

7. Prove:

$$\Box\Box A \leftrightarrow \Box A, \quad (4.9)$$

$$\Diamond\Diamond A \leftrightarrow \Diamond A, \quad (4.10)$$

$$\Diamond\Box A \leftrightarrow \Box\Diamond A, \quad (4.11)$$

$$\Box\Diamond A \leftrightarrow \Diamond\Box A. \quad (4.12)$$

It follows that strings of unary temporal operators “collapse” to a single operator or to a pair of distinct operators.

8. Using the operator \mathcal{U} , modify Equation 4.1 (page 79) so that it also specifies freedom from starvation for process p .

9. The temporal operator *leads to*, denoted \leadsto , is defined as: $A \leadsto B$ is true in a state s_i if and only if for all states s_j , $j \geq i$, if A is true s_j , then B is true in some state s_k , $k \geq j$. Express \leadsto in terms of the other temporal operators.

10. Prove the correctness of Peterson’s algorithm, repeated here for convenience:

Algorithm 4.3: Peterson’s algorithm	
boolean wantp \leftarrow false, wantq \leftarrow false integer last \leftarrow 1	
p	q
loop forever	loop forever
p1: non-critical section	q1: non-critical section
p2: wantp \leftarrow true	q2: wantq \leftarrow true
p3: last \leftarrow 1	q3: last \leftarrow 2
p4: await wantq = false or last = 2	q4: await wantp = false or last = 1
p5: critical section	q5: critical section
p6: wantp \leftarrow false	q6: wantq \leftarrow false

First show that

$$(p4 \wedge q5) \rightarrow (wantq \wedge last = 1), \quad (4.13)$$

$$(p5 \wedge q4) \rightarrow (wantp \wedge last = 2) \quad (4.14)$$

are invariant, and then use them to prove that mutual exclusion holds. To prove liveness for process p , prove the following formulas:

$$p4 \wedge \Box \neg p5 \rightarrow \Box\Diamond(wantq \wedge (last \neq 2)), \quad (4.15)$$

$$\Diamond\Box(\neg wantq) \vee \Diamond(last = 2), \quad (4.16)$$

$$p4 \wedge \Box \neg p5 \wedge \Diamond(last = 2) \rightarrow \Diamond\Box(last = 2), \quad (4.17)$$

and deduce a contradiction.