Advanced Signal Processing

Course Introduction

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Aims:

- To introduce the fundamentals of **statistical signal processing**
- The emphasis will be upon:
  - ⊛ stochastic models
  - ⊛ classical and modern estimation theory
  - ⊛ parametric and nonparametric modelling
  - ⊛ the class of least squares methods
  - ⊛ adaptive estimation $\rightsquigarrow$ suitable for nonstationary data
- **Practical experience** in utilising statistical signal processing on real world signals:
  - ⊛ multimedia (your own speech recorded via PC)
  - ⊛ financial data (from *yahoo finance*)
  - ⊛ some biomedical data
- To introduce the concept of **estimation** from real world data, as opposed to the **analysis** of signals, transfer functions, and power spectra

So what is Advanced Signal Processing?

Advanced Signal Processing is not a field *per se* but a set of advanced DSP theories.

- We will introduce a family of algorithms for:
  - ⊛ creation
  - ⊛ efficient representation
  - ⊛ effective modelling

  of real world signals.

- For this purpose we will employ:
  - ⊛ parametric models
  - ⊛ non-parametric models
  - ⊛ statistical techniques

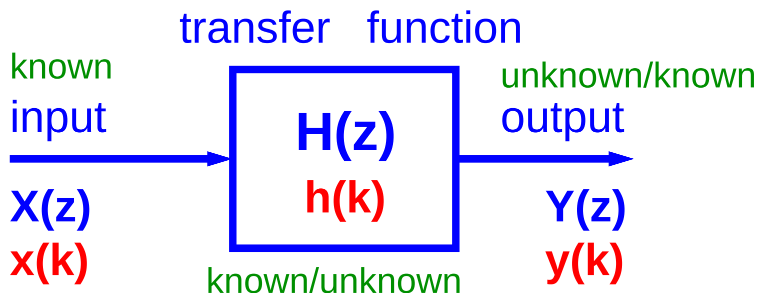
  in order to extract information from data corrupted by noise.

The simultaneous account of signals and noise leads to rigorous solutions!

The difference in this course

So far, you are familiar with problems which:

- Have a **well defined transfer function** in the form



$$H(z) = \frac{Y(z)}{X(z)}$$

- Are **deterministic** (assuming a mathematically tractable signal model)
- There exist notions of **impulse response**, **step response**, **frequency response**
- **Operate in a noise-free & stationary environment**

In this course we will consider more realistic situations where:

⊛ **Signals are random**, and we only known their statistics

⊛ **Models are adaptive**, and operate for nonstationary data

These are huge advantages that allow such models to perform **detection**, **estimation**, *and* **prediction** *for real world data.*

Illustration: From deterministic to random

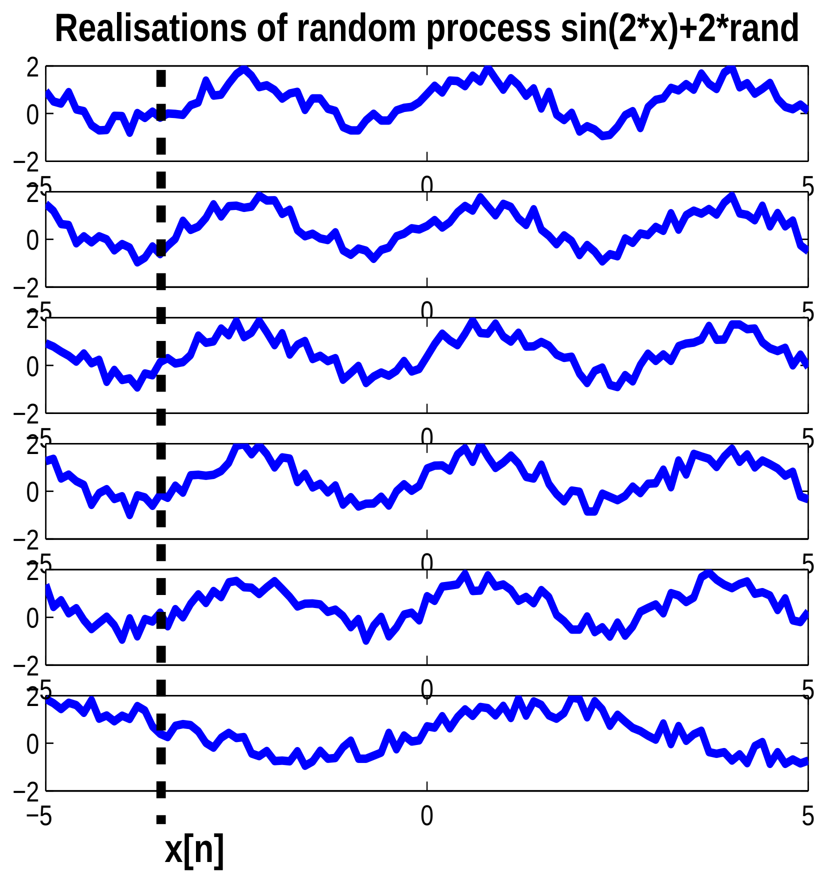
Ensemble $\looparrowright$ collection of **all possible realisations** of a **random signal**

Consider 6 realisations of the process

$$y = \sin(x) + rand \;\Leftrightarrow\; '\mathrm{det}' + '\mathrm{stoch}'$$

- our aim is to **estimate** frequency f
- sinusoid $\looparrowright$ ***deterministic***
- noise $\looparrowright$ ***stochastic***

We need to use a **statistical** estimator, which will be ***unbiased*** and will have ***minimum variance***



Can we average both **along** one and **across** all realisations?

Casting this into a mathematical formalism

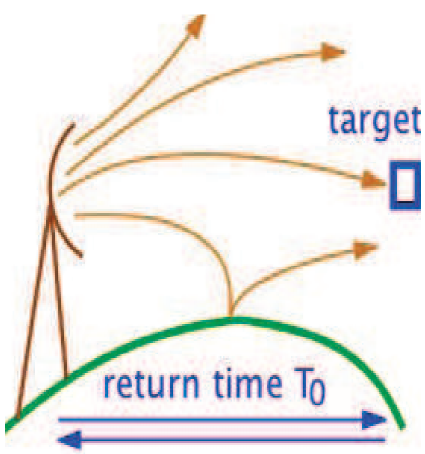
Problem: Based on an N-point dataset $\mathbf{x} = \big[x[0], x[1], \ldots, x[N-1]\big]^T$

Find an unknown parameter, θ, based on the data $\mathbf{x}$ in order to define an *estimator* (e.g. $\hat{\theta}$ can be sinewave frequency)

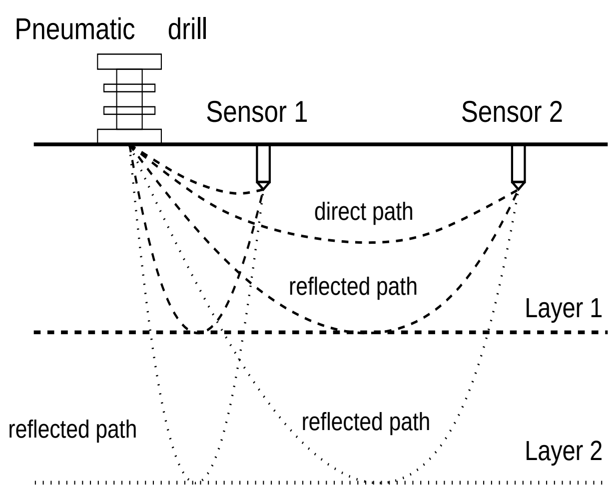
$$\hat{\theta} = g\big(x[0], x[1], \ldots, x[N-1]\big), \qquad g \text{ is some function}$$

The problem becomes that of **parameter estimation** from **random** signals

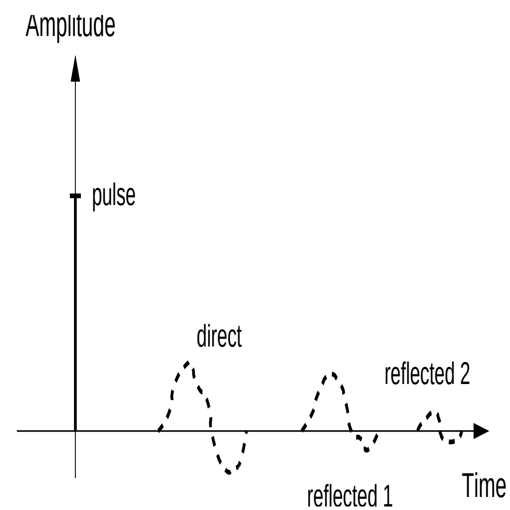
Depending on the choice of g we have: $\circledast$ linear, $\circledast$ nonlinear, $\circledast$ maximum likelihood, $\circledast$ least squares, ... estimation



Radar



Seismics



Seismic impulse response

Two illustrative examples

Recall that shape of the autocorrelation mimics shape of the original signal

Detection of Tones in Noise:

Consider a noisy tone $x = A\cos(\omega n + \theta)$

$$y[n] = A\cos(\omega n + \theta) + w[n]$$

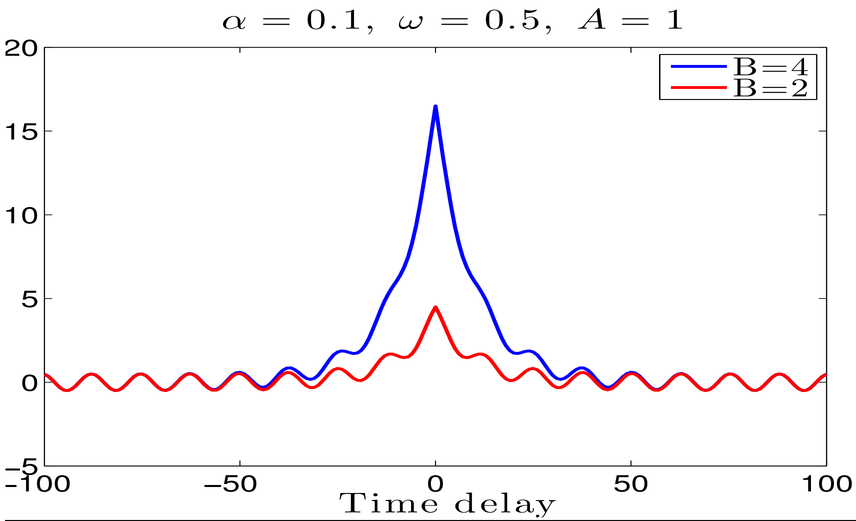
$$\mathbf{ACF}: \;\; R(m) = E\big[y[n]y[n+m]\big] =$$

$$= R_x(m) + R_w(m) + R_{xw}(m) + R_{wx}(m)$$

For $R_w = B^2 \exp(-\alpha|m|)$ & $x \perp w$, then

$$R_y(m) = \frac{1}{2}A^2\cos(\omega m) + B^2\exp(-\alpha|m|)$$

- for large m, the ACF $\propto$ the signal
- $\exists$ extract tiny signal from large noise



Principle of Radar:

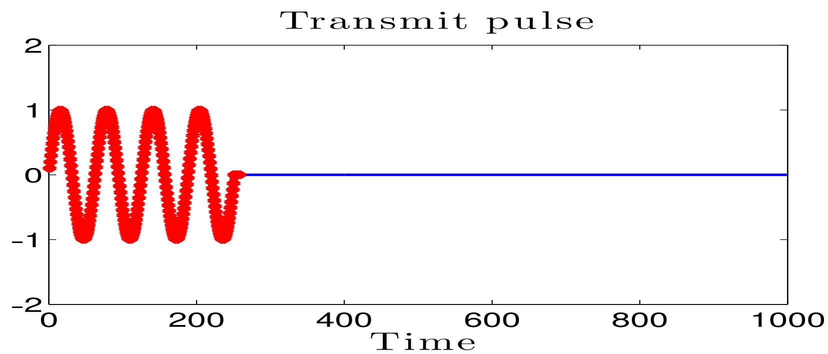
The received signal (see previous slide)

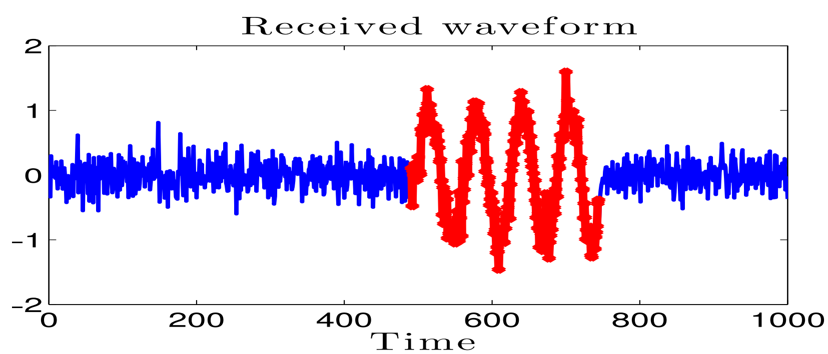
$$y[n] = ax[n - T_0] + w[n], \quad \text{so that}$$

$$\begin{aligned} R_{xy}(\tau) &= E\{x(n)y(n+\tau)\} \\ &= aR_x(\tau - T_0) + R_{xw}(\tau) \end{aligned}$$

Since

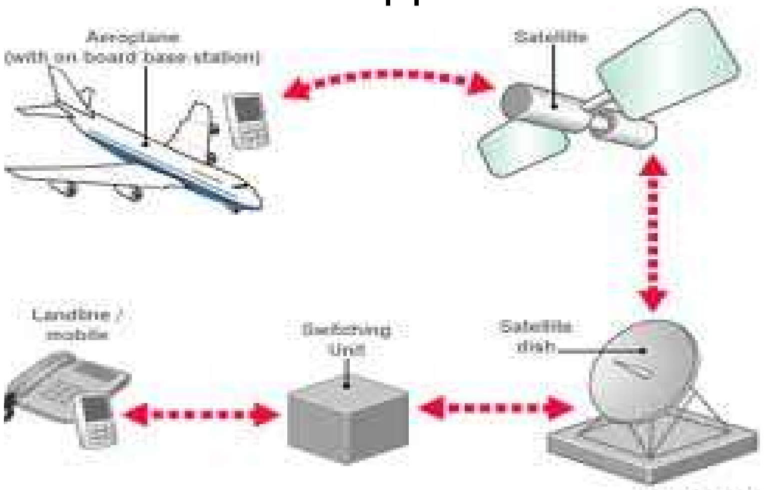
$$\boldsymbol{x \perp w \rightsquigarrow R_{xy}(\tau) = aR_x(\tau - T_0)}$$

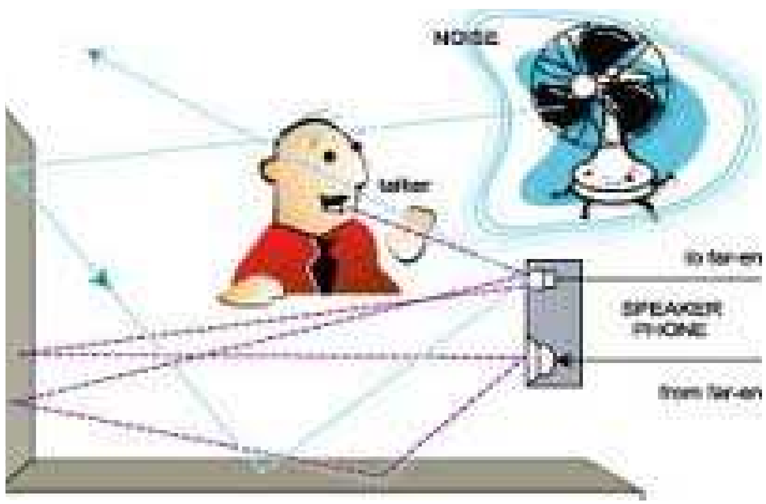




Course aims: Estimation theory perspective of DSP

To highlight the need for **estimation** when processing real world random signals, consider some applications:





Course aims: More specifically

- To introduce discrete-time random signals, their properties and representations
- To introduce linear stochastic models and illustrate the importance of correlation structure in deriving the parameters of such models
- To provide a grounding in the fundamentals of linear estimation theory and optimal filtering to facilitate the design of advanced signal processing algorithms
- Based upon these concepts, we will:
  - ⊛ Explain the notion of signal modelling, its applications, and its relations to parametric spectral estimation
  - ⊛ Describe the need for adaptive signal processing
- To illuminate the application of estimation theory in prediction, equalisation, echo and noise cancellation
- To verify theoretical and practical bounds for the performance of the estimation algorithms involved

Course structure

The course is divided roughly into four parts:

1. Introduction to Statistical Estimation Theory

   discrete random signals, moments, bias-variance, curse of dimensionality

2. Signal Modelling and Estimators

   linear stochastic models, ARMA modeling, properties of estimators bias/variance, Cramer Rao, MVU estimator, BLUE, ML estimation, Bayesian estimation (optional)

3. Least Squares Estimation (block and adaptive)

   orthogonality principle, block and sequential forms, Wiener filter, adaptive filtering, signal modelling, concept of an artificial neuron

4. Coursework

   assignments based on the above - using Matlab

Course format

Lecture notes with problem/answer sets and coursework.

- coursework involves the implementation of the algorithms we discuss in the class
- we will regularly discuss coursework and Matlab implementation

Prerequisites:

- ⊛ there are no prerequisites, although DSP and basic probability would be useful
- ⊛ the course is aimed to be self-contained
- ⊛ due to algorithm implementation, knowledge of Matlab is important

Assessment:

100% Coursework assignments. There are 5 Assignments (from random signals to audio denoising) $\looparrowright$ **Matlab based**

Feedback: $\looparrowright$ **after completing Assignment 1**

Reference material

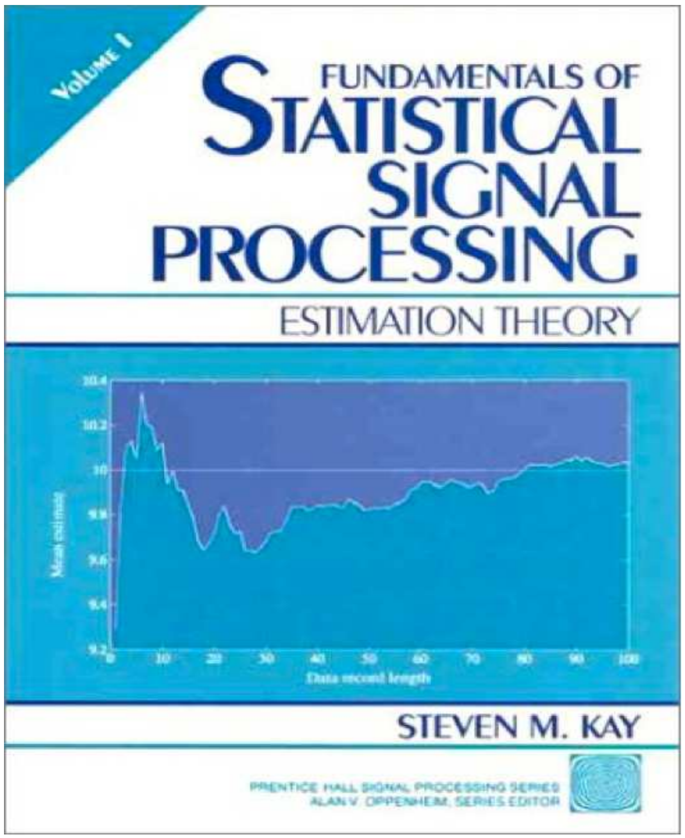
Course notes and problem sets: Prof D. Mandic

- There is no single textbook that covers all the material in the course
- We will use S. Kay's book for the first part of the course (an excellent text, covers most of the estimation theory, well worked-out examples, highly recommended)
- For parametric modelling we will use the Box & Jenkins book (a 'bible' for time series analysis, easy to read, excellent examples, used by people in engineering, physics, finance)
- For the least squares part, we will use M. Hayes' book (wider scope than Kay's book, less detailed derivations, a must have for practitioners)
- For more background and further reading, the book by S. Haykin (Adaptive Filters) and D. Mandic & J. Chambers (Recurrent Neural Networks)

The course is self-contained: most of the material is already in course notes

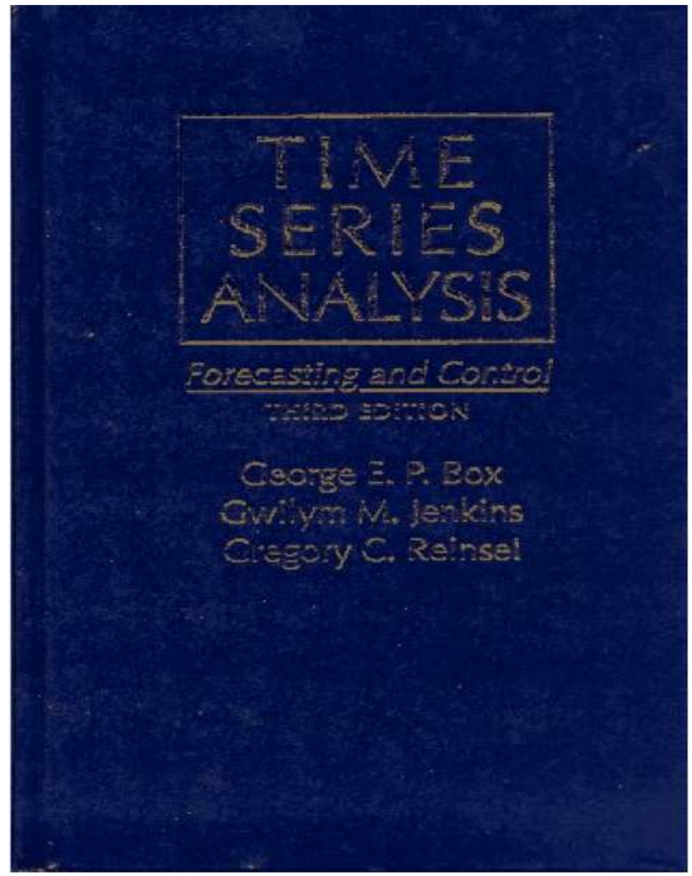
Textbooks: Recommended

S. Kay (*Estimation Theory*, several editions)



a comprehensive account of estimation theory

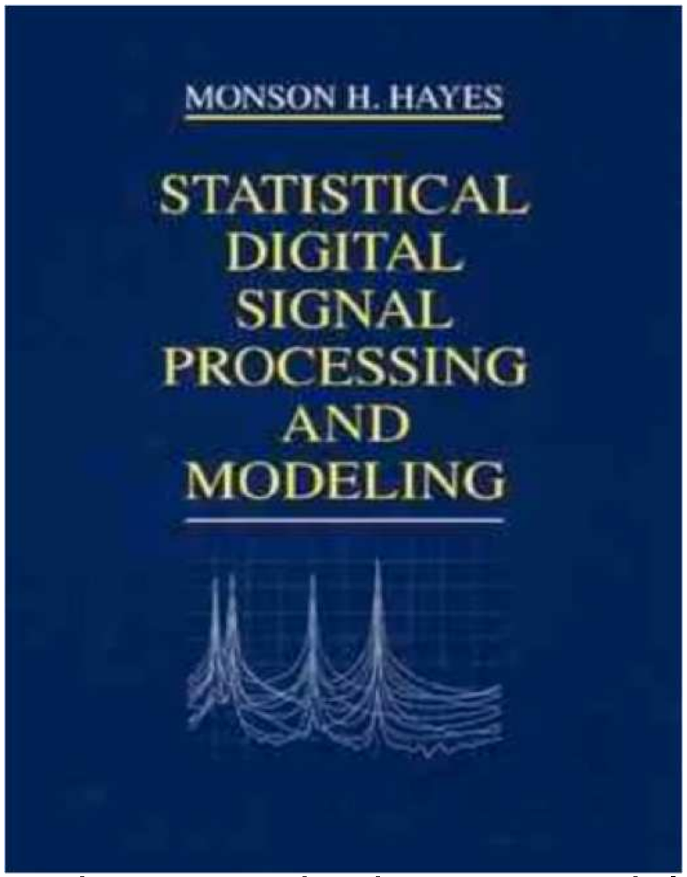
G. Box and G. Jenkins (*Time Series Analysis*, several editions)



linear stochastic models

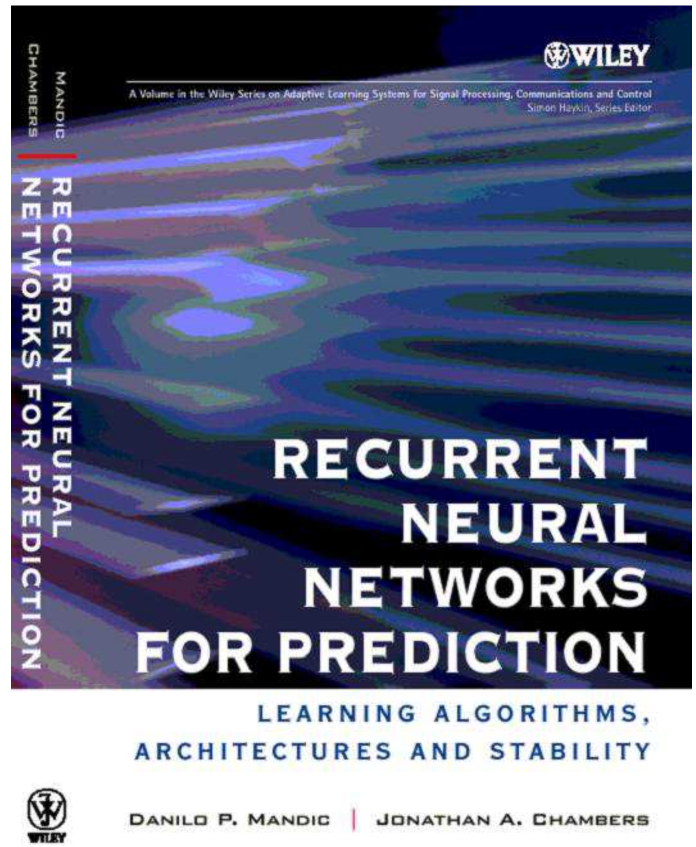
Textbooks: Additional reading (optional)

M. Hayes (*Statistical Signal Processing and Modeling*, several editions)



stochastic and adaptive models

D. Mandic and J. Chambers (*Recurrent Neural Networks*, Wiley 2001.)



(what can I say) - neural models

Course plan

1 Lect: Week 2, Course introduction and motivation

2 Lect: Week 2-3, Discrete time random signals

4 Lect: Week 3-5, Linear estimation theory

3 Lect: Week 6-7, Signal modelling and basic spectral estimation

3 Lect: Week 8-9, Adaptive signal processing

s Lect: Week 9-10, Consolidation and research directions

Course web page: www.commsp.ee.ic.ac.uk/∼mandic/Teaching

Lectures, additional reading, homework, problem sets, and other material will be put on course webpage

Notes:

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