

1.a.

The two columns on the very left should have the same T/F assignments

A	B	C	D	$(A \wedge B \rightarrow C \wedge D)$	$(A \wedge B \rightarrow C)$
T	T	T	T	T	T
T	T	T	F	F	F
T	T	F	T	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	T	F	T	T
T	F	F	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	T	F	T	T
F	T	F	T	T	T
F	T	F	F	T	T
F	F	T	T	T	T
F	F	T	F	T	T
F	F	F	T	T	T
F	F	F	F	T	T

All assignments that satisfied $(A \wedge B \rightarrow C \wedge D)$ also satisfied $(A \wedge B \rightarrow C)$. (All instances in which the left side were true also led to the right side being true as well)

1.b. $(A \wedge B \rightarrow C \wedge D) \models (A \wedge B \rightarrow C)$ using **Natural Deduction**.

1. b. $A \wedge B \rightarrow C \wedge D$ ← premises.

1. $\neg(A \wedge B) \vee (C \wedge D)$ Implication Elimination.

2. $(C \vee \neg(A \wedge B)) \wedge (D \vee \neg(A \wedge B))$ Distributivity.

3. $(A \wedge B \rightarrow C) \wedge (A \wedge B \rightarrow D)$ Implication elimination (Done on both sides).

4. $(A \wedge B \rightarrow C)$ And Elimination.

1. c. $A \wedge B \rightarrow C \wedge D$ premise.

$(\neg A \vee \neg B) \vee (C \wedge D)$ Implication Elimination

$((\neg A \vee \neg B \vee C) \wedge (\neg A \vee \neg B \vee D))$ Distribution / pushing negation inwards.

$(\neg A \vee \neg B \vee C), (\neg A \vee \neg B \vee D)$ ← KB

Query = $Q = A \wedge B \rightarrow C = \neg(A \wedge B) \vee C = (\neg A \vee \neg B) \vee C = \neg A \vee \neg B \vee C$

$\neg Q = \neg(\neg A \vee \neg B \vee C) = \overline{A \wedge B \wedge \neg C}$

So clauses are

$(\neg A \vee \neg B \vee C), (\neg A \vee \neg B \vee D), A, B, \neg C$

These clauses contradict each other

So proof is done.

2.a

$O1Y \rightarrow \text{not}(L1Y)$

$O1W \rightarrow \text{not}(L1W)$

$O1W \rightarrow \text{not}(C1Y)$

$O1Y \rightarrow \text{not}(C1W)$

$O1Y \wedge O1W \rightarrow C1B$

$O1Y \wedge O1W \rightarrow \text{not}(L1B)$

$O2Y \rightarrow \text{not}(L2Y)$

$O2W \rightarrow \text{not}(L2W)$

$O2W \rightarrow \text{not}(C2Y)$

$O2Y \rightarrow \text{not}(C2W)$

$O2Y \wedge O2W \rightarrow C2B$

$O2Y \wedge O2W \rightarrow \text{not}(L2B)$

$O3Y \rightarrow \text{not}(L3Y)$

$O3W \rightarrow \text{not}(L3W)$

$O3W \rightarrow \text{not}(C3Y)$

$O3Y \rightarrow \text{not}(C3W)$

$O3Y \wedge O3W \rightarrow C3B$

$O3Y \wedge O3W \rightarrow \text{not}(L3B)$

$O1W \wedge O2W \rightarrow C3Y$

$O1Y \wedge O2Y \rightarrow C3W$

$O2W \wedge O3W \rightarrow C1Y$

$O2Y \wedge O3Y \rightarrow C1W$

$O1W \wedge O3W \rightarrow C2Y$

$O1Y \wedge O3Y \rightarrow C2W$

2.b

Facts $\{O1Y, L1W, O2W, L2Y, O3Y, L3B\}$

$O1Y$ and $O3Y$ from knowledge base gives us $C2W$.

This is from the rule

$O1Y \wedge O3Y \rightarrow C2W$

From the knowledge base

3.

- a. $\text{CanBikeToWork} \rightarrow \text{CanGetToWork}$
- b. $\text{CanDriveToWork} \rightarrow \text{CanGetToWork}$
- c. $\text{CanWalkToWork} \rightarrow \text{CanGetToWork}$
- d. $\text{HaveBike} \text{ WorkCloseToHome} \wedge \text{Sunny} \rightarrow \text{CanBikeToWork}$
- e. $\text{HaveMountainBike} \rightarrow \text{HaveBike}$
- f. $\text{HaveTenSpeed} \rightarrow \text{HaveBike}$
- g. $\text{OwnCar} \rightarrow \text{CanDriveToWork}$
- h. $\text{OwnCar} \rightarrow \text{MustGetAnnualInspection}$
- i. $\text{OwnCar} \rightarrow \text{MustHaveValidLicense}$
- j. $\text{CanRentCar} \rightarrow \text{CanDriveToWork}$
- k. $\text{HaveMoney} \text{ CarRentalOpen} \rightarrow \text{CanRentCar}$
- l. $\text{HertzOpen} \rightarrow \text{CarRentalOpen}$
- m. $\text{AvisOpen} \rightarrow \text{CarRentalOpen}$
- n. $\text{EnterpriseOpen} \rightarrow \text{CarRentalOpen}$
- o. $\text{CarRentalOpen} \rightarrow \text{IsNotAHoliday}$
- p. $\text{HaveMoney} \text{ TaxiAvailable} \rightarrow \text{CanDriveToWork}$
- q. $\text{Sunny} \wedge \text{WorkCloseToHome} \rightarrow \text{CanWalkToWork}$
- 3
- r. $\text{HaveUmbrella} \wedge \text{WorkCloseToHome} \rightarrow \text{CanWalkToWork}$
- s. $\text{Sunny} \rightarrow \text{StreetsDry}$

Facts: {Rainy, HaveMountainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen}

Facts serve as agenda

Facts: {Rainy, HaveMountainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen}

Worked out (highlighted words are “crossed out”):

m. $\text{AvisOpen} \rightarrow \text{CarRentalOpen}$

So we add carRentalOpen to facts

Facts: {Rainy, HaveMountainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, carRentalOpen}

HaveMoney and CarRentalOpen mean we can rent a car:

k. $\text{HaveMoney} \text{ CarRentalOpen} \rightarrow \text{CanRentCar}$

Facts: {Rainy, HaveMountainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, carRentalOpen, canRentCar}

CanRentCar means j. $\text{CanRentCar} \rightarrow \text{CanDriveToWork}$

Facts: {Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, carRentalOpen, canRentCar, canDriveToWork}

canDriveToWork means we can get to work:

b. $\text{CanDriveToWork} \rightarrow \text{CanGetToWork}$

Facts: {Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen, carRentalOpen, canRentCar, canDriveToWork, canGetToWork}

canGetToWork was generated, so we're done.

4.

FirstElement is the top of the stack

Stack: firstElement, secondElement,

Working out the problem steps:

Stack: canGetToWork

canGetToWork has canDriveToWork as antecedent (rule b)

Stack: canDriveToWork

canDriveToWork has canRentCar as antecedent (rule j)

Stack: CanRentCar

CanRentCar has two antecedents: HaveMoney and CarRentalOpen (rule k)

Stack: HaveMoney, CarRentalOpen

HaveMoney is a known fact

Stack: CarRentalOpen

CarRentalOpen has AvisOpen as an antecedent (rule m)

AvisOpen is a fact

Stack is empty, so canGetToWork is true.

5. It seems that backward chaining should be used in the instance that the rules in the knowledge base generate inferences that make it difficult to reach a conclusion from. Forward chaining also provides rules in the form of definite clauses, so ambiguity in the clauses through the use of or (like one of these clauses might work) will not be permitted to using forward chaining with if that is the case. Additionally, if the clauses seem that they will be easy to generate, maybe they are generic, then it could be of advantage to use backward chaining. Also if the knowledge base is really large, backward chaining might be better given there are so many choices to work from if you do forward chaining.