Assignment tariff plan non-life insurance |

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Chapter 1

Introduction

1.1 Binning of Spatial Data

Spatial data was represented as a pair of coordinates in the dataset. These correspond to the center of each of the Belgian regions. In this way they could be linked to the postal codes, which is a known variable for each policy holder in the dataset. This allows us to make predictions of the claim frequency and claim severity based on the region that a policy holder lives in. Then, based on these predictions the postal codes can be binned in an optimal number of factors.

To start, a model to estimate the predicted frequency needs to be constructed. A Generalized Additive Model will be used for this purpose. A base model to model spatial data is $y \sim s(long, lat, bs = "tp")$. Note that a smoother was used for longitude and latitude coordinates, which makes us able to see regions with higher expected claim frequency. If only postal codes were used, these regional effect would not be visible and all cities would be seen as independent of their location with respect to each other.

We use a Poisson-family with a log-link function which is a logical choice when modeling claim frequency in an insurance context. Also, the exposure needs to be added as an offset due to a not all policy holders being covered for a full year and due to modifications (eg. people that move change their postal code, people that buy a more powerful car). The spatial model is then extended with several other variables. By comparing the log-likelihood and the Akaiki Information Criterion (AIC) of different GAM-models, the most optimal GAM for frequency data was found: $freq \sim s(ageph) + s(long, lat) + power + cover + fleet + split + fuel + sexph + agecar$. Also, this model was constructed with a Restricted Maximum Likelihood (REML), while still applying the offset and the same family and link function.

The model was then used to predict the expected claim frequency. On a continuous scale, this would yield results presented in the figure 1.1:
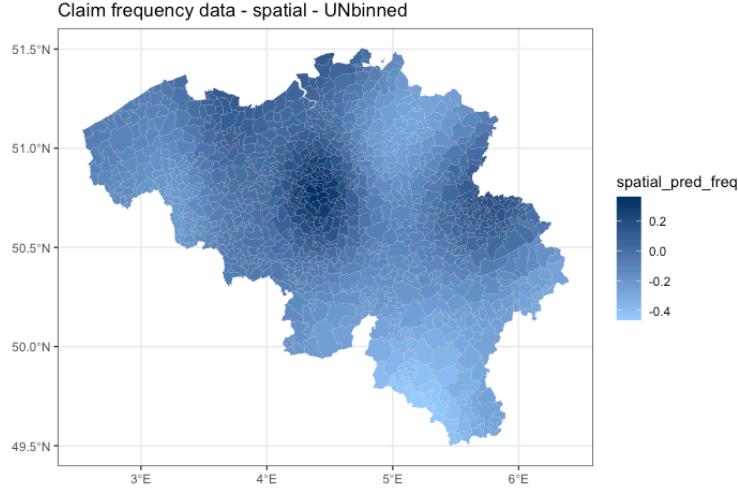
Figure 1.1: Unbinned spatial GAM

Linking each policy holder's postal code to its predicted frequency, makes us able to bin the continuous variable using Fisher's method. This method bins based on the steepness of the cumulative distribution function of the predicted values.

A follow-up problem of binning is choosing the optimal number of bins over which the observations need to be distributed. By comparing the BIC's of models with 2 to 20 bins. This yields a factor variable that approximates the continuous spatial variables. The factor variable is added to the dataset to be used in further modeling (see figure 1.2).

(a) Fisher frequency binning
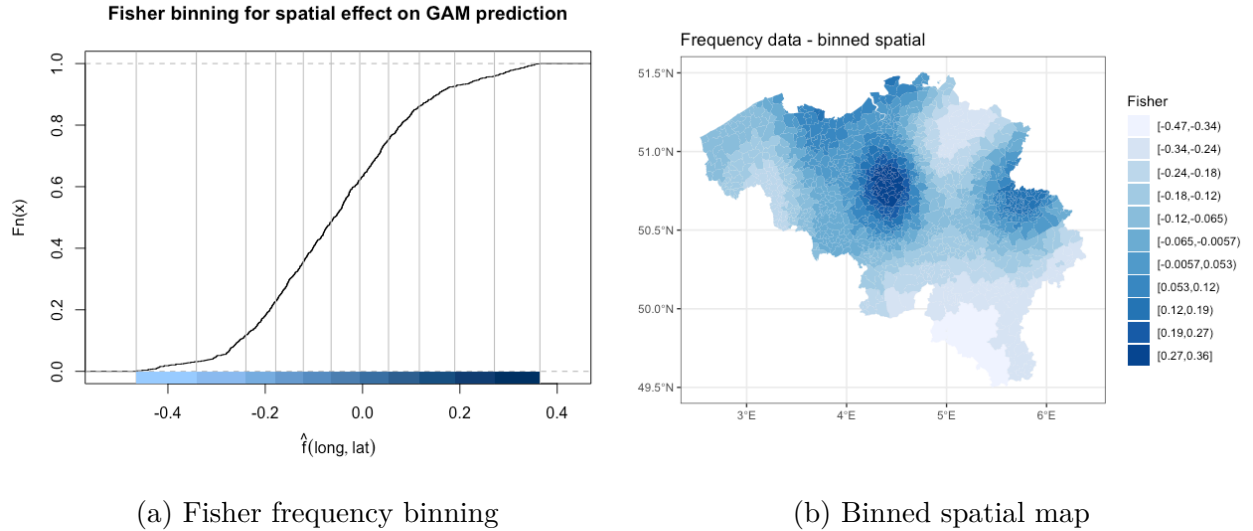
(b) Binned spatial map

Figure 1.2: Binning spatial variable

The same binned variables were used for the severity analysis for two main reasons. The first one is that the same dataset us then used for both analysis, making the process more structured. The second reason is that the binning of severity would deviate from frequency,

creating an unwanted irregularity between the two datasets.For example (mathijs jouw mooie voorbeeld)

1.2 Binning of Age Variable

In the dataset used to price our insurance products, age is recorded as a continuous variable with ages ranging from 17 to 95 years old. To optimally use a GLM, it is advised to use only categorical variables, and thus age is best converted to an ordered factor variable.

For claim frequency, the GAM-model used for the binning of spatial data was reused, but with the binned version of the spatial data as a replacement for its continuous counterpart, all other aspects identical. Thus, the used formula is: $freq \sim s(ageph) + geo + power + cover + fleet + split + fuel + sexph + agecar$, with 'geo' being the binned spatial variable. A new dataset was constructed (GAM_data) with a frequency prediction for every value of age and their observed count in the training dataset. This is possible because 'age' is not really a continuous variable, it can only take positive integer values. In this new dataset, the coefficient of the smoother of 'age' is also included. Based on GAM_data, the evtree( )-function in R can be used to construct a regression tree based on an evolutionary algorithm and with the counts per age as weights. In this function, it is useful to include an evtree.control( ), which controls the complexity of the constructed tree. 4 Parameters were used for this goal, with the first being the alpha, which was set to 100 to penalize complexity. Also, the maximum depth of the tree was set to 5 to control the size of the tree. The two last control parameters were set to control the choice wether to split or not. 'minbucket' Sets the minimum sum of weights in a terminal note, here set to 5% of the total weights in the training set. 'minsplit' Determines the needed minimum sum of weights in a node to consider a split, here set to 10% of the total weights in the training set.

The constructed tree yields the following breakpoints: 17, 26, 29, 32, 35, 38, 51, 55, 59, 63, 73, 95, which makes for 11 bins. This tree has been plotted in figure 1.3.
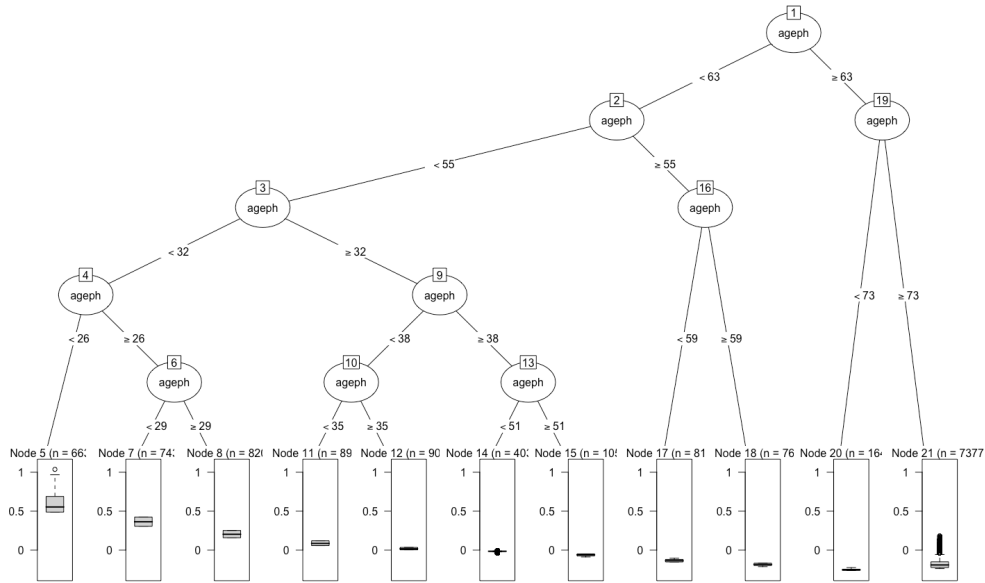

Figure 1.3: Binning age variable

Clearly, the differences in expected claim frequency per bin can be seen in the boxplots below each bin in the tree. Policy holders younger than 26yo have a higher expected claim frequency compared to for example a 55yo policy holder. The binned variable is added to the original dataset under 'agephGR'.

1.3 Exploratory Analysis

Before the technical tariff structure can be constructed, an exploratory analysis of the given data has to be done. The exploratory analysis will be split up in two parts. The first part will explore the data for the frequency information and the second part will explore the data for the severity information.

1.3.1 Frequency

To get a feel for the data some basic R-functions (str, names, head, summary) are used on the original dataset and the sampled training set. There are no noteworthy findings after running these functions. Only some variables where you may expect another type, like the variables long, lat where you could expect type long instead of chr. This should be kept in mind when using these variables. After inspecting the data, the variables 'lnexpo' and 'freq_ann' are removed from the dataset. They were considered irrelevant, because they could easily be reproduced with other variables in the dataset.

To obtain further understanding of the dataset, a grid of relative frequency plots is constructed based on the training data (figure**??**) . Some interesting points can be noted when observing these graphs. A first example was the number of claims: most policy holders report zero claims, while almost no policy holders report more than 2 claims over the exposed period. Exposure too, has an interesting shape: most people are covered for a full year. Values below one for exposure seem to be about uniformely distributed. Most policy holders have minimal coverage (MTPL) and only few have the maximum coverage offered (MTPL+++). Also, most policy holders drive weaker vehicles with less than 66 horsepower. Lastly, but probably most importantly, note the very uneven distribution for 'use', 'fleet', 'sportc'. This will have an effect on modeling discussed further into the paper.

To make sure the training dataset is a good sample for the full datasetfull dataset, the empirical frequency of all variables is compared between both. The empirical mean and variance of claim frequency are very comparable across the full and training dataset respectively. The same comparison was made for all other variables which let us to the conclusion that the sampled training set indeed is a representation of the actual dataset.
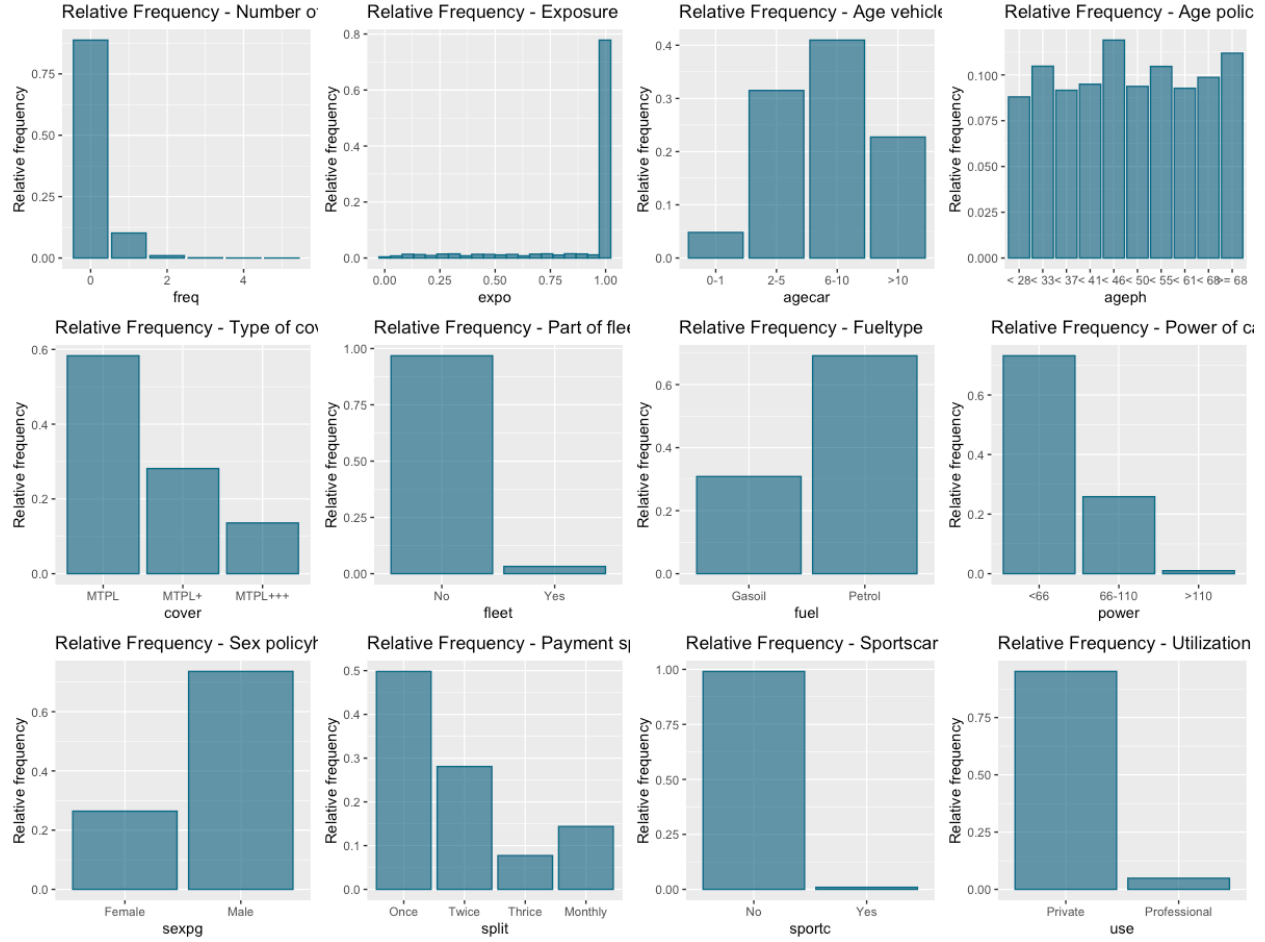
Figure 1.4: relative frequency

1.3.2 Severity

We started the exploratory analysis on severity by looking at the density of logarithmic claim amounts(see figure 1.5. From this, we can clearly see that the data is not following any well-behaved distribution and we thus opted to use the Gamma distribution as this is more versatile and flexible compared to another often used distribution for such problems: log-normal distribution.From the graph we can also deduce that there are a lot of outliers, which is expected for claim amounts as they tend to deviate from the mean. Next, the mean, variance, skewness and kurtosis are 1866.57, 19338.67, 78.84, 7701.894 respectively. This also indicated a lot of deviation on the positive side and frequent outliers as the kurtosis is substantial. This result also emphasizeq the conclusion we made from the graph. The following figure is a representation of the relative claim amounts per region where we subdivide the claim amounts per region according to the following quantiles: 0.2, 0.8 , 0.9, 0.95 and 0.99 . From the map of Belgium we can say that most of the claim amounts occur in the cities like Brussels, Antwerp and Liege. Although, there are some regions where the claim amount is also a sizable amount, for example in the Hainaut Province. This probably indicates that some outliers might be located in these regions.

(a) density of claim amount
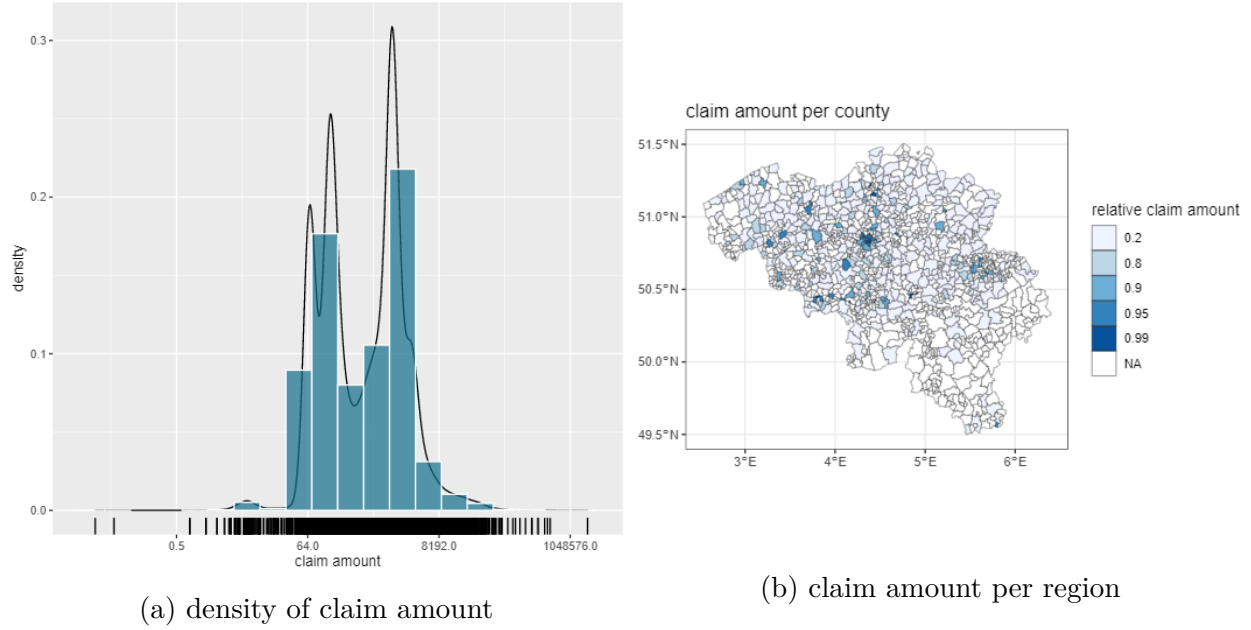
(b) claim amount per region

Figure 1.5: exploratory analysis of severity

Now that the claim amounts have been plotted according to the postal codes, we can subsequently plot the claim amounts based on the other independent variables (see figure 1.6). Here it is clearly visible that there are some variables which could be redundant and thus have less explanatory power than the others. For example the variables 'use', 'fleet' and 'sport car' are highly homogeneous and likely not able to explain a lot of the variation in our dependent variable. Also, the variables 'sexph', "fuel" and 'power' do not have a visible impact on claim amounts. On the other hand, the variables 'agecar', 'cover', 'split' and the binned version of 'ageph' create a detectable change in the claim amounts and will thus be more likely to explain our dependent variable. Note, that the outliers are primarily located in cars with six to ten years of age from males between 38 and 51 having a one time split of the premium with an MTPL cover. But we also need to be careful of this explanation as this is also the largest subgroup in the population and might as well just posses these outliers in proportion.
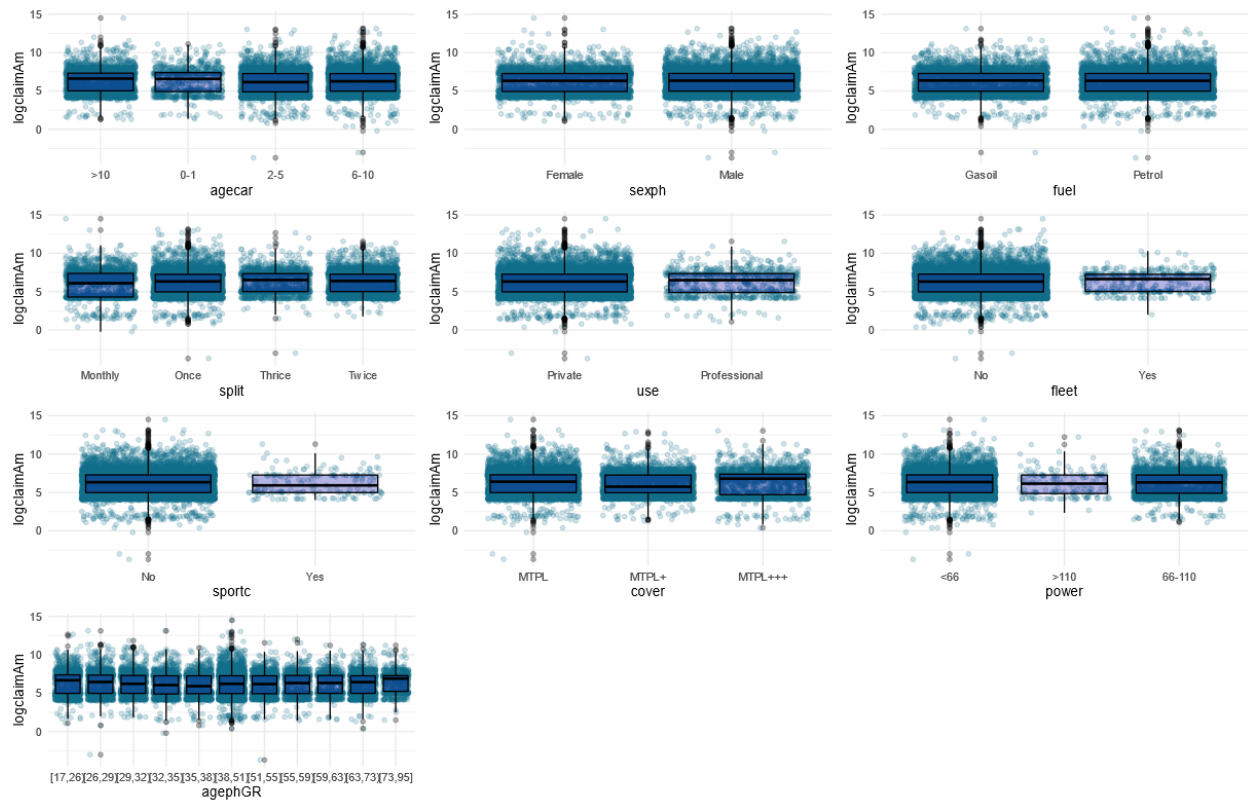
Figure 1.6: boxplots: log claim amount and factor variables

1.4 Interaction Effects

Now that we have looked at the independent variables with respect to the dependent variable we want to have a closer look at the interaction effects between the independent variables. To do this we follow the paper of Chavent et al. [2011]. Chavent et al. [2011] measured the homogeneity between variables by looking at the squared Pearson correlation r^2 and the correlation ratio η^2 between the variables. They defined the homogeneity of a cluster of variables C_k as:

$$H(C_k) = \sum_{x_j \in C_k} r^2_{x_j, y_k} + \sum_{z_j \in C_k} \eta_{y_k | z_j} = \lambda^k$$

Where x_j are the quantitative variables, z_j are the qualitative variables and y_k is the first principle component (PCA) of the cluster (or the synthetic variable of the cluster). Then, Chavent et al. [2011] devised an algorithm which chooses the partition which maximizes the homogeneity of the cluster. This is done in nested approach giving us the dendograms in figure 1.7.

The interaction between the variables is shown in the dendogram. Clearly, 'sportc' and 'power' are closely linked, which can perfectly be explained by the logic that sportscars mostly have a more powerful engine. Also the stronger correlation between the age of the car and the type of cover can be explained: most newer cars have an MTPL-coverage, while older cars tend to go for MTPL+ or MTPL+++. Next, the link between 'use' and 'fleet' can be explained by the fact that most professional-use cars are company cars which are part of a fleet. Lastly, an interaction between the age of the policyholder and the payment method ('split') is studied based on the logic that younger people prefer to split their premium payments due to them having less money to spend compared to older people (see figure 1.8. Although, looking at the graph, this relation is not observable in the data. Note that our depended variable is also included and we can already see which variables might be important in explaining the claim size. For claim amount 'ageph' , 'split' and 'geo' will probably have an significant impact.
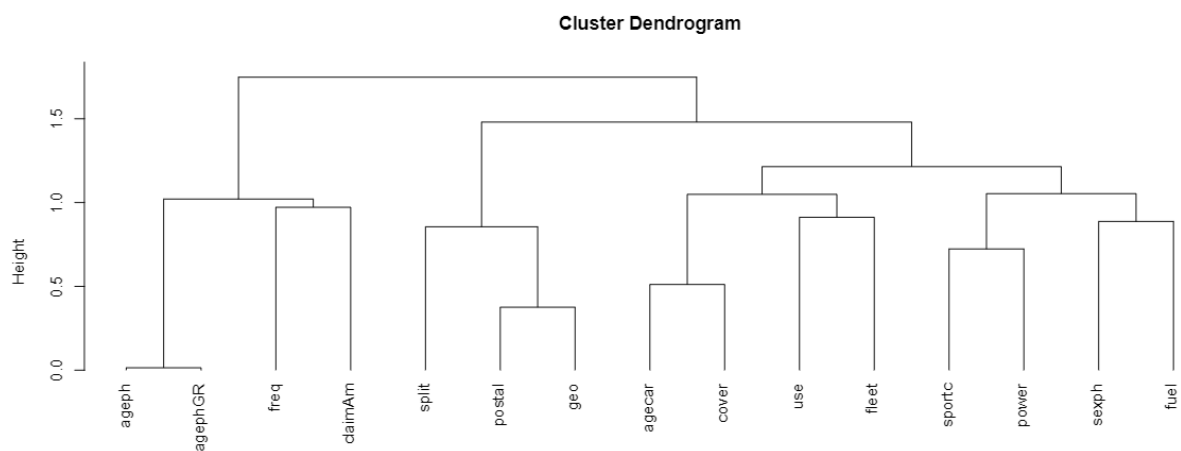
Cluster Dendrogram

Figure 1.7: dendogram: relations between variables
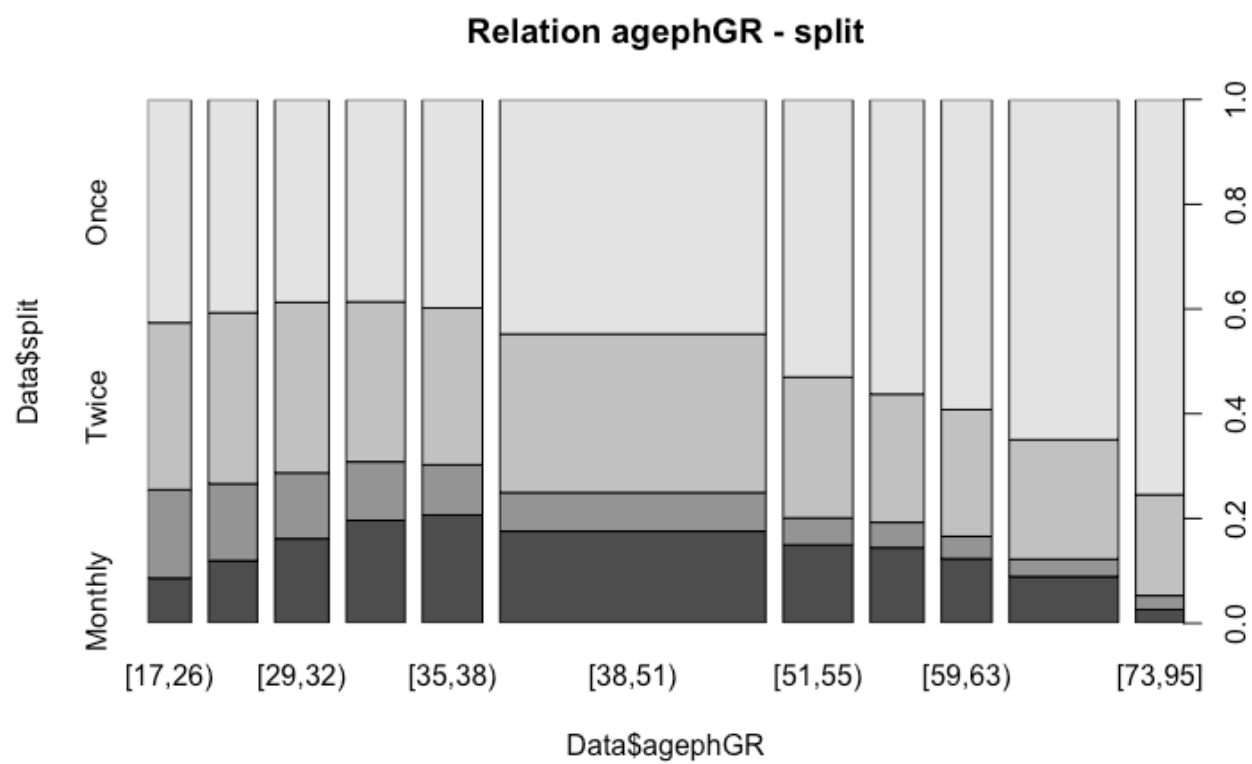
Relation agephGR - split

Figure 1.8: relation between the age of the policy holder and the split of the premium

Chapter 2

Model Building

2.1 Frequency

Frequency analysis will be done using two models. On the one hand a Generalized Linear Model and on the other hand a Gradient Boosting Machine. Both will be compared whether they have sufficient predictive power, given their complexity. The final model will be chosen based on the higher accuracy on unseen data, while maintaining a model as simple as possible. Note a GLM is much simpler compared to a GBM, but is a lot less flexible.

2.1.1 Generalized Linear Model (GLM)

A GLM was chosen because it is one of the most interpretable models of all possible models, which is very important when explainability is an important factor as it is in the insurance industry. A simple model is easy to explain to the legislator and other stakeholders who may not be an expert in this field. As already mentioned, a GLM is not the most flexible model with respect to other models. It requires linear relations and is preferably modelled with categorical independent variables. Because of this the spatial variable and the variable of the age of the policyholder were binned into two factor variables, as discussed earlier.

To find the optimal GLM, the simplest possible model was used as a starting point: freq ~ 1. Because Poisson is a good distribution for frequency data, the Poisson family was used when constructing GLM's with the logarithm of exposure as an offset to correct for non-full years of coverage. On this base case, other variables were added one by one and their drop in deviance was compared. The added variable which constituted the biggest drop in deviance, given this drop was significant compared to a Chisq. critical value, was kept. Once all singular variables were exhausted, interaction terms were considered, limited to two-way interactions. Not all possible interactions were tested, but only the logical ones and the ones based on a Dendogram (see Interaction Effects).

In the added Excel-file (tab 'GLM'), the reader can follow and compare all the added variables in each step. As an example, the first box is given in figure **??** which is used to find the first variable to add to the model. The first line is the base case (glm0), respresented by the

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formula $ freq ~ 1$, with a deviance of 72.239 and 130.926 degrees of freedom. Each line in the second box represents a different added variable. Every model has its own deviance and degrees of freedom. By comparing these with the base case, the model with the biggest drop in deviance can be selected. In this case adding 'agephGR' gives the biggest significant drop in deviance, while 'use' gives the smallest. The p-values of each added variable were calculated using the Chi-squared distribution. Following these findings, the model to proceed with is $freq \sim 1 + agephGR$, which will be used as the new base case for adding a second variable.

Deviance	DF	DELTA DEV	DELTA DF	pvalue	test	formula	
72239	130926	/	/	/		1	
71196	130916	1042,700	10	< 2.2E-16	***	1	agephGR
71793	130916	446,020	10	< 2.2E-16	***	1	geo
72230	130924	9,255	2	0,00978	**	1	power
72179	130924	60,273	2	0,00000	***	1	cover
72232	130925	7,287	1	0,00695	**	1	sportc
72213	130925	26,292	1	0,00000	***	1	fleet
72239	130925	0,355	1	0,55140		1	use
71615	130923	623,540	3	< 2.2E-16	***	1	split
72112	130925	126,990	1	< 2.2E-16	***	1	fuel
72215	130925	24,154	1	0,00000	***	1	sexph
72187	130923	52,483	3	0,00000	***	1	agecar

Figure 2.1: screenshot results GLM

Applying this method repeatedly it can be shown that the most optimal GLM, given the observed training dataset is:

$$freq \sim 1 + agephGR + geo + split + fuel + cover + power + agecar + agecar : cover$$

As a last check whether the aded variables are indeed significant and meaningful, the optimal GLM is compared to the initial base case (glm0). The output below (anova with Chi-squared test) shows a drop in deviance of 2163,7, a difference of 37 degrees of freedom which lead to a p-value small enough to make almost-certain conclusions (***). (see appendix for output optimal GLM)

2.1.2 Gradient Boosting Machine (GBM)

A second approach to modelling frequency data, was by applying a Gradient Boosting Machine. Gradient Boosting is a machine learning technique for regression and classification problems to build a predictive model as an ensemble of weak predictive models. It builds sequential models, in contrast to random forest which builds in parallel, and optimizes an arbitrary differentiable loss function Hastie et al. [2009]. This differentiable loss function is the Poisson deviance for this application, due to it being used widely when modelling claim frequency. In short, GBM combines weak learners into a single strong learner in an iterative way. By doing this, the model becomes rather flexibel compared to simpler models as a GLM. But this approach has a much lower interpretability and explainability (see figure 2.2. Just imagine having to provide every tree in the sequence to explain why a certain prediction comes about. Obviously this is much harder than just offering a Least Squares model that can be interpreted by almost anyone with basic statistical knowledge.
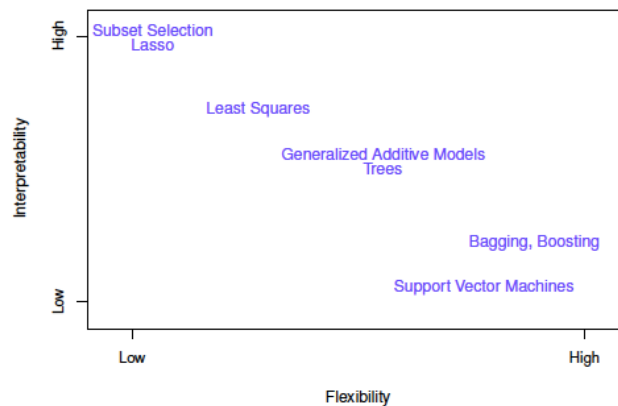

Figure 2.2: Interpretability and flexibility

Gradient Boosting has two tuning paramters which need to be optimized. On the one hand the number of sequential trees (n.trees, T) needs to be determined, and on the other hand the interaction depth (interaction.depth, d) needs to be determined. T is rather self explanatory in the sense that more trees make for a complexer, but a more flexible model. The interaction depth might need some added explanation. When d is 1, an additive model is made. When d is 2, the algorithm allows up to two-way interaction. Also, note that when d increases, the complexity of the model will also increase.

Besides tuning parameters, also two hyperparameters need a value assigned to them. Firstly a shrinkage paramter $(0 < \lambda < 1)$ needs to be assigned, which determines the learning rate or step-size reduction of each iterative tree. Assigning a higher value to this parameter will result in better performance, but will also increase the number of trees and thus increase the computational time. For this application, a value of 0,01 is used based on the finding of Henckaerts et al. [2020] . The second hyperparameter also is based on the finding of this same paper, namely the bag fraction $(0 < \delta < 1)$, which gets the value 0,75 assigned to it. This means that in each iteration 75% of the training set is randomly selected and used to determine the following step. By introducing randomness, it is usefull to set a seed in R so that the results can be reproduced. Another choice that has to be made before modelling is the number of folds in the cross-validation (cv) of the GBM. Tests have been conducted with cv set to 5 and 10, but the added computational cost of a 10-fold cross-validation was not worth the marginal gain in accuracy. Because of this reason a 5-fold CV was deemed to be sufficient. Lastly, there was opted to set the minimum amount of observations in a terminal node equal to 10.000 to set a certain constrain on the size and complexity of the model.

The results for different values of 'interaction.depth' and 'n.trees' were conventiently summarized in figure 2.3a. To optimize the numbers of iterative trees two methods were used. First the out-of-bag estimate was calculated and secondly cross-validation was used. Although the OOB-method gives a warning of underestimation in R, there was opted to proceed using the findings of this method.

Note in the table that an increase of the interaction depth from d = 2 to d = 3 does not significantly decrease the training error, while this change increases the complexity by quite a bit. The training error is calculated on the last iteration in the model, being the value of n.trees. Comparing the values of OOB n.trees for d = 2 and cv = 5, yields an optimal GBM with d = 2 and n.trees = 681 as shown in figure 2.3b of the Poisson deviance and in figure 2.4 the OOB changes in this deviance.

Using the optimal Gradient Boosted model (gbm_perf), partial dependence plots can be constructed. These depict the functional relationship between an input variable and the predictions and they show how predictions depend on these inputs individually. In this way the relevance of different input varaibles can be determined. The PDP's of all individual variables of the optimal GBM are tabulated below. From figures 2.6, ref{fig: VI GBM} and 2.7, it is clear that 'geo', 'ageph', 'agecar', 'sexph', 'fuel', 'split', 'cover' and 'power' have a significant influence on the predictions made. In contrast, 'use', 'fleet' and 'sportc' do not have this important role when predicting claim frequency. Although a certain comment has to be made with respect to these last three variables. When looking at the relative frequency of these three variables, it is clear that there were only very few observations of sportscars,

Interaction Depth	Cross Folds	n.trees	train.error	OOB n.trees	CV n.trees
2	5	500	0.7515610	500	500
		1000	0.7505098	681	1000
		1500	0.7502090	692	1479
		2000	0.7500273	718	2000
	10	500	0.7515611	500	500
		1000	0.7504996	658	1000
		1500	0.7501954	672	1500
		2000	0.7500185	676	1997
3	5	500	0.7510068	500	500
		1000	0.7501817	553	1000
		1500	0.7498289	559	1500
		2000	0.7496027	560	1922
	10	500	0.7510112	500	500
		1000	0.7501880	538	1000
		1500	0.7498340	550	1500
		2000	0.7496349	564	1960

(a) Results GBM
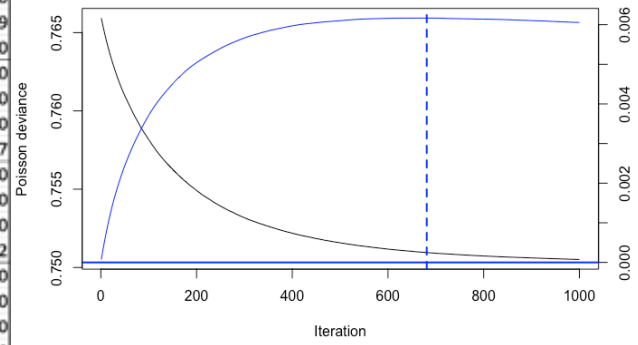
(b) Poisson deviance

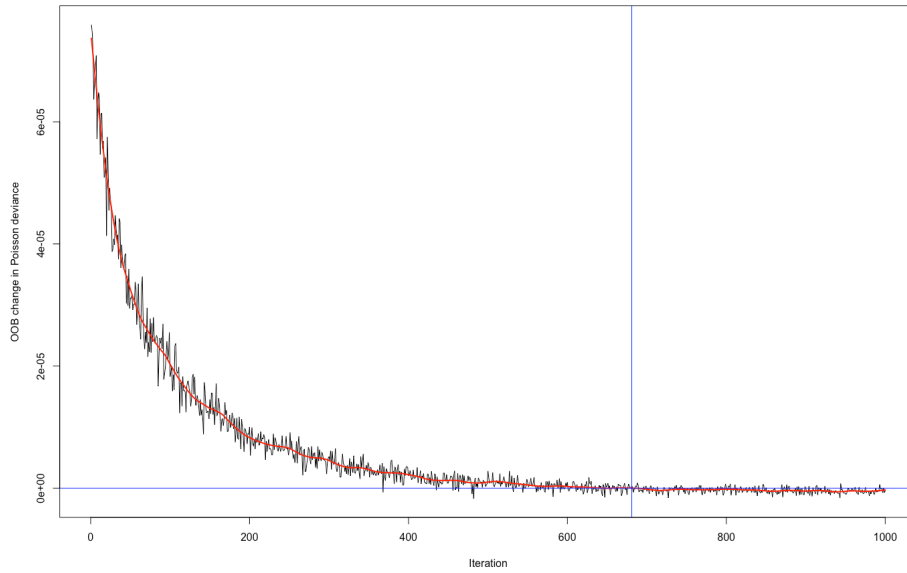Figure 2.3: The results and deviance

Figure 2.4: OOB chnages in poisson deviance

professional use cars and cars that were part of a fleet. This could be a (partial) explanation of the findings of an insignificant influence.
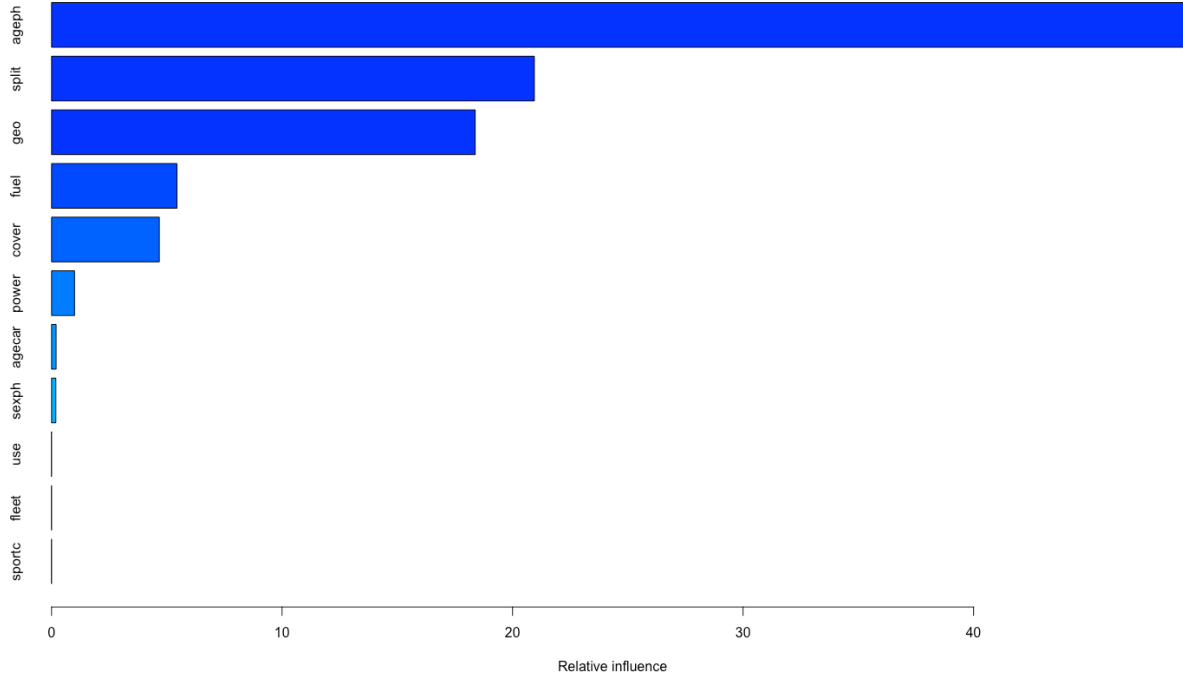
Figure 2.5: variable importance

2.1.3 Conclusion

Two models were contructed and optimized, a GLM on the one hand and a GBM on the other hand. Both of these models serve a predictive models for claim frequency, and both yield satisfactory results. Although, one of both has to be chosen. This choice will be based on predictive power and complexity of the model. In this stage the test set will be put to use, which is the dataset with unseen data for both models. Based on the predictive power on this set, the right model will be chosen. Predictive power is measured with the Root Mean Squared Error (RMSE) and the calculations are shown below.

$$RMSE = \sqrt{\frac{1}{n}\sum(\hat{y}_i - y_i)^2}$$

model	training set	test set
Generalized Linear Model	0,1331316	0,1319949
Gradient Boosting Model	0,1333529	0,1320059

Table 2.1: Root Mean Square Error

Notice that the RMSE of the test set is comparable for both models with only marginal differences. With these findings in mind, there can be concluded that the optimal GLM is the model to use. This because the RMSE's are so close to each other that the added
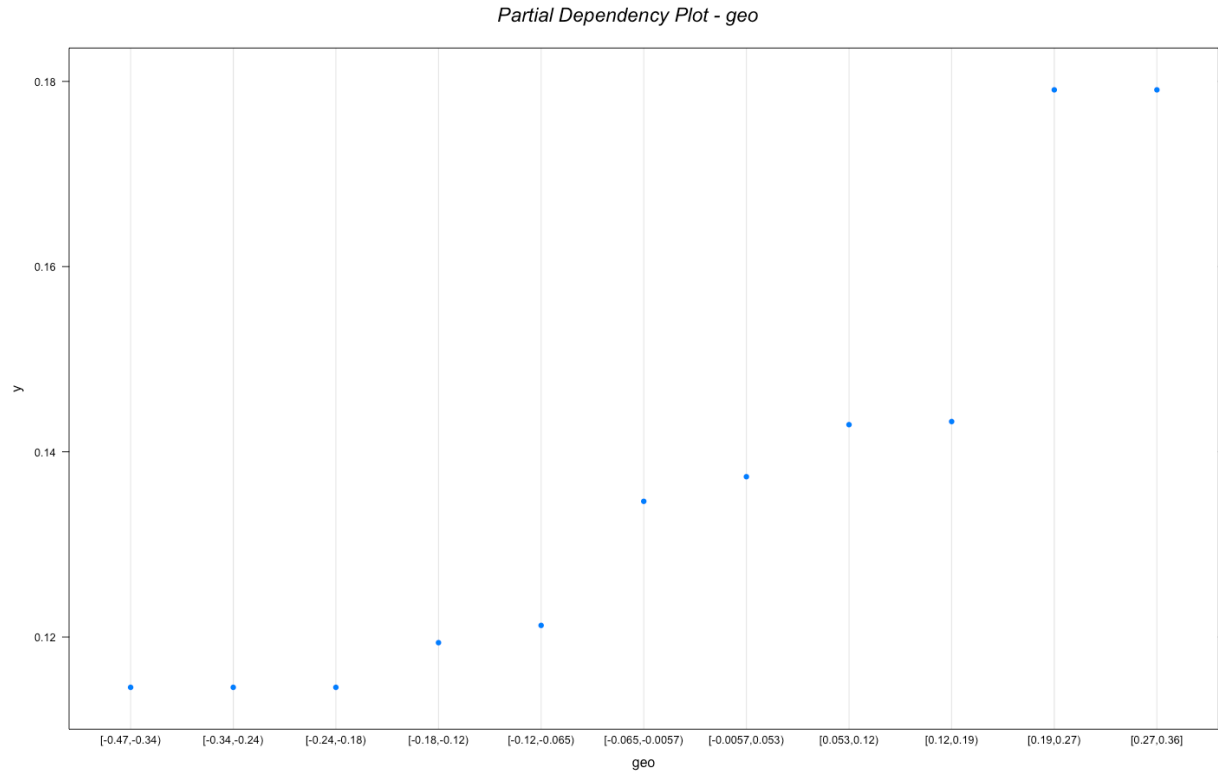
Figure 2.6: Partial dependency plot 'geo'
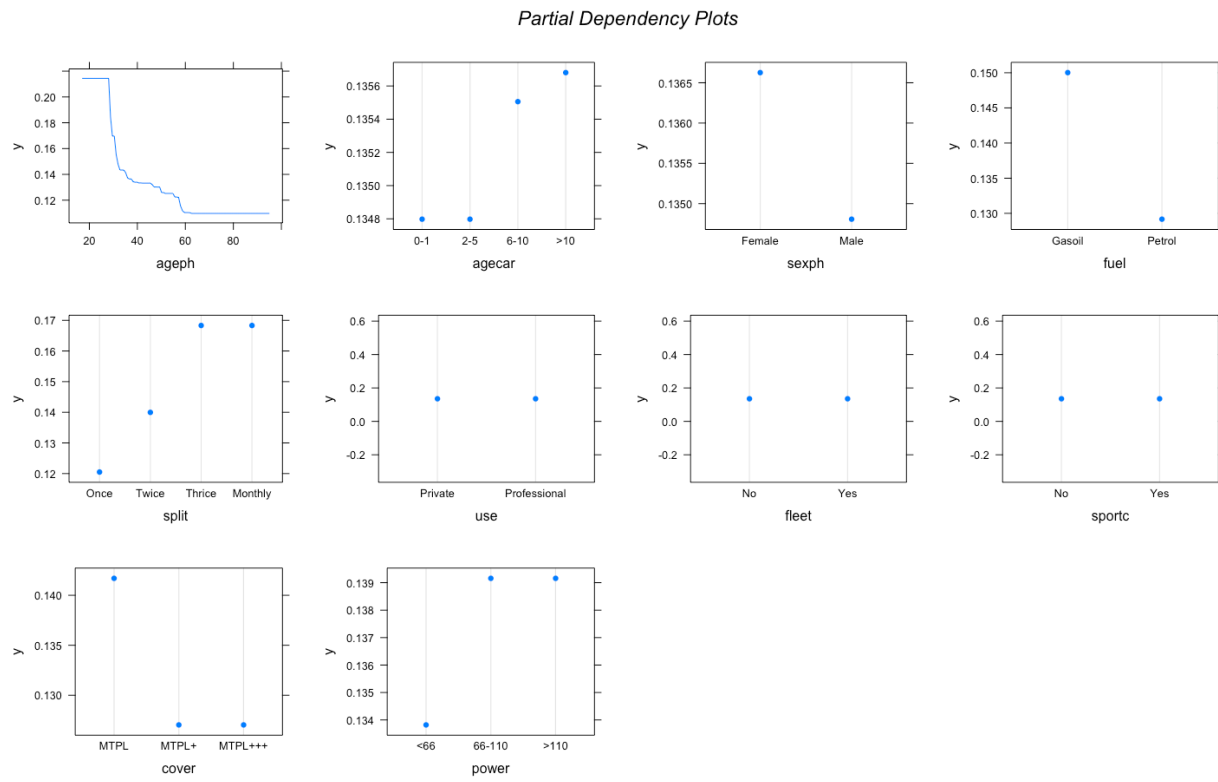
Partial Dependency Plots

Figure 2.7: pdp other variables

complexity of the optimal GBM just is not worth it. As a conclusion of frequency modeling, to final model will be repeated below:

model	Generalized Linear Model
family	Poisson
offset	log(exposure)
formula	$freq \sim 1 + agephGR + geo + split + fuel + cover +$ $power + agecar + agecar : cover$

Table 2.2: chosen model

2.2 Severity

For modeling the severity, we opted for a GLM and a Random Forest. This chose was based on the fact that GLMs are still the most interpretable models currently available and thus a good way to give meaning to the results. The Random Forest was chosen because they require no fine tuning like GBM and are thus less susceptible to overfitting, which in the severity case is better to avoid.

2.2.1 Generlized Linear Model

The most important aspect of fitting the correct model with a GLM is including the right covariates. To do this we follow Calcagno and de Mazancourt [2010], which have build an R-package for variable selection in GLM models. The package of Calcagno and de Mazancourt [2010] is a wrapper around the GLM function. It does variable selection in two ways: fitting all models and a genetic algorithm approach. In our case the first option is used. As the name of the method explains we will fit all possible models and then select the best model based on an information criteria. We will only allow the interaction effect seen in dendogram at height one (see exploratory analysis), so that the comparison between models is manageable. two exception have been made: the interaction effect of 'group_ageph'[1] with 'geo' and 'group_ageph' with 'split' as these are more closely related to the dependent variable in the dendogram. This gives us a total of 8125 possible models to compare with. Two information criteria will be used to select the best model: Aikake Information Criteria (AIC) and Bayesian Information Criteria (BIC). The Gamma distribution with a log link is chosen for modeling the claim amount in the glm. The AIC and BIC are also calculated with the loglikelihood of the gamma distribution as we model the GLM with the Gamma distribution. The results of the variable selection procedure can be found in the following graphs and will be discussed in the nect paragraphs.

In figure 2.8 the two Information Ciriteria (IC) profiles are plotted of each model. The horizontal line is according to Calcagno and de Mazancourt [2010] a rule of thumb to indicate which models are worth considering as optimal model. For the AIC there are three potential models, while for the BIC there is only one optimal model.

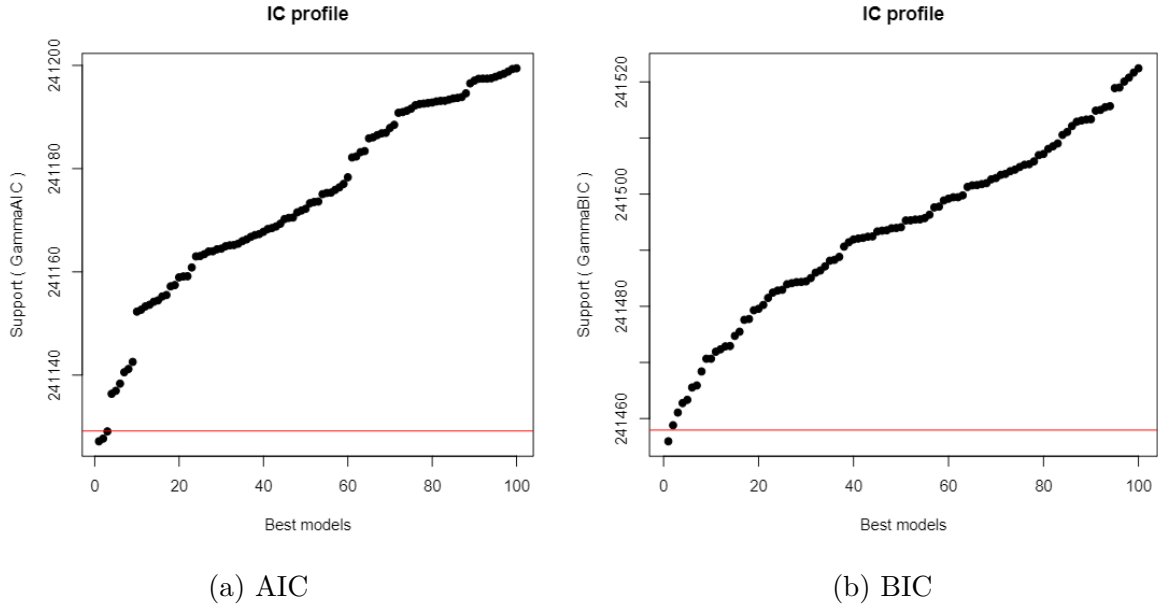[1]'group_ageph' and 'agephGR' are the same variable

(a) AIC

(b) BIC

Figure 2.8: Best 100 models of 8125 possible models

Figure 2.9 plots the Ranked Relative Evidence Weight (RREW) of the models for the two IC: $exp(\frac{-\Delta IC}{2})$. Where ΔIC is the difference between a model and the best model. The RREW is subsequently normalized and then plotted. The interpretation of the RREW is that they are the probability of a model for being the optimal model in the set. The red vertical line indicates the 95% quantile of the cumulated evidence weight. For the AIC there are three possible models which could be the optimal model. Similarly, for the BIC there are also three possible models. Although the last model can be neglectable.

Figure 2.10 shows estimated importance of each variable, computed as the sum of the relative evidence weights of all models in which the variable appears. It is noticeable that the BIC selects fewer variables than the AIC, but by doing so it disregards important variables like 'group_age'. This might indicate that AIC is the preferable IC for our analysis.

(a) AIC   (b) BIC

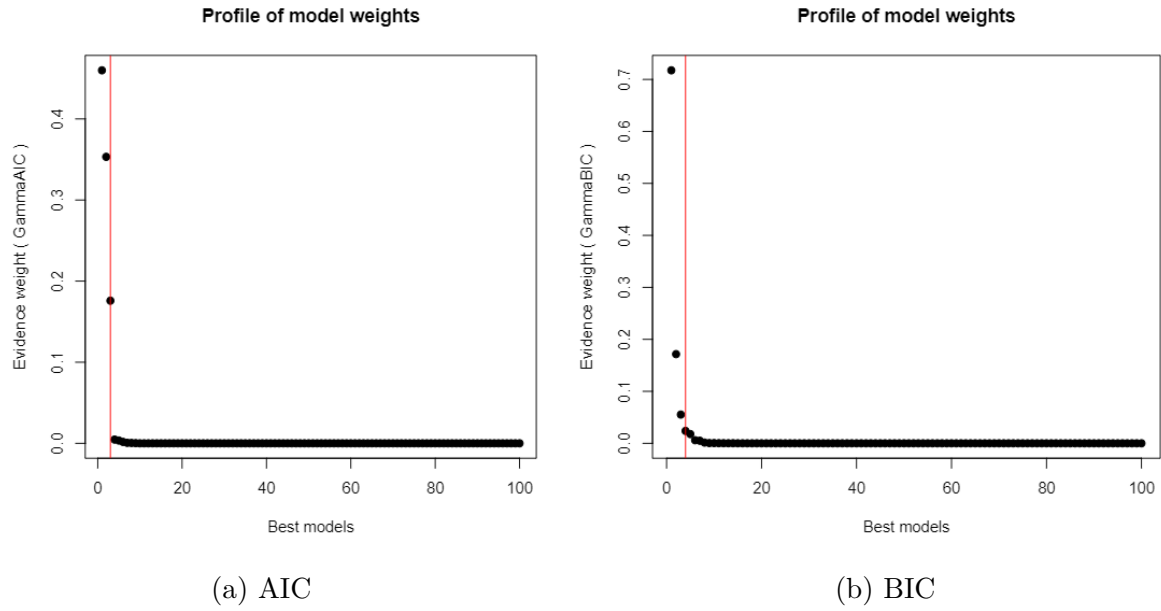Figure 2.9: Ranked Relative Evidence Weight (RREW)

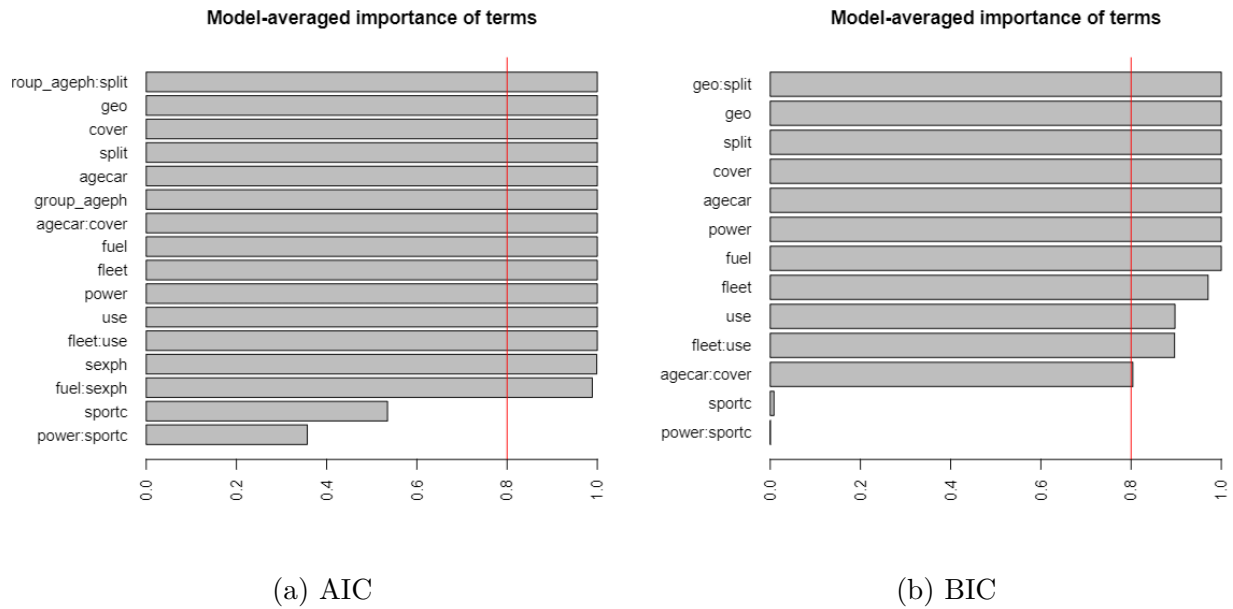(a) AIC   (b) BIC

Figure 2.10: Variable importance (VI)

The following optimal models based on the variables selection are :

Aikake Information Criteria:

1. $claimAm \sim groupageph + agecar + sexph + fuel + split + use + fleet + cover + power + geo + fuel : sexph + split : groupageph + fleet : use + cover : agecar$

2. $$claimAm \sim groupageph + agecar + sexph + fuel + split + use + fleet$$
$$+sportc + cover + power + geo + fuel : sexph + split : groupageph+$$
$$fleet : use + cover : agecar + power : sportc$$

3. $$claimAm \sim groupageph + agecar + sexph + fuel + split + use + fleet+$$
$$sportc + cover + + power + geo + fuel : sexph + split : groupageph + fleet : use+$$
$$cover : agecar$$

Bayesian Information Criteria:

1. $$claimAm \sim agecar + fuel + split + use + fleet + cover + power + geo + fleet : use+$$
$$cover : agecar + geo : split$$

The final model we chose was the first model of AIC without the interaction effect 'sexph:fuel'. We did not include the interactions effects because it did not make any practical sense and it was also not important in the BIC analysis.

2.2.2 Random Forest

The last model used is the Random Forest (RF) model. The general concept of RF is that you build a substantial amount of base learners (here gamma regression trees) on a bootstrapped sample and that you combine their predictions. This approach helps avoid overfitting and it reduces variance. The dominant variables in the dataset can cause d tree correlation. Therefore, the RF only selects from a subset of variables at each split and by doing so it avoids the tree correlation and further decreases the variance.

The Random Forest is implemented with the package distRforest, which is based on the rpart package. The distRforest builds a gamma regression forest with the following model specification. The model is build with 500 trees, five randomly chosen variable candidates at each split, a maximum depth of five nodes for each tree, a splitting criteria of twenty observation in the existing node and each terminal node needs to have at least twenty observations. All the trees use a subsample of 80% to construct the gamma regression. The formula used, is $claimAm \sim ageph + agecar + sexph + fuel + split + use + fleet + sportc + cover + power + geo$. Note that the continuous counterpart of the binned 'ageph' is implemented. in figure 2.11 the out of bag (OOB) errors and the variable importance are plotted. from the fig we can infer that the errors quickly stabilize after 100 trees and that the most important variables are 'ageph', 'split' and 'agecar'. Unfortunately the partial differential plots could not be made from RF as the distRforest package is not yet compatible with the pdp package in R.
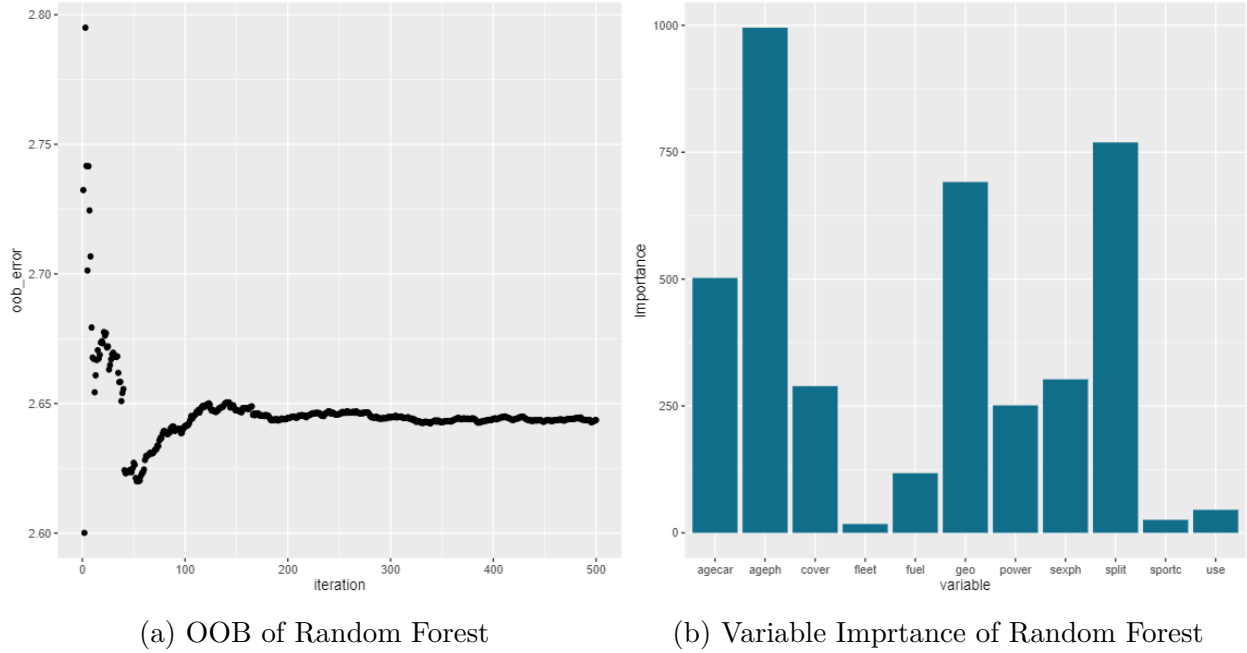
(a) OOB of Random Forest

(b) Variable Imprtance of Random Forest

Figure 2.11: analysis Random Forest

2.2.3 Conclusion

To decide which model to chose from, Random Forest or Generlized Linear Model, we predict both models on our test and train set and measure the predictive power of both models using the Root Mean Square Error:

$$RMSE = \sqrt{\frac{1}{n} \sum (\hat{y}_i - y_i)^2}$$

model	training set	test set
Generalized Linear Model	18326.9	13551.58
Random Forest	17974.3	13581.56

Table 2.3: Root Mean Square Error

From these results we can conclude that the RF preforms better on the train set than on the test set if we compare it to the GLM. We thus opted for the GLM as our final model:

model	Generalized Linear Model
family	Gamma
formula	$claimAm \sim groupageph + agecar + sexph + fuel + split + use + fleet$ $+cover + power + geo + fuel : sexph + split : groupageph+$ $fleet : use + cover : agecar$

Table 2.4: chosen model

Chapter 3

Loadings

The risk premium of a policy can be split up in two main parts, a pure premium and a risk loading. The pure premium which is calculated in the previous part, is used to pay the future losses of the policy. The risk loading which will be discussed in this part, has the purpose of covering excess future losses that are not covered by the pure premium. Yang et al. [2020] The most used approach for calculating the risk premium is by separately analysing the pure premium and the risk loading. Traditionally generalized linear models or GLMs are used for this analysis. Risk loadings can be derived in this traditional way by using the expected value premium principle or standard deviation premium principle. (Yang et al., 2020)Yang et al. [2020]

Expected value premium principle:

$$H(Y_i) = E(Y_i) + \phi E(Y_i)$$

Standard deviation premium principle:

$$H(Y_i) = E(Y_i) + \phi\sqrt{VAR(Y_i)}$$

GLMs are considered the industry standard Baione and Biancalana [2019] although there are some downsides. Traditional regression models often have to rely on assumptions Kudryavtsev [2009]. According to Kudryavtsev the following problems can occur with GLMS. An Inaccurate estimate of loss distribution may occur. This estimation of the loss distribution could be very different from the real one. It could be difficult to give larger weights to extreme values, thus making it difficult to work with loss distributions that have heavy tails. Working with a number of outliers in the sample and dependence structure of the data also could cause problems.

From the previous part there can be concluded that traditional regression methods like GLMs are not always the ideal match for real word situations. Therefore there should be looked at (to?) some alternative approaches Kudryavtsev [2009]. Kudryavtsev and Yang both propose a quantile regression method. (maybe state advantages of Kudryavstev paper) But because GLMs are well known and frequently used in actuarial sciences this will be used

for calculation of the safety loading Baione and Biancalana [2019]. Keeping in mind the criticism on these GLMs.

To calculate the risk loading the risk loading parameters need to be determined. In practice the risk loading parameters need to be determined in advance. This can be done in several ways. This can be done in a top-down way as proposed by Buhlmann. Were premiums are related to stability criterion of portfolio risks and dividend requirements for the capital invested into the insurance operation Frees et al. [2013]. The risk loading parameters can also be determined by a down-top-down. Here they first calculate the risk premium for each policy with the expected value premium principle at the individual level. They then obtain the total risk premium by putting together these risk premiums. The risk loading parameter can then be determined by making sure that the total of al risk premiums is big enough to cover the total expected losses Yang et al. [2020].

The top down approach for calculating the risk loading parameters, suggested by Yang in the paper "Risk loadings in classification ratemaking" seems to be the best method. First the risk premium of the entire portfolio will be calculated by applying the bootstrap method. This enables the possibility to get the total risk premium at the collective level. The risk premiums of different tariff classes can then be calculated by GLMs or quantile regressions. Like stated earlier this discussion will only focus on GLMs.

For the calculation of the risk premium the expected value premium principle will be used. This was chosen because it is easy to understand and calculate and often used in actuarial practice Heras et al. [2018]. Before the expected value premium principle can be used the risk loading parameter phi has to be calculated. To calculate phi the earlier mentioned top down approach suggest by Yang will be used. First a predictive distribution of the future losses of the policy will be calculated by using a bootstrap with 2500 repetitions. Yang suggests to use 10000 repetitions this was not possible with the used PC. The bootstrap was run over the pure premium which was calculated in the previous parts. These pure premiums were calculated with a two-part GLM regression. The 99.5% quantile of the predictive distribution of the bootstrap equals 36 214 173. The 99.5% quantile was used, because this is conform the Solvency II regulation Yang et al. [2020]. This 99.5% quantile and the sum of all pure premiums calculated in the previous part can then be used to calculate ϕ .

$$\phi = \frac{C - \sum E(Y_i)}{\sum E(Y_i)}$$

ϕ can then be filled in in the expected value premium principle, so that the risk loading and total risk premium can be calculated for each individual policy.

Chapter 4

Appendix

4.1 Output Optimal GLM -Frequency -

	Est.	S.E.	z val.	p
(Intercept)	-1.63	0.04	-44.30	0.00
agephGR.L	-0.77	0.03	-24.05	0.00
agephGR.Q	0.27	0.03	8.83	0.00
agephGR.C	-0.08	0.03	-2.36	0.02
agephGR^4	0.11	0.03	3.83	0.00
agephGR^5	0.06	0.03	1.91	0.06
agephGR^6	0.02	0.03	0.87	0.39
agephGR^7	-0.02	0.03	-0.72	0.47
agephGR^8	-0.02	0.03	-0.80	0.42
agephGR^9	-0.06	0.03	-1.92	0.06
agephGR^10	-0.02	0.03	-0.64	0.52
geo.L	0.59	0.07	8.69	0.00
geo.Q	0.08	0.07	1.13	0.26
geo.C	0.08	0.06	1.33	0.18
geo^4	0.19	0.05	3.43	0.00
geo^5	0.08	0.05	1.77	0.08
geo^6	0.10	0.04	2.38	0.02
geo^7	-0.04	0.03	-1.28	0.20
geo^8	0.02	0.03	0.79	0.43
geo^9	-0.01	0.03	-0.50	0.62
geo^10	-0.04	0.02	-2.12	0.03
split.L	0.27	0.02	15.53	0.00
split.Q	-0.12	0.02	-6.61	0.00
split.C	-0.07	0.02	-3.57	0.00
fuelPetrol	-0.20	0.02	-11.68	0.00
coverMTPL+	-0.10	0.03	-3.43	0.00
coverMTPL+++	-0.01	0.04	-0.34	0.73
power.L	0.20	0.05	3.66	0.00
power.Q	0.04	0.03	1.11	0.27
agecar.L	0.07	0.05	1.54	0.12
agecar.Q	-0.00	0.04	-0.09	0.93
agecar.C	-0.02	0.02	-0.95	0.34
coverMTPL+:agecar.L	-0.28	0.08	-3.64	0.00
coverMTPL+++:agecar.L	0.01	0.11	0.13	0.90
coverMTPL+:agecar.Q	0.10	0.06	1.58	0.12
coverMTPL+++:agecar.Q	0.35	0.09	4.13	0.00
coverMTPL+:agecar.C	-0.02	0.04	-0.48	0.63
coverMTPL+++:agecar.C	-0.05	0.06	-0.92	0.36

```
MODEL INFO:
Observations: 130927
Dependent Variable: freq
Type: Generalized linear model
  Family: poisson
  Link function: log

MODEL FIT:
χ²(37) = 2163.67, p = 0.00
Pseudo-R² (Cragg-Uhler) = 0.03
Pseudo-R² (McFadden) = 0.02
AIC = 100517.55, BIC = 100889.78

Standard errors: MLE
```

```
> anova(glm0, glm_opt, test = "Chisq") # ***, p-value: < 2.2e-16
Analysis of Deviance Table

Model 1: freq ~ 1
Model 2: freq ~ 1 + agephGR + geo + split + fuel + cover + power + agecar +
    agecar:cover
  Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
1    130926      72239
2    130889      70075 37   2163.7 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

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