$$\frac{\sigma}{\sigma - 1} E \ln \left[\sum C_i^{1-1/\sigma} \right] \to \max \tag{1}$$
$$\text{s.t. } C_i = A_i L_i$$

The expectation is over productivity shocks, taking labor allocation as given. Labor demand across sectors has to add up to total fixed labor supply. We are in autarky.

Let $w = 1$. Then real income is P^{-1} and $P_i = A_i^{-1}$. How do productivity shocks affect the CES price index P?

$$P = \left[\sum_i P_i^{1-\sigma} \right]^{1/(1-\sigma)}$$

so that

$$U = \left[\sum_i A_i^{\sigma-1} \right]^{1/(\sigma-1)}.$$

Do a log-linear approximation so that we can apply the delta method for calculating variance

$$d \ln U \approx \sum_i U^{1-\sigma} A_i^{\sigma-1} d \ln A_i =$$

$$\sum_i \frac{A_i^{\sigma-1}}{\sum_j A_j^{\sigma-1}} d \ln A_i$$

Suppose variance of $d \ln A_i = \Sigma$, iid across sectors. Then, by the delta method,

$$\text{Var d} \ln U \approx \Sigma \sum_i s_i^2,$$

where

$$s_i = \frac{A_i^{\sigma-1}}{\sum_j A_j^{\sigma-1}}.$$

Clearly, when $\sigma = 1$, $\sum s_i^2 = 1/n$,the lowest possible value for a Herfindahl index. If $\sigma > 1$ or $\sigma < 1$, the volatility is larger, because the cost share of sectors is more unequal. It also means there is more response of volatility to changes in the parameters across sectors.