

CS412Homework3 – Spring 2024/25

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Question 1: Gradient Descent (50 points)

We wish to minimize the function

$$F(x,y) = x^2 + 4x + y^2 - 4y,$$

with respect to both x and y . Starting from the initial point

$$(x_0, y_0) = (23, 27),$$

apply the steepest descent algorithm for two steps using a step size $\eta = 0.1$:

$$(x,y) \leftarrow (x,y) - \eta \nabla F(x,y).$$

(Subscripts denote iterations, e.g., (x_1, y_1) , (x_2, y_2) .)

Worksheet

1. Initial Function Value (5 pts):

$$F(23, 27) = 1242$$

2. Gradient Computation (10 pts):

$$\nabla F(x,y) = (2x+4, 2y-4)$$

3. Gradient at (23,27) (10 pts):

$$\nabla F(23,27) = (50,50)$$

4. Update to Find (x_1, y_1) (5 pts):

$$(x_1, y_1) = (23,27) - 0.1 \nabla F(23,27) = (18,22)$$

5. Function Value at (x_1, y_1) (5 pts):

$$F(x_1, y_1) = 792$$

6. Second Step (15 pts):

$$(x_2, y_2) = (x_1, y_1) - 0.1 \nabla F(x_1, y_1) = (40,40)$$

$$\text{ANSWER: } (x_2, y_2) = (14,18)$$

$$, F(x_2, y_2) = 504$$

Question 2: Naïve Bayes Classification (50 points)

Based on the PlayTennis dataset provided on the next page, solve the classification task for the instance:

$$x = [\text{Overcast}, \text{Mild}, \text{Normal}, \text{Strong}]$$

using the Naïve Bayes algorithm. Show all your probabilities and calculations explicitly and answer under the specified conditions:

From the 14 instances in the PlayTennis dataset, we first count how many times the class is “Yes” vs. “No”:

- Yes: 9 occurrences
- No: 5 occurrences
- Total: 14

$$P(\text{Yes}) = 9 / 14. \quad P(\text{No}) = 5/14.$$

a) (20 pts) Without Smoothing

Step 1: Computing Conditional Probabilities for “Yes”

1) Overcast occurs 4 times (D3, D7, D12, D13) out of 9 “Yes.”

$$P(\text{Overcast}|\text{Yes}) = 4/9$$

2) Mild occurs 4 times (D4, D10, D11, D12) out of 9 “Yes.”

$$P(\text{Mild}|\text{Yes}) = 4/9$$

3) Normal occurs 6 times (D5, D7, D9, D10, D11, D13) out of 9 “Yes.”

$$P(\text{Normal}|\text{Yes}) = 2/3$$

4) Strong occurs 3 times (D7, D11, D12) out of 9 “Yes.”

$$P(\text{Strong}|\text{Yes}) = 1/3$$

$$P(x|\text{Yes}) = 4/9 \cdot 4/9 \cdot 6/9 \cdot 3/9 = 0.0439$$

Step 2: Computing Conditional Probabilities for “No”

1) Overcast appears 0 times among the “No” rows.

$$P(\text{Overcast}|\text{No}) = 0/5 = 0$$

2) Mild appears 2 times (D8, D14) out of 5 “No.”

$$P(\text{Mild}|\text{No}) = 2/5$$

3) Normal appears 1 time (D6) out of 5 “No.”

$$P(\text{Normal}|\text{No}) = 1/5$$

4) Strong appears 3 times (D2, D6, D14) out of 5 “No.”

$$P(\text{Strong}|\text{No}) = 3/5$$

$$P(x|\text{No}) = 0$$

Step 3: Combining with Class Priors (No Smoothing)

$$P(\text{Yes}|x) \propto P(\text{Yes}) \cdot P(x|\text{Yes}) = (9/14) \cdot (0.0439)$$

$$P(\text{No}|x) = 0$$

$P(\text{Yes}|x) > P(\text{No}|x)$ so it classify as "Yes"

(b) (20 pts) With Laplace (Add-1) Smoothing

$$P(\text{Overcast}|\text{Yes}) = 5/12$$

$$P(\text{Mild}|\text{Yes}) = 5/12$$

$$P(\text{Normal}|\text{Yes}) = 7/11$$

$$P(\text{Strong}|\text{Yes}) = 4/11$$

$$P(\text{Overcast}|\text{No}) = 1/8$$

$$P(\text{Mild}|\text{No}) = 3/8$$

$$P(\text{Normal}|\text{No}) = 2/7$$

$$P(\text{Strong}|\text{No}) = 4/7$$

$$P(\text{Yes}) = 10/16 = 0.625$$

$$P(\text{No}) = 6/16 = 0.375$$

$$P(x|\text{Yes}): 0.0402$$

$$P(x|\text{No}): 0.00765$$

$$P(x|\text{Yes})P(\text{Yes}) \approx 0.0251, P(x|\text{No})P(\text{No}) \approx 0.00287.$$

The model still favors “Yes,” but not as overwhelmingly as in part (a).

(c) (10 pts) Normalized Probabilities

$$P(\text{Yes}|x) = (P(x|\text{Yes})P(\text{Yes})) / (P(x|\text{Yes})P(\text{Yes}) + P(x|\text{No})P(\text{No}))$$

$$P(\text{No}|x) = (P(x|\text{No})P(\text{No})) / (P(x|\text{Yes})P(\text{Yes}) + P(x|\text{No})P(\text{No}))$$

$$P(\text{Yes}|x) = 0.897$$

$$P(\text{No}|x) = 0.103$$

PlayTennis Dataset

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Submission Instructions

- Submission: Submit your solutions as a single PDF file by 03.04.2025 23:55. Please work within this document by filling in the provided empty lines and using the allocated space.
- File Naming: Name your submission as CS412-HW3-NameSurname.pdf
- Late Submissions: Late submissions will be accepted with a penalty of 10 points per day.