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AN UNCOUPLED PROCEDURE FOR WING FLUTTER ANALYSIS

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ABSTRACT

Flutter analysis of AGARD Wing 445.6 is performed with an uncoupled method. Unsteady modal aerodynamic loads are calculated with an unstructured Euler solver for the first four modes of the wing. ANSYS® is used for modal analysis of the wing. Mode shapes and the modal pressure distributions are transferred between the structural code and the CFD code by surface interpolation. Nodal forces obtained from the pressure distributions are fitted a polynomial in terms of the reduced frequency. The resulting polynomial eigenvalue problem is solved for the flutter frequency. The results indicate that even a 2nd order polynomial is adequate for flutter frequency prediction.

INTRODUCTION

Solution of aeroelastic problems require interaction of two disciplines, namely, structures and fluid mechanics. Iterative procedures are employed even for static aeroelastic problems except for very simple ones. Iteration is necessary to determine the deformed shape of the surface involved in equilibrium under interacting aerodynamic and elastic loads and the aerodynamic loads generated by the deformed surface. Dynamic aeroelasticity is much worse in that iteration is necessary in every time step as the deformation is time-varying.

The level of real time interaction (i.e., coupling) between the two disciplines determines the time required for solution and accuracy of the results. The disciplines may be uncoupled or coupled. Coupled procedures range from fully coupled to weakly coupled ones. In the former, a single code encompasses both structure and fluid solvers. In all other cases, two separate codes exist. Both codes must be run at each time step and data must be interchanged continually. As a result wall time and CPU time requirements are quite high for dynamic aeroelastic problems. The situation may be worse in optimization where aeroelastic characteristics are to be optimized or are the contraints.

Flutter is the most widely studied dynamic aeroelastic phenomenon. The aerodynamic theories used evolved from strip theory to 3D Euler and Navier-Stokes solvers. A recent study used Euler and Navier-Stokes solutions in a fully coupled algorithm [6], [7]. Liu, Cai, Zhu developed a strongly coupled Computational Fluid Dynamics (CFD)/Computational Structural Dynamics (CSD) method for predicting flutter boundaries using unsteady Euler/Navier-Stokes equations [5].

The objective of this study is to perform flutter analysis wherein the structural and CFD codes run independently of each other and data transfer is minimal and unidirectional. To that end, an uncoupled method is used wherein the interaction between the codes need not be in real time so that separate people can run the codes and hand over the results. Data transfer occurs only once. The approach consists of the following steps:

- Compute several lowest mode shapes of the wing in question with a structural code and feed these to a CFD code.
- Compute the unsteady pressure distribution on the lifting surface for each mode shape oscillating at a number of frequencies in a certain range.
- Transform the set of pressure distributions on the CFD mesh to nodal force distributions on the structural mesh via an interface (a surface spline fitting technique here).
- Fit polynomial functions to modal aerodynamic forces in terms of frequency.
- Solve a polynomial eigenvalue problem to compute the flutter frequency.

The methodology is applied to AGARD Aeroelastic Wing 445.6 whose behavior is well documented[8].

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ANSYS® is the structural code used. The unsteady aerodynamic analysis is carried out with USER3D (Three Dimensional Unsteady Parallel Euler Solver)[2]. Computations were carried out with the parallel processing facility of Aerospace Engineering Department of Middle East Technical University.

METHOD

Various procedures exist for flutter analysis. The theoretical basis of the method used here is the same as the P-K method but it differs from the latter in the formulation of the problem and in the way the flutter frequency is obtained[4]. In the absence of damping, as is the condition at the instance of flutter, the equation of motion is given by

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}}) = \mathbf{A}_1\ddot{\mathbf{q}} + \mathbf{A}_2\dot{\mathbf{q}} + \mathbf{A}_3\mathbf{q} \qquad (1)$$

where **M**, **K**, and **q** are the mass matrix, stiffness matrix and displacement vector, respectively. The aerodynamic force vector **f** is expressed in terms of the unsteady aerodynamic load matrices $\mathbf{A}_j$. Equation (1) can be transformed to modal coordinates using a reduced set of modes and assuming

$$\mathbf{q} = \Phi\eta\, e^{st} \qquad (2)$$

where Φ and η are the reduced modal matrix and the amplitude of the modal coordinate vector. Substituting Eq. (2) in (1) and premultiplying by Φ^T,

$$\left(\mathbf{K}_M + s^2\mathbf{M}_M\right)\eta\, e^{st} = \Phi^T \mathbf{A}\Phi\ \eta\, e^{st} \qquad (3)$$

where $\mathbf{A}=\mathbf{A}(s, \mathrm{Ma})$ consists of relevant terms in Eq. (1). $\mathbf{K}_M$ and $\mathbf{M}_M$ are the reduced modal stiffness and mass matrices, respectively. Ma is the Mach number. An aerodynamic load database was generated by imposing the harmonic motion

$$\mathbf{q}_{jk} = a_j\phi_j \sin\omega_k t\,;\ j = 1,2,\ldots,J \text{ and } k = 1,\ldots,K \qquad (4)$$

on the wing in the CFD code, where ϕ_j is the jth mode shape. Four modes were used (J=4) and the loads were computed at 10 frequencies (K=10) for each mode. The CFD code gave the following aerodynamic forces for each mode and frequency:

$$\mathbf{h}_{jk} = \mathrm{Im}\{a_j\mathbf{A}_k\phi_j\, e^{i\omega_k t}\} \qquad (5)$$

with $s_k=i\omega_k$, where $\mathbf{A}_k\equiv\mathbf{A}(\omega_k,\mathrm{Ma})$. Equation (5) can be rewritten as

$$\mathbf{h}_{jk} = \begin{bmatrix} h_{jk,1}\sin(\omega_k t - \theta_{jk,1}) \\ h_{jk,2}\sin(\omega_k t - \theta_{jk,2}) \\ \cdot \\ \cdot \\ h_{jk,N}\sin(\omega_k t - \theta_{jk,N}) \end{bmatrix};\ j = 1,\ldots,J \qquad (6)$$

Letting the scaling coefficient a_j =1 and $\mathbf{r}_{jk} = \mathbf{A}_k\phi_j$,

$$\mathbf{r}_{R_{jk}} \equiv \mathrm{Re}\{\mathbf{r}_{jk}\} = \frac{2}{T}\int_{t_0}^{t_0+T_k}\mathbf{h}_{jk}\sin\omega_k t dt = \begin{bmatrix} h_{jk,1}\cos\theta_{jk,1} \\ \cdot \\ \cdot \\ h_{jk,N}\cos\theta_{jk,N} \end{bmatrix}$$

$$\mathbf{r}_{I_{jk}} \equiv \mathrm{Im}\{\mathbf{r}_{jk}\} = \frac{2}{T_k}\int_{t_0}^{t_0+T_k}\mathbf{h}_{jk}\cos\omega_k t dt = -\begin{bmatrix} h_{jk,1}\sin\theta_{jk,1} \\ \cdot \\ \cdot \\ h_{jk,N}\sin\theta_{jk,N} \end{bmatrix}$$

(7)

where T_k is the period $2\pi/\omega_k$. Thus $\mathbf{r}_{jk}$ are obtained by performing the operations indicated in Eq. (7) on the output of the CFD code. Defining

$$\mathbf{R} \equiv \mathbf{A}\Phi = \left[\mathbf{r}_1 \quad \mathbf{r}_2 \quad \ldots \quad \mathbf{r}_J\right] = \mathbf{R}_R + i\mathbf{R}_I \qquad (8)$$

for an arbitrary frequency of oscillation ω, a polynomial can be fitted to the real and imaginary parts of each term in **R** in terms of the reduced frequency $k=\omega b/U_\infty$ with b being the representative half chord length (here, the value at three quarters of the span) and U_∞ the free stream speed. Hence,

$$\mathbf{R}_R = \sum_{m=0}^{M} k^m\mathbf{T}_m$$
$$\mathbf{R}_I = \sum_{m=0}^{M} k^m\mathbf{Z}_m \qquad (9)$$

where M^{th} order polynomials have been fitted to the real and imaginary parts of the aerodynamic load. Real and imaginary parts of Eq. (3) are separated next and written as an augmented system of twice the size of the complex system. The result is

$$\left[\sum_{m=0}^{M} k^m\overline{\mathbf{Q}}_m\right]\overline{\eta} = \mathbf{0} \qquad (10)$$

which is a polynomial eigenvalue problem, where

$$\overline{\mathbf{Q}}_0 \equiv \mathbf{Q}_0 + \begin{bmatrix} \mathbf{K}_M & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_M \end{bmatrix}$$

$$\overline{\mathbf{Q}}_2 \equiv \mathbf{Q}_2 - \left(\frac{U_\infty}{b}\right)^2 \begin{bmatrix} \mathbf{M}_M & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_M \end{bmatrix} \tag{11}$$

$$\overline{\mathbf{Q}}_m \equiv \mathbf{Q}_m \quad \text{for} \quad m = 1,3,4,\ldots,M$$

with

$$\mathbf{Q}_m \equiv \begin{bmatrix} -\Phi^T \mathbf{T}_m & \Phi^T \mathbf{Z}_m \\ -\Phi^T \mathbf{Z}_m & -\Phi^T \mathbf{T}_m \end{bmatrix} \quad \text{for} \quad m = 0,1,\ldots,M \tag{12}$$

and

$$\overline{\eta} \equiv \begin{bmatrix} \eta_R \\ \eta_I \end{bmatrix} \tag{13}$$

The eigenvalues of Eq. (10) will come in complex conjugate pairs. The one with the smallest positive real part will be the reduced frequency at which flutter occurs.

MODAL ANALYSIS OF THE AGARD WING

Planform geometry of AGARD wing 445.6 is shown in Fig. 1. The streamwise cross-section of the wing, constructed of laminated mahogany, is NACA 65A004 airfoil. Of the two configurations of the wing that exist, namely, the solid and weakened versions, the weakened one is investigated in this study. A finite element model of the wing is created based on the information in [8]. Two hundred orthotropic shell elements (10x20) are used in the model, which is a flat shell model (Fig. 2), yielding 231 nodes. Following the approach in [4], the shell element thicknesses are assigned values in accordance with the real wing thickness distribution and are, therefore, different from each other.

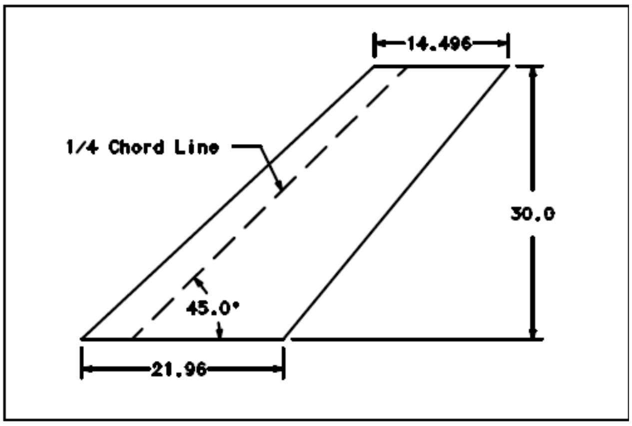

Figure 1: AGARD wing 445.6 (dimensions in inches)

The first four modal frequencies are given in Table 1 along with numerical results from other studies and experimental results. The first four modes are first bending, first torsion, second bending, and second torsion, in ascending order of frequency. The present study gives close results to the experiments[8] and to Reference 3. The first four mode shapes also compare very well with those in [4]. The fourth mode comparison is given in Fig.3 as an example [1].

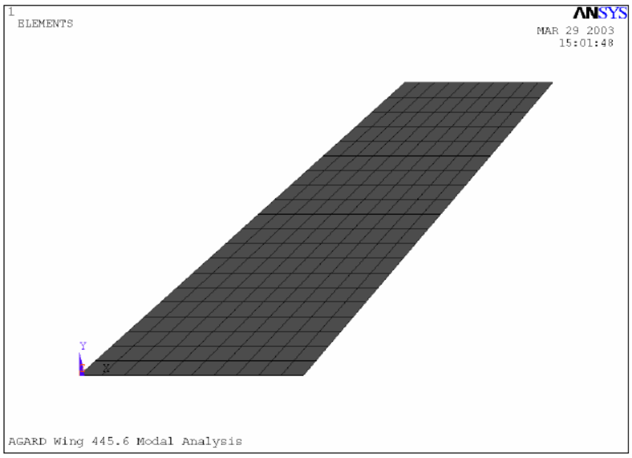

Figure 2: Finite element model of the wing

Natural Frequency (Hz)							
Mode	Exper. [8]	Present Study & % Error		Kolonay[4] & %Error		Li [3] & % Error	
1	*9.60*	9.688	0.92	9.63	0.31	10.85	13.02
2	*38.10*	37.854	-0.65	37.12	-2.57	44.57	16.98
3	*50.70*	50.998	0.59	50.50	-0.39	56.88	12.19
4	*98.50*	92.358	-6.24	89.94	-8.69	109.10	10.76

Table 1: Comparison of modal frequencies

AERODYNAMIC ANALYSIS OF THE WING

An unstructured volume grid consisting of tetrahedral elements was generated in the solution domain (i.e., the fluid domain) using I-DEAS®. Figure 4 shows the CFD grid which has 126380 elements and 26027 nodes. The elements are refined on the wing surface to obtain an accurate solution. The CFD grid at the wing surface is shown in Figure 5 in a non-dimensional form. A total of 13254 elements and 6667 nodes of the CFD domain neighbor the wing surface on the upper and lower side.

The CFD solution of the wing is performed with a 3D parallel finite-volume based unstructured Euler solver (USER3D)[2]. The parallel version of the program was developed in [2]. The program uses arbitrary Lagrangian-Eulerian formulation of the three dimensional inviscid flow equations in integral form. An implicit time integration scheme is used. The solution domain is updated at each time step according to the deflection of the wing by a moving mesh algorithm[2]. In doing that, the nodes of the tetrahedral elements are assumed to be mutually connected by springs whose stiffness is inversely proportional to the relative displacement between the pair of nodes in question. The deformed mesh at

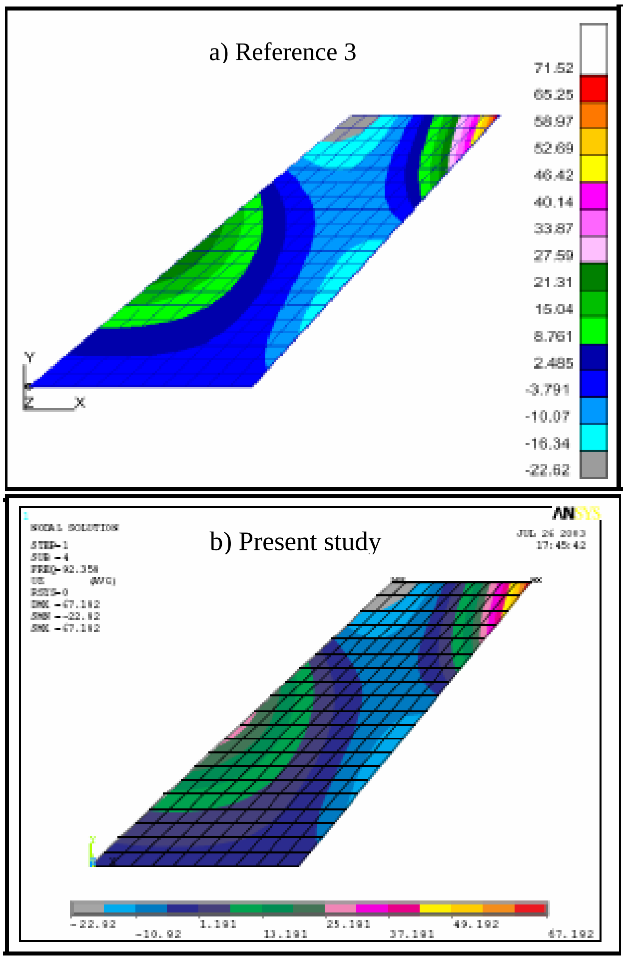

Figure 3: Comparison of mode 4

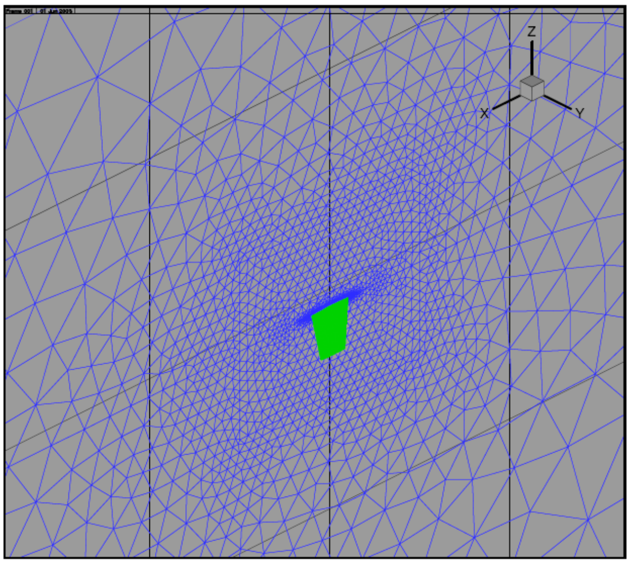

Figure 4: Unstructured volume grid

each time step is computed according to the particular mode shape and the frequency of oscillation prescribed to USER3D. Before unsteady analysis is started, steady flow around the wing is calculated which is then used as the initial condition for the unsteady solution. Because of the difference in structural and CFD meshes at the wing surface, mode shape transfer and pressure data transfer to/from USER3D is made with a surface interpolation routine.

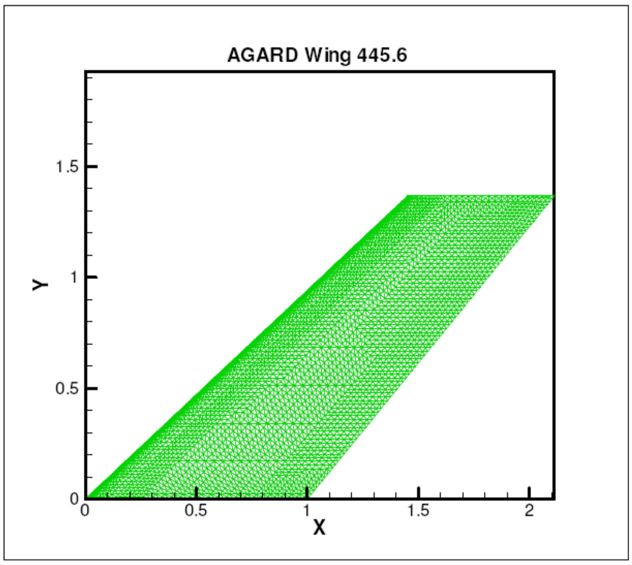

Figure 5: CFD mesh at the wing surface

Unsteady loads at Ma_∞=0.678 and α=0°, caused by harmonic oscillation of the four mode shapes one at a time, each at 10 discrete frequencies in the range 9.688-92.358 Hz (inclusive), are computed, yielding a total of 40 data sets. Time step length for each run is such that one period of oscillation is covered by 120 steps and the code is run for three periods. One such run takes about 40-45 minutes of wall clock time with the eight-node parallel computing facility of the Department of Aerospace Engineering, METU, where each node is a dual PIII 700 MHz processor workstation machine with 512 MB RAM. For a couple of cases, 1200 time steps per period were used and the adequacy of 120 steps was proven.

Figure 6 shows the variation of the total lift coefficient C_L of the wing as the first mode shape oscillates at the first modal frequency. It is noted that the wing

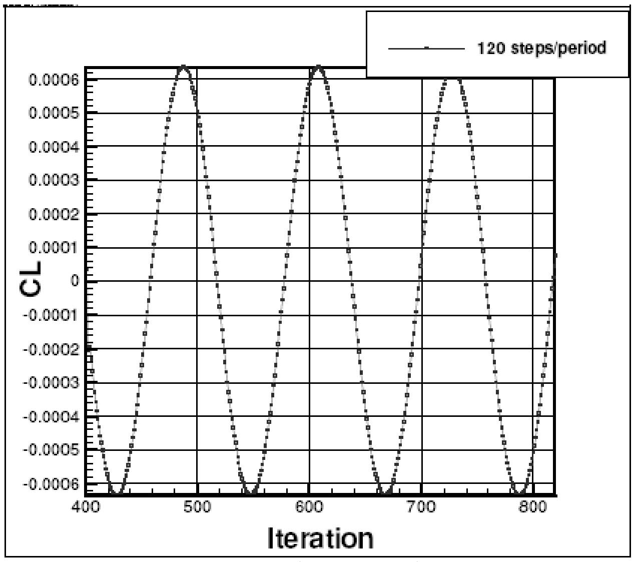

Figure 6: C_L variation, 1st mode, 1st modal frequency

oscillates around 0° and the airfoil profile, which is NACA65A004, is a symmetric one. What is shown in Fig. 6 is the output of USER3D. After the pressure data is transferred to the structural domain, from which nodal forces are computed, and the lift coefficient is re-calculated, there is a loss of nearly 25 % in C_L. This may be a result of the nodal forces being assumed to act normal to the wing symmetry plane whereas the airfoil has a certain profile. Unfortunately the exact cause of this could not be found and the calculations proceeded with the data transferred to the structural domain.

Aeroelastic Phase Angle

Aeroelastic phenomena strongly depend on phase differences between various quantities. Figure 7 shows the phase angle distribution between structural oscillations and aerodynamic forces. The phase angle is that of local aerodynamic load relative to local structural displacement, namely, angle $\theta_{jk,n}$ in Eq. (6). The abscissa shows the node numbers on

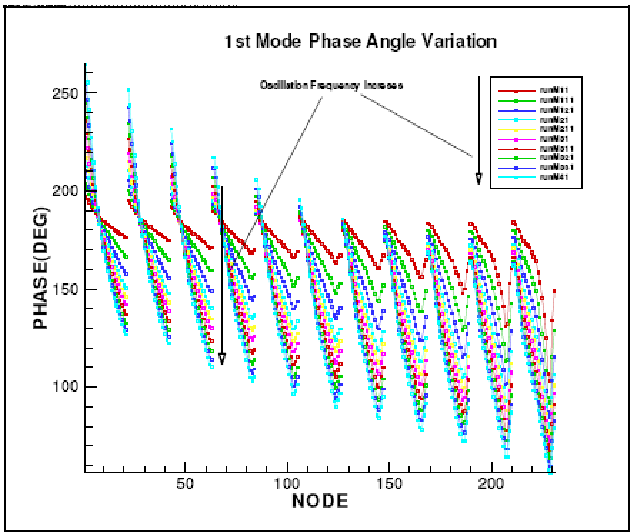

Figure 7: Phase angle distribution for the first mode

the wing surface. There are 21 nodes in spanwise direction and 11 in chordwise direction (Fig. 2). The numbering starts from the root node at the leading edge, proceeds toward the tip, then returns to the root and proceeds along the second line, and so on. The figure shows the phase distribution for the first mode oscillating at ten different frequencies. Phase angle at a given node decreases as the frequency of oscillation increases. For the inboard half of the wing, there is some decrease in the phase angle for a given frequency as one proceeds toward the trailing edge along a chord. Phase angle is nearly uniform for a given frequency along a chord on the outboard half. Phase angle drops toward the tip quite significantly for a given frequency and chordwise location. Hence, for the first mode spanwise variation of the phase angle is much more significant than chordwise variation. Phase angle variation is more complex for higher modes.

FLUTTER ANALYSIS

In order to carry out flutter analysis, modal mass and stiffness matrices of the wing are first extracted from ANSYS. Polynomials are then fitted to nodal aerodynamic forces on the wing (Eqs. 8 and 9). Namely, a polynomial was fit to the ten data points at each structural node and at each mode. 2^{nd}, 3^{rd}, 4^{th}, and 5^{th} order polynomials were tried which gave close results to each other. Figure 8 shows the computed data and the 3^{rd} order polynomial fit at node 11 (leading edge, mid-span) oscillating in mode 2.

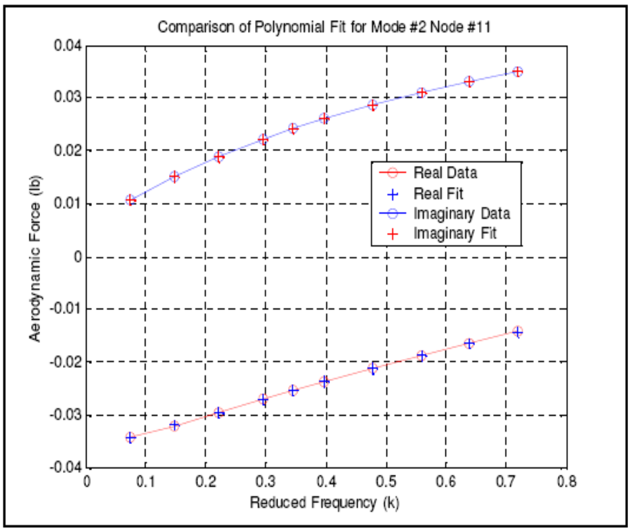

Figure 8: Polynomial fit to nodal force data

Table 2 shows the flutter frequencies at Ma_∞=0.678, computed from Eq. (10) using various order polynomials in comparison with an experimental result from [8]. MATLAB® was used to solve the polynomial eigenvalue problem.

Order of fit	Flutter frequency , rad/s (present study)	Experimental result [8]
2	82.6957	113.0
3	82.6897	
4	82.6741	
5	82.6577	

Table 2: Flutter frequencies with various order fits

The order of the polynomial isn't influential on the flutter result but there is an underprediction by nearly 27 %. This is thought to be due to the inaccuracy introduced as pressure data in the CFD domain is transformed to force data in the structural domain.

CONCLUSIONS

An uncoupled procedure was applied to compute the flutter frequency of AGARD Wing 445.6 at Ma_∞=0.678. The procedure is similar to the P-K method. An uncoupled CFD solver is run as the wing oscillates in various modes in turn at a number of frequencies and a polynomial is fit to the aerodynamic modal forces. The problem is cast as a polynomial eigenvalue problem, the solution of which gives the reduced frequency at flutter. The procedure is efficient but needs to be applied to more cases with a more accurate pressure transfer. The result obtained for the example case underpredicted the flutter frequency given in the literature by 27 %, which is thought to be due to the transfer of data between the CFD and structures domains via interpolation.

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