

1) Examining correlations and convolutions numerically

Define a few short vectors to use to calculate convolutions and correlations:

```
a = mod(1:100,3);  
b = abs(sin(1:100));  
c = randn(100,1);  
d = [-2,-1,0,1,2];  
e = [ 3,-2,4,1,-5];  
f = [-1,3,2,-5,2,3,-1];
```

Use `conv(f,g,'same')` for convolution \otimes and
`conv(f,flip(g),'same')` for correlation $*$

Graph 1: $a, a \otimes d, a \otimes e$
Graph 2: $b, b \otimes d, b \otimes e$
Graph 3: $c, c \otimes d, c \otimes e$
Graph 4: $c, (c \otimes d) \otimes e, c \otimes (d \otimes e)$
Graph 5: $a, a * d, a * e$
Graph 6: $a, (a * d) * e, a * (d * e)$
Graph 7: $c, (c * d) * f, c * (d * f)$
Graph 8: $c, (c * d), c \otimes d, c * f, c \otimes f$

Questions to answer:

- When are correlation and convolutions the same?
- Are correlations associative?
- Are convolutions associative?

2) Membrane potential– filtering with exponentials

Construct a time vector that runs from 0 to 999.9 milliseconds in bins of 1 millisecond. Make another vector, called s that is the same size as t and use what you learnt in the probability lecture to put 1s in s at the times that there would be spikes in a Poisson process with mean rate 40 spikes/sec.

Imagine that each of these spikes produces a sudden influx of electric charge into a postsynaptic membrane. The postsynaptic neuron is far enough below threshold that we'll ignore any active channels in it. In those circumstances, the membrane potential will s be convolved by an exponential filter, with time constant $\tau = RC$ where R is the membrane resistance and C the membrane capacitance.

The exponential filter is $r(t) = \tau e^{-\frac{t}{\tau}}$ for $t > 0$, otherwise it is 0.

1. Assume $\tau = 20\text{ms}$, and convolve s with an exponential filter with that time constant. What does $V(t)$ look like?
2. Now play with different values of τ . What does V look like as τ gets large? What does it look like if τ is small?

Let's reconsider the problem, but let's now assume that each synaptic connection has two components: a fast excitatory component, and a slower inhibitory component. Let's imagine both components are exponential, so we are going to convolve our spike train s with the filter:

$$r(t) = \frac{1}{\tau_1} e^{-\frac{t}{\tau_1}} - \frac{1}{\tau_2} e^{-\frac{t}{\tau_2}} \text{ for } t > 0, 0 \text{ otherwise}$$

3. Let $\tau_1 = 5\text{ ms}$, $\tau_2 = 10\text{ ms}$ and convolve s with this filter. What does $V(t)$ look like now?