Characteristic Analysis

⇒ hyperbolic form of velocity.

$$v(\rho) = \begin{cases} V_{max} & \rho \le \rho_c \\ V_{max}\left[\frac{1}{\rho} - \frac{1}{\rho_{jam}}\right] & \rho_c \le \rho \le \rho_{jam} \\ 0 & \rho \ge \rho_{jam} \end{cases}$$

flow function for above velocity.

$$f(\rho) = \begin{cases} \rho V_{max} & \rho \le \rho_c \\ \rho V_{max}\left[1 - \frac{\rho}{\rho_{jam}}\right] & \rho_c \le \rho \le \rho_{jam} \\ 0 & \rho \ge \rho_{jam} \end{cases}$$

Burger's eqn

$$\rho_t + (f(\rho))_x = 0$$

Case 1: $\rho \le \rho_c$

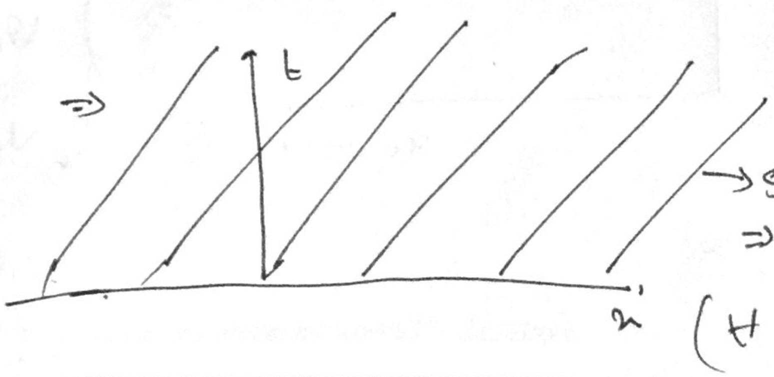
$$\rho_t + (\rho V_{max})_x = 0$$

$$\Rightarrow \rho_t + V_{max}\, \rho_x = 0$$

$$\Rightarrow \frac{\partial \rho}{\partial t} + V_{max} \frac{\partial \rho}{\partial x} = 0$$

$$\Rightarrow \frac{\partial t}{1} = \frac{\partial x}{V_{max}} = \frac{\partial \rho}{0} = C$$

$$\Rightarrow t = \frac{x - x_0}{V_{max}} \Rightarrow$$



→ slope ⇒ $1/V_{max}$ $(\forall\, x_0)$

Case 2

$$\rho_t + \left(\rho V_{max}\left[1-\frac{\rho}{\rho_{jam}}\right]\right)_x = 0 \quad ; \quad \rho(x,0) = \rho_0(x_0)$$

$$\Rightarrow \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho V_{max}\left[1-\frac{\rho}{\rho_{jam}}\right]\right) = 0$$

$$\Rightarrow \frac{\partial t}{1} = \frac{\partial x}{\rho V_{max}\left[1-\frac{\rho}{\rho_{jam}}\right]} = \frac{\partial \rho}{0} \quad \Rightarrow \rho = C$$

$$\Rightarrow \frac{\partial x}{\rho V_m\left[1-\frac{\rho}{\rho_j}\right]} = \frac{\partial t}{1}$$

C → some constant

$$\Rightarrow \frac{\partial x}{\rho_0(x_0) V_m\left[1-\frac{\rho_0(x_0)}{\rho_{jam}}\right]} = \partial t.$$

← ∵ $\rho(x,t)$ is constⁿ & $\rho(x,0) = \rho_0(x_0)$

$$\Rightarrow \boxed{\frac{x - x_0}{\phi(x_0)} = t.}$$

$$\phi(x_0) = \rho_0(x_0) V_{max}\left[1 - \frac{\rho_0(x_0)}{\rho_{jam}}\right]$$

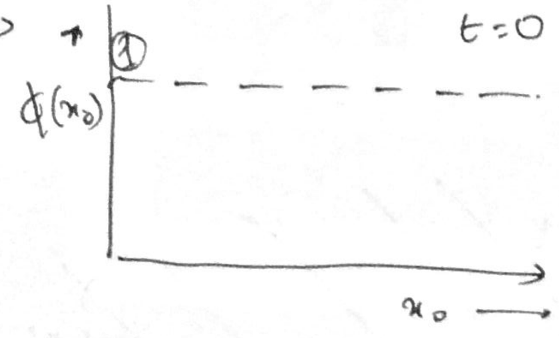
Analysing for multiple initial condition.

condition!

$$\phi(x_0) = 1$$

$$\Rightarrow \rho_0(x_0) V_{max} - \rho_0^2(x_0)\frac{V_{max}}{\rho_{jam}} - 1 = 0$$

$$\Rightarrow \rho_0(x_0) = \frac{V_{max} \pm \sqrt{V_{max}^2 - 4\frac{V_{max}}{\rho_{jam}}}}{2(V_{max}/\rho_{jam})}$$

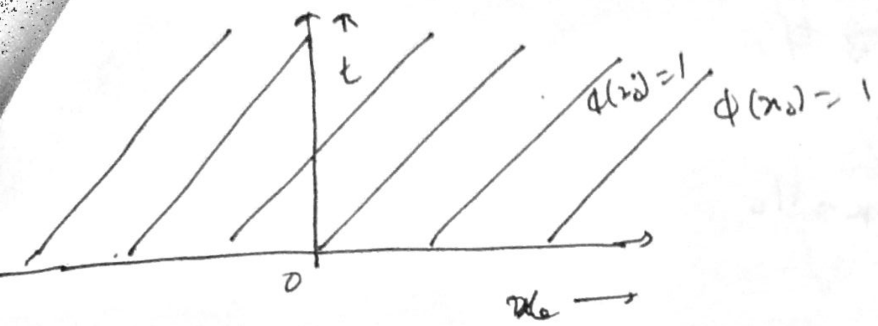


$$\Rightarrow V_{max}^2 - 4V_{max}/\rho_{jam} \geq 0$$

$$\Rightarrow \left\{\begin{array}{l} V_{max} \geq 0 \\ V_{max} \geq 4/\rho_{jam} \end{array}\right\} \text{feasible sol}^n$$

Unique solⁿ if $(V_{max} = 4/\rho_{jam}) \longrightarrow$ imp

Characteristic plot



$$\left(\phi(x,t) = 1. \quad \because \phi(x,t) = C \ \ \& \ \ \phi_0(x_0) = 1. \right)$$

Condition 2

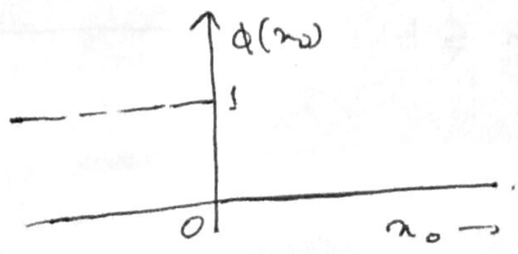
$$\phi(x_0) = \begin{cases} 1 & x < 0 \\ 0 & x \geq 0. \end{cases}$$

$$\Rightarrow \rho_0(x_0) = \begin{cases} \left. \dfrac{V_{max} \pm \sqrt{V_{max}^2 - 4\dfrac{V_{max}}{\rho_{jam}}}}{2(V_{max}/\rho_{jam})} \right\} & x < 0 \\ \left. \begin{matrix} 0 \\ \rho_{jam} \end{matrix} \right\} & x \geq 0 \end{cases}$$

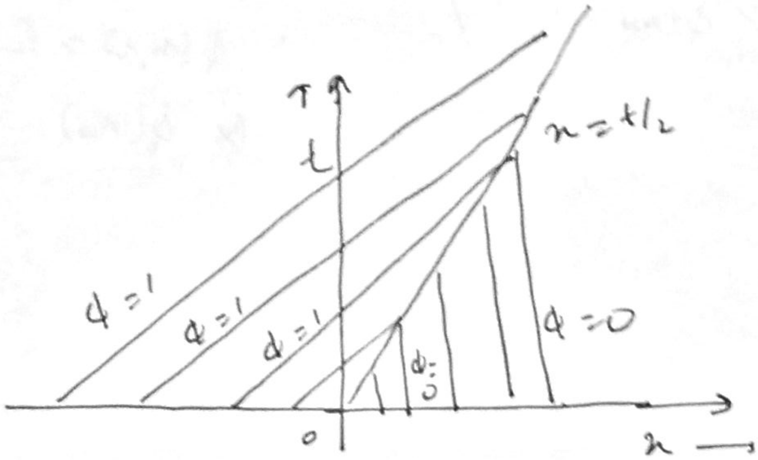
$\Rightarrow$ Assuming continious flow of traffic & $V_{max} = 4/\rho_{jam}$

$$\rho_0(x_0) = \begin{cases} \rho_{jam}/2 & x \leq 0 \\ \rho_{jam} & x \geq 0 \end{cases}$$

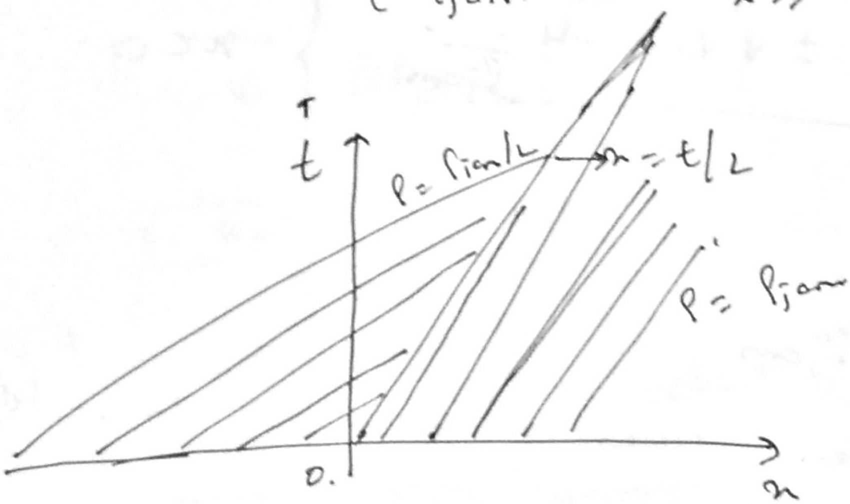
$\phi(x_0)$:



$$\phi(x,t) = \begin{cases} 1 & x < t/L \\ 0 & x \geq t/L \end{cases}$$

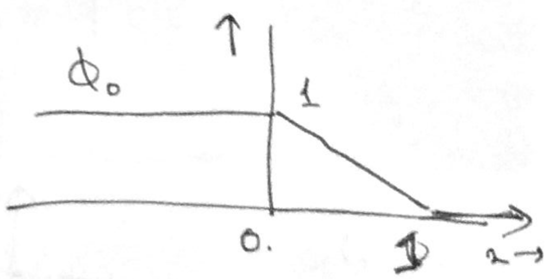


$$\Rightarrow \rho(x,t) = \begin{cases} \rho_{jam}/2 & x < t/L \\ \rho_{jam} & x \geq t/L \end{cases}$$



Condition 3:

$$\phi(x,0) = \begin{cases} 1 & x < 0 \\ 1-x & 0 \leq x < 1 \\ 0 & x \leq 1 \end{cases}$$

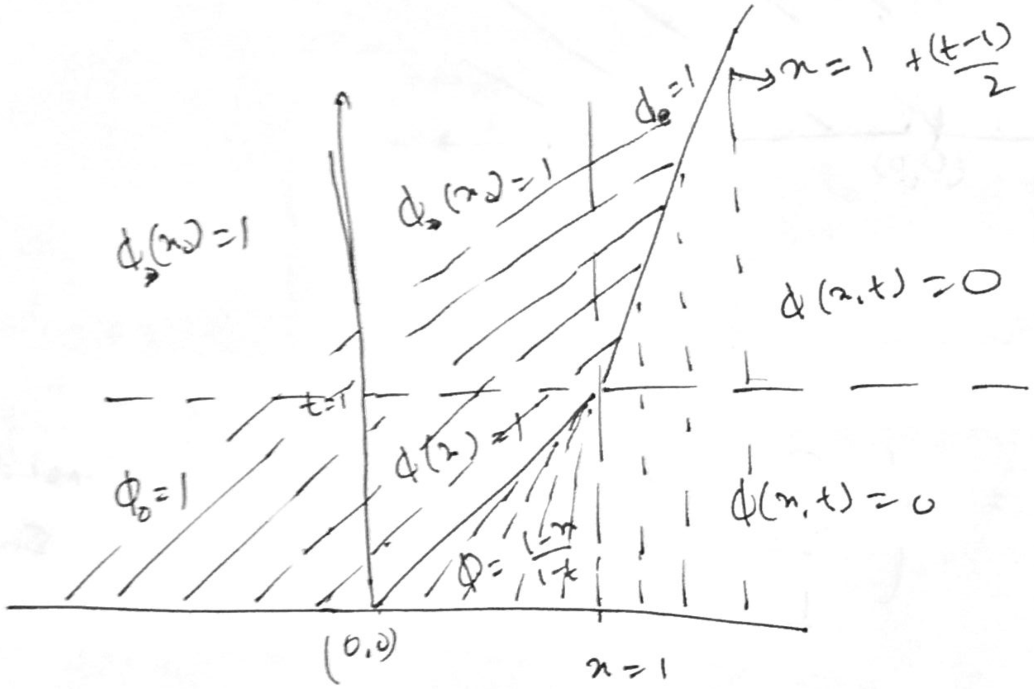


characteristic nature

For $t < 1$,
$$\phi(x,t) = \begin{cases} 1 & x < t \\ \dfrac{1-x}{1-t} & t \le x \le 1 \\ 0 & x > 1 \end{cases}$$

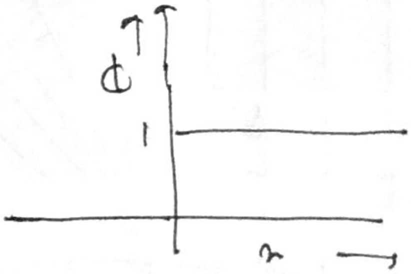
For $t \geq 1$
$$\phi(x,t) = \begin{cases} 1 & x < \tfrac{1}{2}(t-1) + 1 \\ 0 & x > \tfrac{1}{2}(t-1) + 1 \end{cases}$$

Plot



Condition ϕ:

$$\phi(x,0) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

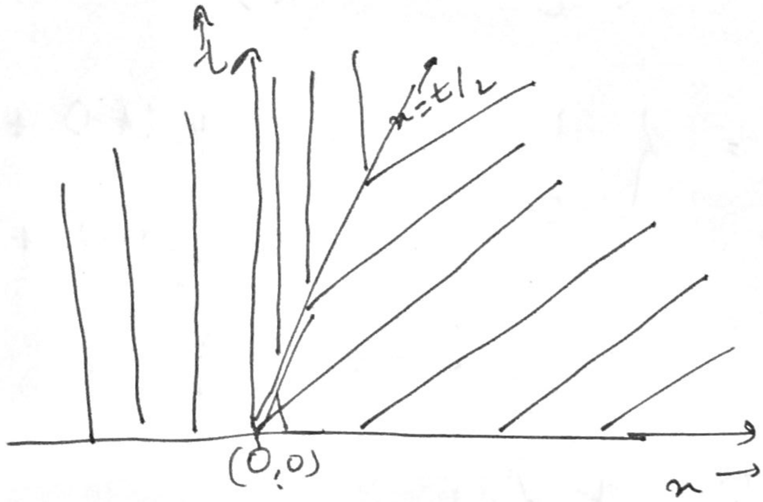


Characteristic plot

To fill missing data their exist 2 sol^n

Sol 1.

$$\phi(x,t) = \begin{cases} 0 & x < t/2 \\ 1 & x \geq t/2 \end{cases}$$



Based on RH condition

Sol 2

$$\phi(x,t) = \begin{cases} 0 & x \leq 0 \\ x/t & 0 < x < t \\ 1 & x \geq t \end{cases}$$

Based on Entropy condition