling Algorithm
  - Weakly Coupled (Separate CFD – CSD)
  - Strongly Coupled (Stability = Least Stable Code)
  - Fully Coupled (Iterative solution at each time step)

$$\ddot{q}_i + 2\zeta_i \omega_i \dot{q}_i + \omega_i^2 q_i = Q_i$$

- 2 SDOF Equations
- Decoupled by Diagonalization
- Solved at each time step

# Flutter Modeling

## Present Approach

- Forced Response Analysis
- Eigenvalue Solution
- Aim is to decouple CSD and CFD

$$[M]\{\ddot{q}\}+[K]\{q\}=\{f(q,\dot{q},\ddot{q},t)\} \longrightarrow \text{EOM for the system}$$

- Harmonic displacement
- Transform to Modal Coordinates

$$\{q\}=[\Phi]\{\eta\}e^{i\omega t}$$

$$\{f_j\}=\mathrm{Im}\left([A]\{\phi_j\}e^{i\omega t}\right)$$

$$\{r_j\}=[A]\{\phi_j\}$$

Unsteady Aerodynamic Force Definition

• Substitute to EOM and get Eigenvalue flutter problem

# Flutter Modeling

$$\left\{[K_M]-\omega^2[M_M]-[\Phi]^T[R]\right\}\{\eta\}=0 \longrightarrow$$ Basic EV Flutter Eqn.

– Similar to P-K method Formulation

– Generalized Aerodynamic Force (GAF) Matrix ( [R] ) is unsymmetric

– [R] = *f(M,k)*  *k : reduced frequency*

$$\left.\begin{aligned}[R]&=[R_R]+i[R_I]\\ [R_R]&=\sum_{p=0}^{P}c^p\left[T_p\right]\\ [R_I]&=\sum_{p=0}^{P}c^p\left[Z_p\right]\end{aligned}\right\}$$

– By time integration get real and imaginary parts of aerodynamic force

– Fit *P^th* order polynomial to aerodynamic force

• Transform into a Polynomial Eigenvalue Problem

# Flutter Modeling

– Polynomial curve fitting in terms of reduced frequency *(k)*

$$\left(\sum_{p=0}^{P} k^p \bar{Q}_p\right)\{\bar{\eta}\} = \{0\} \longrightarrow$$

Polynomial Eigenvalue Flutter Equation

$$\{\bar{\eta}\} = \begin{bmatrix} \{\eta_R\} \\ \{\eta_I\} \end{bmatrix} \longrightarrow$$

Transform to a real augmented system
[Q] is (8x8)

$$\bar{Q}_0 = \begin{bmatrix} [K_M] & [0] \\ [0] & [K_M] \end{bmatrix}$$

$$\bar{Q}_2 = Q_2 - \left(\frac{U_\infty}{b}\right)^2 \begin{bmatrix} [M_M] & [0] \\ [0] & [M_M] \end{bmatrix}$$

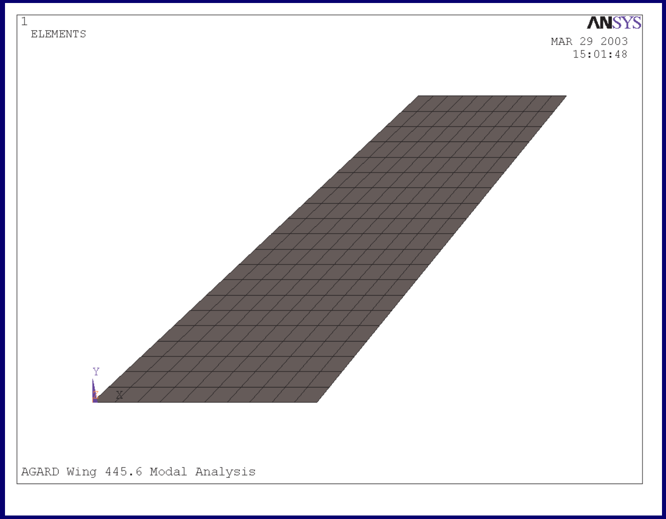
$$\bar{Q}_p = Q_p$$ *for p = 1,3,4 … P*

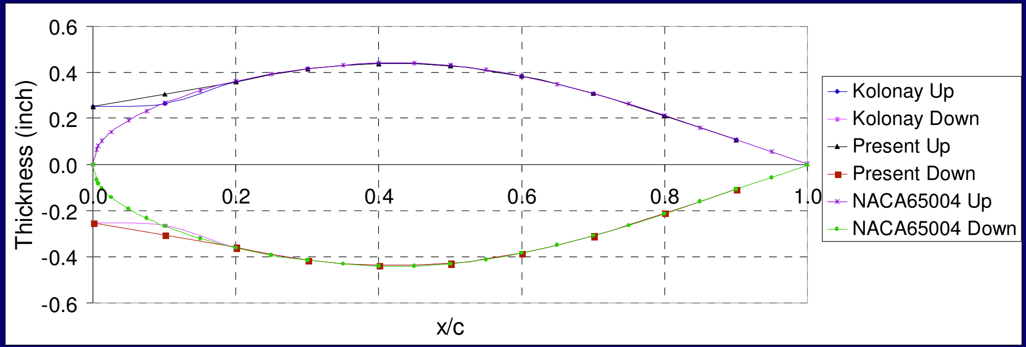
$$Q_p = \begin{bmatrix} -[\Phi]^T\left[T_p\right] & [\Phi]^T\left[Z_p\right] \\ -[\Phi]^T\left[Z_p\right] & -[\Phi]^T\left[Z_p\right] \end{bmatrix}$$

# Modal Analysis of AGARD Wing 445.6

## Finite Element Model

- Shell elements
- Distributed thickness
- Rotated element coordinate system
- In accordance with real wing



# Modal Analysis of AGARD Wing 445.6

## Modal Analysis

- Theoretical Aspects
- Comparison with Experimental Results

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=\{0\}$$

- Undamped System
- Harmonic Free Vibrations

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \quad \{q\}=\{u\}e^{i\omega t} \qquad [C]=0$$

$$-\omega^2[M]\{u\}+[K]\{u\}=\{0\}$$

- Carried out with ANSYS®

# Modal Analysis of AGARD Wing 445.6

**Natural Frequency Comparison**

| Modes | Experiment | Present Study | | Kolonay | | Li | |
|---|---|---|---|---|---|---|---|
| 1 | 9.60 | 9.688 | %0.92 | 9.63 | % 0.31 | 10.85 | %13.0 |
| 2 | 38.10 | 37.854 | -%0.65 | 37.12 | -%2.57 | 44.57 | %16.9 |
| 3 | 50.70 | 50.998 | % 0.59 | 50.50 | -%0.39 | 56.88 | %12.2 |
| 4 | 98.50 | 92.358 | -%6.24 | 89.94 | -%8.69 | 109.10 | %10.8 |

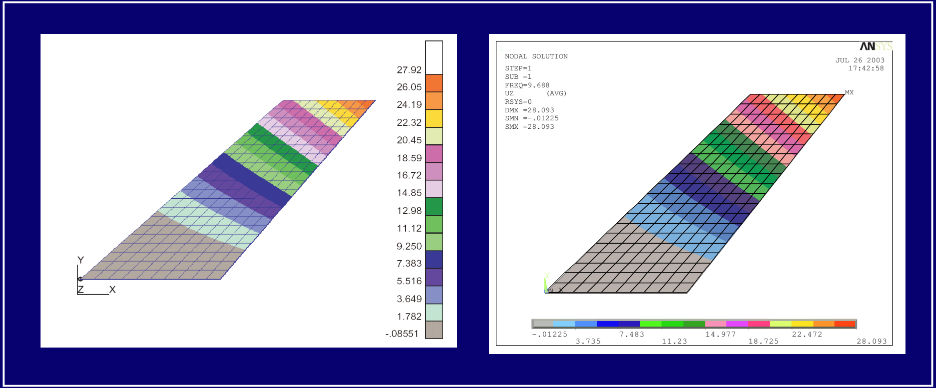
**Deflection Comparison**

| Modes | Present Study | | Kolonay | |
|---|---|---|---|---|
| 1 | -0.0125 | 28.093 | -0.086 | 27.92 |
| 2 | -25.375 | 45.873 | -45.48 | 25.09 |
| 3 | -27.923 | 31.595 | -27.17 | 31.38 |
| 4 | -22.82 | 67.182 | -22.62 | 71.52 |

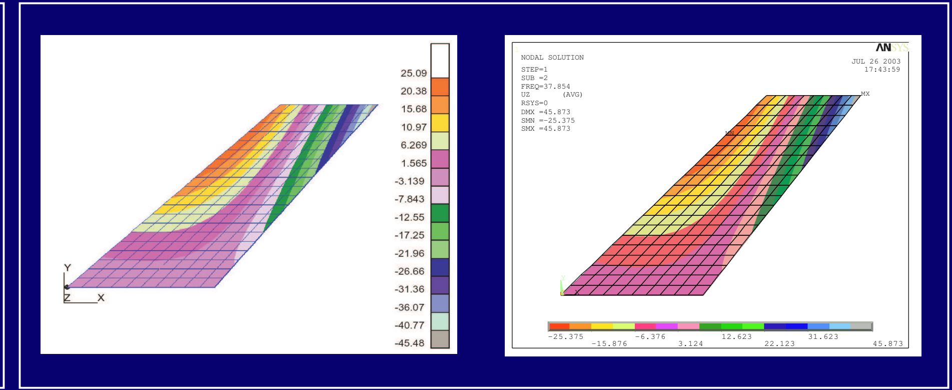
# Modal Analysis of AGARD Wing 445.6

## Comparison of Mode Shapes

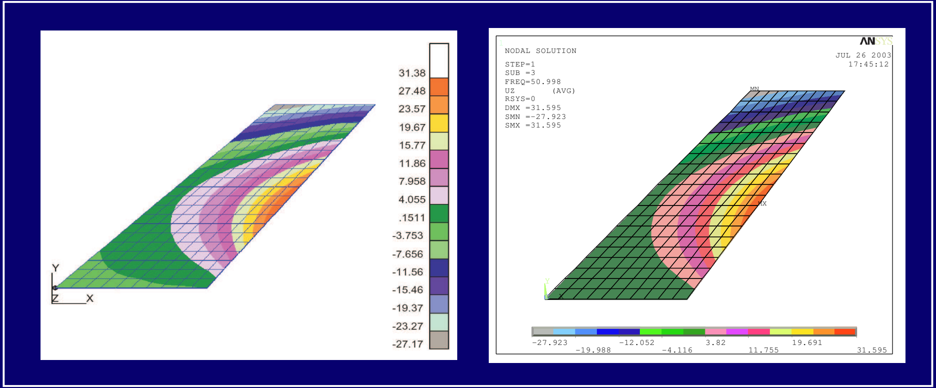
1st Mode



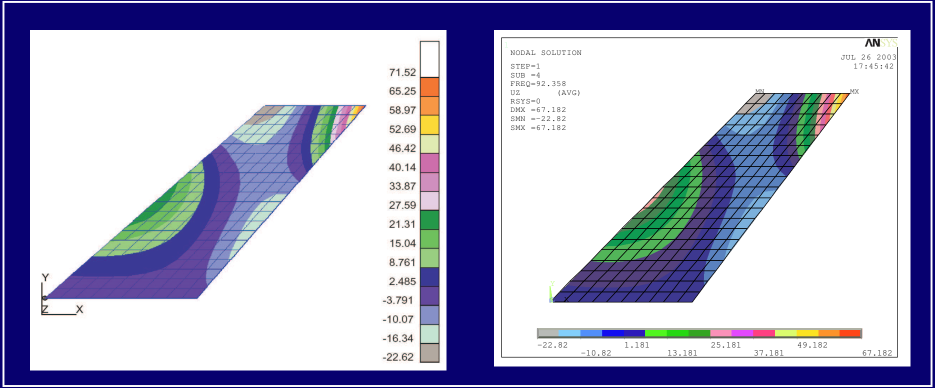

2nd Mode




3rd Mode




4th Mode




– The results of the modal analysis are accepted for further use …

# Aerodynamic Analysis of AGARD Wing 445.6

Aerodynamic Analysis

- USER3D
  - Unstructured 3D Euler Solver
- Harmonic oscillation of the wing
  - 120 time steps per period for each oscillation frequency
- Grid transformation
  - Surface interpolation technique is adopted
- Pressure transformation
  - Surface interpolation technique is adopted
- Force Calculation
  - Using the pressure distribution on structural model

# Aerodynamic Analysis of AGARD Wing 445.6

## USER3D

- Parallel finite volume based unstructured 3D Euler solver
- Uses ALE (Arbitrary Lagrangian-Eulerian) formulation
- The flow variables are non-dimensionalised

$$\rho' = \rho \cdot \rho_0$$
$$u' = u \cdot a_0$$
$$v' = v \cdot a_0$$
$$w' = w \cdot a_0$$
$$a' = a \cdot a_0$$
$$T' = T \cdot T_0$$
$$P' = P \cdot \rho_0 a_0^2 \qquad P' = P \cdot P_0 \gamma$$

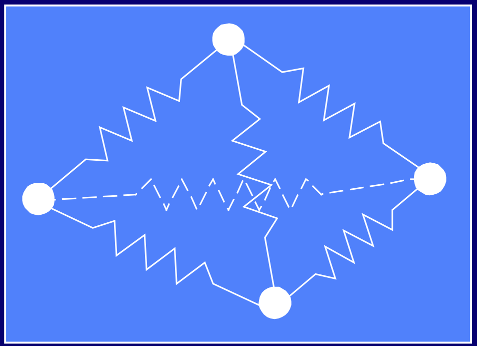
- Pressure is calculated from ideal gas relation

$$p = (\gamma - 1)\left[ e - \frac{1}{2}\rho(u^2 + v^2 + w^2) \right]$$

# Aerodynamic Analysis of AGARD Wing 445.6

## USER3D

- Moving Mesh Algorithm
- Spring analogy



$$k_m = 1 \Big/ \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2 + \left(z_j - z_i\right)^2}$$

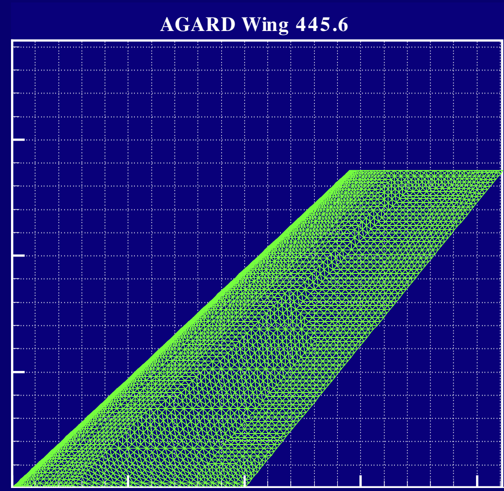
- Geometric Conservation Law

$$\frac{\partial}{\partial t} \iiint_{\Omega} dV = \iint_{\partial\Omega} \vec{W}_s \cdot \vec{n} dS$$

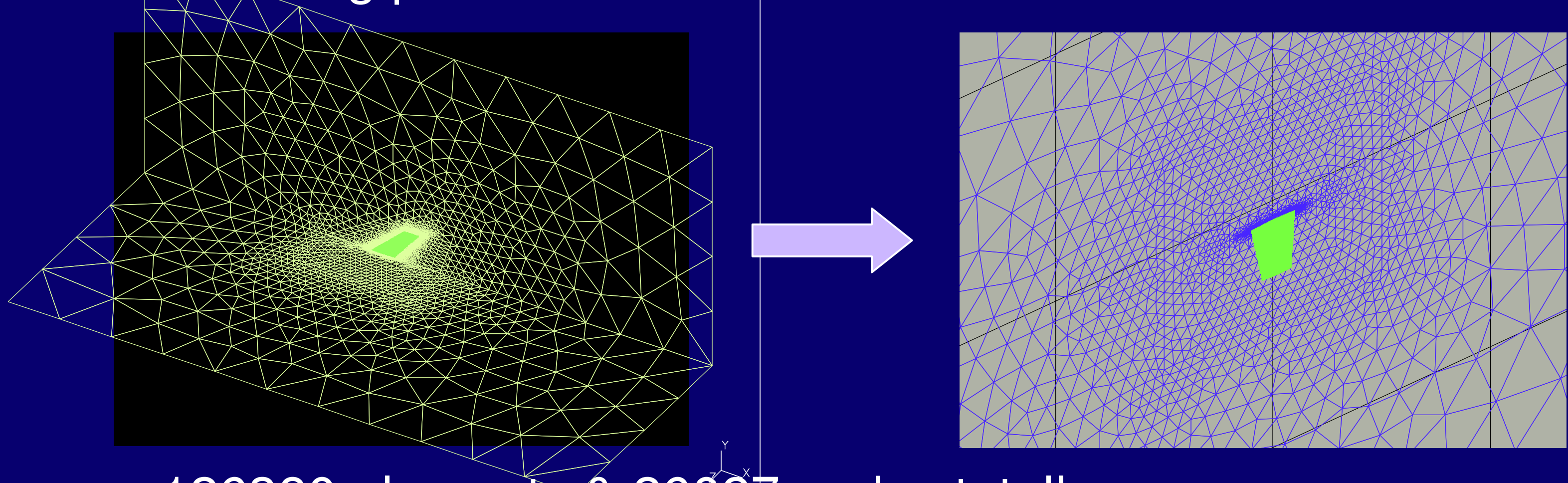
- MPICH library (portable and high performance implementation of MPI) is used in USER3D for parallelization

# Aerodynamic Analysis of AGARD Wing 445.6

## CFD Model




- Non-dimensionalised wrt root chord (21.96”)
- Unstructured grid (tetrahedral elements)
- I-DEAS® is used to generate mesh
- Half-domain is meshed → mirrored full-domain
- The wing profile is NACA 65A004



- 126380 elements & 26027 nodes totally
- 13254 elements & 6667 nodes on the wing

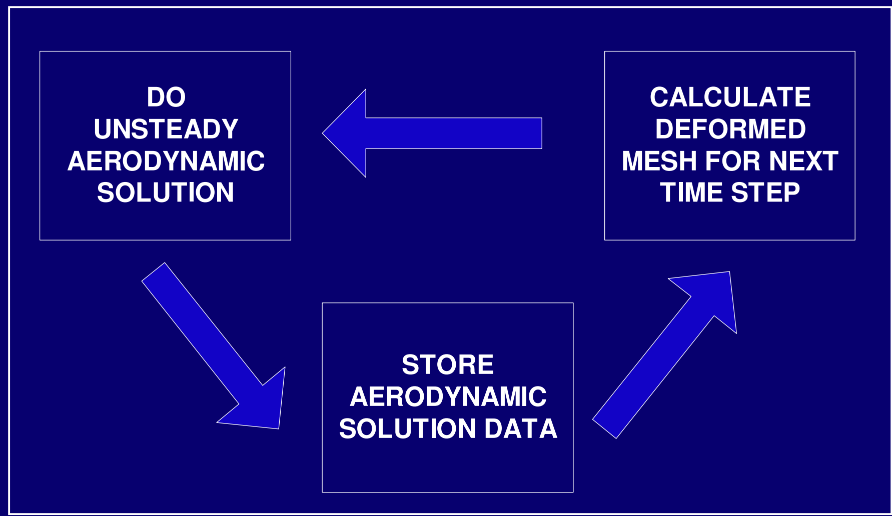
# Aerodynamic Analysis of AGARD Wing 445.6

## Solution Procedure

- Mesh deformation according to mode shapes
- Sinusoidal oscillation for three periods at each frequency
- 10 different frequencies at first 4 mode shapes (40 solution cases)

$$\{Q\} = \{Q_0\} \times sin(\omega t)$$

| Mode # | Maximum Tip Deflection (inch) |
|---|---|
| 1 | 0.015 |
| 2 | 0.03 |
| 3 | 0.004 |
| 4 | 0.0015 |

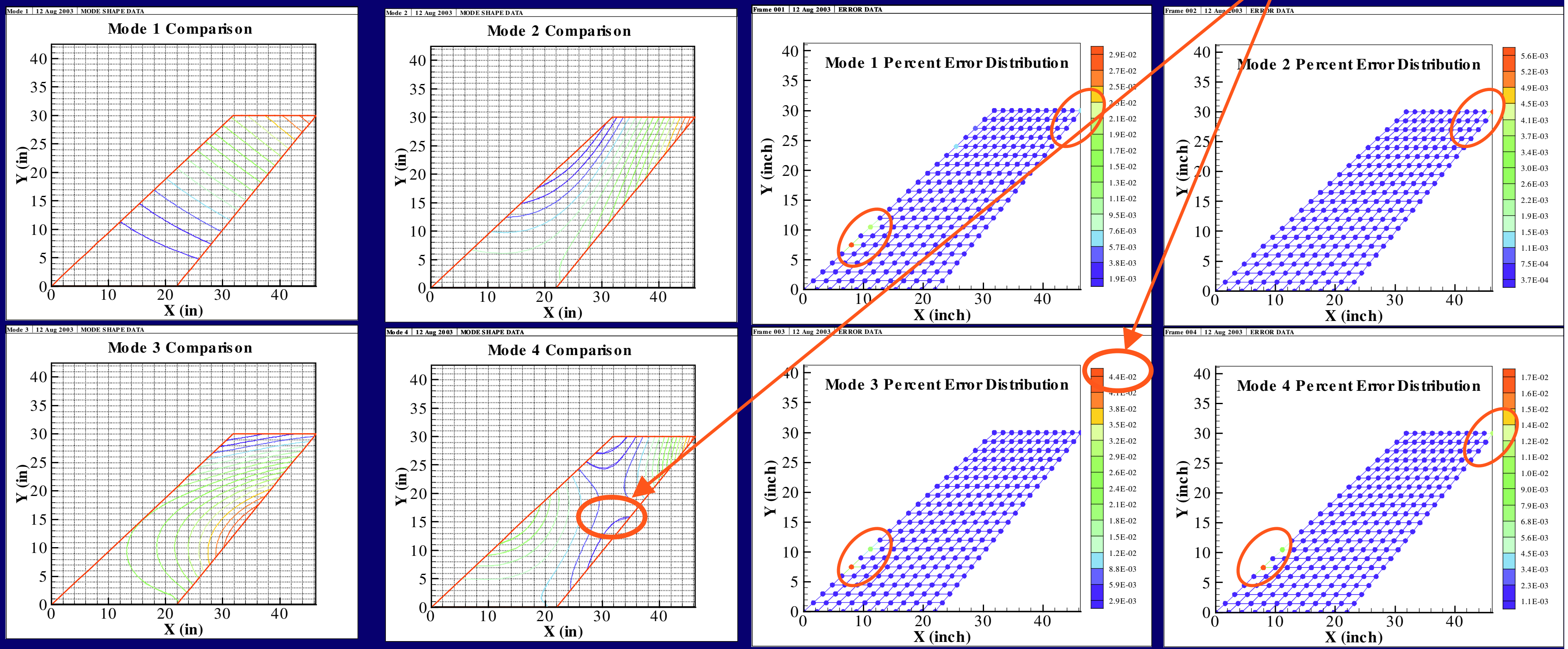


# Aerodynamic Analysis of AGARD Wing 445.6

## Grid Transformation

- 2D Structural Model → 3D CFD Model
- Mode shape transformation
- Surface interpolation by Akima (IMSL)

Max % error

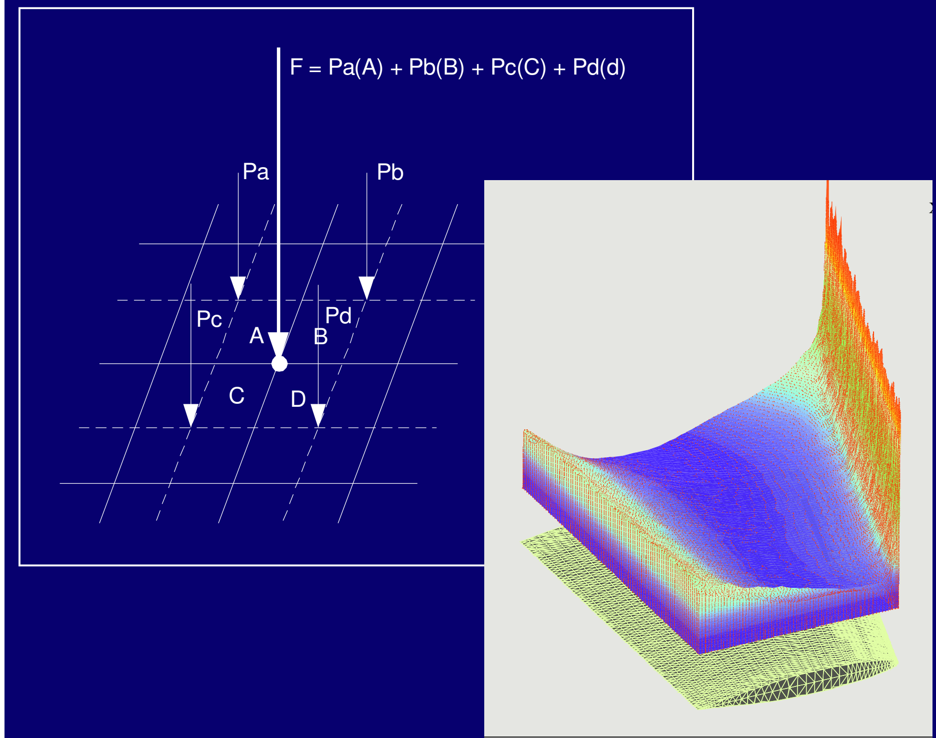


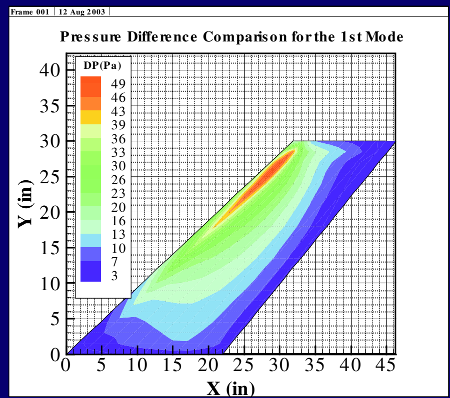
# Aerodynamic Analysis of AGARD Wing 445.6

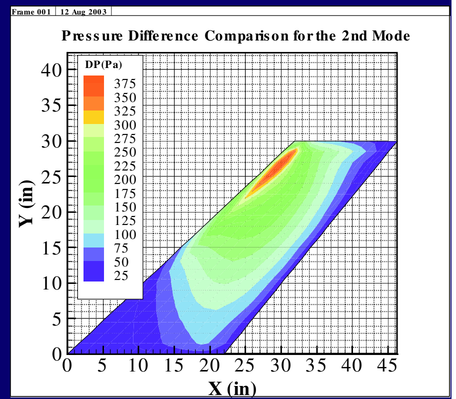
## Force Calculation

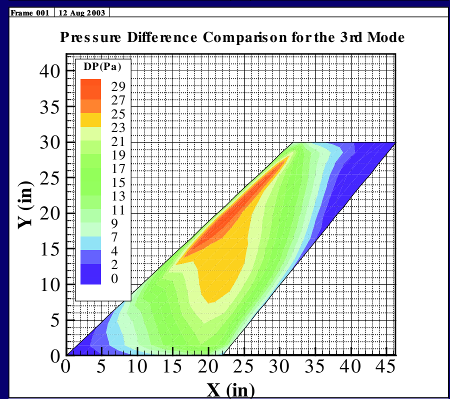
- Calculate dimensional pressure distribution
- Use surface interpolation to transfer pressures

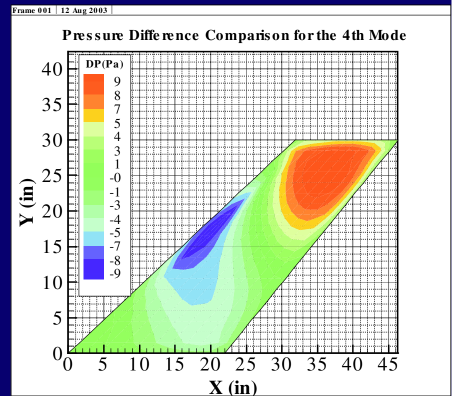
  CFD → Structure mesh at mid points of elements
- Transform pressures at mid-elements to forces at nodes



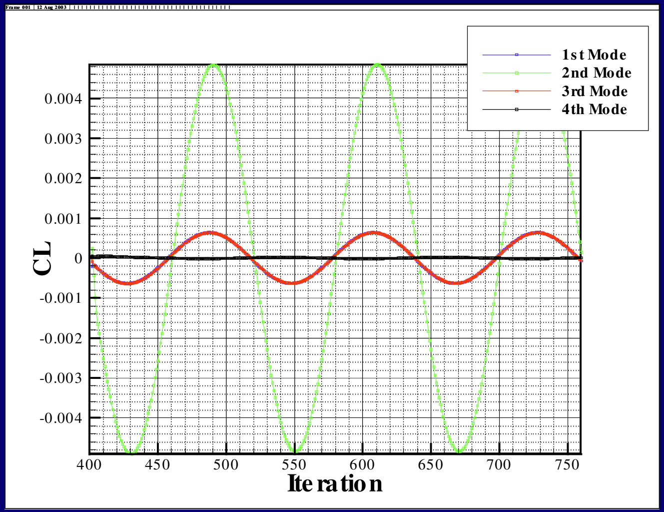



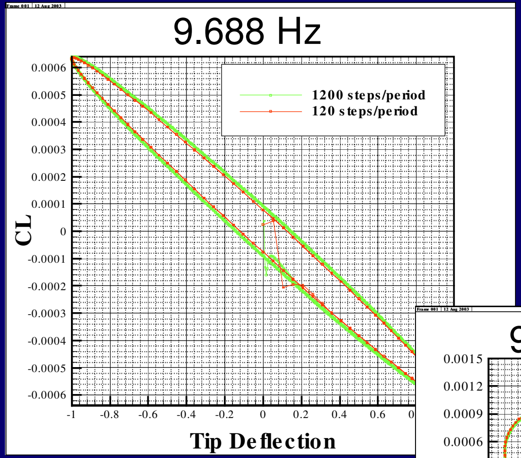



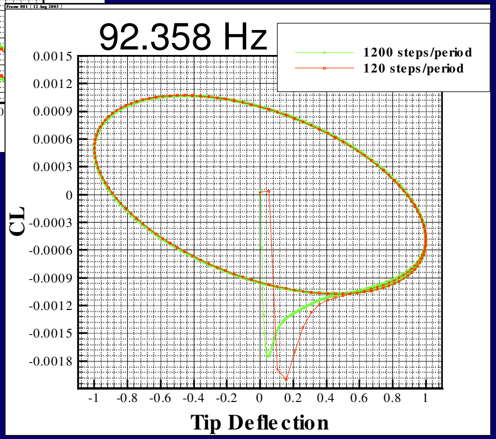
# Aerodynamic Analysis of AGARD Wing 445.6

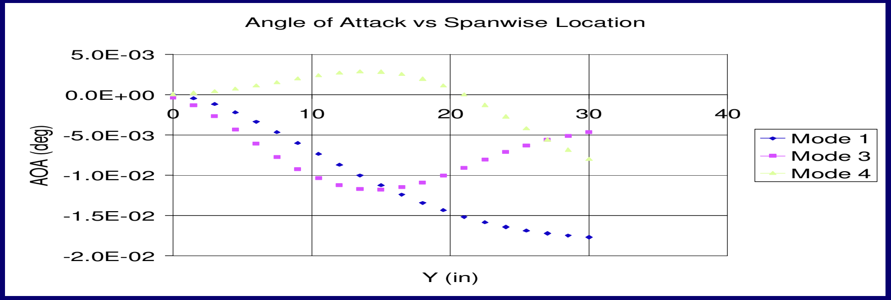
## CFD Results

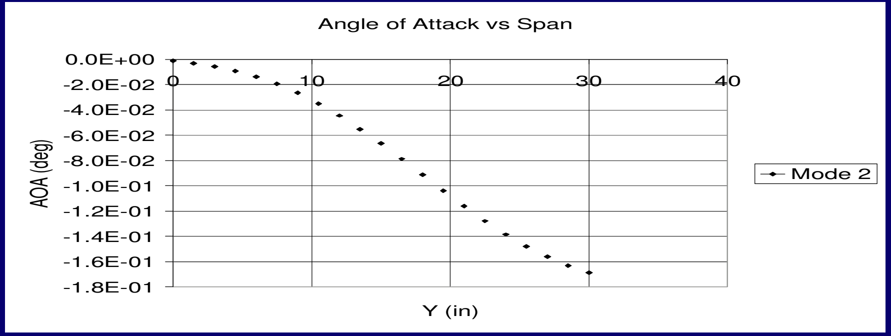
- Total Lift Coefficient Convergence
- Pressure Distribution
- Total Lift Coefficient Comparison
- Phase Difference



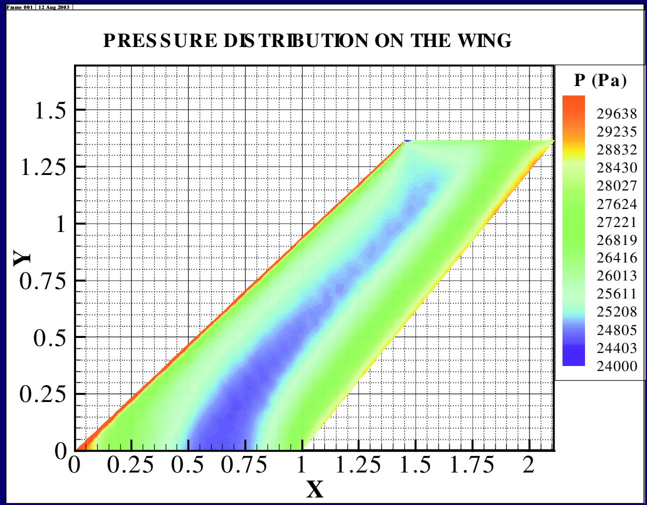



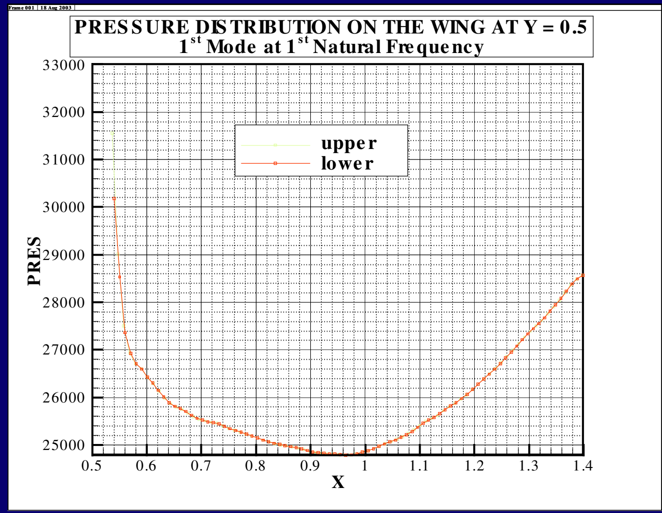



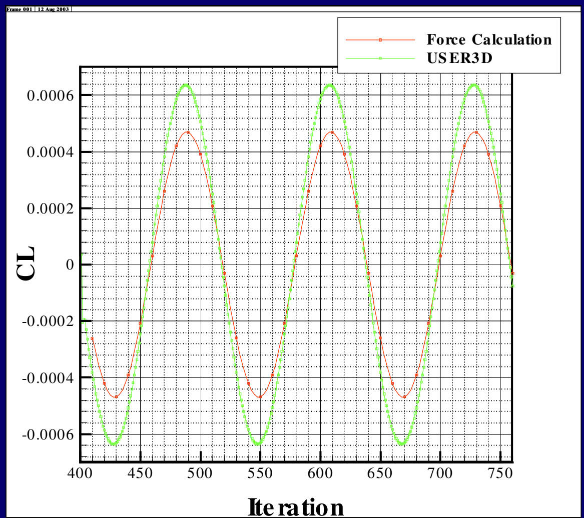
# Aerodynamic Analysis of AGARD Wing 445.6

– Pressure distribution

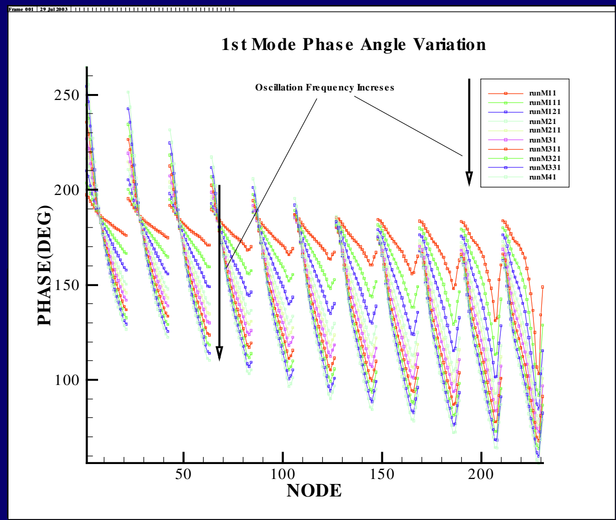



–Total Lift Coefficient Comparison




– Phase Difference



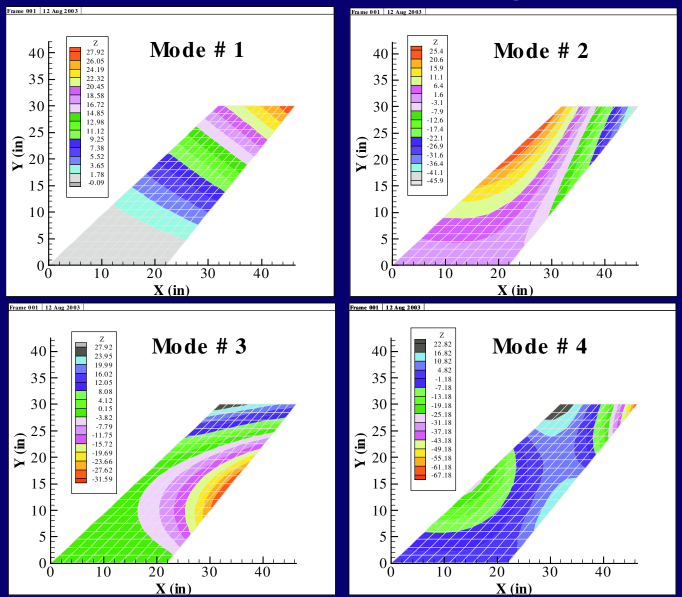
# Aerodynamic Analysis of AGARD Wing 445.6

## CFD Results

- Total Lift Coefficient Convergence
  - Verified through the Total Lift Coefficient Histograms
- Pressure Distribution
  - Order of magnitude comparison to results by simpler methods
- Total Lift Coefficient Comparison
  - Around 25 % error is introduced from force calculation
- Phase Difference
  - Phase difference exists between each node as expected

## Eigenvalue Solution

- Carried out with MATLAB®
- Element Mass and Stiffness Matrices extracted from ANSYS®
  - Substructuring analysis
  - User Programmable Features of ANSYS®
- Modal Analysis repeated with MATLAB ®
  - Mode shapes and natural frequencies are compared




| Mode # | ANSYS | MATLAB |
|---|---|---|
| 1 | 9.688 | 9.688 |
| 2 | 37.854 | 37.854 |
| 3 | 50.998 | 50.998 |
| 4 | 92.358 | 92.358 |

# Numerical Results

## Eigenvalue Solution

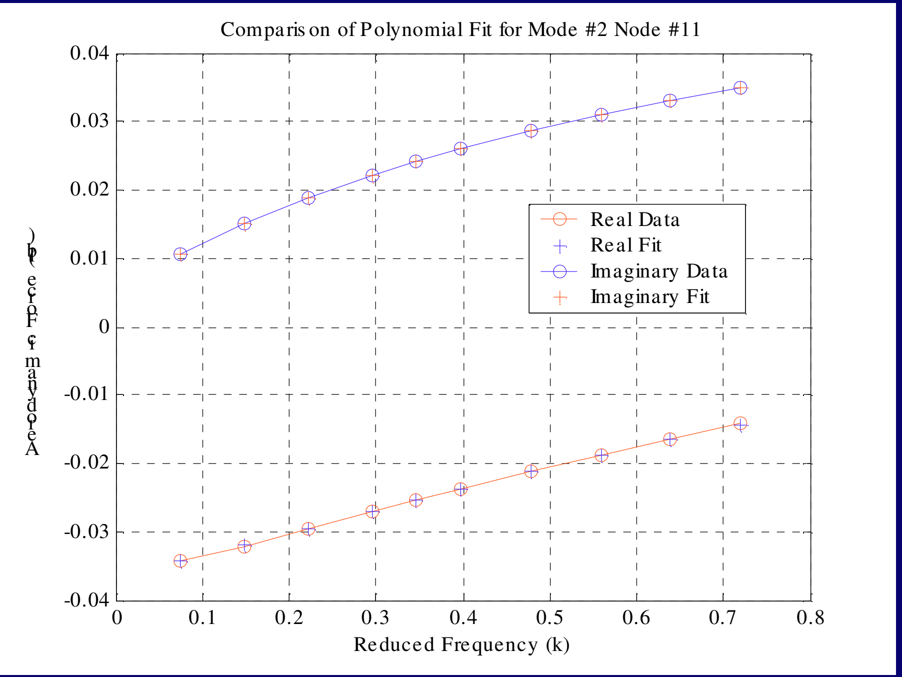
– Polynomial curve fitting to real and imaginary parts of aerodynamic forces

$$\left[T^l\right]=\begin{bmatrix} t^l_{1,1} & \cdots & t^l_{1,n} \\ \vdots & \ddots & \vdots \\ t^l_{231,1} & \cdots & t^l_{231,n} \end{bmatrix}$$

$$\left[Z^l\right]=\begin{bmatrix} z^l_{1,1} & \cdots & z^l_{1,n} \\ \vdots & \ddots & \vdots \\ z^l_{231,1} & \cdots & z^l_{231,n} \end{bmatrix}$$

$$[T]=\left[T^3\right]k^3+\left[T^2\right]k^2+\left[T^1\right]k+\left[T^0\right]$$

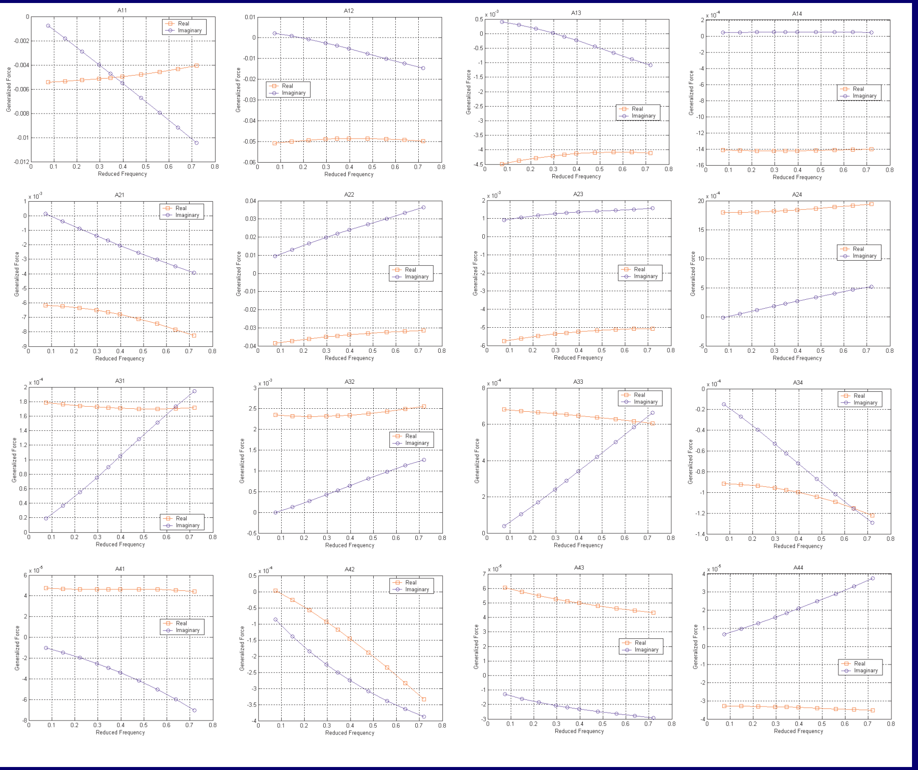
$$[Z]=\left[Z^3\right]k^3+\left[Z^2\right]k^2+\left[Z^1\right]k+\left[Z^0\right]$$



## Eigenvalue Solution

- Calculate generalized aerodynamic forces
- Calculate flutter frequency

| Deflection | Mach # | Present Study | Experiment | Error |
|---|---|---|---|---|
| Normal | 0.678 | 82.6897 (rad/s) | 113.0 (rad/s) | %26.8 |
| High | 0.678 | 82.58 (rad/s) | 113.0 (rad/s) | %26.9 |



For high deflection case, tip deflection values are taken from Reference 8

# Conclusions & Recommendations

- A new approach was applied to AGARD Wing 445.6
- Modal analysis of the wing is performed
- CFD analysis of the wing is performed
- Surface interpolation technique is implemented
  - Grid Transformation
  - Pressure Transformation
- Polynomial curve fitting to aerodynamic forces is implemented
- Significant decrease in computational time for flutter prediction
- Significant error compared to experiment
  - Force calculation technique should be revised
  - All experimental cases should be studied
  - GAF's should be verified with literature