

# On the Combination of Merton and Heston Models in the Theory of Option Pricing

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**Abstract** – This paper presents the combination of Heston and Merton model in the theory of option pricing. Merton model is one of the modern pricing models that allow discontinuous trajectories of the underlying prices of the asset. Heston model is one of the most widely used stochastic volatility models. Its attractiveness lies in the powerful duality of its tractability and robustness relative to other stochastic volatility models. We consider Bates model as the combination of the Merton and Heston models.

**Keywords** – Bates Model, Black-Scholes Model, European Option, Heston Model, Merton Model, Option.

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## I. INTRODUCTION

The Black-Scholes model and its extension comprise one of the major developments in modern finance.

All option pricing models rely upon a risk-neutral representation of the data generating process that includes appropriate compensation for the various risks.

The Bates [1] and Scott [6] option pricing models were designed to capture two features of asset returns: the fact that conditional volatility evolves over time in a stochastic but mean-reverting fashion and the presence of occasional substantial outliers in asset returns. The two models combined the Heston [2] model of stochastic volatility with the Merton [4] model of independent normally distributed jumps in the log asset price. The Bates model ignores interest rate risk, while the Scott Model allows interest rates to be stochastic. Both models evaluate European option prices numerically, using Fourier inversion approach of Heston. The Bates model also includes an approximation for pricing American options. The two models were historically important in showing that the tractable class of affine option pricing models includes jump processes as well as diffusion processes.

Much of the recent literature on option valuation has successfully applied Merton and Heston models for the valuation of options such as ([3], [4], [6], [7], [8]) just to mention a few.

In this paper we shall consider the Bates model as the combination of the Merton and Heston models in the theory of option pricing. The paper is outlined as follows; in Section 2, we discuss how to price option under Bates model. Section 3 concludes the paper.

## II. BATES MODEL IN THE THEORY OF OPTION PRICING

This section presents Bates model in the theory of option pricing as follows. The geometric Brownian motion

(Wiener process) is the building block of modern finance. In particular, the Black-Scholes model, the underlying price of the asset is assumed to follow the dynamics of the geometric Brownian motion of the form:

$$dS_t = rS_t dt + \sigma S_t dW_t \quad (1)$$

where  $S_t$ : the underlying price of the asset,  $r$ : the risk-free interest rate,  $\sigma$ : the volatility,  $W_t$ : the Brownian motion or Wiener process,  $t$ : the maturity time.

The solution to (1) is obtained as follows;

Using Ito's lemma given by

$$du = \left( \frac{\partial u}{\partial t} + \frac{1}{2} f^2 \frac{\partial^2 u}{\partial S_t^2} + g \frac{\partial u}{\partial S_t} \right) dt + f \frac{\partial u}{\partial S_t} dW_t \quad (2)$$

From (1),  $f = rS_t$ ,  $g = \sigma S_t$ , since the underlying price of the asset  $S_t$  is assumed to follow the process in (1) but we are interested in the process followed by  $\log S_t$ . Let

$$u = \log S_t \quad (3)$$

Differentiating  $u$  with respect to the underlying price of the asset  $S_t$  and maturity time  $t$  we have

$$\frac{\partial u}{\partial S_t} = \frac{1}{S_t}, \frac{\partial^2 u}{\partial S_t^2} = -\frac{1}{S_t^2}, \frac{\partial u}{\partial t} = 0 \quad (4)$$

Substituting (3) and (4) into (2) yields

$$d(\ln S_t) = \left( r - \frac{\sigma^2}{2} \right) dt + \sigma dW_t \quad (5)$$

Integrating (5) from 0 to  $t$ , we have that

$$S_t = S_0 \exp \left\{ \left( r - \frac{\sigma^2}{2} \right) t + \sigma W_t \right\} \quad (6)$$

The empirical facts, however, do not confirm model assumptions. Financial returns exhibit much fatter tails than in the Black-Scholes model [8].

Bates [1] proposed a model with stochastic volatility and jumps. This model is the combination of the Merton and Heston models. Now we give a brief overview of Merton and Heston models below:

### A. Merton Model

According to [8] if an important piece of information about a company becomes public it may cause a sudden change in the company's stock price. To cope with this observation, Merton [4] proposed a model that allows discontinuous trajectories of the underlying prices of the asset. Merton model is one of the modern pricing models. This model extends (1) by adding jumps to the stock price dynamics, then (1) becomes

$$dS_t = rS_t dt + \sigma S_t dW_t + dZ_t \quad (7)$$

where  $Z_t$  is a compound Poisson process with a log-normal distribution of jump sizes. The jumps follow the same Poisson process  $N_t$  with intensity  $\lambda$ , which is independent of  $W_t$ . The log-jump sizes  $Y_i \sim N(\mu, \sigma^2)$  are independent, identically distributed random variables with mean  $\mu$  and variance  $\sigma^2$ , which are independent of both  $N_t$  and  $W_t$ . The model becomes incomplete which means that there are many possible ways to choose a risk-neutral measure such that the discounted price process is a martingale. Merton proposed to change the drift of the geometric Brownian motion and to leave the other ingredients unchanged. The underlying price of the asset dynamics is obtained as

$$S_t = S_0 \exp \left( \left( r - \sigma^2 - \lambda \exp \left( \left( \mu + \frac{\sigma^2}{2} \right) - 1 \right) \right) t + \sigma W_t + \sum_{i=1}^{N_t} Y_i \right) \quad (8)$$

Setting

$$\mu^M = r - \sigma^2 - \lambda \exp \left( \left( \mu + \frac{\sigma^2}{2} \right) - 1 \right) \quad (9)$$

Therefore (8) becomes

$$S_t = S_0 \exp \left( \mu^M t + \sigma W_t + \sum_{i=1}^{N_t} Y_i \right) \quad (10)$$

The jump components add mass to the tail of the returns distribution. Increasing  $\delta$  adds mass to both tails, while a negative or positive  $\mu$  implies relatively more mass in the left or right tail. Let the logarithm of the underlying price of the asset process be given by

$$X_t = \log \left( \frac{S_t}{S_0} \right) \quad (11)$$

The characteristic function of  $X_t$  is of the form;

$$\varphi_{x_t}^{\text{Merton}}(z) = \exp \left\{ t \left( -\frac{\sigma^2 z^2}{2} + i\mu^M z + \lambda \exp \left( -\frac{\sigma^2 z^2}{2} + i\mu^M z - 1 \right) \right) \right\} \quad (12)$$

## B. Heston Model

Heston model is one of the most widely used stochastic volatility models today. Its attractiveness lies in the powerful duality of its tractability and robustness relative to other stochastic volatility models.

Equation (1) can be modified by substituting the parameter  $t$  with a stochastic process which leads to stochastic volatility models, and then the price dynamics is given by

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t \quad (13)$$

where  $v_t$  is another unobservable stochastic process. There are many possible ways of choosing the variance process

$v_t$ . Hull and White [3] proposed to use geometric Brownian motion of the form

$$dv_t = c_1 v_t dt + c_2 v_t dW_t \quad (14)$$

However, geometric Brownian motion tends to increase exponentially which is an undesirable property for volatility. Volatility exhibits rather a mean reverting behavior. Therefore a model based on an Ornstein-Uhlenbeck-type process:

$$dv_t = \kappa(\theta - v_t) dt + \beta dW_t \quad (15)$$

Equation (15) was suggested by Stein and Stein [7]. This process, however, admits negative values of the variance  $v_t$ .

Heston [2] eliminated these deficiencies in stochastic volatility model by introducing first Brownian motion,  $W_t^1$  and second Brownian motion  $W_t^2$  for the volatility modeling in (13) and (15) respectively, which means volatility is not considered as constant. Then (13) and (15) become respectively as

$$dS_t = rS_t dt + \sqrt{v_t} S_t dW_t^1 \quad (16)$$

$t \in [0, T]$

$$dv_t = \kappa(\theta - v_t) dt + \alpha dW_t^2 \quad (17)$$

Setting

$$\alpha = \sigma \sqrt{v_t} \quad (18)$$

Substituting (12) into (11) yields

$$dv_t = \kappa(\theta - v_t) dt + \sigma \sqrt{v_t} dW_t^2 \quad (19)$$

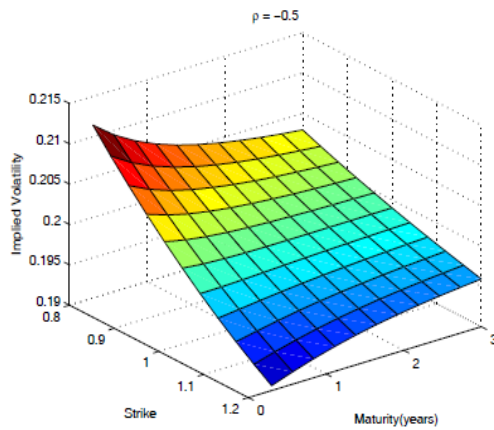
$t \in [0, T]$

With the assumption of:

$$\text{cov}(dW_t^1, dW_t^1) = \rho dt \quad (20)$$

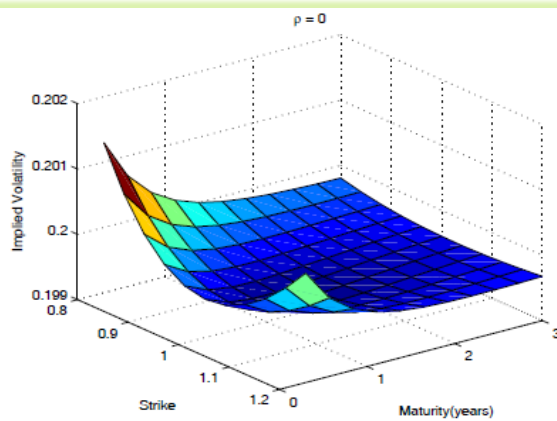
where  $S_t$  and  $v_t$  denote underlying price of the asset and volatility processes respectively,  $W_t^1$  and  $W_t^2$  are correlated with rate  $\rho$ . The term  $\sqrt{v_t}$  in (19) simply ensures positive volatility. When the process touches the zero bound the stochastic part becomes zero and the non-stochastic part will push it up. Parameter  $k$  measures the speed of mean reversion or rate of reversion,  $\theta$  is the long run mean or the average level of volatility and  $\sigma$  is called volatility of volatility.

It is clear that, in Heston model we can imply more than one distribution by changing the value of  $\rho$ . We define  $\rho$  as the correlation between returns and volatility, and hence we can deduce that  $\rho$  affects the heavy tails of the distribution. When  $\rho < 0$ , there is an inverse proportion between underlying price of the asset and volatility, when  $\rho = 0$ , the skewness is close to zero and when  $\rho > 0$ , this means that as underlying price of the asset increases volatility increases which also increase the heaviness of the right tail and squeeze the other one as shown in the Figures 1: (a), (b) and (c) below



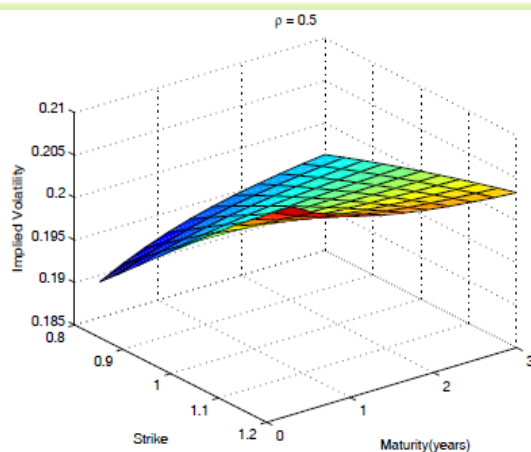
(a)

$S_0 = 1, r = 0.01, v_0 = 0.04, \theta = 0.04, \rho = -0.5, \kappa = 2,$   
 $\sigma = 0.1, K = \{0.8-1.2\}, T = \{0.5-3\}$



(b)

$S_0 = 1, r = 0.01, v_0 = 0.04, \theta = 0.04, \rho = 0, \kappa = 2,$   
 $\sigma = 0.1, K = \{0.8-1.2\}, T = \{0.5-3\}$



(c)

$S_0 = 1, r = 0.01, v_0 = 0.04, \theta = 0.04, \rho = 0.5, \kappa = 2,$   
 $\sigma = 0.1, K = \{0.8-1.2\}, T = \{0.5-3\}$

Fig.1. The Effect of Varying the Correlation Coefficient  $\rho$

Heston's stochastic option pricing model has a closed form solution, the infinite integral is still solved by a numerical method. It is much faster than other stochastic volatility models, it takes into account the leverage effect, its volatility updating structure permits analytical solutions to be generated for standard plain vanilla European options and thus the model allows a fast calibration to given market data [5].

The risk neutral dynamics is given in a similar way as in the Black-Scholes model. For the logarithm of the underlying price of the asset process given by

$$X_t = \log\left(\frac{S_t}{S_0}\right) \text{ in (11) one obtains the equation of the}$$

form:

$$dX_t = \left(r - \frac{1}{2}v_t\right)dt + \sqrt{v_t}dW_t^1 \quad (21)$$

The characteristic function is given by

$$\varphi_{X_t}^{Heston}(z) = \left\{ \frac{\exp\left(\frac{k\theta t(\kappa - i\rho\sigma z)}{\sigma^2} + iztr + izx_0\right)}{\left(\cosh\frac{\gamma t}{2} + \frac{\kappa - i\rho\sigma z}{\gamma} \sinh\frac{\gamma t}{2}\right)^{\frac{2\kappa\theta}{\sigma^2}}} \right\} \quad (22)$$

$$\times \exp\left(-\frac{(z^2 + iz)v_0}{\gamma \cosh\frac{\gamma t}{2} + \kappa - i\rho\sigma z}\right)$$

where  $\gamma = \sqrt{\sigma^2(z^2 + iz) + (\kappa - i\rho\sigma z)^2}$ ,  $x_0$  is the initial value for the log-price process and  $v_0$  is the initial value for the volatility process.

The Merton [4] and Heston [2] models were combined by Bates [1] i.e. (Bates Model  $\varphi_{X_t}^{Bates}(z) \approx \varphi_{X_t}^{Heston}(z)\varphi_{X_t}^{Merton}(z)$ ), who proposed a model with stochastic volatility and jumps. This model, described in Bates [1], adds jumps to the dynamics of the Heston model. For non-dividend paying stock, the stock price  $S_t$  and its variance  $v$  are given by

$$\left. \begin{aligned} dS_t &= rS_t dt + \sqrt{v_t}dW_t^1 + dZ_t, \\ dv_t &= \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dW_t^2 \\ Cov(dW_t^1, dW_t^2) &= \rho dt \end{aligned} \right\} \quad (23)$$

Equation (23) can also be written for the case of dividend paying stock as:

$$\left. \begin{aligned} dS_t &= (r - q)S_t dt + \sqrt{v_t}dW_t^1 + dZ_t, \\ dv_t &= \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dW_t^2 \\ Cov(dW_t^1, dW_t^2) &= \rho dt \end{aligned} \right\} \quad (24)$$

Where  $q$  is the dividend yield paid by underlying price of the asset,  $Z_t$  is a compound Poisson process with Intensity  $\lambda$  and a log-normal distribution of jump sizes independent of  $W_t^1$  and  $W_t^2$ . If  $J$  denotes the jump size then

$$\log(1+J) = N\left(\log(1+\chi) - \frac{1}{2}\delta^2, \delta^2\right) \quad (25)$$

The parameters  $\chi$  and  $\delta$  determine the distribution of the jumps and the Poisson process is assumed to be independent of the Brownian motions. Under the risk neutral probability one obtains the equation for the logarithm of the underlying price of the asset with non-dividend and dividend yields respectively as:

$$dX_t = \left(r - \lambda\chi - \frac{1}{2}v_t\right)dt + \sqrt{v_t}dW_t^1 + \tilde{Z}_t \quad (26)$$

$$dX_t = \left(r - q - \lambda\chi - \frac{1}{2}v_t\right)dt + \sqrt{v_t}dW_t^1 + \tilde{Z}_t \quad (27)$$

where  $\tilde{Z}_t$  is a compound Poisson process with normal distribution of jump magnitudes.

From (23) jumps are independent of the diffusion part then the characteristic function for the log-price process in which the underlying price of the asset pays no dividend can be obtained as:

$$\phi_{X_t}^{Bates}(z) = \left\{ \frac{\exp\left(\frac{k\theta t(\kappa - i\rho\sigma z)}{\sigma^2} + izt(r - \lambda\chi) + izx_0\right)}{\left(\cosh \frac{\gamma t}{2} + \frac{\kappa - i\rho\sigma z}{\gamma} \sinh \frac{\gamma t}{2}\right)^{\frac{2\kappa\theta}{\sigma^2}}} \right\} \times \exp\left(-\frac{(z^2 + iz)v_0}{\gamma \cosh \frac{\gamma t}{2} + \kappa - i\rho\sigma z}\right) \times \exp\left\{t\lambda\left(\exp\left(-\frac{\delta^2 z^2}{2} + \log(1+\chi) - \frac{\delta^2}{2}\right) - 1\right)\right\} \quad (28)$$

$$\phi_{X_t}^{Bates}(z) = \left\{ \frac{\exp\left(\frac{k\theta t(\kappa - i\rho\sigma z)}{\sigma^2} + izt(r - q - \lambda\chi) + izx_0\right)}{\left(\cosh \frac{\gamma t}{2} + \frac{\kappa - i\rho\sigma z}{\gamma} \sinh \frac{\gamma t}{2}\right)^{\frac{2\kappa\theta}{\sigma^2}}} \right\} \times \exp\left(-\frac{(z^2 + iz)v_0}{\gamma \cosh \frac{\gamma t}{2} + \kappa - i\rho\sigma z}\right) \times \exp\left\{t\lambda\left(\exp\left(-\frac{\delta^2 z^2}{2} + \log(1+\chi) - \frac{\delta^2}{2}\right) - 1\right)\right\} \quad (29)$$

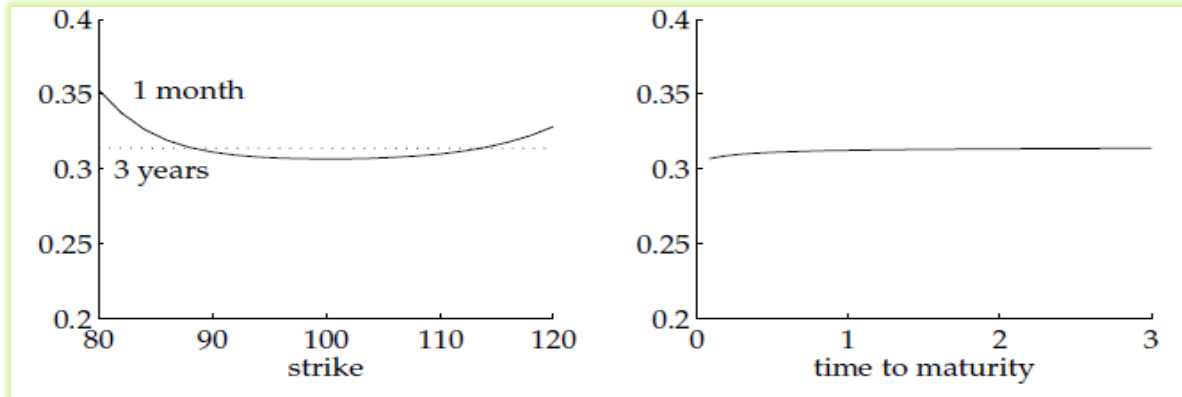
Similarly, for dividend paying stock we have:

(28) and (29) can be written as  $\phi_{X_t}^{Bates}(z) = D + J$  since they can be split into diffusion part,  $D$  and jump part,  $J$ . Therefore, we have respectively for (28) and (29) below

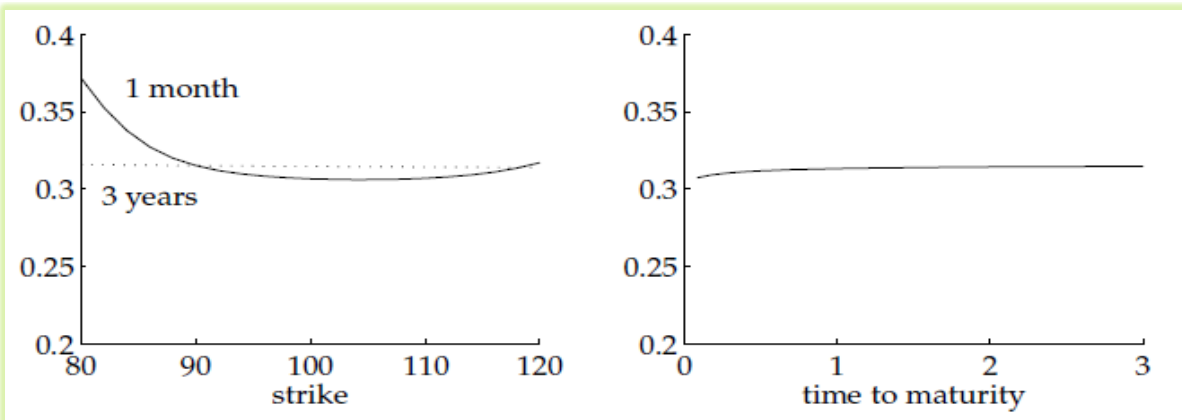
$$\phi_{X_t}^{Bates}(z) = D + J, \quad D = \left\{ \frac{\exp\left(\frac{k\theta t(\kappa - i\rho\sigma z)}{\sigma^2} + izt(r - \lambda\chi) + izx_0\right)}{\left(\cosh \frac{\gamma t}{2} + \frac{\kappa - i\rho\sigma z}{\gamma} \sinh \frac{\gamma t}{2}\right)^{\frac{2\kappa\theta}{\sigma^2}}} \right\} \times \exp\left(-\frac{(z^2 + iz)v_0}{\gamma \cosh \frac{\gamma t}{2} + \kappa - i\rho\sigma z}\right), \quad J = \exp\left\{t\lambda\left(\exp\left(-\frac{\delta^2 z^2}{2} + \log(1+\chi) - \frac{\delta^2}{2}\right) - 1\right)\right\} \quad (30)$$

$$\phi_{X_t}^{Bates}(z) = D + J, \quad D = \left\{ \frac{\exp\left(\frac{k\theta t(\kappa - i\rho\sigma z)}{\sigma^2} + izt(r - q - \lambda\chi) + izx_0\right)}{\left(\cosh \frac{\gamma t}{2} + \frac{\kappa - i\rho\sigma z}{\gamma} \sinh \frac{\gamma t}{2}\right)^{\frac{2\kappa\theta}{\sigma^2}}} \right\} \times \exp\left(-\frac{(z^2 + iz)v_0}{\gamma \cosh \frac{\gamma t}{2} + \kappa - i\rho\sigma z}\right), \quad J = \exp\left\{t\lambda\left(\exp\left(-\frac{\delta^2 z^2}{2} + \log(1+\chi) - \frac{\delta^2}{2}\right) - 1\right)\right\} \quad (31)$$

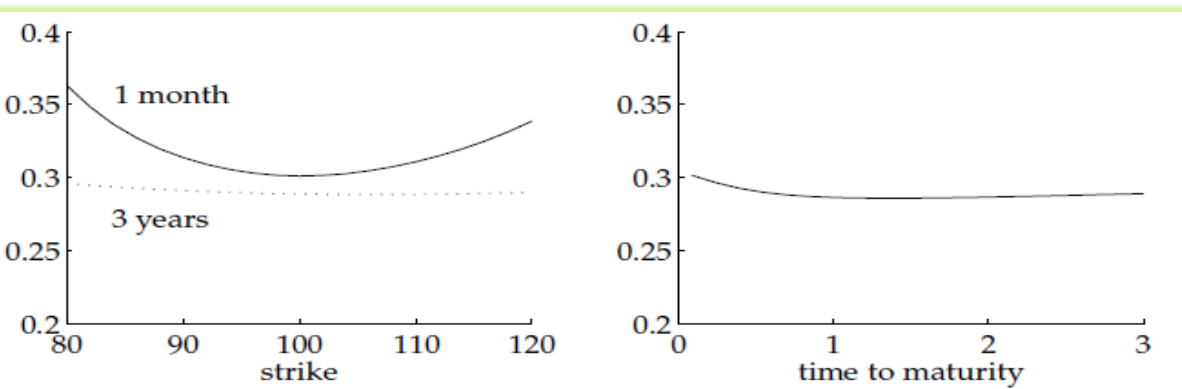
From (30), it can be seen clearly that the diffusion part is similar to (22) with difference of  $\lambda\chi$  called risk neutral correction. Also (12) has a similar structure as the jump part in (30), where  $\mu = \log(1+\chi) - \frac{\delta^2}{2}$ . Since the jumps are assumed independent, the characteristic function is the product of  $\phi_{X_t}^{Heston}(z)$  with the function for the jump part in (30). Figure 2 shows that adding jumps makes it easier to introduce curvature into the volatility surface, at least for short maturities.



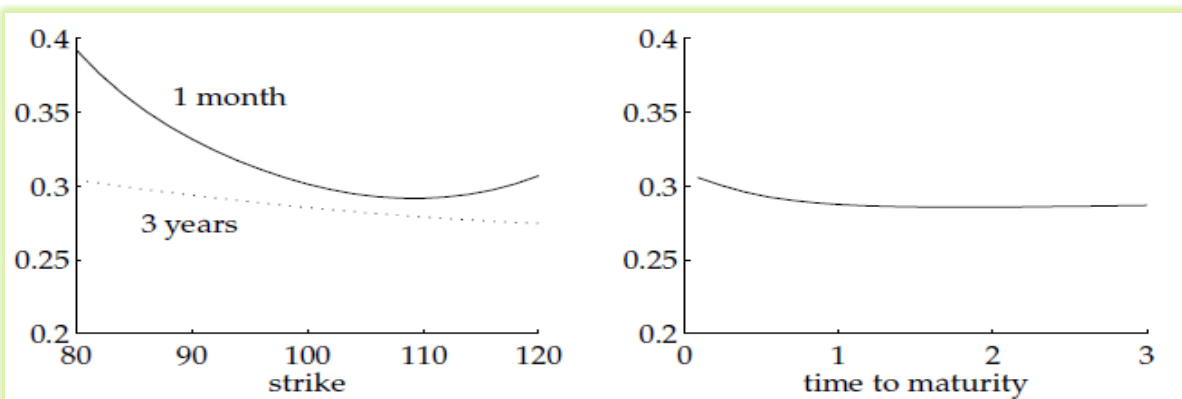
(a)  $S = 100, r = 0.02, q = 0.02, v_0 = 0.09, \theta = 0.09, \rho = 0, \kappa = 1, \sigma = 0.1, \delta = 0.3$  (Volatility of volatility is zero, as the jump mean)



(b)  $S = 100, r = 0.02, q = 0.02, v_0 = 0.09, \theta = 0.09, \rho = 0, \kappa = 1, \sigma = 0.1, \delta = -0.1$  (More Asymmetry)



(c)  $S = 100, r = 0.02, q = 0.02, v_0 = 0.09, \theta = 0.07, \rho = 0, \kappa = 1, \sigma = 0.1, \delta = 0.3$



(d)  $S = 100, r = 0.02, q = 0.02, v_0 = 0.09, \theta = 0.7, \rho = -0.3, \kappa = 1, \sigma = 0.1, \delta = 0.3, \mu = -0.1$

Fig.2 Bates Model: Recreating the Implied Volatility Surface



Bates [1] found that some of the parameters of Heston model need to be implausibly high when fitting the market data. One explanation for this is the absence of price jumps. The correlation parameter,  $\rho$  controls the level of skewness and the volatility of variance,  $\sigma$  controls the level of kurtosis.

**Remarks:**

Assuming that the previous dynamics represent the evolution of the state process  $(S_t, v_t)$  under a risk-neutral measure, then the pricing equation of a European contingent claim  $c$  on  $S$  is given by

$$\left. \begin{aligned} \frac{\partial c}{\partial t} + \frac{1}{2} v S^2 \frac{\partial^2 c}{\partial S^2} + \frac{1}{2} v \sigma^2 \frac{\partial^2 c}{\partial v^2} + \rho \theta v S \frac{\partial^2 c}{\partial S \partial v} \\ + (r - q - \lambda \chi) S \frac{\partial c}{\partial S} + \kappa (\theta - v) \frac{\partial c}{\partial v} - rc + \psi c = 0 \end{aligned} \right\} \quad (32)$$

where

$$\left. \begin{aligned} \psi c(S, v, t) &= \lambda \int_0^\infty [c(S\xi, v, t) - c(S, v, t)] g(\xi) d\xi, \\ g(\xi) &= \frac{1}{\sqrt{2\pi\delta^2}} \exp\left(-\frac{1}{2\delta^2} (\log \xi - m)^2\right), \\ m &:= \log(1 + \chi) - \frac{1}{2} \delta^2, \\ \bar{k} &:= \exp\left(\frac{\delta^2}{2 + m}\right) \end{aligned} \right\} \quad (33)$$

The equation holds for  $S \in [0, \infty)$ , we also assume that  $v \in [0, v_{\max}]$ . The partial differential equation is usually given along with a proper payoff. In the case of vanilla European call option we have that:

$$c(S, v) = (S - K)^+ = \max(S - K, 0) \quad (34)$$

where  $K$  is the strike price or exercise price. It should be noted that closed form solutions of problem (32) for vanilla-option payoff do exist. Nevertheless, direct numerical integration of (32) is important when dealing with non-trivial payoff functions.

### III. CONCLUSION

In this paper we consider the Bates model as the combination of Merton and Heston models for pricing options. Heston model is one of the most popular stochastic volatility option pricing models. This model is motivated by the widespread evidence that volatility is stochastic and that the distribution of risky asset return has tails heavier than that of a normal distribution. The stochastic volatility model incorporates several important features of stock returns. For many stochastic models, closed form solutions are not available. Some numerical methods are used. But usually it is timeconsuming to get the price using these numerical methods. Although Heston's stochastic option pricing model has a closed form solution, the infinite integral is still solved by a numerical method. It is much faster than other stochastic volatility models, it takes into account the leverage effect, its volatility updating structure permits analytical solutions

to be generated for standard plain vanilla European options and thus the model allows a fast calibration to given market data. However, there remain some drawbacks such as; the integral needed for the computation of the option prices do not always converge fast enough. The standard Heston model usually fails to create a short term skew as strong as the one given by the market. In the real financial markets, prices exhibit jumps rather than continuous changes. Large price changes cannot be generated by pure diffusion processes in stochastic volatility models. Some of the parameters of Heston model need to be implausibly high when fitting the market data. One explanation for this is the absence of price jumps [5]. The correlation parameter,  $\rho$  controls the level of skewness and the volatility of variance,  $\sigma$  controls the level of kurtosis. But the ability of Heston's model to generate enough short term kurtosis is limited. And a high level of  $\sigma$  means the short-term kurtosis is very high. Bates model captures both stochastic volatility and jump risk within a tractable affine specification.

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