* 기계학습 3주차 A4 1Page 정리 201610409 컴퓨터공학과 김성수

* 미분: Derivative

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$(f(x)+g(x))' = f'(x) + g(x)$$

$$(g(f(x)))' = g'(f(x))\, f'(x)$$

* Chain Rule

$$F(x) = f(g(h(u(v(x)))))$$

$$\frac{dF}{dx} = \frac{df}{dg}\frac{dg}{dh} \cdots \frac{dv}{dx}$$

Ex) $f(x) = \log(\sqrt{x})$

$$\frac{df}{dx} = \frac{d}{dg}\log(g) \cdot \frac{d}{dx}\sqrt{x}$$

$$= \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2x}$$

* Vector form.

$$\frac{df(v)}{dx} = \frac{\partial f(v)}{\partial v} \cdot \frac{\partial v}{\partial x} = \nabla_v f(v) \cdot \frac{\partial v}{\partial x}$$

* Partial Derivations

$$\frac{\partial f}{\partial x_n} = \lim_{h \to 0} \frac{f(x_1, \cdots, x_n + h) - f(x)}{h}$$

* Jacobian.

$$J(f(x)) = \nabla_x f(x) = \frac{df(x)}{dx}$$

$$\downarrow$$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1} & \cdots & \frac{\partial f_m}{\partial x_n} \end{bmatrix} = (\nabla f_1)^T$$

* Gradient & Hessian.

$$\left.\begin{aligned} \nabla_x b^T x &= b \\ \nabla_x^2 b^T x &= 0 \end{aligned}\right\} \rightarrow \nabla_x A x = \nabla_x \begin{bmatrix} a_1^T x \\ \vdots \\ a_n^T x \end{bmatrix} = A^T$$

* Quadratic Function.

$$\left.\begin{aligned} \nabla_x x^T A x &= 2Ax \\ \nabla_x^2 x^T A x &= 2A \end{aligned}\right\} \rightarrow \begin{aligned} \nabla_x \|x\|_2^2 &= \nabla_x x^T x = 2x \\ \nabla_x (x^T A^T A x) &= 2A^T A x \\ \nabla_x (b^T A x) &= (b^T A)^T \end{aligned}$$

* Iterative Optimization & Gradient Descent

$$\hat{\theta} = \underset{\theta}{\arg\min}\, \underbrace{J(\theta)}_{\text{목적함수}}$$

$D = \langle x^1, y^1 \rangle, \langle x^2, y^2 \rangle \cdots \langle x^n, y^n \rangle$

↳ Training Dataset

* Learning Mechanism

On-Time → 1-batch = 1-training Example (→ Stochastic)

Batch → 1-batch = Training Set

Mini-Batch → Subset of Training Set