Appendix B to "A Balls-and-Bins Model of Trade:" Nesting Balls-and-Bins in a Structural Model

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In this Appendix we show how to nest the balls-and-bins framework in a structural model. The model of choice is a slightly modified version of Helpman, Melitz, and Rubinstein (2008), extended to encompass product heterogeneity as well as comparative-advantage forces. The model also features firm-level fixed costs of exporting to each country. We briefly derive the key equations concerning firm-level and aggregate (country-product) trade flows.

The first step is to generate finite-sample data predictions from the model. The model's market shares inform the likelihood that a shipment/observation belongs to a particular category. Following the same steps as in the main text, the probability distribution over categories can be used to compute the moments of interest, like the expected number of empty country-product trade flows or the share of exporters among all firms, given a number of observations. In the model presented here, some categories may have zero probability due to the fixed costs and a bounded productivity distribution: we call these fundamental zeros.

We also show that the structural model nests nicely our baseline calibration for the balls-and-bins framework, where we assumed no systematic relationship between countries, products, and firms. In short, our baseline calibration corresponds to parametrizations such that the model's trade volumes are multiplicatively separable in country, product, and/or firm fixed-effects. It is typically possible, and quite intuitive, to calibrate a trade model this way. For example, for country-product trade flows to be multiplicatively separable in the model presented here, relative wages across sectors must be equal across countries—this amounts to shutting down traditional comparative advantage forces—and the productivity distribution must be unbounded—isolating aggregate trade flows from firm-level fixed costs.

In addition we also ask whether the fixed costs and the upper bound on the productivity distribution—the key parameters behind the extensive margin at the firm and aggregate level, respectively—are identified in a sparse dataset. We show the fixed costs are readily pinned down by matching the share of exporters observed in the data—a fact that the balls-and-bins model missed. In contrast, the mechanism leading to empty country-product trade flows is poorly identified in a sparse dataset—even if we seek identification within the strict confines of a single structural model.

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1 A simple trade model with economies of scale

We present here a modified version of Helpman, Melitz, and Rubinstein (2008), who in turn took the model's key features from Melitz (2003) and Chaney (2008). The main modification is to introduce product heterogeneity so the model has predictions for the full product-country set of trade flows. Since we check the model exclusively against U.S. export data, we abridge the model description along several dimensions and focus on the equations concerning the foreign demand for U.S. goods. The reader can refer to Helpman, Melitz, and Rubinstein (2008) for a complete description of the model.

There are $j = 0, 1, \ldots, J$ countries with the U.S. indexed by $j = 0$. There is a continuum of firms, each indexed by $\omega \in \Omega$, adding up to a mass N_j in country j and producing each a differentiated commodity with non-mobile labor. We assume each firm's differentiated good belongs to one out of G different product categories or "sectors," with the distribution of firms across sectors given by $\{\mu^g\}$, with $\sum_G \mu^g = 1$. Firms are heterogeneous in their labor productivity, φ_i, distributed according to a truncated Pareto distribution,

$$\Psi(\varphi) = \frac{1 - \varphi_l^k \varphi^{-k}}{1 - \varphi_l^k \varphi_h^{-k}}$$

on the support $[\varphi_l, \varphi_h]$, allowing for the possibility that the support is unbounded above, $\varphi_h = \infty$. The wage rate in country j for sector g is denoted w_j^g and is taken as given. We allow the wage distribution across sectors to vary across countries, introducing comparative advantage as a possible source of trade.

The demand for firm ω, belonging to sector g, located in country i and selling in country j is given by

$$y_{ij}^g(\omega) = \frac{\alpha^g Y_j}{P_j^g} \left(\frac{p_{ij}^g(\omega)}{P_j^g} \right)^{-\sigma} \tag{1}$$

where Y_j is the country j income, α^g is the share of expenditures in good g, $\sigma > 1$ is the elasticity of substitution across goods, and the price index P_j^g is given by

$$P_j^g = \left(\sum_{i \in J} \int_{\omega \in \Omega_{ij}^g} \left(p_{ij}^g(\omega) \right)^{1-\sigma} d\omega \right)^{1/(1-\sigma)}, \tag{2}$$

where Ω_{ij}^g is the set of firms from country i, in product classification g, that sell in country j. This demand system is standard in trade and can be derived from well-known preferences.

To ship a good to country i from country j a firm must incur on "iceberg" trade costs $\tau_{ij} \geq 1$, with $\tau_{ii} = 1$. Firms operate under monopolistic competition, resulting in the familiar markup-over-marginal-cost pricing,

$$p_{ij}^g(\omega) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i^g}{\varphi(\omega)} \tag{3}$$

if firm ω sells in country j. If it does, its revenues are

$$r_{ij}^g(\omega) = p_{ij}^g(\omega)\, y_{ij}^g(\omega) = \alpha^g Y_j \left(\frac{p_{ij}^g(\omega)}{P_j^g} \right)^{1-\sigma}. \tag{4}$$

Subtracting the variable costs, the firm would make net revenues equal to $r_{ij}^{g}(\omega)/\sigma$.

But not all firms export to all destinations.[1] There is a fixed cost F for exporting to each destination. The decision rules for export participation are quite simple. Firm ω will export to country j if and only if

$$r_{ij}^{g}(\omega) - \sigma F \geq 0.$$

The export-participation decisions can be solved for in terms of productivity thresholds, which in turn makes it quite simple to solve for the set of firms exporting a product type to a destination. Since the only idiosyncratic attributes of a firm in country i are its productivity level and the product category it belongs to, we can rewrite the (potential) revenues in terms of a simple function $r_{ij}^{g}(\varphi)$, where we substitute the optimal price (3) in the revenue equation (4). This can be used to characterize the vector of thresholds φ_{ij}^{g} for each good g as

$$r_{ij}^{g}\left(\varphi_{ij}^{g}\right) = \sigma F. \tag{5}$$

If $\varphi_{ij}^{g} < \varphi_l$, then all firms in product g in country i export to country j. Conversely, if $\varphi_{ij}^{g} > \varphi_h$ then no firm in product g in country i export to country j. For threshold values in the interior of the support, then only a fraction of the firm export, namely those with productivity above the threshold, $\varphi \geq \varphi_{ij}^{g}$.

To characterize bilateral trade volumes, let

$$V_{ij}^{g} = \int_{\varphi_{ij}^{g}}^{\varphi_h} \varphi^{\sigma-1} d\Psi(\varphi)$$

with the understanding that $V_{ij}^{g} = 0$ if $\varphi_{ij}^{g} > \varphi_h$. For the predicted export flows in good g from country i to country j we just aggregate the sales revenues across the firms active in that market,

$$X_{ij}^{g} = \int_{\varphi \geq \varphi_{ij}^{g}} r_{ij}^{g}(\varphi)\, d\Psi(\varphi). \tag{6}$$

With some simple algebra we obtain that

$$X_{ij}^{g} = \left(\frac{\sigma \tau_{ij} w_i^{g}}{(\sigma-1)P_j^{g}}\right)^{1-\sigma} \alpha^{g} Y_j \mu^{g} N_i V_{ij}^{g} \tag{7}$$

where the price index for good g in country j is

$$P_j^{g} = \left(\sum_{i \in J} \left(\frac{\sigma \tau_{ij} w_i^{g}}{(\sigma-1)}\right)^{1-\sigma} \mu^{g} N_i V_{ij}^{g}\right)^{\frac{1}{1-\sigma}}. \tag{8}$$

Note that the aggregate bilateral trade flow between country i to j in the product classification g will be zero if no firm exports to that market, that is, $V_{ij}^{g} = 0$ or $\varphi_{ij}^{g} > \varphi_h$.

To complete the model we just need income-expenditure equality conditions in each country. However, we will not solve the general equilibrium and instead focus on the model's

[1] We assume there are no fixed costs associated with selling domestically, so all firms are active in their own country.

implications for U.S. firms and exports. Empirically, several equilibrium variables will simply be captured by country or product fixed-effects, as in Helpman, Melitz, and Rubinstein (2008) and other empirical applications.

We will also be interested in firm-level facts, like the fraction of exporters or their relative size. For example, the predicted revenues of a U.S. firm with productivity φ in sector g are

$$r^g(\varphi) = \sum_{j=0}^{J} 1\left[\varphi \geq \varphi_j^g\right] r_j^g(\varphi)$$

where $1\left[\varphi \geq \varphi_j^g\right]$ is the indicator function and whenever we omit the subscript for the country of origin the latter is understood to be the United States or $j = 0$. It is also straightforward to compute the total sales distribution. The fraction of exporters among U.S. firms in product sector g is simply $1 - \Psi\left(\min_{j\geq 1} \varphi_j^g\right)$. It is also quite trivial to characterize the distribution of domestic and foreign sales among exporters, and the share of multi-destination exporters.

We note that the predicted flow for good g to country j may be zero if and only if (i) the productivity distribution is bounded above, since then it is possible that the threshold φ_j^g is outside the support for the labor productivity, $\varphi_j^g > \varphi_h$, and (ii) there are fixed costs at the firm level, $F > 0$. This is actually the key mechanism in Helpman, Melitz, and Rubinstein (2008).

2 Finite-sample data predictions

The previous model, and indeed most trade models, takes the form of a set of continuous trade flows. That is, if we were to evaluate the models at different frequencies, the predicted export flows would just scale up or down proportionately with the frequency. The set of traded good-country destinations, for example, will be invariant. In this sense trade in the models is similar to oil flowing through a pipeline at a constant rate.

The data, however, consists of a finite number of observations, corresponding to the transactions in a given time period, usually a year. We bridge the gap between the theory and the data by sampling the model, mapping the model's market shares into the likelihood that a given transaction belongs to a particular category. In doing so, we effectively nest the balls-and-bins framework with the underlying structural model.

Let $n \in \mathbb{N}$ be the number of observations. This can be set to reproduce the number of observations in the data but we can also explore the model implications in a dense dataset by letting n tend to infinity. It is also possible to "discretize" the revenues flows in the model (or the data) assuming a shipment size, say ξ. This is particularly useful when the number of shipments is unknown or the researcher does not want to use the observed shipment distribution.

The key step is to derive the likelihood a shipment belongs to a given category from the model-implied market shares. The probability that a shipment from the firm has country j as a destination is given by

$$s_j^g(\varphi) = \frac{1\left[\varphi \geq \varphi_j^g\right] r_j^g(\varphi)}{r^g(\varphi)}.$$

That is, the likelihood of a shipment sent to country j is given by the share of that destination within the firm's total sales. The probability then that *any* shipment from the firm reaches destination j, that is, the probability that the firm is exporting to country j, is simply

$$1 - \left(1 - s_j^g(\varphi)\right)^{n^g(\varphi)},$$

where $n^g(\varphi)$ is the number of shipments/observations assigned to the firm. The formulas from the main text carry on, making it possible to compute the expected number of destinations a firm will serve, expected relative size, and so on.

In order to obtain predictions for the aggregate country-product flows, we also need to characterize the probability that a shipment belongs to a given firm i. Again, this is simply the firm's share as predicted by the model,

$$s^g(\varphi) = \frac{r^g(\varphi)}{\bar{R}}$$

where $\bar{R}$ is the U.S. total sales as predicted by the model. Now adding up all firms, we would obtain that the probability that a single shipment belongs to a given country-product pair is

$$s_j^g = \frac{X_j^g}{\bar{R}}.$$

The above probability distribution is defined for all $j = 0, 1, 2, ..., J$ countries, including the home country. By contrast, most of our facts concern only export flows. This is not a problem, as shipments are assumed to be i.i.d., then the distribution conditional on the shipment being shipped abroad is simply

$$s_j^g = \frac{X_j^g}{\bar{X}}. \tag{9}$$

where $\bar{X}$ is total U.S. exports as predicted by the model.

Summarizing, we let the structural model determine the bin-size distribution and replicate the number of shipments or observations in the data. The market shares in the underlying model become the likelihood that a single shipment belongs to the category of interest. A market share can be zero in the model: a fundamental zero as the particular category will not be observed to receive shipments. These fundamental zeros, though, will coexist with sample zeros if the data are sparse. If instead the data are dense, the realized frequency of shipments across categories will converge almost surely to the probability distribution, and thus we recover exactly the predicted market shares in the structural model. In particular, only fundamental zeros will remain.

We should emphasize that it is possible to create a finite-sample data set prediction for *any* structural model simply following the steps presented here.

3 Nesting the baseline balls-and-bins calibration

In the main text, we constructed our baseline calibration for the balls-and-bins by matching the distribution across product classifications and countries—the marginal distributions—

and constructing the probability of a particular country-good pair as the product of probabilities, that is, abstracting from any systematic relationship between product and countries and, by extension, firms.

We nest this baseline assumption in the structural model presented above as a special case. From (9) it should be clear that we can write $s_j^g = s_g s_j$, where s_g and s_j are the probabilities of a shipment being of product g and being sent to destination j, respectively, if and only if we can express the trade flow in the underlying model as

$$X_j^g = d_g d_j$$

for all g and j. That is, the trade flow is multiplicatively separable in two "fixed effects," one for the product and one for the country of destination.[2] The fixed effects clearly allow the model to capture perfectly the marginal distributions of trade across products and countries.

When is it possible to express trade flows as $X_j^g = d_g d_j$ in the underlying model? By simple inspection of the demand function (7) it is clear that most of the terms depend on the product or destination, but not both. The exceptions are the price level of the good g in country j, P_j^g, and the composition term, V_j^g.[3] Inspecting the price level, the composition term reappears, now for every country pair, V_{ij}^g, and we now have the vector of wages for good g across origin countries, w_i^g. Note that the former term is closely related to the economies of scale in exporting, while the latter is tied to comparative advantage forces. In order to obtain the desired separability in trade flows $X_j^g = d_g d_j$, we require a condition on each:

1. The relative wages across sectors are identical in every country, i.e.,

$$w_i^g = \omega_g \omega_i.$$

2. The productivity distribution is unbounded, $\varphi_h = \infty$.

The first condition is not surprising: if some countries can produce certain products (relatively) cheaper than other countries, that is, have a comparative advantage on these goods, there will be a systematic relationship between destinations and products. Mathematically, it is clear that the unit cost must be itself multiplicatively separable if we want the predicted flows to satisfy $X_j^g = d_g d_j$. With $w_i^g = \omega_g \omega_i$, the price index can be re-written as

$$P_j^g = \omega_g \left(\mu^g\right)^{\frac{1}{1-\sigma}} \left(\sum_{i \in J} \left(\frac{\sigma \tau_{ij} \omega_i}{(\sigma - 1)} \right)^{1-\sigma} N_i V_{ij}^g \right)^{\frac{1}{1-\sigma}},$$

and we are left only with V_{ij}^g as the only joint country-product term.

The second condition is closely related to the underlying model's ability to generate zero trade flows *in the aggregate*. It does not impose any condition on the fixed cost parameter,

[2] The decomposition does not need to be unique, as some terms may not be indexed to either product or country. The choice of decomposition does not matter as the resulting distribution of probabilities s_j^g will be invariant. A similar exercise can be done for firm-level flows.

[3] Recall we have fixed the U.S. as the country of origin and omitted the corresponding subscript.

F, so it is possible to have firms that do not export yet the condition be satisfied. Formally, the steps are as follows. A key property of the Pareto distribution when $\varphi_h = \infty$ is that

$$V_{ij}^g = \varphi_{ij}^g \frac{k}{1+k-\sigma}.$$

From the entry condition $r_{ij}^g \left(\varphi_{ij}^g\right) = \sigma F$ we can solve for the threshold using the revenue function (4). The latter is multiplicatively separable in origin, destination, and product but for the price index itself, which allows us to express the threshold as such as well, $\varphi_{ij}^g = \rho_g \rho_i \rho_j \left(P_j^g\right)^{-1}$. Substitute back in (8) and there are no joint product-country terms left.

We should note that the second condition is sufficient, but not necessary. For example, it is possible to have $\varphi_h < \infty$ and $X_j^g = d_g d_j$ if there are no fixed costs, $F = 0$. This alternative, though, is not really satisfactory since it completely shuts down the extensive margin in the model.

The structural model allows many extensions and yet our baseline balls-and-bins calibration remains nested as a special case. For example, fixed costs can be specified to depend on the country of origin, country of destination, and product. Once again, imposing that the heterogeneity is multiplicatively separable, $F_{ij}^g = \phi_g \phi_i \phi_j$, would recover the property behind our baseline calibration. Similarly it would be possible to encompass differences in the productivity distribution across sectors or countries, additional factors of production, or sector-specific trade costs.

4 Identification in a sparse dataset

In the main text we showed the difficulties at distinguishing different model classes in a sparse data set. Unfortunately, there are also difficulties within a structural model when it comes to identify its parameters. As an example, we document the model's prediction, evaluated for a sparse data set, for the share of exporters and positive country-product trade flows across a series of calibrations. We start from the baseline balls-and-bins case, i.e., a calibration satisfying conditions 1 and 2 above. We then explore how the model's predictions vary with the fixed costs, F. Finally we work with a truncated Pareto distribution and explore different values for the upper bound parameter, φ_h.

4.1 Set parameters

We refer to the literature to pin down the elasticity of substitution σ to 6. The lower bound for the support φ_l can be normalized to 1 and the slope k is set to match the estimates for the tail distribution of firm size, $k\left(\sigma-1\right)^{-1} = 1.06$, as it is common in the literature.

We use product and country fixed-effects to capture the heterogeneity across products and countries in the model, respectively. Note that, it is in principle possible, and perhaps even preferable, to complete the calibration by using GDP or similar for each country's income Y_j, distance and other proxies for the matrix of trade costs, τ_{ij}, PPP exchange rates for relative wages, as well as proxies for product market shares. However, for the purposes here, the fixed-effect approach is better suited since it ensures the distribution of trade across products and countries—the marginal distributions—exactly matches the data. It has the additional advantage that the fixed-effects can be readily computed for most cases by simply equating them to the respective product and country market shares in U.S. exports. The only difficulty is in the case of a truncated Pareto distribution, which requires to solve a large non-linear system to obtain the fixed-effects. We do find, however, that for the range of parameters explored below the fixed-effects vary little and actually simply track the observed country and product shares in total U.S. exports very closely. Finally, evaluating the model under a sparse data requires deciding on the the number of shipments observed in the data, about 22 million.

We are thus left with the two parameters of interest: the fixed cost of exporting to one destination, F, as well as the upper bound of the support, φ_h. These govern fundamental zeroes at the firm and country-product level, respectively. Thus the natural empirical targets the fraction of traded country-good pairs and the fraction of exporters among all firms.

4.2 Identifying fixed costs of exporting at the firm level

We start from the balls-and-bins framework, with $F = 0$ and an unbounded productivity support, as well as no comparative advantage trade due to differences in relative wages across countries. As discussed above, the structural model does not introduce any systematic relationship between countries and products under these conditions.

The results are, not surprisingly, as in the main text. The model generates 72 percent empty trade flows in the country-product matrix and greatly overstates the share of exporters, close to 75 percent in the model versus 18 percent in the data. Exporters in the model, despite being more frequent than in the data, are also larger than their data counterparts.

In the main text we claimed that the share of exporters is indeed a useful stylized fact to identify the underlying theory of the extensive margin of interest, that is, export participation at the firm level. In the simple model presented here, the extensive margin is clearly tied to the fixed costs of exporting, so we explore whether parameter values $F > 0$ help the model match the share of exporters.

Indeed they do. As we increase F, we reduce the share of exporters, as expected. The relationship is sharp, and we easily obtain a point estimate such that the model predicts about 18 percent of the firms exporting, as in the data.[4] Increasing or decreasing the fixed cost by ten percent drives the share of exporters in the model to 16 and 20 percent, respectively. The exporter size premium, though, remains substantially above the data, about 16 versus $4 - 5$, respectively.

[4] The fixed cost parameters comes close to \$1 million, substantially above estimates elsewhere using a richer model of export participation.

4.3 The extensive margin at the country-product level

In contrast, the fraction of traded country-product pairs does not bulge at all as F is increased. As argued earlier, economies of scale at the firm level do not introduce a systematic relationship between countries and products. Thus the baseline calibration for the balls-and-bins model remains in place and as long as the fixed-effects are set to match the aggregate distribution of trade across products and countries, we will obtain the exact same predictions for any parameter value $F > 0$. Because of this, we can target the two stylized facts separately, as we did in the main text.

What are the effects of the upper bound in the productivity distribution, $\varphi_h < \infty$? The structural model only predicts zero-probability trade flows when there is an upper bound. The model with unbounded support underpredicts the share of zeros in country-product flows, so there is room for improvement. However, we find that imposing an upper bound, even a tight one, has a muted impact on the model's predictions. As we reduce φ_h we start "closing" trade flows that were very small to start with, and thus predicted to be zero with very high probability anyway in the sparse dataset.

Of course, the support for the productivity distribution can be set small enough that eventually the predicted fraction of empty trade flows increases. Unfortunately, it is impossible to do so without shutting down trade all together in many product categories as well as to several destinations. Decreasing the upper bound φ_h also decreases the share of exporters, in a rather mechanical way: as the support is reduced, there is a larger mass of firms close to the lower bound—which are less likely to export. If we decrease F then to keep the share of exporters constant at 18 percent, the model actually cannot increase the fraction of empty trade flows beyond about 75 percent.

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