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Hw7 1) A CFG is called right linear if all productions are of the form $A \to aB$ or $A \to e$ and called left linear if all productions are of the form $A \to Ba$ or $A \to e$ where $A, B \in V$ and $a \in T$ and e is the empty string. Show that both right linear and left linear grammars generate regular languages. Specify finite state machines corresponding respectively to right and left linear grammars.

- For showing right linear CFG generates regular language:

For every right linear CFG $G = (V, T, R, S)$ there exists a DFA $D = (Q, \Sigma, \delta, q_0, F)$ accepting the same language as G.

Right linear CFG:

$S \to aA$

$A \to bB$

$B \to c$

Constructed DFA:

$Q = \{S, A, B, q_f\}$

$\Sigma = \{a, b, c\}$

$q_0 = S \quad F = \{q_f\}$

- $\delta(S, a) = A$
- $\delta(A, b) = B$
- $\delta(B, c) = q_f$

We constructed a ~~FSM~~ DFA from given CFG. So that we conclude every right-linear CFG generates a regular language.

• For showing left linear CFG generates regular language:

Left linear CFG's also generates regular languages. But it is hard to directly show it with DFA.

Assume we have left-linear CFG:

$S \to Aa$

$A \to Bb$

$B \to \epsilon$

what does this produce?

$S \to Aa \to Bba \to \epsilon ba \to$ "ba" → to be inverse of this

$L = \{ba\}$

inverse: "ab"

→ Find right linear CFG for this:

$S \to aA$

$A \to b$

→ So we can convert to DFA

So this DFA accepts the language L^R

• if L^R is regular, then L is regular as well!

→ Since we say L is regular, left linear CFG also generates regular languages.

$\boxed{7.1.3}$ $S \to 0A0 \mid 1B1 \mid BB$

$A \to C$

$B \to S \mid A$

$C \to S \mid \epsilon$ $(C \to \epsilon)$

a) Eliminate ϵ productions

After eliminating $C \to e$:

$S \to 0A0 \mid 1B1 \mid BB$

$A \to C \mid e$ $(A \to e)$

$B \to S \mid A$

$C \to S$

After eliminating $A \to e$:

$S \to 0A0 \mid 1B1 \mid BB \mid 00$

$A \to C$

$B \to S \mid A \mid e$ $(B \to e)$

$C \to S$

loop

After eliminating $B \to e$:

$S \to 0A0 \mid 1B1 \mid BB \mid 00 \mid 11 \mid B$

$A \to C$

$B \to S \mid A$

$C \to S$

b) Eliminate any unit products in the resulting grammar:

$S \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$A \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$B \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$C \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$) useless

c) Eliminate any useless symbols in the resulting grammar.

$S \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$A \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$B \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

we removed C

we dont even achive to "C" from "S".

d) Put the resulting grammar into Chomsky Normal Form.

$S \to XK \mid XX \mid YL \mid YY \mid BB$

$A \to XK \mid XX \mid YL \mid YY \mid BB$

$B \to XK \mid XX \mid YL \mid YY \mid BB$

$X \to 0$ $\quad K \to AX$

$Y \to 1$ $\quad L \to BY$

7.1.4

$S \to AAA \mid B$

$A \to aA \mid B$

$B \to \epsilon$ → gidecek direkt

a) ① after eliminating $B \to \epsilon$:

$S \to AAA \mid B \mid \epsilon$

$A \to aA \mid B \mid \epsilon$

② after eliminating $S \to \epsilon$:

$S \to AAA \mid B$

$A \to aA \mid B \mid \epsilon$

③ After eliminating $A \to \epsilon$:

$S \to AAA \mid B \mid AA \mid A \mid \epsilon$

$A \to aA \mid B \mid a$

④ After eliminating $S \to \epsilon$

$S \to AAA \mid B \mid AA \mid A$

$A \to aA \mid B \mid a$

Just remove the epsilon

Final

b) Eliminate Unit Products

$S \to AAA \mid AA \mid aA \mid a$

$A \to aA \mid a$

We need to get rid of "B" because there is no rule for B (i.e $B \to$ ~~~)

Same

c) $S \to AAA \mid AA \mid aA \mid a$

$A \to aA \mid a$

d) Chomsky form:

$S \to AB \mid AA \mid XA \mid a$

$A \to XA \mid a$

$B \to AA$

$X \to a$

7.2.1 Use the CFL pumping lemma to show each of these languages not to be context-free:

b) $\{a^n b^n c^i \mid i \leq n\} = L$ Assume that L is context free.

By our assumption, there should be a CFG $G = (V, T, R, S)$ accepting language L.

if L is context free;
for $|z| \geq n$, $z = uvwxy$

① $|vwx| \leq n$

② $vx \neq \epsilon$

③ $\forall i \geq 0$, $uv^iwx^iy \in L$

$z = a^n b^n c^n \in L$

$|z| = 3n \geq n$ and
$uvwxy = a^n b^n c^n$

we chose this string which is inside of the language L.

By PL $|vwx| \leq n$, hence (i) $vwx = a^k$ where $k \leq n$ or,

(ii) $vwx = a^k b^m$ where $k+m \leq n$ or,

(iii) $vwx = b^k$ where $k \leq n$ or,

(iv) $vwx = b^k c^m$ where $k+m \leq n$ or,

(v) $vwx = c^k$ where $k \leq n$.

$k = n$ olduğunda "v" boş olabilir. Bu kurallara aykırı değildir. ama vx boş olamaz.

υx peşo a'lardan ol-şuyorlar onu çıkarıyoz $|\upsilon x| = p$

- if (i) holds, $0 < p = |\upsilon x| \leq k \leq n$

$\upsilon w y = a^{n-p} b^n c^n \notin L$ (because $n-p \neq n$

- if (ii) holds, $\upsilon w y = a^{n-i} b^{n-j} c^n \notin L$ (because c is more than either a or b.)

- if (iii) holds, $\upsilon w y = a^n b^{n-p} c^n \notin L$ (because a's and b's are not equal amount)

- if (iv) holds, $u\upsilon^2 w x^2 y = a^n b^{n+i} c^{n+j} \notin L$

$|\upsilon x| \leq p$

υx

$($ cc $)$ 3

$\upsilon^2 x^2$ $) +3 = p$

cc ccc $)$ 6

- if (v) holds, $u\upsilon^2 w x^2 y = a^n b^n c^{n+p} \in L$

In all cases, pumping lemma contradicts. Thus L is not context free

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7.2.1 c) $\{0^p \mid p$ is prime$\} = L$ show it's not context free.

For every $z \in L$, $|z| \geq n$; $z = u\upsilon w x y$

① $|\upsilon w x| \leq n$

② $\upsilon x \neq e$ $(|\upsilon x| > 0)$

③ $\forall i \geq 0$, $u\upsilon^i w x^i y \in L$

Let n be as in Pumping Lemma, and chose a prime number p, where $p > n$. Chose a $z = 0^p \in L$, then $|z| = p > n$, and by PL $uvwxy = 0^p$, and show that $uv^{(p+1)}wx^{(p+1)}y \notin L$

① $|vwx| \leq n$

② $|vx| = k > 0$

→ we assume these are true and trying to show:

$$uv^{(p+1)}wx^{(p+1)}y \overset{?}{\in} L$$

$$uv^{(p+1)}wx^{(p+1)}y = 0^{p(k+1)} \notin L$$

$p(k+1)$ cannot be prime.

$p+(p+1)\cdot k$

$p+pk+k$

$\boxed{i = p+1 \text{ seçilmiş}}$

$|vx| = k > 0$ = pump ettiğimiz bölümün uzunluğu: k

$uv^iwx^iy = 0^{(p+(i-1)\cdot k)} = 0^{p+pk}$

$\boxed{i = 2 \text{ olsa}}$

$uv^iwx^iy = 0^{p+k}$

önceki soru

$|vx| = p$) pump ettiğimiz bölümün uzunluğu: p

If (v) holds, $(vwx = c^n)$ i = 2 olursa

$uv^iwx^iy = c^{(n+(i-1)\cdot p)} = c^{n+p}$