Filter Assignment

- **Issue**

  If the sampling frequency is fs, the cutoff frequency is fc, and the quality factor is Q, the following bi-quadratic filter is implemented

  Please do so.

  1. High Pass Filter

$$b_0 = a_0^{-1}(1 + c)$$
$$b_1 = -2b_0$$
$$b_2 = b_0$$
$$a_1 = 4ca_0^{-1}$$
$$a_2 = a_0^{-1}(s - 2)$$

  2. Band Pass Filter

$$b_0 = sa_0^{-1}$$
$$b_1 = 0$$
$$b_2 = -b_0$$
$$a_1 = 4ca_0^{-1}$$
$$a_2 = a_0^{-1}(s - 2)$$

  3. Band Stop Filter

$$b_0 = 2a_0^{-1}$$
$$b_1 = -4ca_0^{-1}$$
$$b_2 = b_0$$
$$a_1 = 4ca_0^{-1}$$
$$a_2 = a_0^{-1}(s - 2)$$

  However, c, s, and a0 shall be as follows

$$c = \cos(2\pi f_c f_s^{-1})$$
$$s = \sin(2\pi f_c f_s^{-1}) \times Q^{-1}$$
$$a_0 = s + 2$$

- **implementation procedure**

  1. Adding Filter Types

     Modify filterTypes in app/src/parameterDescriptor.js

  2. Processing Implementation

     Add a case in updateFilterCoefficients() in app/public/SynthesizerWorklet.js to implement the calculation process of 5 coefficients (b0, b1, b2, a1, a2)

     Please refer to the commit (URL) when Low Pass Filter is added as a concrete example.

(Supplemental)

- ## bi-quadratic filter

  The bi-quadratic filter is a digital filter represented by the following blocks, which can be transformed into a filter with various characteristics by adjusting each coefficient.
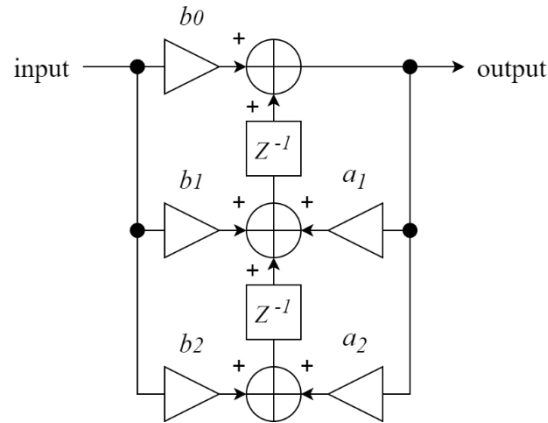
  The transfer function of the above filter is expressed as follows

$$H(\mathrm{z}) = \frac{b_0 + b_1 \mathrm{z}^{-1} + b_2 \mathrm{z}^{-2}}{1 - a_1 \mathrm{z}^{-1} - a_2 \mathrm{z}^{-2}}$$

  There are various ways to derive the coefficients of each filter, but in this case, the coefficients are derived from the transfer function of the analog filter using the bilinear transform.

- ## biquadratic transformation

  In the bi-linear conversion, the transfer function of the digital filter is derived by substituting the following equation for the transfer function of the analog filter.

$$s = \frac{1 - \mathrm{z}^{-1}}{1 + \mathrm{z}^{-1}} \cdot \frac{1}{\tan(\pi f_c f_s^{-1})}$$

  The transfer functions for each analog filter are as follows. If you are confident in your calculations, please try to find the formulas for the coefficients by yourself.

  1. Low Pass Filter

$$H(s) = \frac{1}{1 + Q^{-1}s + s^2}$$

  2. High Pass Filter

$$H(s) = \frac{s^2}{1 + Q^{-1}s + s^2}$$

3. Band Pass Filter

$$H(s) = \frac{Q^{-1}s}{1 + Q^{-1}s + s^2}$$

4. Band Stop Filter

$$H(s) = \frac{1 + s^2}{1 + Q^{-1}s + s^2}$$

... and upwards