기계학습 Probability 1Page 정리 2016/04/09 컴퓨터공학과 김성수

Terminology

- experiment : 관측의 프로세스
- outcomes : 관측의 결과.
- Sample Space : 가능한 결과의 집합 S
- event : Sample Space의 부분집합
- event space : 가능한 event의 집합 E
- probability measure : E에서 정의된 function
- probability space : (S, E, P)

* Conditional Probability & Bayes' Theorem

- $P(A|B) = \dfrac{P(A\cap B)}{P(B)}$ $P(B) > 0$
- $P(A|B) = \dfrac{P(B|A)P(A)}{P(B)}$

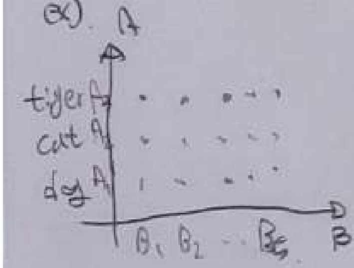
expand. $A_i \cap A_j = \emptyset$ for $\forall i \neq j$

$\bigcup_{i=1}^{n} A_i = S$.

$$P(B) = \sum_{i=1}^{n} P(B\cap A_i) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$$

$$\rightarrow P(A_i|B) = \frac{P(B|A_i)P(A_i)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}$$

ex)



$P(A_i|B)$: 꼬리길이 알 때, 동물의 확률

$P(B|A_i)$: 각 동물의 꼬리길이 분포

$P(A_i)$: dog, cat, tiger의 개체수 비율

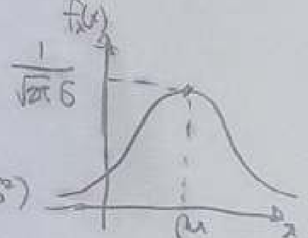
- Independence : $P(A\cap B) = P(A)P(B)$
- Mean : $\mu_X = E(X) = \begin{cases} \sum_k x_k p_X(x_k) & X: \text{discrete} \\ \int_{-\infty}^{\infty} x f_X(x)\,dx & X: \text{continuous} \end{cases}$
- Variance : $\sigma_X^2 = Var(X) = E\{[X - E(X)]^2\}$

$= E(X^2) - E(X)^2$.

- Correlation : $E(XY)$.
  - → orthogonal : $E(XY) = 0$
  - → uncorrelated : $E(XY) = E(X)E(Y)$
- Covariance : $Cov(X,Y) = E(XY) - E(X)E(Y)$
  - → uncorrelated : $\sigma_{XY} = 0$

* Gaussian Distribution.

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/(2\sigma^2)}$$



$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_x\sigma_y(1-\rho^2)^{\frac{1}{2}}} \exp\left[-\frac{1}{2}q(x,y)\right]$$

$$q(x,y) = \frac{1}{1-\rho^2}\left[\left(\frac{x-m_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-m_x}{\sigma_x}\right)\left(\frac{y-m_y}{\sigma_y}\right) + \left(\frac{y-m_y}{\sigma_y}\right)^2\right]$$

→ Correlation Coefficient $\rho = 0$, X, Y = independent

$f_{XY}(x,y) = f_X(x)f_Y(y)$.

$$f_X(x) = \frac{1}{(2\pi)^{\frac{n}{2}}|\det K|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}(x-m)^T K^{-1}(x-m)\right]$$