

→ kalıp

**Hw 7** 1) A CFG is called right linear if all productions are of the form  $A \rightarrow aB$  or  $A \rightarrow \epsilon$  and called left linear if all productions are of the form  $A \rightarrow Ba$  or  $A \rightarrow \epsilon$  where  $A, B \in V$  and  $a \in T$  and  $\epsilon$  is the empty string.

Show that both right linear and left linear grammars generate regular languages. Specify finite state machines corresponding respectively to right and left linear grammars.

• For showing right linear CFG generates regular language:

For every right linear CFG  $G = (V, T, R, S)$  there exists a DFA  $D = (Q, \Sigma, \delta, q_0, F)$  accepting the same language as  $G$ .

Right linear CFG:

$S \rightarrow aA$

$A \rightarrow bB$

$B \rightarrow \epsilon$

Constructed DFA:

$Q = \{S, A, B, q_f\}$

$\Sigma = \{a, b, c\}$

$q_0 = S$   $F = \{q_f\}$

We constructed a FSM DFA from given CFG. So that

$\delta(S, a) = A$

$\delta(A, b) = B$

$\delta(B, c) = q_f$

We conclude every

right-linear CFG generates a regular language



For showing left linear CFG generates regular language

Left linear CFG also generates regular languages. But it is hard to directly show it with DFA.

Assume we have left-linear CFG:

$S \rightarrow Aa$   
 $A \rightarrow Bb$   
 $B \rightarrow \epsilon$

what does this produce?

$S \rightarrow Aa \rightarrow Bba \rightarrow \epsilon ba \rightarrow ba$

inverse of "ab" → "ba"

to be inverse of this

$L = \{ba\}$

Find right linear CFG for this

$S \rightarrow aA$   
 $A \rightarrow b$

so we can convert to DFA

So this DFA accepts the language  $L^R$

if  $L^R$  is regular, then  $L$  is regular as well

Since we say  $L$  is regular, left linear CFG also generates regular languages.

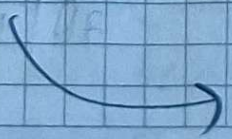
7.1.3  $S \rightarrow aAO \mid bB \mid BB$

$A \rightarrow \epsilon$

$B \rightarrow S \mid A$

$(\rightarrow S \mid \epsilon) \quad (C \rightarrow \epsilon)$

a) Eliminate  $\epsilon$  productions





A) eliminating  $C \rightarrow \epsilon$

A) eliminating  $A \rightarrow \epsilon$

$S \rightarrow 0A0 \mid 1B1 \mid BB$

$S \rightarrow 0A0 \mid 1B1 \mid BB \mid 00$

$A \rightarrow C \mid \epsilon$  ( $A \rightarrow \epsilon$ )

$A \rightarrow C$

$B \rightarrow S \mid A$

$B \rightarrow S \mid A \mid \epsilon$  ( $B \rightarrow \epsilon$ )

$C \rightarrow S$

$C \rightarrow S$

loop

A) eliminating  $B \rightarrow \epsilon$

b) eliminate any unit products in the resulting grammar

$S \rightarrow 0A0 \mid 1B1 \mid BB \mid 00 \mid 11 \mid B$

$S \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$A \rightarrow C$

$A \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$B \rightarrow S \mid A$

$B \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$C \rightarrow S$

$(C \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB)$  useless

c) eliminate any useless symbols in the resulting grammar

$S \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$A \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

$B \rightarrow 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

we removed  $C$

we don't even achieve to "C" from "S"

d) put the resulting grammar into Chomsky Normal Form.

$S \rightarrow XK \mid XX \mid YL \mid YY \mid BB$

$A \rightarrow XK \mid XX \mid YL \mid YY \mid BB$

$B \rightarrow XK \mid XX \mid YL \mid YY \mid BB$

$X \rightarrow 0 \quad K \rightarrow AX$

$Y \rightarrow 1 \quad L \rightarrow BY$



7.1.4

$$S \rightarrow AAA|B$$

$$A \rightarrow aA|B$$

$$B \rightarrow \epsilon \Rightarrow \text{delete rule}$$

a) after eliminating  $B \rightarrow \epsilon$ :

$$S \rightarrow AAA|B|\epsilon$$

$$A \rightarrow aA|B|\epsilon$$

after eliminating  $S \rightarrow \epsilon$ :

$$S \rightarrow AAA|B$$

$$A \rightarrow aA|B|\epsilon$$

After eliminating  $A \rightarrow \epsilon$ :

$$S \rightarrow AAA|B|AA|A|\epsilon$$

$$A \rightarrow aA|B|a$$

After eliminating  $S \rightarrow \epsilon$ :

$$S \rightarrow AAA|B|AA|A$$

$$A \rightarrow aA|B|a$$

Final

Just remove the epsilon.

Some

b) Eliminate Unit Product

$$S \rightarrow AAA|AA|aA|a$$

$$A \rightarrow aA|a$$

$$S \rightarrow AAA|AA|aA|a$$

$$A \rightarrow aA|a$$

We need to

get rid of "B" because

there is no rule for B

(i.e.  $B \rightarrow \dots$ )

d) (homsky form?)

$$S \rightarrow AB|AA|XA|a$$

$$A \rightarrow XA|a$$

$$B \rightarrow AA$$

$$X \rightarrow a$$



**7.2.1** Use the (FL pumping lemma to show each of these languages not to be context-free:

b)  $\{a^n b^n c^n \mid n \leq n\} = L$  Assume that  $L$  is context free.

By our assumption, there should be a CFG  $G = (V, T, R, S)$  accepting language  $L$ .

if  $L$  is context free;  
for  $|z| \geq n$ ,  $z = uvwxy$

①  $|vwx| \leq n$

②  $v \neq \epsilon$

③  $\forall i \geq 0, uv^iwx^iy \in L$  which is inside of the language  $L$ .

$z = a^n b^n c^n \in L$

$|z| = 3n \geq n$  and  
 $uvwxy = a^n b^n c^n$  ✓

→ we chose this string

By PL,  $|vwx| \leq n$ , hence (i)  $vw = a^k$  where  $k \leq n$  or,

$k = n$  ol duğunda  
"v" boylolabilir. Bu  
küratllara aykırı degillir.  
ama  $\forall x$  boylolamaz.

(ii)  $vw = a^k b^m$  where  $k+m \leq n$  or,

(iii)  $vw = b^k$  where  $k \leq n$  or,

(iv)  $vw = b^k c^m$  where  $k+m \leq n$  or,

(v)  $vw = c^k$  where  $k \leq n$ .





if (i) holds,  $0 < p = |ux| \leq k \leq n$

if  $p \leq n$  then  $0 < p \leq n$   
 or  $(k+1) \leq p \leq n$

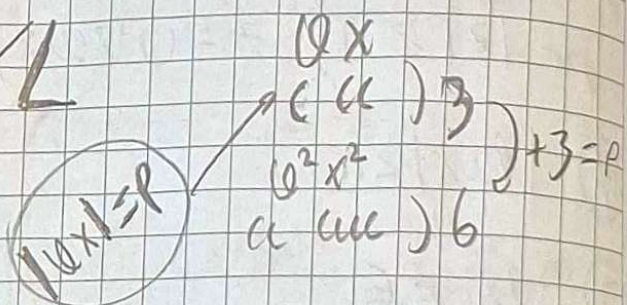
$$vwy = a^{n-p} b^n c^n \notin L \text{ (because } n-p \neq n)$$

if (ii) holds,  $vwy = a^{n-i} b^{n-j} c^n \notin L$  (because  $c$  is more than either  $a$  or  $b$ .)

if (iii) holds,  $vwy = a^n b^{n-p} c^n \notin L$  (because  $a$ 's and  $b$ 's are not equal amount)

if (iv) holds,  $uv^2wx^2y = a^n b^{n+i} c^{n+j} \notin L$

if (v) holds,  $uv^2wx^2y = a^n b^n c^{n+p} \in L$



In all cases, pumping lemma contradicts. Thus  $L$  is not context free

7.2.1 (c)  $\{a^p \mid p \text{ is prime}\} = L$  Show it's not context free.

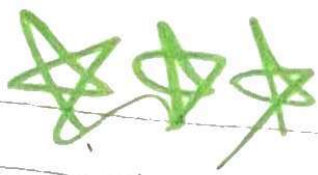
For every  $z \in L$ ,  $|z| \geq n$ ;  $z = uv^iwx^iy$

(1)  $|vwx| \leq n$

(2)  $v \neq \epsilon$  ( $|v| > 0$ )

(3)  $\forall i \geq 0, uv^iwx^iy \in L$





Let  $n$  be as in Pumping Lemma, and choose a prime number  $p$  where  $p > n$ . (Choose a  $z = 0^p \in L$ ), then  $|z| = p > n$ , and by PL  $\exists uwx y = 0^p$ , and show that  $\forall u^{(p+1)} w x^{(p+1)} y \notin L$

①  $|uwx| \leq n$

②  $|ux| = k > 0$

→ we assume these are true and trying to show:

$\forall u^{(p+1)} w x^{(p+1)} y \stackrel{?}{\in} L$

$\forall u^{(p+1)} w x^{(p+1)} y = 0^{p(p+1)} \notin L$

$p(p+1)$  cannot be prime

$p + (p+1) \cdot k$   
 $p + p + k$

$[i = p+1 \text{ seçilmiş}]$

$|ux| = k > 0$  = pump ettiğimiz bölümün uzunluğu  $k$

$\forall u^i w x^i y = 0^{(p+(i-1) \cdot k)} = \underline{\underline{0^{p+pk}}}$

$[i = 2 \text{ olsa}]$

$\forall u^i w x^i y = \underline{\underline{0^{p+k}}}$

önceki işer

$|ux| = p$

) pump ettiğimiz bölümün uzunluğu  $p$

if (V) holds,  $(uwx = c^n)$   $i = 2$  olursa

$\forall u^i w x^i y = c^{(n+(i-1) \cdot p)} = \underline{\underline{c^{n+p}}}$