

# 1. Bias-Variance Tradeoff

```
In [2]: import numpy as np
import pickle
import math
from torch import nn
import torch
from torch.optim import SGD, Adam
import torch.nn
from sklearn.model_selection import KFold
import torch.nn.functional as F
import random
from tqdm import tqdm
import math
import matplotlib.pyplot as plt

from functools import wraps
from time import time
def timing(f):
    @wraps(f)
    def wrap(*args, **kw):
        ts = time()
        result = f(*args, **kw)
        te = time()
        print('func:%r took: %2.4f sec' % (f.__name__, te-ts))
        return result
    return wrap
```

1a

```
In [3]: # load dataset
(train_X_raw, train_y), (test_X_raw, test_y) = pickle.load(open("./mnist.pkl", 'rb'))

# normalize features (not labels)
train_X_norm = train_X_raw / train_X_raw.max()
test_X_norm = test_X_raw / test_X_raw.max()
```

1b

```
In [4]: def create_chunks(complete_list, chunk_size=None, num_chunks=None):
    """
    Cut a list into multiple chunks, each having chunk_size (the last chunk
    may be smaller)
    """
    chunks = []
    if num_chunks is None:
        num_chunks = math.ceil(len(complete_list) / chunk_size)
    elif chunk_size is None:
        chunk_size = math.ceil(len(complete_list) / num_chunks)
    for i in range(num_chunks):
```

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        chunks.append(complete_list[i * chunk_size: (i + 1) * chunk_size])
    return chunks

# Shuffle the training data
# define permutation index to make sure x values (features) are shuffled with y values
perm_index = np.random.permutation(len(train_X_norm))
# permute to predetermined indices
train_X_perm = train_X_norm[perm_index]
train_y_perm = train_y[perm_index]
# split into 3 chunks
chunks_X = create_chunks(train_X_perm, num_chunks=3)
chunks_y = create_chunks(train_y_perm, num_chunks=3)
# make test data by combining two chunks
test_X1 = np.concatenate(chunks_X[0:2])
test_y1 = np.concatenate(chunks_y[0:2])
# validation data is wan chunk
validate_X1 = chunks_X[2]
validate_y1 = chunks_y[2]

```

```

In [10]: class Trainer():
    def __init__(self, model, optimizer_type, learning_rate, epoch, batch_size, input_transform):
        """
        A class for training the model
        model: nn.Module
            A pytorch model
        optimizer_type: 'adam' or 'sgd'
        learning_rate: float
        epoch: int
        batch_size: int
        input_transform: func
            Transforms the input data. Can be used to reshape data.
        """
        self.model = model
        if optimizer_type == "sgd":
            self.optimizer = SGD(model.parameters(), learning_rate, momentum=0.9)
        elif optimizer_type == "Adam":
            self.optimizer = Adam(model.parameters(), lr=learning_rate)
        self.epoch = epoch
        self.batch_size = batch_size
        self.input_transform = input_transform

    @timing
    def train(self, inputs, outputs, val_inputs, val_outputs, early_stop=False, l2=False, silent=False):
        """ train self.model with specified arguments
        inputs: np.array, The shape of input_transform(input) should be (n, n_features)
        outputs: np.array shape (n, n_outputs)
        val_inputs: np.array, The shape of input_transform(val_input) should be (n_val, n_features)
        val_outputs: np.array shape (n_val, n_outputs)
        early_stop: bool
        l2: bool. Whether or not to use L2 regularization.
        silent: bool. Controls whether or not to print the train and val errors
        """
        @return
        a dictionary of arrays with train and val losses and accuracies
        """
        ### convert data to tensor of correct shape and type here ###

```

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inputs = torch.tensor(inputs, dtype=torch.float)
outputs = torch.tensor(outputs, dtype=torch.int64)

#inputs = inputs.reshape(-1, 1024)

losses = []
accuracies = []
val_losses = []
val accuracies = []
weights = self.model.state_dict()
lowest_val_loss = np.inf
loss_fn = nn.CrossEntropyLoss()

for n_epoch in tqdm(range(self.epoch), leave=False):
    self.model.train()
    # batch indices is number of input entries
    batch_indices = list(range(inputs.shape[0]))
    # shuffle batch indices
    random.shuffle(batch_indices)
    # create chunks
    batch_indices = create_chunks(batch_indices, chunk_size=self.batch_size)
    epoch_loss = 0
    epoch_acc = 0
    # Batch
    for batch in batch_indices:
        # proportion of the total output that is represented by that batch
        batch_importance = len(batch) / len(outputs)
        batch_input = inputs[batch]
        batch_output = outputs[batch]
        ### make prediction and compute loss with loss function of y
        batch_predictions = self.model(batch_input)
        loss = loss_fn(batch_predictions, batch_output)
        if l2:
            ### Compute the loss with L2 regularization ###
            self.optimizer = Adam(self.model.parameters(), weight_decay=l2)
        self.optimizer.zero_grad() # sets the gradients of all model parameters to zero
        loss.backward() # backpropagates
        self.optimizer.step() # updates parameters based on gradient
        ### Compute epoch_loss and epoch_acc
        # number of accurately predicted points / num points in batch
        acc = torch.argmax(batch_predictions, dim=1).eq(batch_output).sum().item()
        epoch_loss += loss.item() * batch_importance
        epoch_acc += acc
    val_loss, val_acc = self.evaluate(val_inputs, val_outputs, print_preds=False)
    if n_epoch % 10 == 0 and not silent:
        print("Epoch %d/%d - Loss: %.3f - Acc: %.3f" % (n_epoch + 1, self.epoch, epoch_loss, epoch_acc))
        print("Val_loss: %.3f - Val_acc: %.3f" % (val_loss, val_acc))
    losses.append(epoch_loss)
    accuracies.append(epoch_acc)
    val_losses.append(val_loss)
    val accuracies.append(val_acc)
    if early_stop:
        if val_loss < lowest_val_loss:
            lowest_val_loss = val_loss
            # saves current state of model's parameters to dict weights
            weights = self.model.state_dict()

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if early_stop:
    # loads saved parameters back into model
    self.model.load_state_dict(weights)

# plot training and validation losses
plt.figure(figsize=(12,4))
plt.subplot(1,2,1)
plt.plot(losses, label='Training Loss')
plt.plot(val_losses, label='Validation Loss')
plt.title('Training vs Validation Loss')
plt.xlabel('Epochs')
plt.ylabel('Loss')
plt.legend()

# plot training and validation accuracy
plt.subplot(1,2,2)
plt.plot(accuracies, label='Training Accuracy')
plt.plot(val_accuracies, label='Validation Accuracy')
plt.title('Training vs Validation Accuracy')
plt.xlabel('Epochs')
plt.ylabel('Accuracy')
plt.legend()

plt.show()

return {"losses": losses, "accuracies": accuracies, "val_losses": val_losses, "val_accuracies": val_accuracies}

def evaluate(self, inputs, outputs, print_acc=True):
    """
    Evaluate model on provided input and output
    inputs: np.array, The shape of input_transform(input) should be (n_data, n_features)
    outputs: np.array shape (n_data, n_classes)
    print_acc: bool

    @return
    losses: float
    acc: float
    """
    inputs = torch.tensor(inputs, dtype=torch.float)
    outputs = torch.tensor(outputs, dtype=torch.int64)
    #inputs = inputs.reshape(-1, 1024)
    loss_fn = nn.CrossEntropyLoss()

    self.model.eval()
    batch_indices = list(range(inputs.shape[0]))
    batch_indices = create_chunks(batch_indices, chunk_size=self.batch_size)
    acc = 0
    loss = 0

    for batch in batch_indices:
        batch_importance = len(batch) / len(outputs)
        batch_input = inputs[batch]
        batch_output = outputs[batch]
        batch_predictions = self.model(batch_input)
        with torch.no_grad():

```

```

        # compute prediction and loss
        batch_acc = torch.argmax(batch_predictions, dim=1).eq(batch_
        # how much
        loss += loss_fn(batch_predictions, batch_output) * batch_imp
        acc = acc + batch_acc

    if print_acc:
        print("Accuracy: %.3f" % acc)
    return loss, acc

```

```

In [6]: class ANN(torch.nn.Module):
        """
        This class defines an ANN.
        """
        def __init__(self, input_size, hidden_size, output_size, dropout_rate=0):
            super().__init__()
            self.hidden_layer = torch.nn.Linear(input_size, hidden_size)
            self.output_layer = torch.nn.Linear(hidden_size, output_size)
            self.activation = torch.nn.Sigmoid()
            self.dropout = nn.Dropout(p = dropout_rate)

        def forward(self, x0):
            x1 = self.hidden_layer(x0)
            x2 = self.activation(x1)
            x2 = self.dropout(x2)
            x3 = self.output_layer(x2)
            x4 = self.activation(x3)

            return x4

```

## 1c

Devise an ANN that has 2 computing layers: a hidden layer of size 3 neurons and the final output layer of 10 output neurons and use a sigmoid activation function. Use the ADAM optimizer with learning rate of  $2e-3$ , batchsize of 128, and 50 epochs.

```

In [6]: kf = KFold(3, shuffle=True, random_state=49)

        for idx, (train_index, val_index) in enumerate(kf.split(train_X_norm)):
            X_train_fold, X_val_fold = train_X_norm[train_index], train_X_norm[val_index]
            y_train_fold, y_val_fold = train_y[train_index], train_y[val_index]

            model1 = ANN(1024, 3, 10)
            train_model1 = Trainer(model1, "Adam", 2e-3, 50, 128)
            model1_results = train_model1.train(X_train_fold, y_train_fold, X_val_fold)

```

```

4%|██████████| 2/50 [00:00<00:10, 4.62it/s]

```

```

Epoch 1/50 - Loss: 2.209 - Acc: 0.290
          Val_loss: 2.135 - Val_acc: 0.469

```

```

22%|██████████| 11/50 [00:02<00:06, 5.61it/s]

```

Epoch 11/50 - Loss: 1.779 - Acc: 0.548  
Val\_loss: 1.779 - Val\_acc: 0.551

44% | ██████████ | 22/50 [00:04<00:05, 5.27it/s]

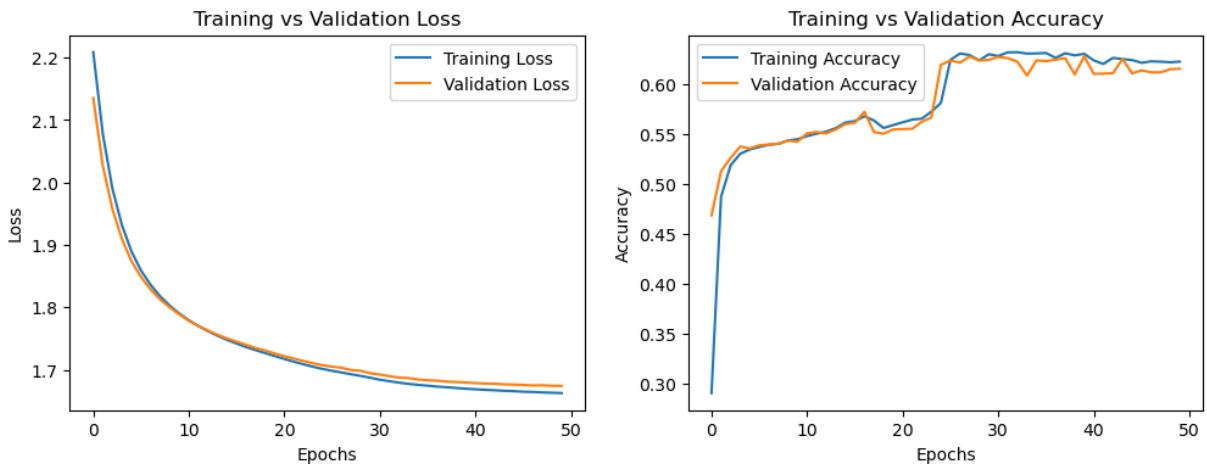
Epoch 21/50 - Loss: 1.717 - Acc: 0.562  
Val\_loss: 1.722 - Val\_acc: 0.555

64% | ██████████ | 32/50 [00:05<00:03, 5.72it/s]

Epoch 31/50 - Loss: 1.684 - Acc: 0.628  
Val\_loss: 1.693 - Val\_acc: 0.627

84% | ██████████ | 42/50 [00:07<00:01, 5.71it/s]

Epoch 41/50 - Loss: 1.669 - Acc: 0.624  
Val\_loss: 1.679 - Val\_acc: 0.610



func:'train' took: 9.2838 sec

4% | ██████ | 2/50 [00:00<00:08, 5.74it/s]

Epoch 1/50 - Loss: 2.190 - Acc: 0.333  
Val\_loss: 2.117 - Val\_acc: 0.375

24% | ████████ | 12/50 [00:02<00:06, 5.68it/s]

Epoch 11/50 - Loss: 1.792 - Acc: 0.542  
Val\_loss: 1.793 - Val\_acc: 0.539

44% | ██████████ | 22/50 [00:04<00:05, 5.10it/s]

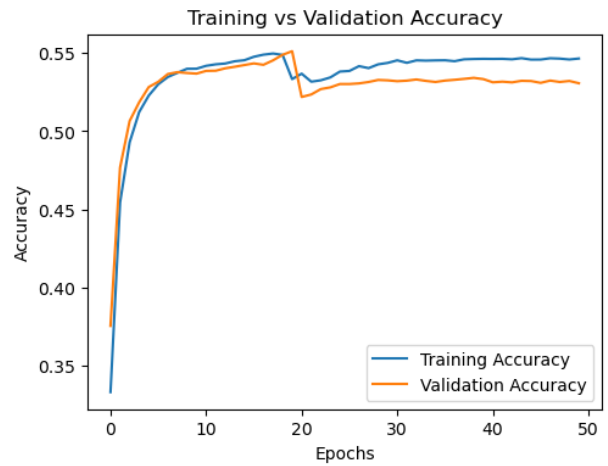
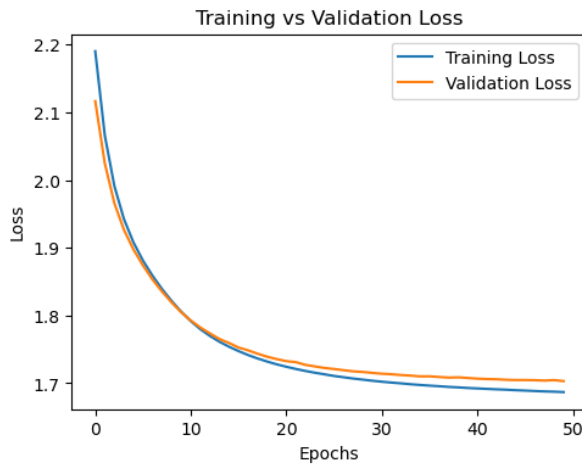
Epoch 21/50 - Loss: 1.724 - Acc: 0.537  
Val\_loss: 1.733 - Val\_acc: 0.522

62% | ██████████ | 31/50 [00:05<00:03, 5.00it/s]

Epoch 31/50 - Loss: 1.702 - Acc: 0.545  
Val\_loss: 1.714 - Val\_acc: 0.532

84% | ██████████ | 42/50 [00:07<00:01, 5.36it/s]

Epoch 41/50 - Loss: 1.692 - Acc: 0.546  
Val\_loss: 1.707 - Val\_acc: 0.531



func:'train' took: 9.4869 sec

4% | ██████████ | 2/50 [00:00<00:08, 5.38i t/s]

Epoch 1/50 - Loss: 2.197 - Acc: 0.326  
Val\_loss: 2.124 - Val\_acc: 0.499

24% | ██████████ | 12/50 [00:02<00:06, 5.43i t/s]

Epoch 11/50 - Loss: 1.794 - Acc: 0.589  
Val\_loss: 1.791 - Val\_acc: 0.573

44% | ██████████ | 22/50 [00:04<00:05, 5.33i t/s]

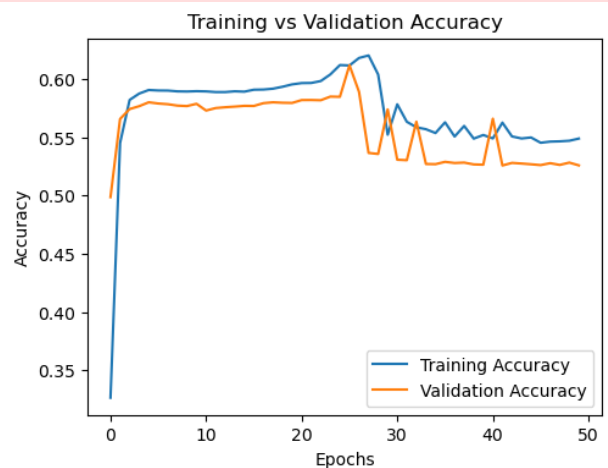
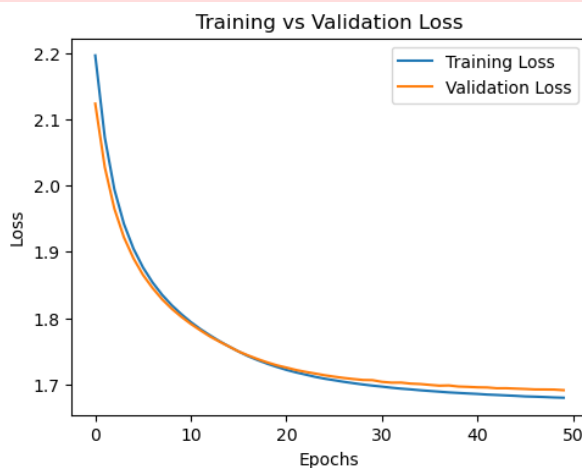
Epoch 21/50 - Loss: 1.722 - Acc: 0.596  
Val\_loss: 1.725 - Val\_acc: 0.582

64% | ██████████ | 32/50 [00:05<00:03, 5.48i t/s]

Epoch 31/50 - Loss: 1.696 - Acc: 0.578  
Val\_loss: 1.704 - Val\_acc: 0.531

84% | ██████████ | 42/50 [00:07<00:01, 5.55i t/s]

Epoch 41/50 - Loss: 1.685 - Acc: 0.549  
Val\_loss: 1.695 - Val\_acc: 0.566



func:'train' took: 9.3534 sec

1d

Devise another ANN with hidden layer of size 50. Do the same as in (1c). Plot your training and validation curve, and comment on the bias-variance tradeoff with this choice.

```
In [7]: kf = KFold(3, shuffle=True, random_state=49)

for idx, (train_index, val_index) in enumerate(kf.split(train_X_norm)):
    X_train_fold, X_val_fold = train_X_norm[train_index], train_X_norm[val_index]
    y_train_fold, y_val_fold = train_y[train_index], train_y[val_index]

    model2 = ANN(1024, 50, 10)
    train_model2 = Trainer(model2, "Adam", 2e-3, 50, 128)
    model2_results = train_model2.train(X_train_fold, y_train_fold, X_val_fold,
```

```
2%|██████████| 1/50 [00:00<00:13, 3.68it/s]
```

```
Epoch 1/50 - Loss: 1.828 - Acc: 0.804
           Val_loss: 1.649 - Val_acc: 0.890
```

```
22%|██████████| 11/50 [00:02<00:09, 3.97it/s]
```

```
Epoch 11/50 - Loss: 1.511 - Acc: 0.953
           Val_loss: 1.520 - Val_acc: 0.940
```

```
42%|██████████| 21/50 [00:05<00:07, 4.08it/s]
```

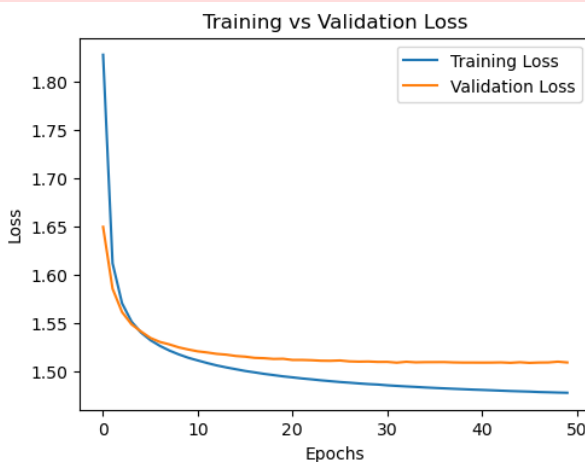
```
Epoch 21/50 - Loss: 1.493 - Acc: 0.967
           Val_loss: 1.511 - Val_acc: 0.947
```

```
62%|██████████| 31/50 [00:07<00:04, 4.10it/s]
```

```
Epoch 31/50 - Loss: 1.485 - Acc: 0.975
           Val_loss: 1.509 - Val_acc: 0.951
```

```
82%|██████████| 41/50 [00:10<00:02, 4.04it/s]
```

```
Epoch 41/50 - Loss: 1.480 - Acc: 0.979
           Val_loss: 1.509 - Val_acc: 0.952
```



```
func:'train' took: 12.7032 sec
```

```
2%|██████████| 1/50 [00:00<00:14, 3.47it/s]
```



Epoch 1/50 - Loss: 1.828 - Acc: 0.799  
Val\_loss: 1.654 - Val\_acc: 0.882

22%|██████████| 11/50 [00:02<00:10, 3.89i  
t/s]

Epoch 11/50 - Loss: 1.509 - Acc: 0.954  
Val\_loss: 1.521 - Val\_acc: 0.942

42%|██████████| 21/50 [00:05<00:07, 4.02i  
t/s]

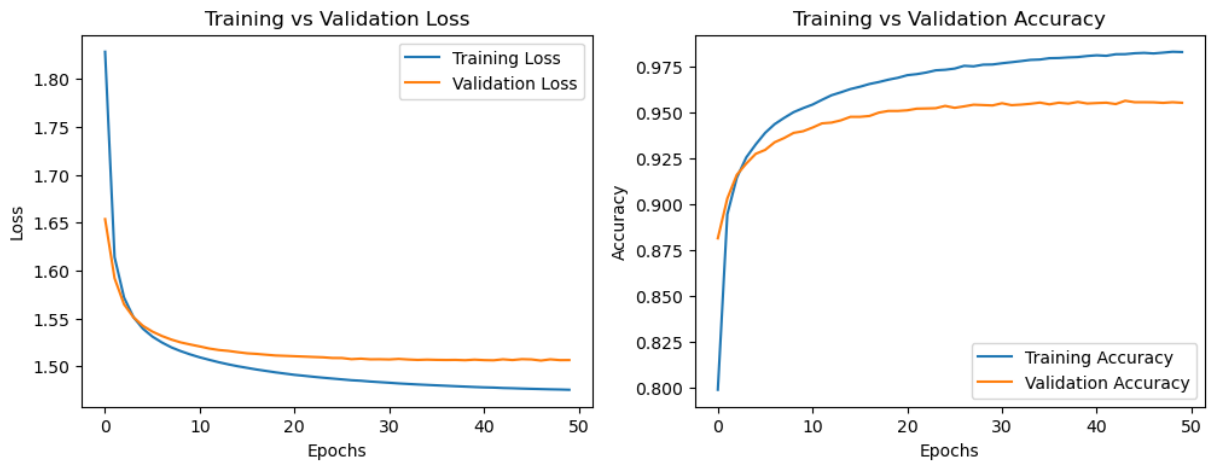
Epoch 21/50 - Loss: 1.491 - Acc: 0.970  
Val\_loss: 1.510 - Val\_acc: 0.951

62%|██████████| 31/50 [00:07<00:04, 4.06i  
t/s]

Epoch 31/50 - Loss: 1.483 - Acc: 0.977  
Val\_loss: 1.507 - Val\_acc: 0.955

82%|██████████| 41/50 [00:10<00:02, 4.05i  
t/s]

Epoch 41/50 - Loss: 1.478 - Acc: 0.981  
Val\_loss: 1.506 - Val\_acc: 0.955



func:'train' took: 12.6804 sec

2%|██████| 1/50 [00:00<00:13, 3.70i  
t/s]

Epoch 1/50 - Loss: 1.827 - Acc: 0.795  
Val\_loss: 1.648 - Val\_acc: 0.892

22%|██████████| 11/50 [00:02<00:10, 3.78i  
t/s]

Epoch 11/50 - Loss: 1.511 - Acc: 0.953  
Val\_loss: 1.518 - Val\_acc: 0.945

42%|██████████| 21/50 [00:05<00:08, 3.62i  
t/s]

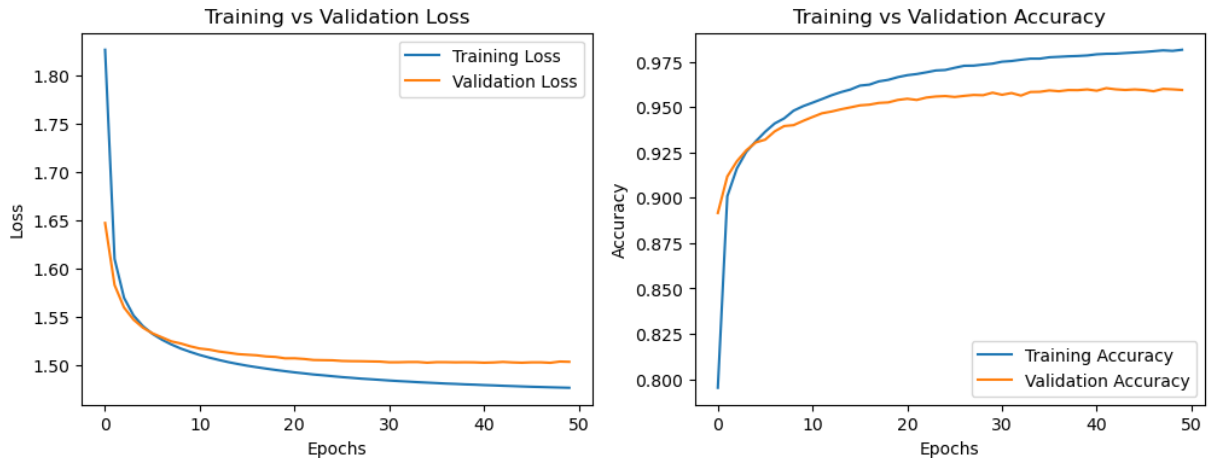
Epoch 21/50 - Loss: 1.493 - Acc: 0.968  
Val\_loss: 1.507 - Val\_acc: 0.955

62%|██████████| 31/50 [00:08<00:04, 3.94i  
t/s]

Epoch 31/50 - Loss: 1.484 - Acc: 0.975  
Val\_loss: 1.503 - Val\_acc: 0.957

82%|██████████| 41/50 [00:10<00:02, 4.08i  
t/s]

Epoch 41/50 – Loss: 1.480 – Acc: 0.979  
 Val\_loss: 1.503 – Val\_acc: 0.959



func: 'train' took: 13.0566 sec

With 50 hidden neurons, accuracy is significantly higher (~0.95 in the validation set of model2 as opposed to ~0.60 in the validation set of model1). However, the validation loss is higher than the training loss for model2, which suggests some overfitting to the training data. In this case, I think the higher variance is hurting a bit. Maybe there's a sweet spot between 3 neurons and 50 neurons where we have a lower variance and higher bias that avoids underfitting.

## 2. Deep Learning and Regularization

### 2a

Using the ANN from 1(d), utilize dropout with 15%. Compare your training and test accuracy to results in (d).

```
In [8]: kf = KFold(3, shuffle=True, random_state=49)

for idx, (train_index, val_index) in enumerate(kf.split(train_X_norm)):
    X_train_fold, X_val_fold = train_X_norm[train_index], train_X_norm[val_index]
    y_train_fold, y_val_fold = train_y[train_index], train_y[val_index]

    model3 = ANN(1024, 50, 10, dropout_rate=0.15)
    train_model3 = Trainer(model3, "Adam", 2e-3, 50, 128)
    model3_results = train_model3.train(X_train_fold, y_train_fold, X_val_fold, y_val_fold)
```

2%|██████████ | 1/50 [00:00<00:13, 3.60it/s]

Epoch 1/50 – Loss: 1.841 – Acc: 0.770  
 Val\_loss: 1.655 – Val\_acc: 0.884

22%|██████████ | 11/50 [00:02<00:10, 3.76it/s]

Epoch 11/50 - Loss: 1.524 - Acc: 0.941  
Val\_loss: 1.522 - Val\_acc: 0.939

42% | ██████████ | 21/50 [00:05<00:07, 3.90i  
t/s]

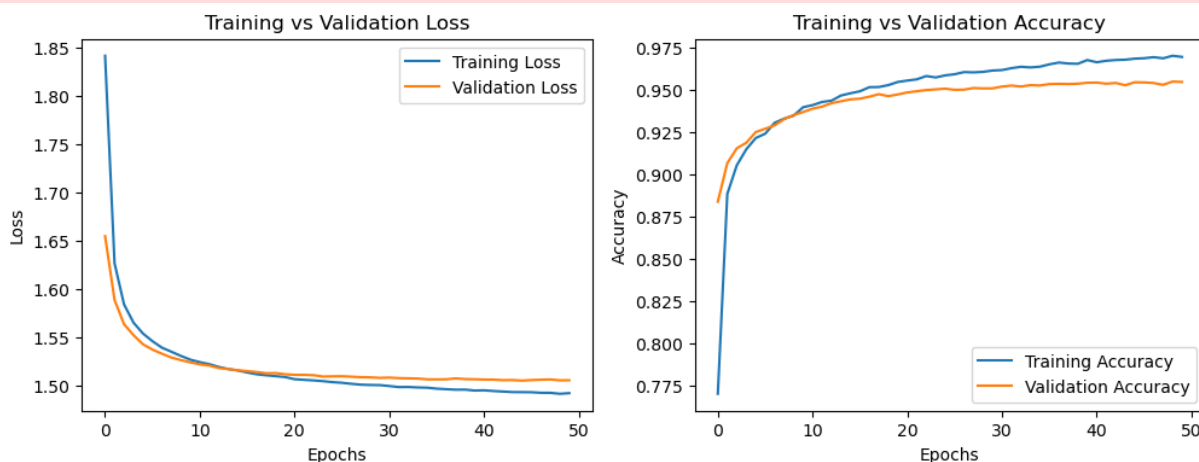
Epoch 21/50 - Loss: 1.507 - Acc: 0.955  
Val\_loss: 1.511 - Val\_acc: 0.948

62% | ██████████ | 31/50 [00:08<00:04, 3.96i  
t/s]

Epoch 31/50 - Loss: 1.500 - Acc: 0.961  
Val\_loss: 1.508 - Val\_acc: 0.952

82% | ██████████ | 41/50 [00:10<00:02, 3.94i  
t/s]

Epoch 41/50 - Loss: 1.495 - Acc: 0.966  
Val\_loss: 1.506 - Val\_acc: 0.954



func:'train' took: 12.9944 sec

2% | ██████ | 1/50 [00:00<00:12, 3.78i  
t/s]

Epoch 1/50 - Loss: 1.843 - Acc: 0.771  
Val\_loss: 1.660 - Val\_acc: 0.879

22% | ████████ | 11/50 [00:02<00:09, 3.91i  
t/s]

Epoch 11/50 - Loss: 1.523 - Acc: 0.941  
Val\_loss: 1.523 - Val\_acc: 0.936

42% | ██████████ | 21/50 [00:05<00:07, 3.75i  
t/s]

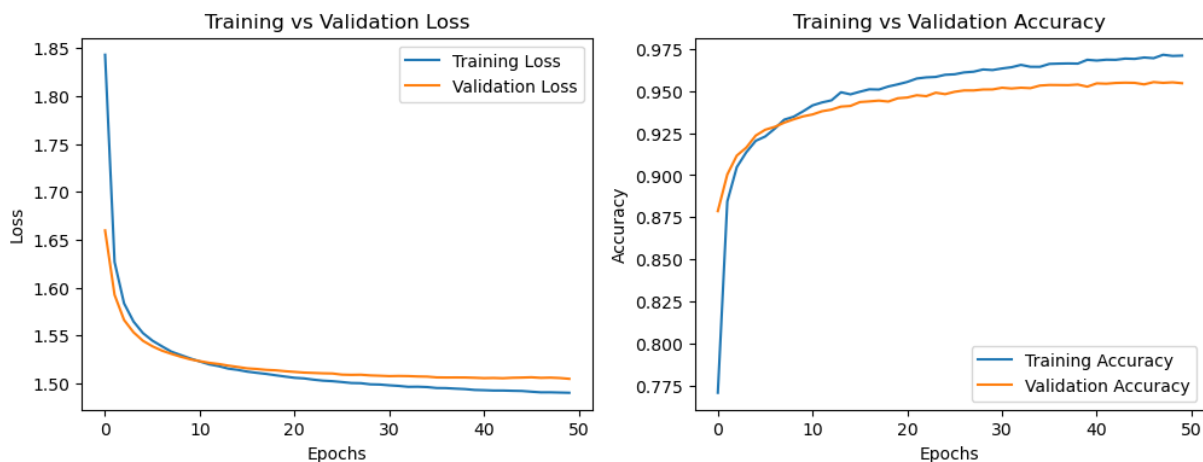
Epoch 21/50 - Loss: 1.506 - Acc: 0.955  
Val\_loss: 1.512 - Val\_acc: 0.946

62% | ██████████ | 31/50 [00:08<00:05, 3.74i  
t/s]

Epoch 31/50 - Loss: 1.498 - Acc: 0.963  
Val\_loss: 1.508 - Val\_acc: 0.952

82% | ██████████ | 41/50 [00:10<00:02, 3.90i  
t/s]

Epoch 41/50 - Loss: 1.493 - Acc: 0.968  
Val\_loss: 1.505 - Val\_acc: 0.954



func:'train' took: 13.1199 sec

2% | ██████████ | 1/50 [00:00<00:13, 3.60i  
t/s]

Epoch 1/50 - Loss: 1.837 - Acc: 0.779  
Val\_loss: 1.653 - Val\_acc: 0.888

22% | ██████████ | 11/50 [00:02<00:10, 3.85i  
t/s]

Epoch 11/50 - Loss: 1.524 - Acc: 0.941  
Val\_loss: 1.519 - Val\_acc: 0.942

42% | ██████████ | 21/50 [00:05<00:07, 3.85i  
t/s]

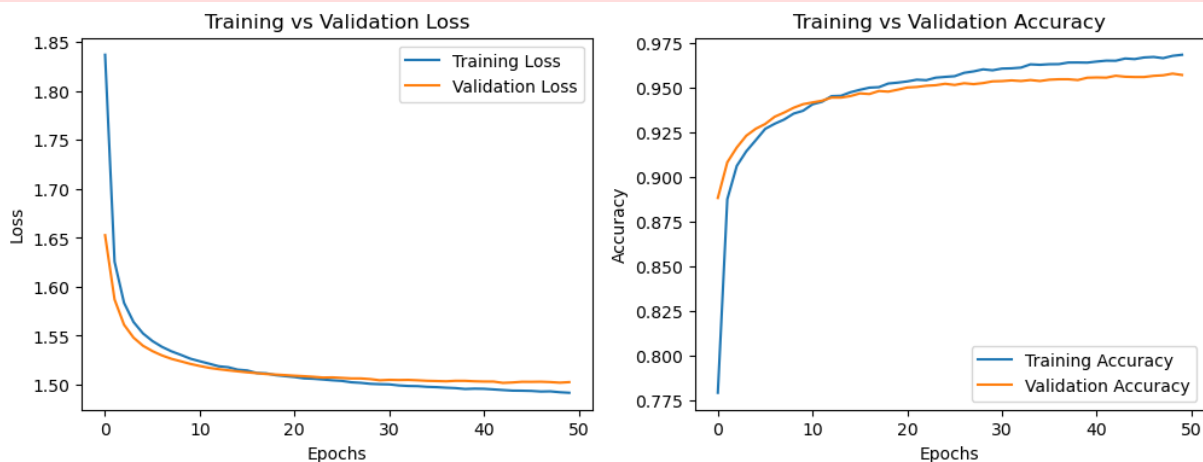
Epoch 21/50 - Loss: 1.508 - Acc: 0.954  
Val\_loss: 1.509 - Val\_acc: 0.950

62% | ██████████ | 31/50 [00:08<00:05, 3.74i  
t/s]

Epoch 31/50 - Loss: 1.500 - Acc: 0.961  
Val\_loss: 1.505 - Val\_acc: 0.954

82% | ██████████ | 41/50 [00:10<00:02, 3.80i  
t/s]

Epoch 41/50 - Loss: 1.496 - Acc: 0.965  
Val\_loss: 1.503 - Val\_acc: 0.956



func:'train' took: 13.2643 sec

With a 15% dropout, the training and validation losses are much closer to one another (though there is still overfitting). This is expected, as dropout is supposed to reduce

overfitting. The accuracies for both are about the same.

## 2b

Using the ANN from 1(d), utilize L2 regularization with  $\lambda=1e-5$ . How does the result compare to (d)?

```
In [9]: kf = KFold(3, shuffle=True, random_state=49)

for idx, (train_index, val_index) in enumerate(kf.split(train_X_norm)):
    X_train_fold, X_val_fold = train_X_norm[train_index], train_X_norm[val_index]
    y_train_fold, y_val_fold = train_y[train_index], train_y[val_index]

    model4 = ANN(1024, 50, 10)
    train_model4 = Trainer(model4, "Adam", 2e-3, 50, 128)
    model4_results = train_model4.train(X_train_fold, y_train_fold, X_val_fold,
```

2% | ██████████ | 1/50 [00:00<00:15, 3.24it/s]

Epoch 1/50 - Loss: 1.943 - Acc: 0.758  
Val\_loss: 1.738 - Val\_acc: 0.855

22% | ██████████ | 11/50 [00:03<00:11, 3.43it/s]

Epoch 11/50 - Loss: 1.536 - Acc: 0.930  
Val\_loss: 1.539 - Val\_acc: 0.925

42% | ██████████ | 21/50 [00:06<00:08, 3.52it/s]

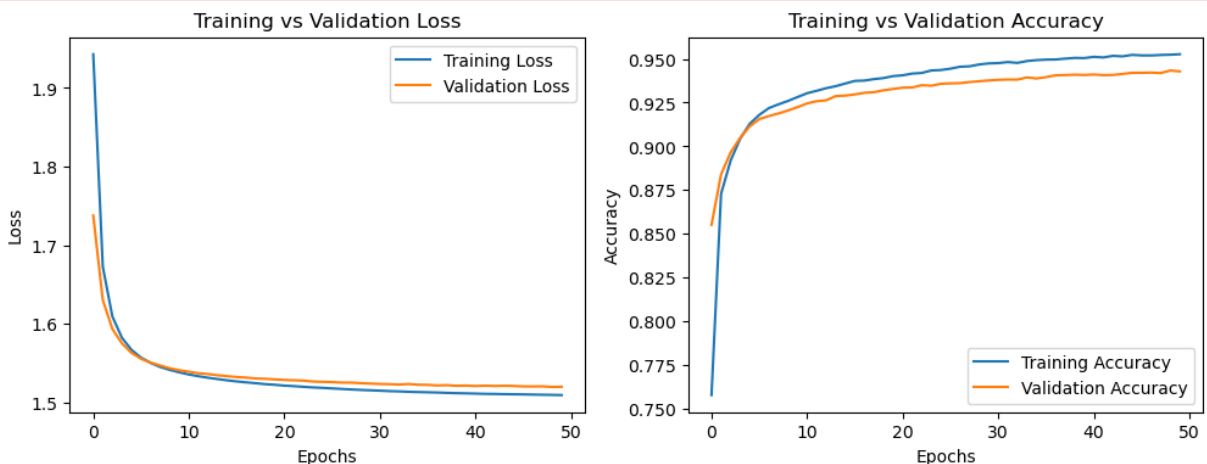
Epoch 21/50 - Loss: 1.522 - Acc: 0.941  
Val\_loss: 1.529 - Val\_acc: 0.934

62% | ██████████ | 31/50 [00:08<00:05, 3.59it/s]

Epoch 31/50 - Loss: 1.515 - Acc: 0.948  
Val\_loss: 1.524 - Val\_acc: 0.938

82% | ██████████ | 41/50 [00:11<00:02, 3.58it/s]

Epoch 41/50 - Loss: 1.511 - Acc: 0.951  
Val\_loss: 1.521 - Val\_acc: 0.941



func:'train' took: 14.3644 sec

2%|██████████| 1/50 [00:00<00:14, 3.47i  
t/s]

Epoch 1/50 - Loss: 1.946 - Acc: 0.751  
Val\_loss: 1.739 - Val\_acc: 0.861

22%|██████████| 11/50 [00:03<00:10, 3.61i  
t/s]

Epoch 11/50 - Loss: 1.536 - Acc: 0.928  
Val\_loss: 1.541 - Val\_acc: 0.924

42%|██████████| 21/50 [00:05<00:08, 3.57i  
t/s]

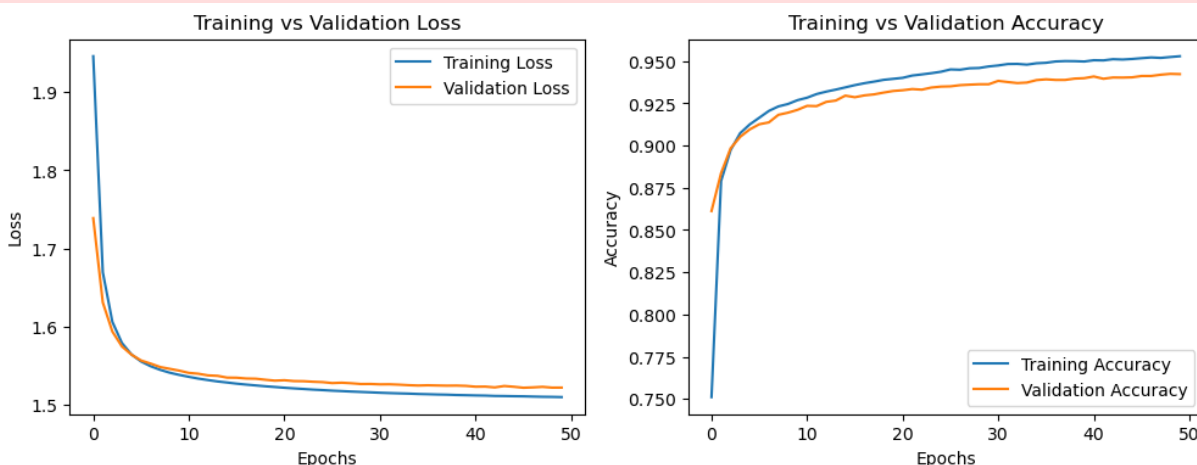
Epoch 21/50 - Loss: 1.522 - Acc: 0.940  
Val\_loss: 1.531 - Val\_acc: 0.933

62%|██████████| 31/50 [00:08<00:05, 3.65i  
t/s]

Epoch 31/50 - Loss: 1.515 - Acc: 0.947  
Val\_loss: 1.526 - Val\_acc: 0.938

82%|██████████| 41/50 [00:11<00:02, 3.64i  
t/s]

Epoch 41/50 - Loss: 1.512 - Acc: 0.951  
Val\_loss: 1.523 - Val\_acc: 0.941



func:'train' took: 13.9580 sec

2%|██████████| 1/50 [00:00<00:14, 3.32i  
t/s]

Epoch 1/50 - Loss: 1.947 - Acc: 0.721  
Val\_loss: 1.735 - Val\_acc: 0.855

22%|██████████| 11/50 [00:03<00:10, 3.64i  
t/s]

Epoch 11/50 - Loss: 1.537 - Acc: 0.928  
Val\_loss: 1.538 - Val\_acc: 0.926

42%|██████████| 21/50 [00:05<00:07, 3.66i  
t/s]

Epoch 21/50 - Loss: 1.523 - Acc: 0.939  
Val\_loss: 1.527 - Val\_acc: 0.935

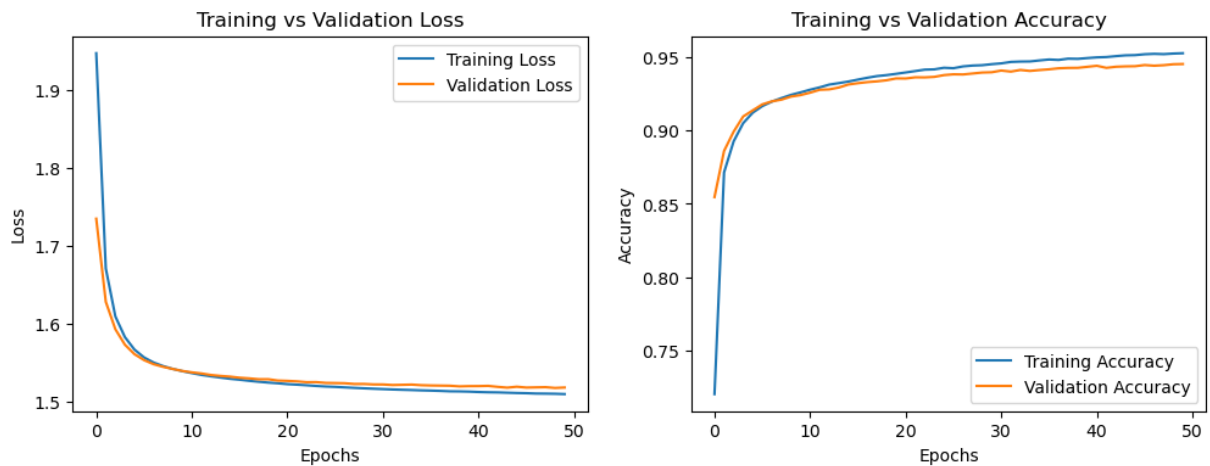
62%|██████████| 31/50 [00:08<00:05, 3.67i  
t/s]

Epoch 31/50 - Loss: 1.516 - Acc: 0.946  
Val\_loss: 1.522 - Val\_acc: 0.941

82% | ██████████ | 41/50 [00:11<00:02, 3.64i  
t/s]

Epoch 41/50 - Loss: 1.513 - Acc: 0.950

Val\_loss: 1.520 - Val\_acc: 0.944



```
func:'train' took: 13.8213 sec
```

Using L2 loss, the training loss is lower than the validation loss. This is good, because L2 loss is also meant to help reduce overfitting by making weights more evenly distributed. However, the model accuracy is a bit lower than that of the model in 1d (~0.93 in 2b model vs ~0.95 in 1d model).

Of the three models in 1d, 2a and 2b, 2b's model has the least difference between training and validation loss without overfitting.

2c

Use principal component analysis on the input, to create a reduced set of input features, keeping 99% of the variance. This is a type of data transformation! How many parameters do you have in this case and how does it compare to the original model (i.e. the ANN in 1(d))?

```
In [8]: from sklearn.decomposition import PCA

# keep 99% of the variance
pca = PCA(n_components=0.99)
# reshape test_X data
train_X_flat = train_X_norm.reshape(-1, 1024)
# PCA fit and transform flattened X
X_pca = pca.fit_transform(train_X_flat)
```

The reduced model for PCA has 331 parameters, which is much fewer than the original model which had 1024 ( $32 \times 32$ ).

## 2d

Use the regularization settings (2a or 2b) that give the best result so far, and using the reduced input space from (c) run the model again. Is the training faster and better this time?

```
In [11]: kf = KFold(3, shuffle=True, random_state=49)

for idx, (train_index, val_index) in enumerate(kf.split(X_pca)):
    X_train_fold, X_val_fold = X_pca[train_index], X_pca[val_index]
    y_train_fold, y_val_fold = train_y[train_index], train_y[val_index]

    model5 = ANN(331, 50, 10)
    train_model5 = Trainer(model5, "Adam", 2e-3, 50, 128, input_transform=la
    model5_results = train_model5.train(X_train_fold, y_train_fold, X_val_fold,
```

2% | ██████████ | 1/50 [00:00<00:10, 4.58i  
t/s]

Epoch 1/50 - Loss: 2.118 - Acc: 0.630  
Val\_loss: 1.893 - Val\_acc: 0.841

22% | ██████████ | 11/50 [00:02<00:08, 4.67i  
t/s]

Epoch 11/50 - Loss: 1.536 - Acc: 0.931  
Val\_loss: 1.542 - Val\_acc: 0.923

42% | ██████████ | 21/50 [00:04<00:06, 4.49i  
t/s]

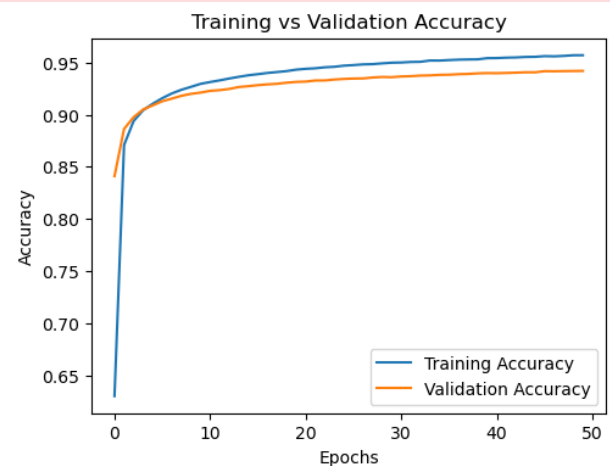
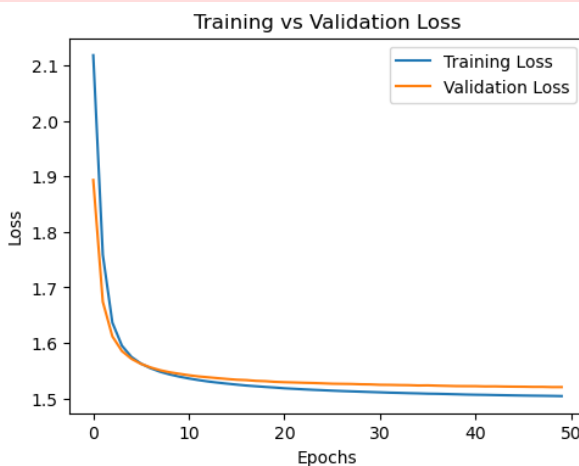
Epoch 21/50 - Loss: 1.518 - Acc: 0.944  
Val\_loss: 1.529 - Val\_acc: 0.932

62% | ██████████ | 31/50 [00:06<00:04, 4.26i  
t/s]

Epoch 31/50 - Loss: 1.511 - Acc: 0.950  
Val\_loss: 1.525 - Val\_acc: 0.937

82% | ██████████ | 41/50 [00:09<00:02, 4.38i  
t/s]

Epoch 41/50 - Loss: 1.507 - Acc: 0.954  
Val\_loss: 1.522 - Val\_acc: 0.940



func:'train' took: 11.6765 sec

2% | ██████████ | 1/50 [00:00<00:10, 4.50i  
t/s]



Epoch 1/50 - Loss: 2.111 - Acc: 0.671  
 Val\_loss: 1.882 - Val\_acc: 0.855

22%|██████████| 11/50 [00:02<00:08, 4.62i  
 t/s]

Epoch 11/50 - Loss: 1.537 - Acc: 0.930  
 Val\_loss: 1.546 - Val\_acc: 0.922

42%|██████████| 21/50 [00:04<00:06, 4.26i  
 t/s]

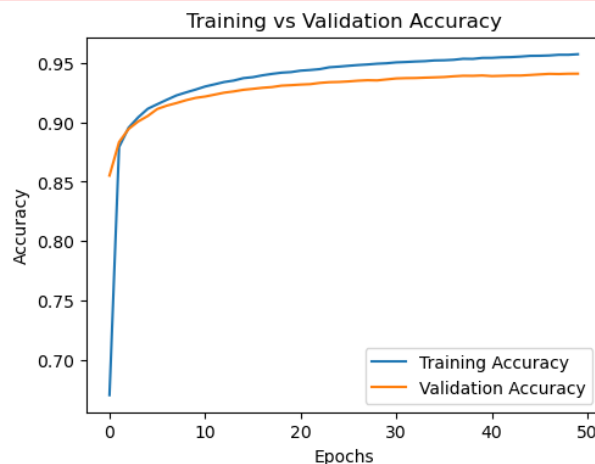
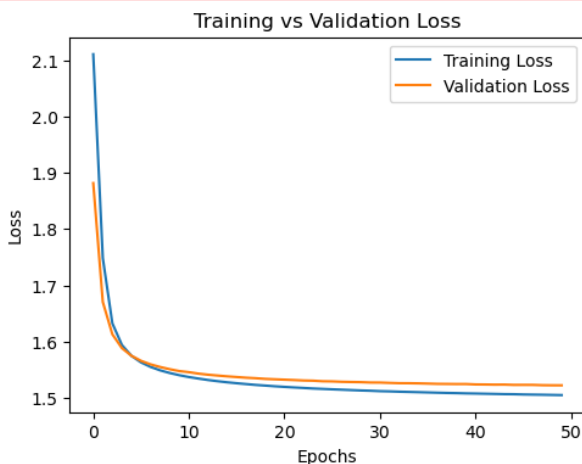
Epoch 21/50 - Loss: 1.519 - Acc: 0.943  
 Val\_loss: 1.532 - Val\_acc: 0.932

62%|██████████| 31/50 [00:07<00:04, 4.63i  
 t/s]

Epoch 31/50 - Loss: 1.512 - Acc: 0.950  
 Val\_loss: 1.527 - Val\_acc: 0.937

82%|██████████| 41/50 [00:09<00:01, 4.75i  
 t/s]

Epoch 41/50 - Loss: 1.507 - Acc: 0.954  
 Val\_loss: 1.524 - Val\_acc: 0.939



func:'train' took: 11.1777 sec

2%|██| 1/50 [00:00<00:10, 4.58i  
 t/s]

Epoch 1/50 - Loss: 2.109 - Acc: 0.640  
 Val\_loss: 1.883 - Val\_acc: 0.857

22%|██████████| 11/50 [00:02<00:07, 4.97i  
 t/s]

Epoch 11/50 - Loss: 1.538 - Acc: 0.929  
 Val\_loss: 1.542 - Val\_acc: 0.924

44%|██████████| 22/50 [00:04<00:05, 5.00i  
 t/s]

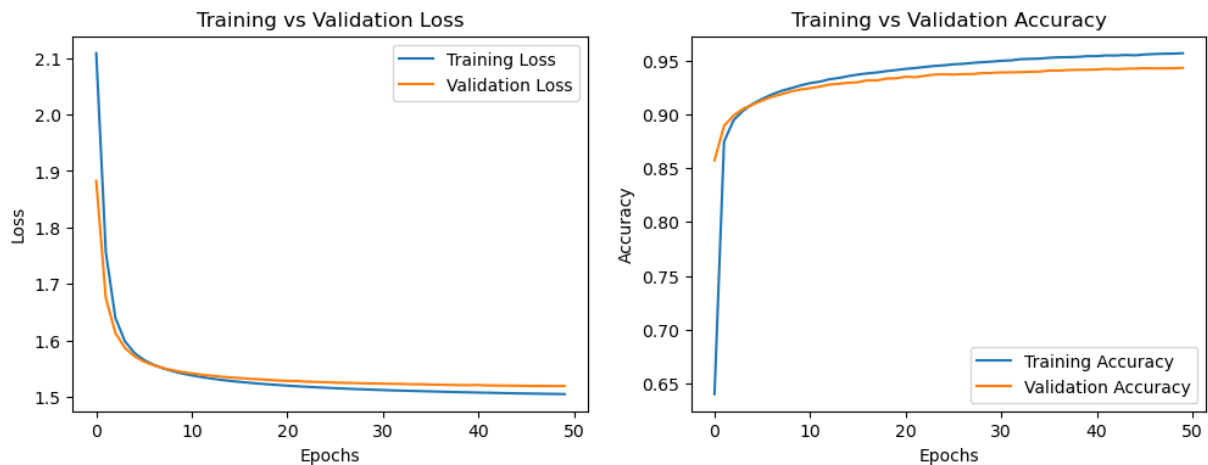
Epoch 21/50 - Loss: 1.520 - Acc: 0.942  
 Val\_loss: 1.528 - Val\_acc: 0.935

62%|██████████| 31/50 [00:06<00:03, 4.80i  
 t/s]

Epoch 31/50 - Loss: 1.512 - Acc: 0.950  
 Val\_loss: 1.524 - Val\_acc: 0.939

82%|██████████| 41/50 [00:08<00:01, 4.65i  
 t/s]

Epoch 41/50 - Loss: 1.508 - Acc: 0.954  
Val\_loss: 1.521 - Val\_acc: 0.941



func: 'train' took: 10.5063 sec

The model in 2d is trained a bit faster than the previous ones, but only by 1-2 seconds on average. I guess on a large enough scale this would make a difference, but the time difference is not significant in this assignment.

The accuracy is not much better really, it's more like the accuracy of the previous model that used L2 loss (2b)