

Efficiency costs of misallocating government contracts

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Introduction

Motivation

- ▶ Misallocation of resources is a candidate cause of income differences across countries.
- ▶ Much research focused on supply-side misallocation: distortions in firms' factor choices.
 - ▶ Banerjee and Duflo (2005), Restuccia and Rogerson (2008), Hsieh and Klenow (2009)

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- ▶ What about demand-side misallocation?
 - ▶ Feenstra (1994), Broda and Weinstein (2006): spending “too much” on old goods
 - ▶ Arkolakis-Costinot-Rodriguez-Clare (2010): spending “too much” on domestic goods

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- ▶ Does the government—biggest buyer—misallocate demand across firms?

Favoritism and its welfare effects

- ▶ Political connections help firms win government contracts.
 - ▶ Italy (Cingano-Pinotti 2013)
 - ▶ US (Goldman et al 2013)
 - ▶ Russia (Mironov-Zhuravskaya 2014)
- ▶ Resulting in
 - ▶ less innovation (Brogaard-Denes-Duchin 2015)
 - ▶ less competition (Coviello-Gagliarducci 2012)
 - ▶ lower productivity (Mironov-Zhuravskaya 2014)
 - ▶ more delays (Schoenherr 2016)

Our contribution

- ▶ The literature documents inefficiencies, but focuses on particular consequences like delays or TFP.
- ▶ We build a general model of demand misallocation, generalizing Feenstra (1994), Broda-Weinstein (2006) and ACR (2010)
- ▶ Our contribution is to compare allocations to an estimated benchmark.
 - ▶ Key idea: firm-level demand should be similar across buyers. This can be used to substitute out unobserved firm productivity.
- ▶ Our measure is more comprehensive: we can capture all inefficiencies.
 - ▶ lack of information
 - ▶ pork barrel politics
 - ▶ corruption

Inefficiency from misallocation

$$\frac{G}{G^*} = \left[\int_{\omega} s^*(\omega)^{1/\sigma} s(\omega)^{1-1/\sigma} e^{-\theta(\omega)(1-1/\sigma)} d\omega \right]^{\frac{\sigma}{\sigma-1}} < 1$$

- ▶ $s(\omega)$: actual market share
- ▶ $s^*(\omega)$: optimal market share
- ▶ $\theta(\omega)$: overpricing

Application

- ▶ We estimate our model on Hungarian public procurement data.
- ▶ Two sets of estimates:
 - ▶ firm-level, using domestic private sales as benchmark
 - ▶ contract-level, using average government agency as benchmark
- ▶ Efficiency losses from demand misallocation are large:
 - ▶ Government could save up to 35 percent.
 - ▶ Political connections of firms explain up to 6 percent.
- ▶ By contrast, “misallocation” to exports are much smaller.

Government misallocation in practice



Outline

1. A model of misallocation
2. An empirical model of buyer-seller relations
3. Application to Hungarian public procurement
4. Counterfactual analysis

A model of misallocation

A model of misallocation

- ▶ We follow the wedge approach of Chari-Kehoe-McGrattan (2007) and Hsieh-Klenow (2009).
- ▶ What are the minimal deviations from a standard industry model (Melitz 2003) to explain patterns of demand?
- ▶ Two key indicators:
 - ▶ ratio of public to private revenue
 - ▶ high G/C : “favored firm”
 - ▶ price-cost markup
 - ▶ high markup: “overpricing”

Production and preferences

Industry production:

$$X_j = \left[\int \nu_j(\omega)^{1/\sigma_j} x_j(\omega)^{1-1/\sigma_j} d\omega \right]^{\frac{\sigma_j}{\sigma_j-1}}. \quad (1)$$

Consumers:

$$U^C(C_1, \dots, C_n) = \Pi_j C_j^{\gamma_j}. \quad (2)$$

Government:

$$U^G(G_1, \dots, G_n) = \Pi_j G_j^{\delta_j}. \quad (3)$$

Cobb-Douglas weights for private and government consumption can differ ($\delta_j \neq \gamma_j$). But CES weights ($\nu_j(\omega)$) within an industry cannot. Welfare:

$$U = U^C(C_1, \dots, C_n) + U^G(G_1, \dots, G_n).$$

Government allocation

$$\frac{g(\omega)}{c(\omega)} = k \cdot e^{\mu(\omega)} \quad (4)$$

Can be microfounded in a model of bribery (Grossman-Helpman 1994) with $b(\omega)$ fraction as kick-back.

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With no favoritism, $\mu(\omega) = 0$.

Overpricing

$$p_g(\omega) = p_c(\omega) \cdot e^{\theta(\omega)}. \quad (5)$$

Microfoundation: firms price in bribes.

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Model solution

Pricing

$$p_c(\omega) = \frac{\sigma}{\sigma - 1} \cdot \phi(\omega)w \quad (6)$$

$$p_g(\omega) = \frac{\sigma}{\sigma - 1} \cdot \phi(\omega)w \cdot e^{\theta(\omega)}. \quad (7)$$

Private sector

$$\frac{c(\omega)}{C} = \nu(\omega) \left[\frac{p_c(\omega)}{P_C} \right]^{-\sigma}.$$

This can be used to substitute out productivity and quality.

Computing the wedges

Overpricing

$$e^{\theta(\omega)} = \frac{p_g(\omega)g(\omega)}{\frac{\sigma}{\sigma-1}wl - p_c(\omega)c(\omega)}.$$

Preferences

$$e^{\mu(\omega)} = \frac{\frac{\sigma}{\sigma-1}wl - p_c(\omega)c(\omega)}{p_c(\omega)c(\omega)} \frac{1}{k}$$

Efficiency loss

- ▶ Counterfactual: Spend amount $P_G G$ according to $\mu \equiv \theta \equiv 0$.

$$\frac{G}{G^*} = \left[\int_{\omega} s^*(\omega)^{1/\sigma} s(\omega)^{1-1/\sigma} e^{-\theta(\omega)(1-1/\sigma)} d\omega \right]^{\frac{\sigma}{\sigma-1}} < 1$$

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- ▶ Special case: $s(\omega) = 1$, $\theta(\omega) = 0$
 - ▶ ω = old goods (Feenstra 1994, Broda and Weinstein 2006)
 - ▶ ω = domestic goods (ACR 2010)

Efficiency loss

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- ▶ Special case: $s(\omega) = 1$, $\theta(\omega) = 0$
 - ▶ ω = old goods (Feenstra 1994, Broda and Weinstein 2006)
 - ▶ ω = domestic goods (ACR 2010)
- ▶ As a function of wedges,

$$\log \frac{G}{G^*} \approx -E_{\Phi} \theta - \frac{1}{2} \text{Var}_{\Phi}(\theta) - \frac{1}{2\sigma} \text{Var}_{\Phi}(\mu) - \text{Cov}_{\Phi}(\mu, \theta)$$

Caveats

- ▶ Government preference may be genuinely different, even within narrow sectors.
 - ▶ Model preference deviations with observables.

$$\nu_j(\omega) = \exp(\beta_j X_j(\omega))$$

- ▶ Does not allow for heterogeneity across buyers. Which buyers “misallocate” the most?
 - ▶ Richer, contract-level specification.
- ▶ No general equilibrium effects of government.
 - ▶ crowding out
 - ▶ competition for rents (Melitz)

Why no Melitz effect?

- ▶ Suppose the government buys goods from set of firms to produce utility G .
- ▶ If it buys from the best firms, will welfare not increase?

Why no Melitz effect?

- ▶ Suppose the government buys goods from set of firms to produce utility G .
- ▶ If it buys from the best firms, will welfare not increase?
- ▶ No, it could achieve the same utility G with fewer workers, buying from everyone in proportion to $c(\omega)$.
- ▶ Importantly, there are no fixed costs of serving the government.

An empirical model of buyer-seller relations

An empirical model of buyer-seller relations

- ▶ For each procurement tender, the procuring agency solves a discrete choice problem: who should win this contract?
- ▶ Using tender-level data, we can control for characteristics of buyers, sellers, tenders.
 - ▶ precise product purchased
 - ▶ buyer-seller distance
- ▶ We derive similar welfare losses from misallocation in a multinomial logit.

Misallocation in multinomial logit

- ▶ Random utility model: *true* utility of choice i

$$\tilde{V}_i^* = u_i^* - \ln p_i^* + \varepsilon_i$$

- ▶ Deterministic utility: $u_i^* - \ln p_i^*$ (may be modeled as βx_i)
- ▶ Random utility: ε_i (type-I extreme value) with scale parameter μ

Choice

Choice probabilities given by multinomial logit

$$\pi_i^* \equiv \Pr(\tilde{V}_i^* \geq \tilde{V}_j^* | \{u_j^*, p_j^*\}) = \frac{e^{(u_i^* - \ln p_i^*)/\mu}}{\sum_j e^{(u_j^* - \ln p_j^*)/\mu}}$$

Welfare

Welfare is CES

$$V^* = E(\max \tilde{V}_i^* | \{u_i^*, p_i^*\}) = \mu \ln \sum_j e^{(u_j^* - \ln p_j^*)/\mu}$$

$$e^{V^*} = \left(\sum_j e^{u_j^*/\mu} p_j^{*-1/\mu} \right)^\mu$$

equivalent to CES with $1/\mu = \sigma - 1$.

Misallocation

Suppose decisionmaker uses u_i, p_i , rather than u_i^*, p_i^* to evaluate alternatives.

Choice

$$\pi_i \equiv \Pr(\tilde{V}_i \geq \tilde{V}_j | \{u_j, p_j\}) = \frac{e^{(u_i - \ln p_i)/\mu}}{\sum_j e^{(u_j - \ln p_j)/\mu}}$$

Welfare

$$\begin{aligned} V_{\{u_j\}}^* &\equiv E(\tilde{V}_i^* | \tilde{V}_i \geq \tilde{V}_j, \{u_j, u_j^*, p_j\}) \\ &= \mu \ln \sum_j e^{(u_j - \ln p_j)/\mu} + \sum_j \pi_j (u_j^* - u_j) - \sum_j \pi_j (\ln p_j^* - \ln p_j) \end{aligned}$$

Efficiency loss

With no mispricing and normalization

$$\sum_j e^{(u_j - \ln p_j)/\mu} = \sum_j e^{(u_j^* - \ln p_j)/\mu} = 1,$$

$$V_{\{u_j\}}^* - V \equiv \Delta V = \mu \sum_j \pi_j (\ln \pi_j^* - \ln \pi_j),$$

proportional to the Kullback-Leibler divergence between $\{\pi_j\}$ (actual choice) and $\{\pi_j^*\}$ (desired choice).

Estimation

- ▶ π_i^* can be estimated in a logit model capturing true preferences,

$$\pi_i^* = \frac{e^{\beta^* x_i}}{\sum_j e^{\beta^* x_j}}$$

- ▶ π_i can be estimated as
 - ▶ alternative logit model: $\hat{\pi}_i = e^{\hat{\beta} x_i} / \sum_j e^{\hat{\beta} x_j}$
 - ▶ observed choice frequency: $\hat{\pi}_i = n_i / \sum_j n_j$
 - ▶ actual choice: $\hat{\pi}_i = \{0, 1\}$
 - ▶ Bayesian estimate (with Dirichlet prior)

Application: Hungarian public procurement

Application: Hungarian public procurement

- ▶ Estimate misallocation of about 200,000 Hungarian public procurement contracts, 1998-2014.
- ▶ Construct variables of political connections.
- ▶ Measure the amount and number of government contracts by winning firm.
- ▶ Relate to domestic private sales and other observables.
- ▶ (Assume no mispricing.)

Data

Managers

- ▶ Firm representatives, 1990-2014 (Complex, 900,000 firms).
- ▶ Name, home address, mother's name, date of birth.

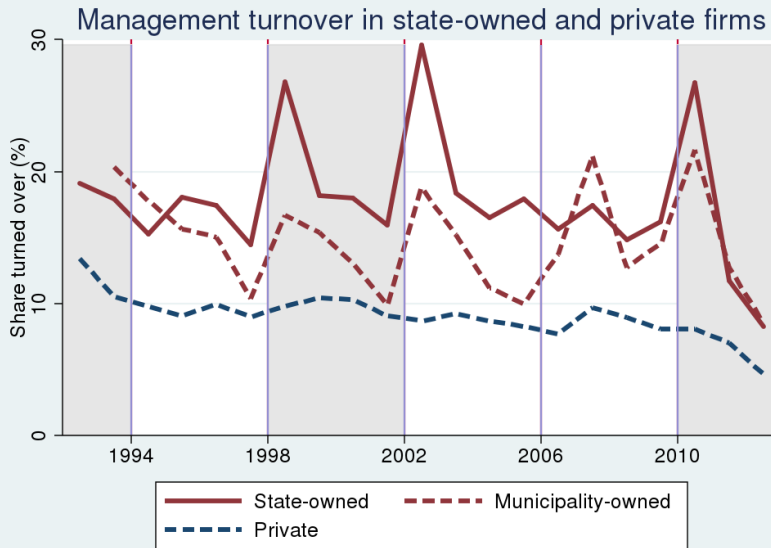
Politicians

- ▶ Candidates on national and local elections (1990-2014, 100,000 names), government officials, executives of important state-owned enterprises and public agencies.
- ▶ Name, city, party affiliation, year of election or appointment.

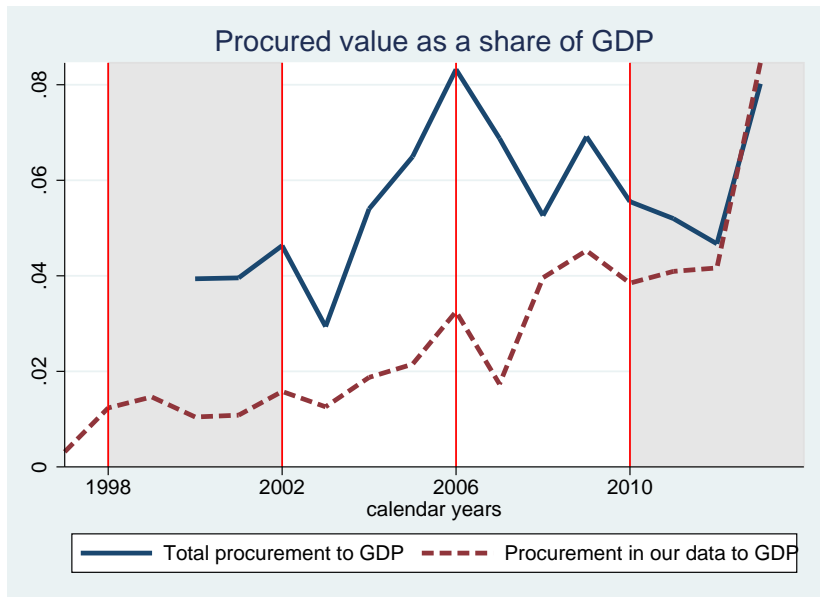
Procurement

- ▶ kozbeszerzes.ceu.hu (1997-2014, 200,000 tenders).
- ▶ Unique identifiers or procuring agency and winning (bidding) firm, product codes, amount won, decision date.

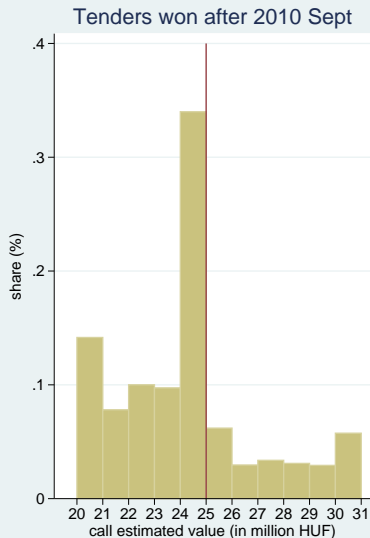
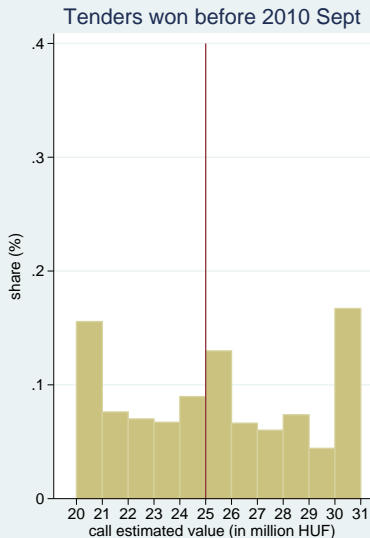
State-owned firms are political



Public procurement represents a large and increasing share of GDP



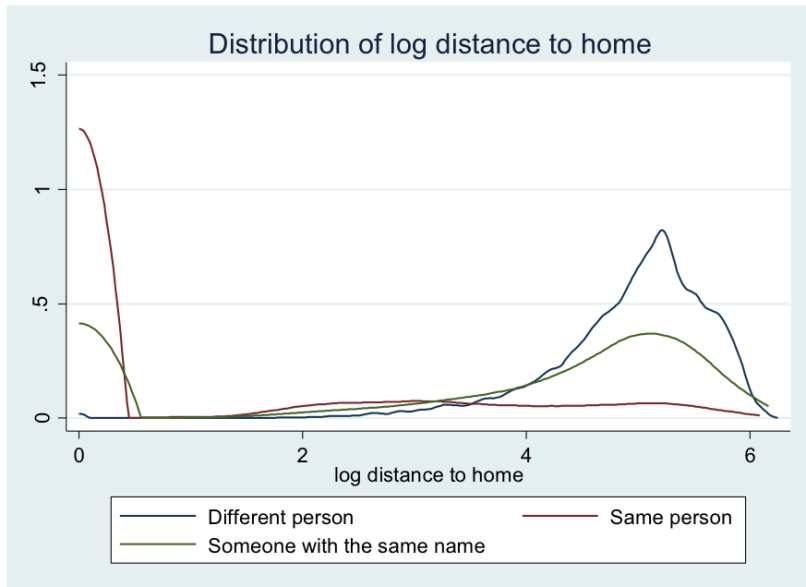
Procurers tend to avoid competition (Szucs, 2017)



Finding politicians in firms

- ▶ Key challenge: two different people can have the same name.
- ▶ Candidate match: (firm person, politician) pair with the same first name and last name.
- ▶ Two approaches:
 1. structural estimate based on commonality of name and distance between firm and electoral district.
 2. machine learning (regression) based on same plus city size

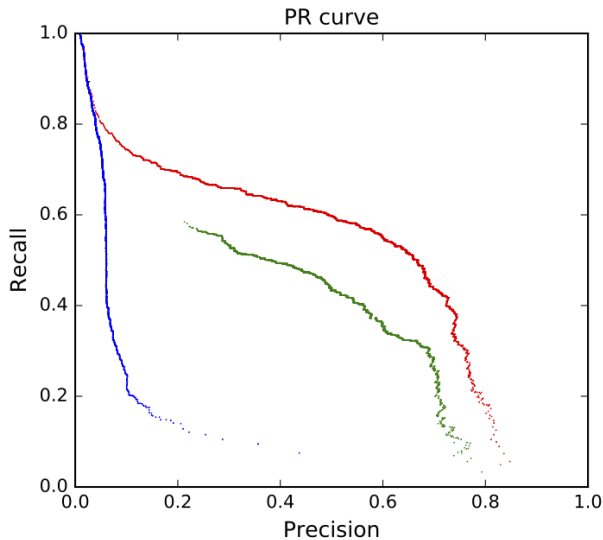
Kernel density of $\ln(1 + \text{distance})$ for different person, same person, and candidate match



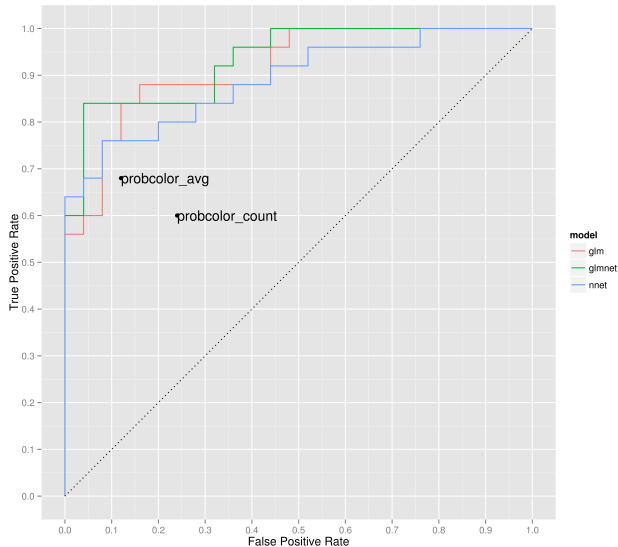
Machine learning

- ▶ Training data:
 - ▶ Outcome variable: Politically connected firms mentioned by media (1,300) plus a random sample of the same size, assumed to be unconnected.
 - ▶ Explanatory variables: number of politicians in firm and connected businesses (same owner, same manager, same address), distance to “oligarchs.
- ▶ Testing data: 100 hand-checked firms.

We can find about 60 percent of politicians



Connected firms can be identified with 80 percent precision



Measurement error

It is important to emphasize that our methodology for identifying connected firms suffers from considerable measurement error. We highlight three main sources of error.

1. We do not color connected firms. Perhaps because they are connected through a relative.
 2. We assign the wrong color to a connected firm. Perhaps because it has multiple politicians.
 3. We color an unconnected firm. Perhaps because the manager participated in a small local election many years earlier.
- In spite of these problems, the large sales share of connected firms we document is consistent with the idea that political connections play a significant role in the Hungarian economy. We also note that in the analysis, such measurement error is likely to bias our estimates towards zero. We are working on improving our measurement of connections.

Trends in political favoritism

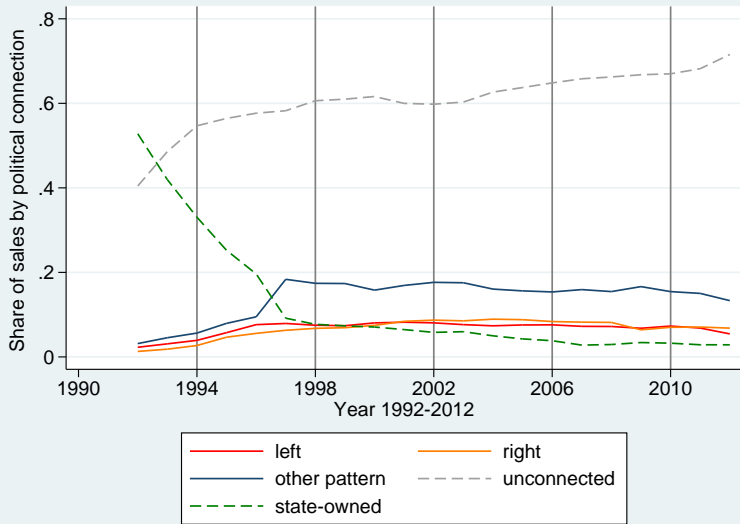
Political cycle in Hungary

	Share in parliament of			Central government
	Left	Right	Far right	
1994-1998	72%	28%	0%	left
1998-2002	42%	55%	4%	right
2002-2006	52%	48%	0%	left
2006-2010	54%	46%	0%	left
2010-2014	20%	68%	12%	right
2014-2018	23%	65%	12%	right

Connected firms are different

Connection status	Number of firms	Employment	Sales per worker	Share of firms foreign	Share of firms state owned
Left	9,547	47.3	32,743	4.9%	7.3%
Right	13,759	37.8	29,497	4.1%	7.2%
Other	37,307	24.8	25,592	4.0%	3.2%
Unconnected	847,275	5.7	23,144	9.1%	0.4%
All firms	907,888	8.0	23,579	8.6%	0.8%

Connected firms capture a large share of the economy



Procurement data

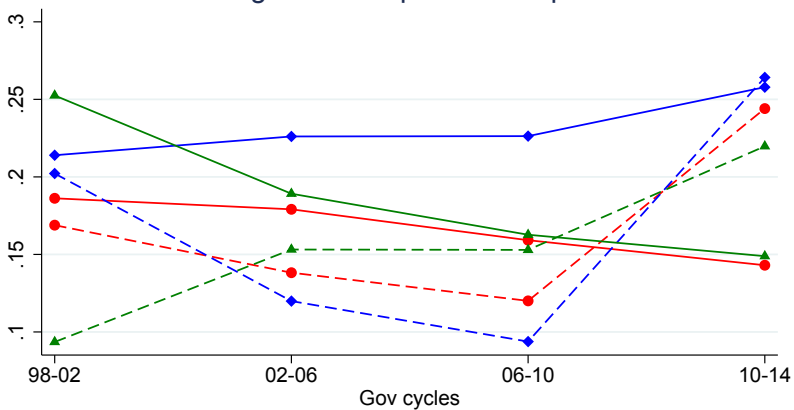
Procurement data

- ▶ Buyer: procuring agency (e.g., municipal government, school)
- ▶ Supplier: firm winning contract
 - ▶ Potential suppliers: all similar firms (Very few firms actually bid.)
- ▶ Additional controls: product being procured (Common Procurement Vocabulary), industry of winner, distance, size measures

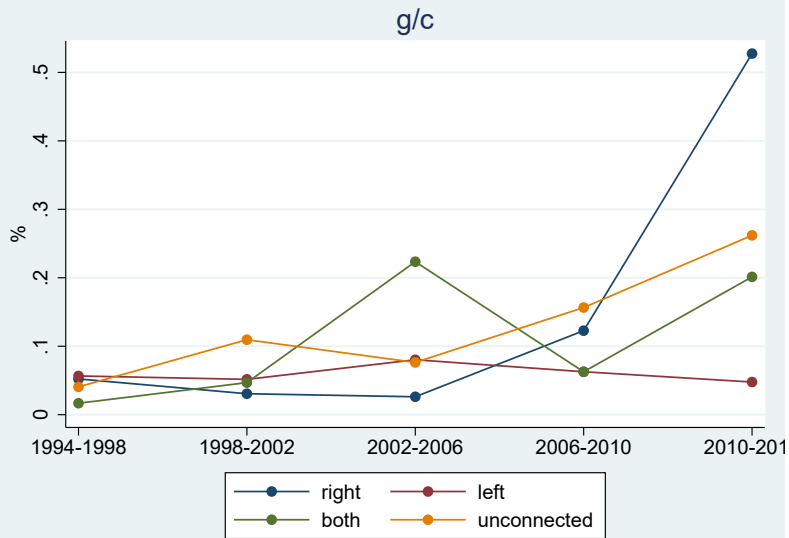
Data cleaning

- ▶ Identify buying agencies, contract winners (Entity Resolution).
- ▶ Remove duplicates.
- ▶ Parse value of contract.

Share of right firms in private and public sales



Public wedge in construction



Counterfactual scenarios

Counterfactual scenarios

We calculate welfare losses from misallocation

1. relative to private sales benchmark
2. relative to average government purchase

Private benchmark

Ignoring mispricing,

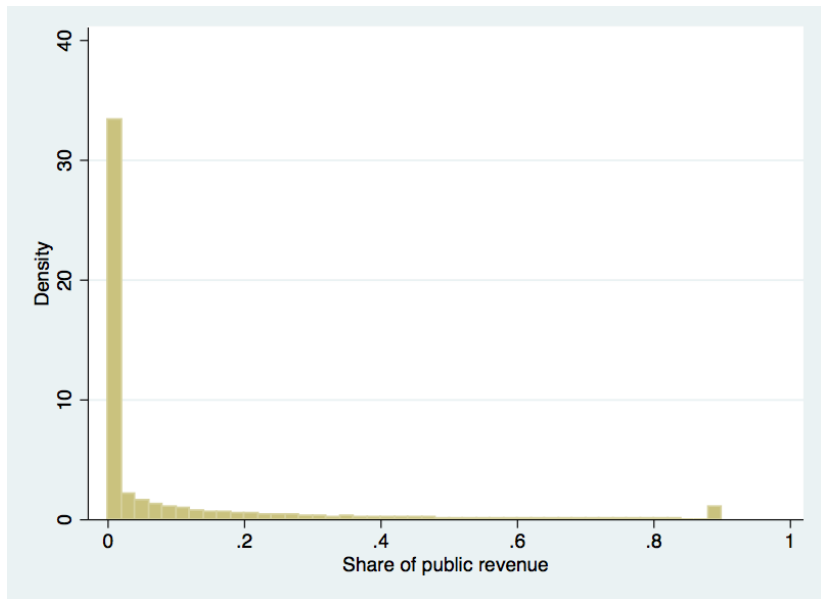
$$\frac{G}{G^*} = \left[\int_{\omega} s^*(\omega)^{1/\sigma} s(\omega)^{1-1/\sigma} d\omega \right]^{\frac{\sigma}{\sigma-1}} < 1$$

- ▶ models for $s^*(\omega)$:
 - ▶ market share among domestic private consumers
 - ▶ public market share predicted by non-political observables (size, productivity)
- ▶ models for $s(\omega)$:
 - ▶ actual public market share (upper bound on misallocation)
 - ▶ change in shares due to politically motivated preferences

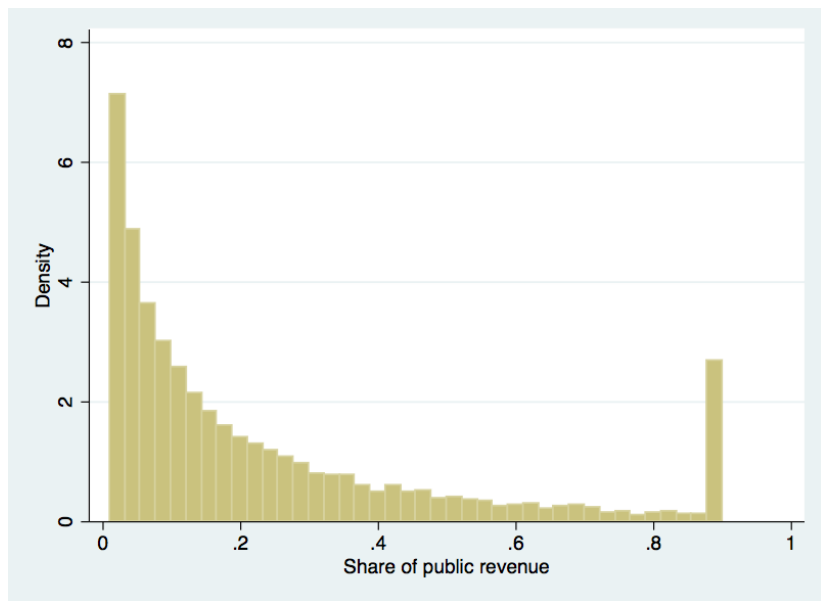
Data

- ▶ Only study firms that ever bid for government contracts.
- ▶ 500 biggest winners and non-winners of similar size (above 5th percentile in sales): 12,000 firms.
- ▶ $g(\omega)$ = total amount of government contracts won in a 4-year electoral cycle
- ▶ $r(\omega)$ = total revenue in same
- ▶ $x(\omega)$ = total export in same (as placebo)
- ▶ $c(\omega) = r(\omega) - g(\omega) - x(\omega)$ (winsorized below at 10 percent of domestic sales)

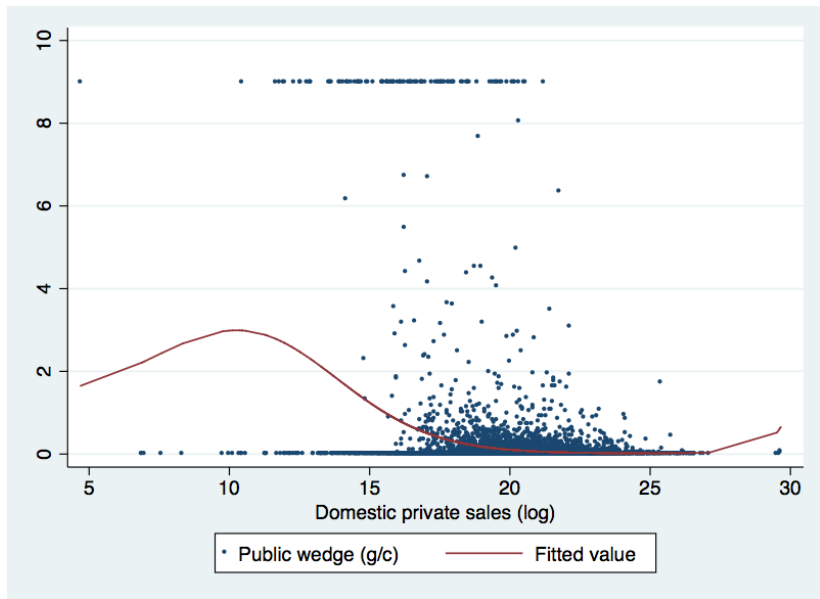
Most firms do not sell to government



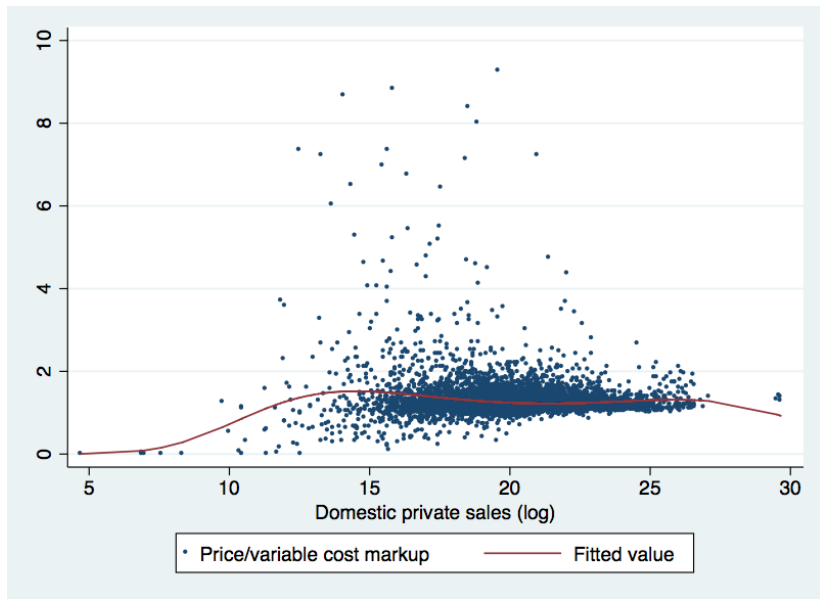
...or sell very little



Public wedge is highest for small firms



Markup does not vary much with size



Large losses from public misallocation ($\sigma = 5$)

$$\frac{G}{G^*} = \left[\int_{\omega} \left(\frac{p(\omega)c(\omega)}{PC} \right)^{1/\sigma} \left(\frac{p(\omega)g(\omega)}{PG} \right)^{1-1/\sigma} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

Utility of government spending relative to optimal

Cycle	Optimal market share	
	Private	Predicted
1998-2002	0.65	0.64
2002-2006	0.69	0.68
2006-2010	0.73	0.70
2010-2014	0.74	0.73

Contribution of preference components

- ▶ Estimate

$$s(\omega) = e^{\beta_0 + \sum_k \beta_k X_k(\omega) + \gamma \ln s^*(\omega) + \varepsilon(\omega)}$$

- ▶ For each preference component m , calculate

$$\tilde{s}_m(\omega) = e^{\tilde{\beta}_0 + \sum_{k \neq m} \beta_k X_k(\omega) + \gamma \ln s^*(\omega) + \varepsilon(\omega)}$$

(with $\tilde{\beta}_0$ recalibrated).

- ▶ This removes market share difference due to X_m .
 - ▶ Keeping all other differences the same.
- ▶ Compare welfare under $s(\omega)$ and $\tilde{s}_m(\omega)$.

Contribution of preference components ($\sigma = 5$)

$$\left[\frac{\int_{\omega} s^*(\omega)^{1/\sigma} \tilde{s}_m(\omega)^{1-1/\sigma} d\omega}{\int_{\omega} s^*(\omega)^{1/\sigma} s(\omega)^{1-1/\sigma} d\omega} \right]^{\frac{\sigma}{\sigma-1}} \cdot 100 - 100$$

Percentage change in welfare

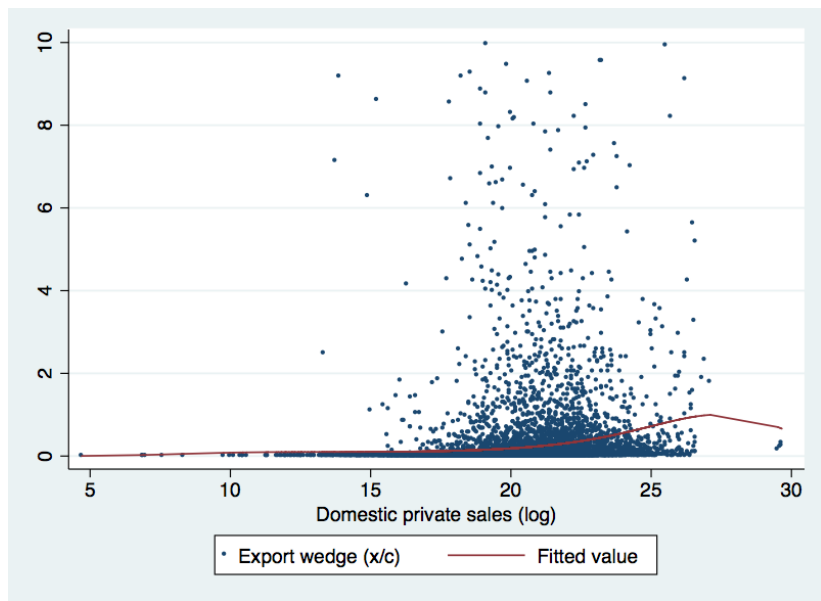
Cycle	Preference component			
	Left	Right	Foreign	Age
1998-2002	0.1%	0.0%	3.1%	1.3%
2002-2006	0.2%	0.2%	3.4%	2.1%
2006-2010	0.1%	0.0%	6.2%	3.2%
2010-2014	-0.0%	6.3%	6.6%	5.5%

Placebo

Placebo

- ▶ We redo the same counterfactuals with exports, $x(\omega)$.
- ▶ We expect misallocation to be smaller.
 - ▶ No political favoritism.
 - ▶ But: Melitz-skewness of exports.

Export wedge is larger for large firms



Smaller losses from export misallocation ($\sigma = 5$)

Utility of exports relative to optimal		
Cycle	Optimal market share	
	Private	Predicted
1998-2002	0.74	0.92
2002-2006	0.72	0.90
2006-2010	0.73	0.84
2010-2014	0.78	0.89

- ▶ Negligible contribution of political preference variables.
- ▶ Smaller contribution of foreign ownership.
- ▶ Larger contribution of size.

Misallocation in multinomial logit

Misallocation in multinomial logit

For buyer i procuring product p from potential seller j in tender t ,

$$\Delta V_{ip} = \mu \sum_j \frac{n_{ijp}}{n_{ip}} [\ln \hat{\pi}_{ijp}^* - \ln(n_{ijp}/n_{ip})]$$

$$\Delta V = \mu \sum_j \frac{n_{ijp}}{n} [\ln \hat{\pi}_{ijp}^* - \ln(n_{ijp}/n_{ip})]$$

- ▶ n_{ip} : number of product- p tenders (assumed identical)
- ▶ $\hat{\pi}_{ijp}^*$: multinomial logit with size, industry, product and geographic controls
- ▶ n_{ijp} : actual number of product- p contracts allocated to seller j

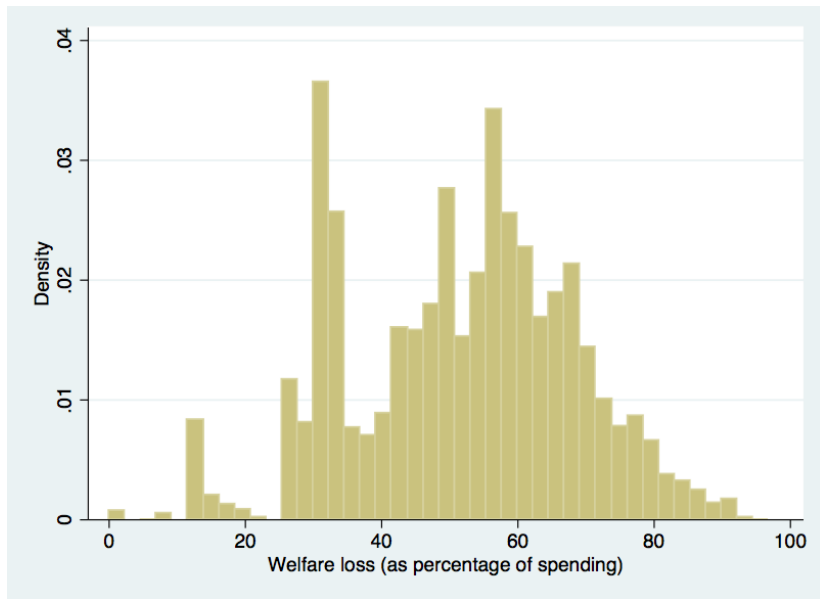
Sample

- ▶ Contracts in construction, manufacturing and business services.
- ▶ 500 largest buying agencies.
- ▶ 1,000 largest winners.
- ▶ 19,500 contracts.

Sample

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- ▶ 500 largest buying agencies.
- ▶ 1,000 largest winners.
- ▶ 19,500 contracts.
- ▶ Sparse buyer-supplier network: only 1% of 500,000 potential links exist.

Welfare loss is heterogeneous across buying agencies



Conclusion

Conclusion

- ▶ We developed a framework to study misallocation of demand.
- ▶ Efficiency losses can be quantified in both CES and logit.
- ▶ We estimated these losses in Hungarian public procurement.
- ▶ Efficiency losses from demand misallocation are large:
 - ▶ Government could up to 35 percent.
 - ▶ Political connections of firms explain up to 6 percent.

Next steps

- ▶ different demand systems
- ▶ general equilibrium feedback
- ▶ richer calibration

Appendix

Microfoundations of government bias

A transactional model of corruption

- ▶ Suppose government receives bribe in return for contract.
- ▶ Percentage of bribe varies exogenously across firms.
 - ▶ differences in bribing technology
 - ▶ politically organized and unorganized firms
- ▶ This leads to both preference and price variation.

Consumers

Consumers maximize

$$U = \left[C^{1-1/\theta} + G^{1-1/\theta} \right]^{\frac{\theta}{\theta-1}} \quad (8)$$

subject to

$$\int c(\omega) p_c(\omega) = I - T.$$

Government

The government maximizes

$$\alpha U + \int b(\omega) p_g(\omega) g(\omega) \frac{1}{P} d\omega \quad (9)$$

subject to

$$\int g(\omega) p_g(\omega) = T$$

- ▶ P : price index of the composite good
- ▶ $b(\omega)$: kickback from firm ω

Preference wedge

$$\frac{g(\omega)}{c(\omega)} = \left[\frac{\lambda_c}{\lambda_g - b(\omega)} \right]^\sigma$$

Pricing wedge

$$\frac{p_g(\omega)}{p_c(\omega)} = \frac{1}{1 - b(\omega)}$$